IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICS
HIGHER LEVEL
PAPER 1

Tuesday 7 May 2002 (afternoon)

2 hours

Name

Number

INSTRUCTIONS TO CANDIDATES

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /120	TOTAL /120	TOTAL /120

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive ***no*** *marks.*

1. Consider the arithmetic series 2 + 5 + 8 + ...

(a) Find an expression for S_n, the sum of the first n terms.

(b) Find the value of n for which $S_n = 1365$.

Working:

Answers:

(a) ______________________

(b) ______________________

2. A particle is projected along a straight line path. After t seconds, its velocity v metres per second is given by $v = \dfrac{1}{2+t^2}$.

(a) Find the distance travelled in the first second.

(b) Find an expression for the acceleration at time t .

Working:

Answers:
(a) ____________________
(b) ____________________

3. (a) Express the complex number 8i in polar form.

(b) The cube root of 8i which lies in the first quadrant is denoted by z . Express z

(i) in polar form;

(ii) in cartesian form.

Working:

Answers:
(a) ____________________
(b) (i) ____________________
(ii) ____________________

4. The matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & k \\ 1 & k & -1 \\ 3 & 4 & 2 \end{pmatrix}.$$

Find the values of k for which $\boldsymbol{A}$ is singular.

Working:

Answers:

5. Find the angle between the vectors $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{w} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$. Give your answer in radians.

Working:

Answer:

6. (a) Use integration by parts to find $\int x^2 \ln x \, dx$.

(b) Evaluate $\int_1^2 x^2 \ln x \, dx$.

Working:

Answers:

(a) ______________________________

(b) ______________________________

7. The probability that it rains during a summer's day in a certain town is 0.2 . In this town, the probability that the daily maximum temperature exceeds 25 °C is 0.3 when it rains and 0.6 when it does not rain. Given that the maximum daily temperature exceeded 25 °C on a particular summer's day, find the probability that it rained on that day.

Working:

Answer:

8. The vector equations of the lines L_1 and L_2 are given by

$$L_1: \boldsymbol{r} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k} + \lambda(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})\ ;$$
$$L_2: \boldsymbol{r} = \boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k} + \mu(2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k})\ .$$

The two lines intersect at the point P . Find the position vector of P .

Working:

Answer:

9. When John throws a stone at a target, the probability that he hits the target is 0.4 . He throws a stone 6 times.

(a) Find the probability that he hits the target **exactly** 4 times.

(b) Find the probability that he hits the target for the first time on his third throw.

Working:

Answers:

(a)

(b)

10. The angle θ satisfies the equation $\tan\theta + \cot\theta = 3$, where θ is in degrees. Find all the possible values of θ lying in the interval $]0°, 90°[$.

Working:

Answers:

11. The weights of a certain species of bird are normally distributed with mean 0.8 kg and standard deviation 0.12 kg. Find the probability that the weight of a randomly chosen bird of the species lies between 0.74 kg and 0.95 kg.

Working:

Answer:

12. The function f is defined on the domain $[0, \pi]$ by $f(\theta) = 4\cos\theta + 3\sin\theta$.

(a) Express $f(\theta)$ in the form $R\cos(\theta - \alpha)$ where $0 < \alpha < \frac{\pi}{2}$.

(b) Hence, or otherwise, write down the value of θ for which $f(\theta)$ takes its maximum value.

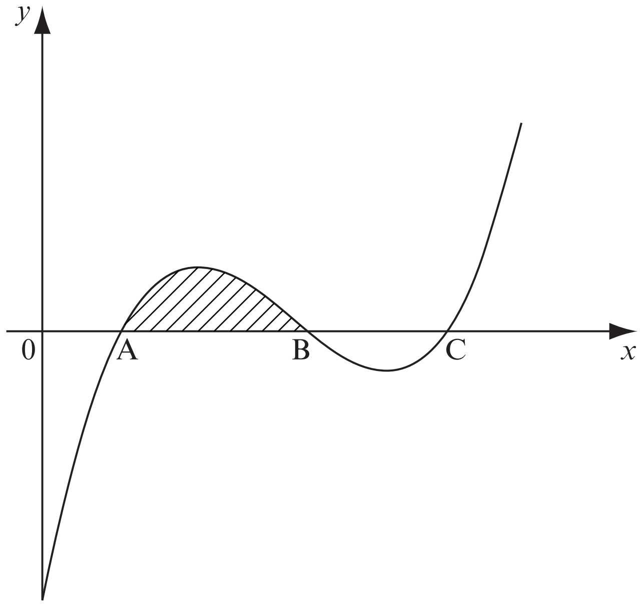
Working:

Answers:

(a) ______________________

(b) ______________________

13. The figure below shows part of the curve $y = x^3 - 7x^2 + 14x - 7$. The curve crosses the x-axis at the points A , B and C .



(a) Find the x-coordinate of A .

(b) Find the x-coordinate of B .

(c) Find the area of the shaded region.

Working:

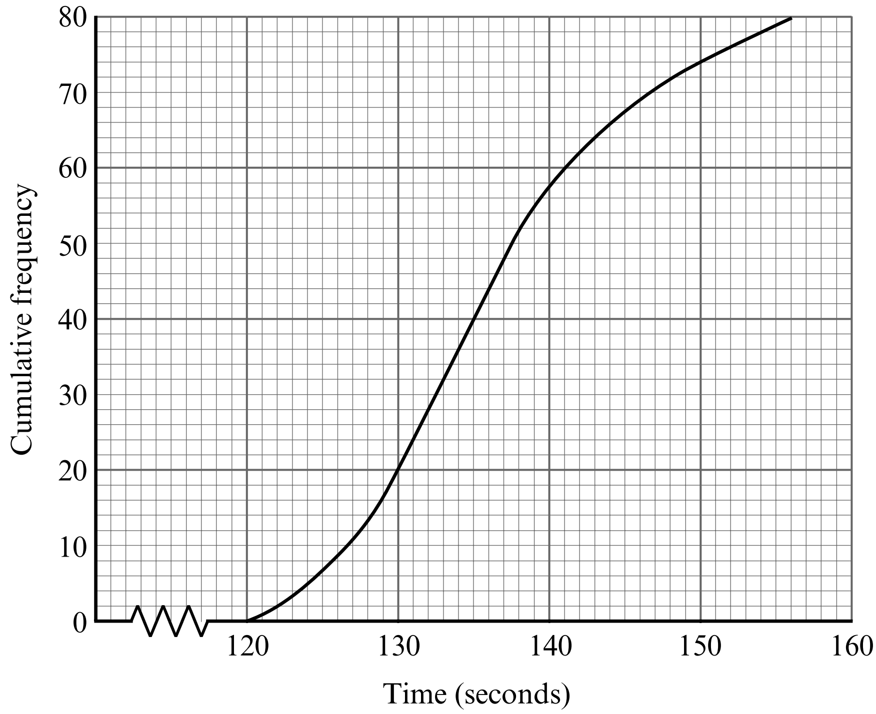
Answers:

(a) ______________________

(b) ______________________

(c) ______________________

14. The 80 applicants for a Sports Science course were required to run 800 metres and their times were recorded. The results were used to produce the following cumulative frequency graph.



Estimate

(a) the median;

(b) the interquartile range.

Working:

Answers:

(a) ____________________

(b) ____________________

15. The one–one function f is defined on the domain $x > 0$ by $f(x) = \dfrac{2x-1}{x+2}$.

(a) State the range, A, of f.

(b) Obtain an expression for $f^{-1}(x)$, for $x \in A$.

Working:

Answers:

(a) ______________________________

(b) ______________________________

16. Find the set of values of x for which $(\mathrm{e}^x - 2)(\mathrm{e}^x - 3)$ ″ $2\mathrm{e}^x$.

Working:

Answer:

17. A curve has equation $xy^3 + 2x^2y = 3$. Find the equation of the tangent to this curve at the point $(1, 1)$.

Working:

Answer:

18. A transformation $\boldsymbol{T}$ of the plane is represented by the matrix

$$\boldsymbol{T} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}.$$

(a) $\boldsymbol{T}$ transforms the point P to the point $(8, 5)$. Find the coordinates of P.

(b) Find the coordinates of all points which are transformed to themselves under $\boldsymbol{T}$.

Working:

Answers:

(a) ______________________________

(b) ______________________________

19. A rectangle is drawn so that its lower vertices are on the x-axis and its upper vertices are on the curve $y = \mathrm{e}^{-x^2}$. The area of this rectangle is denoted by A .

(a) Write down an expression for A in terms of x .

(b) Find the maximum value of A .

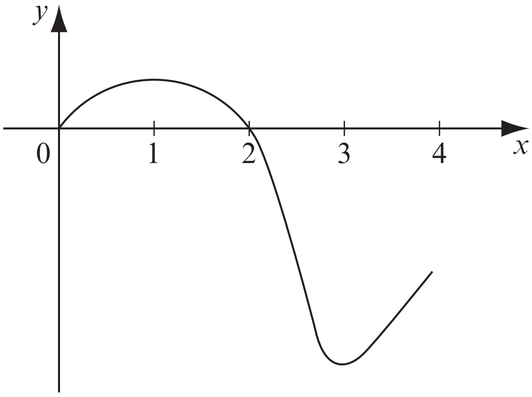
Working:

Answers:

(a) ______________________________

(b) ______________________________

20. The diagram below shows the graph of $y_1 = f(x)$, $0 \leqslant x \leqslant 4$.



On the axes below, sketch the graph of $y_2 = \int_0^x f(t)\,\mathrm{d}t$, marking clearly the points of inflexion.