M05/5/MATHL/HP1/ENG/TZ1/XX

22057204

MATHEMATICS
HIGHER LEVEL
PAPER 1

Tuesday 3 May 2005 (afternoon)

2 hours

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answers.

1.   The vectors *a*, *b* and *c* are defined by $a = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$, and $c = \begin{pmatrix} 2 \\ y \\ 3 \end{pmatrix}$.

Given that *c* is perpendicular to $2a - b$, find the value of *y*.

Working:

Answer:

2.   Find the coefficient of *x* in the expansion of $\left(3x - \dfrac{2}{x} \right)^5$.

Working:

Answer:

3. The line $\dfrac{x-3}{2} = y+1 = \dfrac{5-z}{3}$ and the plane $2x - y + 3z = 10$ intersect at the point P. Find the coordinates of P.

Working:

Answer:

4. The matrices A, B are such that $\det A = \det B$ where $A = \begin{pmatrix} 1 & -3 \\ x+1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & x & -2 \\ -2 & 1 & 0 \\ 0 & -1 & x \end{pmatrix}$.

Find the values of x.

Working:

Answer:

....

5. Find the area of the region enclosed by the graphs of $f(x) = 4 - x^2$ and $g(x) = (x+1)\cos x$.

Working:

Answer:

....

6.   Let X be a normal random variable with mean 25 and variance 4. Find $P\left(\left|X-25\right|<3\right)$.

Working:

Answer:

7. In an examination of 20 multiple-choice questions each question has four possible answers, only one of which is correct. Robert randomly guesses the answer to each question.
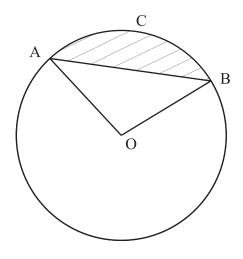
(a) Find his expected number of correct answers.

(b) Find the probability that Robert obtains this expected number of correct answers.

Working:

Answers:

(a) _____

(b) _____

0616

8.   The following diagram shows a circle centre O and radius r. The length of the arc ACB is $2r$.

The area of the shaded segment may be expressed as kr^2. Find the value of k.

Working:

Answer:

0716

9. Consider the function $f(x) = \ln \sqrt{x^2 + 4}$.

(a) Find $f'(x)$.

(b) Find $f''(x)$.

Working:

Answers:

(a) _____

(b) _____

10. In a survey of 50 people it is found that 40 own a television and 21 own a computer. Four do not own either a computer or a television. A person is chosen at random from this group.

(a) Find the probability that this person owns both a television and a computer.

(b) Given that this person owns a computer, find the probability that he also owns a television.

Working:

Answers:

(a) _____

(b) _____

0816

11.   Solve the following equation for z, where z is a complex number.

$$\frac{z}{3+4i} + \frac{z-1}{5i} = \frac{5}{3-4i}$$

Give your answer in the form $a + bi$ where $a, b \in \mathbb{Z}$.

Working:

Answer:

12. Find the gradient of the curve $2\sin(xy) = 1$ when $y = \dfrac{1}{2}$ and $\pi < x < 2\pi$.

Working:

Answer:

13. In a rental property business, the profits in Euros per year for 50 properties are shown in the following cumulative table.

Profit (x)	Number of properties with profit less than x
– 10 000	0
– 5 000	3
0	7
5 000	22
10 000	39
15 000	44
20 000	50

For this population of 50 properties, calculate an estimate for the standard deviation of the profit.

Working:

Answer:

1116

14. Consider the functions $f(x) = e^{2x}$ and $g(x) = \sin\dfrac{\pi x}{2}$.

    (a)   Find the period of the function $f \circ g$.

    (b)   Find the intervals for which $(f \circ g(x)) > 4$.

Working:

Answers:

(a) _____

(b) _____

15. Solve the equation $2\log_3(x-3) + \log_{\left(\frac{1}{3}\right)}(x+1) = 2$.

Working:

Answer:

16. (a) Sketch the graph of $y = \dfrac{x-12}{\sqrt{x^2-4}}$.

    (b) Write down

        (i) the *x*-intercept;

        (ii) the equations of all asymptotes.

Answer: (a)

Working:

Answers:

(b) (i) _____

    (ii) _____

17. Let $f(x) = \dfrac{x^2 + 3x + 12}{x(x+2)^2}$. Find $\displaystyle\int f(x)\,\mathrm{d}x$.

Working:

Answer:

18. A function f is defined by $f(x) = ax^3 + bx^2 + 30x + c$ where a, b and c are constants. The graph of f has a maximum at $(1, 7)$ and a point of inflexion when $x = 3$. Find the value of a, of b and of c.
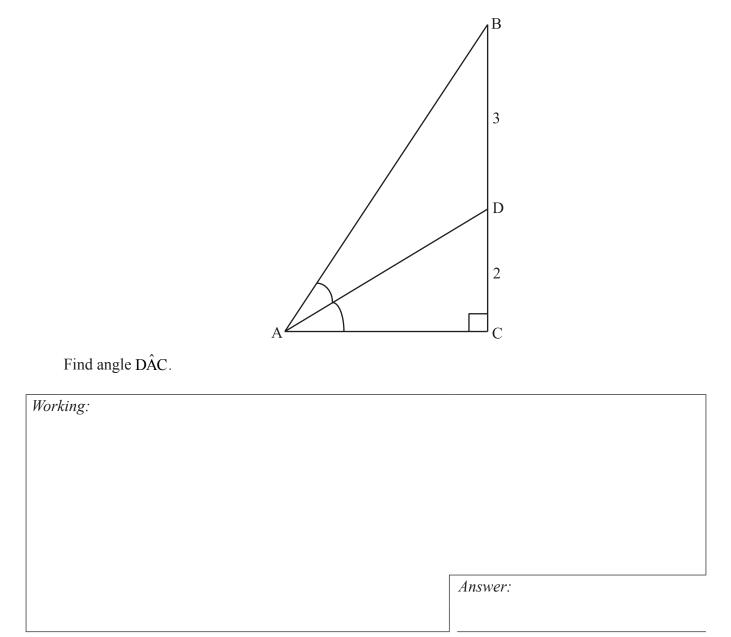
Working:

Answers:

$a =$

$b =$

$c =$

19. Let ABC be a right-angled triangle, where $\hat{C} = 90°$. The line (AD) bisects $B\hat{A}C$, $BD = 3$, and $DC = 2$, as shown in the diagram.

Find angle $D\hat{A}C$.

Working:

Answer:

1516

Turn over

20. A conical tank with vertex down is 8 metres in diameter and 12 metres deep. Water flows into the tank at 10 m^3 per minute. Find the rate of change of the depth of the water at the instant when the water is 6 metres deep.

Working:

Answer: