



**MATHEMATICS
HIGHER LEVEL
PAPER 2**

Friday 8 May 2009 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

In a class of 20 students, 12 study Biology, 15 study History and 2 students study neither Biology nor History.

- (a) Illustrate this information on a Venn diagram. [2 marks]

- (b) Find the probability that a randomly selected student from this class is studying both Biology and History. [1 mark]

- (c) Given that a randomly selected student studies Biology, find the probability that this student also studies History. [1 mark]

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2. [Maximum mark: 5]

Given that $\mathbf{a} = 2 \sin \theta \mathbf{i} + (1 - \sin \theta) \mathbf{j}$, find the value of the acute angle θ , so that \mathbf{a} is perpendicular to the line $x + y = 1$.

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5. [*Maximum mark: 5*]

An influenza virus is spreading through a city. A vaccination is available to protect against the virus. If a person has had the vaccination, the probability of catching the virus is 0.1; without the vaccination, the probability is 0.3. The probability of a randomly selected person catching the virus is 0.22.

(a) Find the percentage of the population that has been vaccinated. *[3 marks]*

(b) A randomly chosen person catches the virus. Find the probability that this person has been vaccinated. *[2 marks]*

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6. [Maximum mark: 6]

The acceleration in ms^{-2} of a particle moving in a straight line at time t seconds, $t \geq 0$, is given by the formula $a = -\frac{1}{2}v$. When $t = 0$, the velocity is 40 ms^{-1} .

Find an expression for v in terms of t .

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7. [Maximum mark: 8]

The cubic curve $y = 8x^3 + bx^2 + cx + d$ has two distinct points P and Q, where the gradient is zero.

(a) Show that $b^2 > 24c$. [4 marks]

(b) Given that the coordinates of P and Q are $\left(\frac{1}{2}, -12\right)$ and $\left(-\frac{3}{2}, 20\right)$ respectively, find the values of b , c and d . [4 marks]

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8. [Maximum mark: 5]

Six people are to sit at a circular table. Two of the people are not to sit immediately beside each other. Find the number of ways that the six people can be seated.

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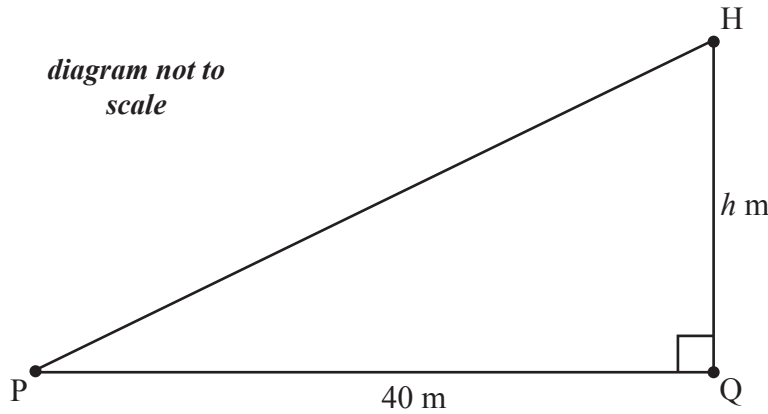
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10. [Maximum mark: 7]

A helicopter H is moving vertically upwards with a speed of 10 ms^{-1} . The helicopter is $h \text{ m}$ directly above the point Q which is situated on level ground. The helicopter is observed from the point P which is also at ground level and $PQ = 40 \text{ m}$. This information is represented in the diagram below.



When $h = 30$,

- (a) show that the rate of change of \widehat{HPQ} is 0.16 radians per second; [3 marks]
- (b) find the rate of change of \widehat{HPQ} . [4 marks]

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SECTION B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

11. [Maximum mark: 20]

Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of 998 ml and a standard deviation of 2.5 ml.

- (a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212. [1 mark]

- (b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water. [3 marks]

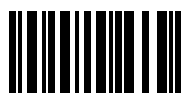
- (c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water, is greater than 0.99. [4 marks]

- (d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B. [6 marks]

- (e) The company that makes the mineral water receives, on average, m phone calls every 10 minutes. The number of phone calls, X , follows a Poisson distribution such that $P(X = 2) = P(X = 3) + P(X = 4)$.
 - (i) Find the value of m .

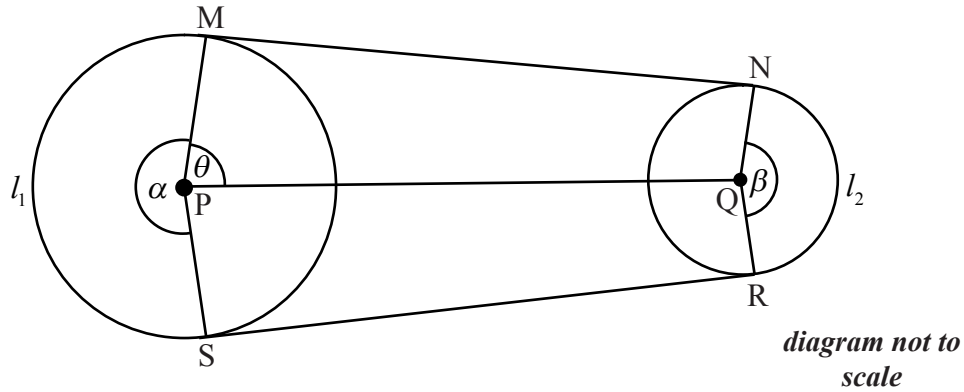
 - (ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period. [6 marks]

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12. [Maximum mark: 18]

Two non-intersecting circles C_1 , containing points M and S, and C_2 , containing points N and R, have centres P and Q where $PQ = 50$. The line segments [MN] and [SR] are common tangents to the circles. The size of the reflex angle MPS is α , the size of the obtuse angle NQR is β , and the size of the angle MPQ is θ . The arc length MS is l_1 and the arc length NR is l_2 . This information is represented in the diagram below.



The radius of C_1 is x , where $x \geq 10$ and the radius of C_2 is 10.

- (a) Explain why $x < 40$. [1 mark]
- (b) Show that $\cos \theta = \frac{x-10}{50}$. [2 marks]
- (c) (i) Find an expression for MN in terms of x .
- (ii) Find the value of x that maximises MN. [2 marks]
- (d) Find an expression in terms of x for
 - (i) α ;
 - (ii) β . [4 marks]
- (e) The length of the perimeter is given by $l_1 + l_2 + MN + SR$.
 - (i) Find an expression, $b(x)$, for the length of the perimeter in terms of x .
 - (ii) Find the maximum value of the length of the perimeter.
 - (iii) Find the value of x that gives a perimeter of length 200. [9 marks]



13. [Maximum mark: 22]

Consider the graphs $y = e^{-x}$ and $y = e^{-x} \sin 4x$, for $0 \leq x \leq \frac{5\pi}{4}$.

- (a) On the same set of axes draw, on graph paper, the graphs, for $0 \leq x \leq \frac{5\pi}{4}$.
Use a scale of 1 cm to $\frac{\pi}{8}$ on your x -axis and 5 cm to 1 unit on your y -axis. [3 marks]
- (b) Show that the x -intercepts of the graph $y = e^{-x} \sin 4x$ are $\frac{n\pi}{4}$, $n = 0, 1, 2, 3, 4, 5$. [3 marks]
- (c) Find the x -coordinates of the points at which the graph of $y = e^{-x} \sin 4x$ meets the graph of $y = e^{-x}$. Give your answers in terms of π . [3 marks]
- (d) (i) Show that when the graph of $y = e^{-x} \sin 4x$ meets the graph of $y = e^{-x}$, their gradients are equal.

(ii) Hence explain why these three meeting points are not local maxima of the graph $y = e^{-x} \sin 4x$. [6 marks]
- (e) (i) Determine the y -coordinates, y_1, y_2 and y_3 , where $y_1 > y_2 > y_3$, of the local maxima of $y = e^{-x} \sin 4x$, for $0 \leq x \leq \frac{5\pi}{4}$. You do not need to show that they are maximum values, but the values should be simplified.

(ii) Show that y_1, y_2 and y_3 form a geometric sequence and determine the common ratio r . [7 marks]

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