

22057207

**MATHEMATICS
HIGHER LEVEL
PAPER 1**

Tuesday 3 May 2005 (afternoon)

Candidate session number

2 hours

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.



Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answers.

1. The position vectors of points P and Q are $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix}$ respectively. The origin is at O. Find
- (a) the angle \widehat{POQ} ;
 - (b) the area of the triangle OPQ.

Working:

Answers:

- (a) _____
- (b) _____

2. Solve the equation $\left| e^{2x} - \frac{1}{x+2} \right| = 2$.

Working:

Answer:



3. The table below shows the probability distribution of a discrete random variable X .

x	0	1	2	3
$P(X = x)$	0.2	a	b	0.25

- (a) Given that $E(X) = 1.55$, find the value of a and of b .
- (b) Calculate $\text{Var}(X)$.

Working:

Answers:

(a) _____

(b) _____



4. Given that $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$, find X if $BX = A - AB$.

Working:

Answer:

5. Consider the 10 data items x_1, x_2, \dots, x_{10} . Given that $\sum_{i=1}^{10} x_i^2 = 1341$ and the standard deviation is 6.9, find the value of \bar{x} .

Working:

Answer:



6. The function f is given by $f(x) = \frac{x^5 + 2}{x}$, $x \neq 0$. There is a point of inflexion on the graph of f at the point P. Find the coordinates of P.

Working:

Answer:



7. Let $P(z) = z^3 + az^2 + bz + c$, where a, b , and $c \in \mathbb{R}$. Two of the roots of $P(z) = 0$ are -2 and $(-3 + 2i)$.
Find the value of a , of b and of c .

Working:

Answer:

8. A team of five students is to be chosen at random to take part in a debate. The team is to be chosen from a group of eight medical students and three law students. Find the probability that
- (a) only medical students are chosen;
 - (b) all three law students are chosen.

Working:

Answers:

(a) _____

(b) _____



9. The probability density function $f(x)$ of the continuous random variable X is defined on the interval $[0, a]$ by

$$f(x) = \begin{cases} \frac{1}{8}x & \text{for } 0 \leq x \leq 3, \\ \frac{27}{8x^2} & \text{for } 3 < x \leq a. \end{cases}$$

Find the value of a .

Working:

Answer:



10. Given that $a \sin 4x + b \sin 2x = 0$, for $0 < x < \frac{\pi}{2}$, find an expression for $\cos^2 x$ in terms of a and b .

Working:

Answer:

11. Given that $|z| = 2\sqrt{5}$, find the complex number z that satisfies the equation

$$\frac{25}{z} - \frac{15}{z^*} = 1 - 8i.$$

Working:

Answer:



12. (a) Express as partial fractions $\frac{2x+4}{(x^2+4)(x-2)}$.

(b) Hence or otherwise, find $\int \frac{2x+4}{(x^2+4)(x-2)} dx$.

Working:

Answers:

(a) _____

(b) _____



13. An experiment is carried out in which the number n of bacteria in a liquid, is given by the formula $n = 650 e^{kt}$, where t is the time in minutes after the beginning of the experiment and k is a constant. The number of bacteria doubles every 20 minutes. Find
- (a) the **exact** value of k ;
 - (b) the rate at which the number of bacteria is increasing when $t = 90$.

Working:

Answers:

- (a) _____
- (b) _____

14. Let $f(x) = \frac{x^2 + 5x + 5}{x + 2}$, $x \neq -2$.

- (a) Find $f'(x)$.
- (b) Solve $f'(x) > 2$.

Working:

Answers:

- (a) _____
- (b) _____



15. The normal to the curve $y = \frac{k}{x} + \ln x^2$, for $x \neq 0$, $k \in \mathbb{R}$, at the point where $x = 2$, has equation $3x + 2y = b$, where $b \in \mathbb{R}$. Find the **exact** value of k .

Working:

Answer:



16. Given that $(A \cup B)' = \emptyset$, $P(A'|B) = \frac{1}{3}$ and $P(A) = \frac{6}{7}$, find $P(B)$.

Working:

Answer:

17. The triangle ABC has an obtuse angle at B, $BC = 10.2$, $\hat{A} = x$ and $\hat{B} = 2x$.

(a) Find AC, in terms of $\cos x$.

(b) Given that the area of triangle ABC is $52.02 \cos x$, find angle \hat{C} .

Working:

Answers:

(a) _____

(b) _____



18. The sum of the first n terms of an arithmetic sequence $\{u_n\}$ is given by the formula $S_n = 4n^2 - 2n$. Three terms of this sequence, u_2 , u_m , and u_{32} , are consecutive terms in a geometric sequence. Find m .

Working:

Answer:



19. The function f is defined for $x > 2$ by $f(x) = \ln x + \ln(x - 2) - \ln(x^2 - 4)$.

(a) Express $f(x)$ in the form $\ln\left(\frac{x}{x+a}\right)$.

(b) Find an expression for $f^{-1}(x)$.

Working:

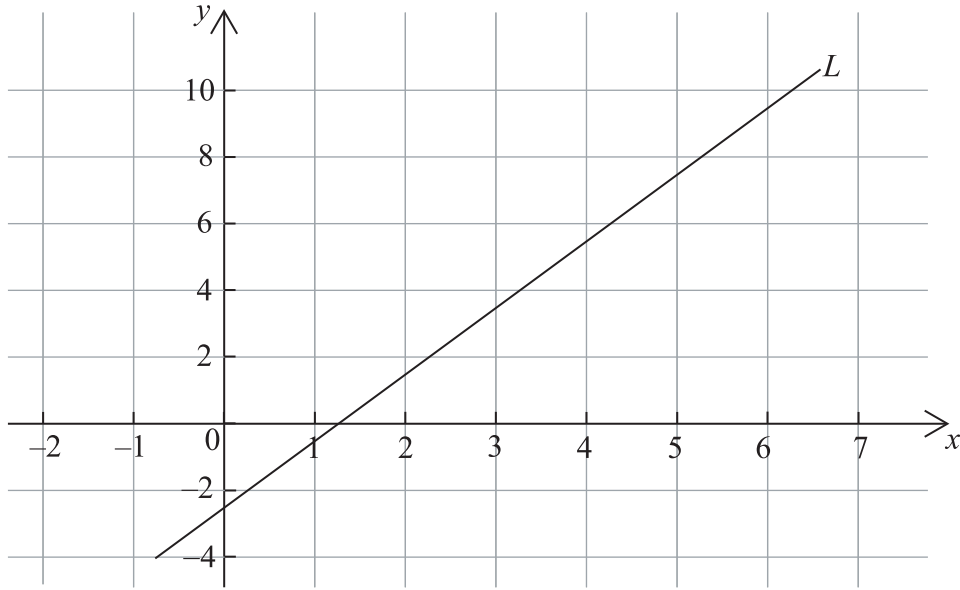
Answers:

(a) _____

(b) _____



20. Let $y = \log_3 z$, where z is a function of x . The diagram shows the straight line L , which represents the graph of y against x .



- (a) Using the graph or otherwise, estimate the value of x when $z = 9$.
- (b) The line L passes through the point $\left(1, \log_3 \frac{5}{9}\right)$. Its gradient is 2. Find an expression for z in terms of x .

Working:

Answers:

(a) _____

(b) _____

