



**MATHEMATICS  
 HIGHER LEVEL  
 PAPER 1**

Thursday 2 November 2000 (afternoon)

2 hours

Name

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Number

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**INSTRUCTIONS TO CANDIDATES**

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80.

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /60	TOTAL /60	TOTAL /60

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.

1. Find the values of the real number  $k$  for which the determinant of the matrix  $\begin{pmatrix} k-4 & 3 \\ -2 & k+1 \end{pmatrix}$  is equal to zero.

*Working:*

*Answers:*

2. Given functions  $f: x \mapsto x + 1$  and  $g: x \mapsto x^3$ , find the function  $(f \circ g)^{-1}$ .

*Working:*

*Answer:*

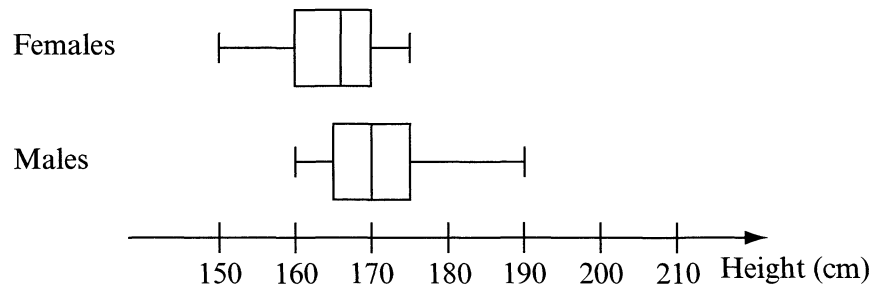
3. For the function  $f: x \mapsto x^2 \ln x$ ,  $x > 0$ , find the function  $f'$ , the derivative of  $f$  with respect to  $x$ .

*Working:*

*Answer:*

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4. The box-and-whisker plots shown represent the heights of female students and the heights of male students at a certain school.



- (a) What percentage of female students are shorter than any male students?
- (b) What percentage of male students are shorter than some female students?
- (c) From the diagram, **estimate** the mean height of the male students.

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_

5. Calculate the area bounded by the graph of  $y = x \sin(x^2)$  and the  $x$ -axis, between  $x = 0$  and the smallest positive  $x$ -intercept.

*Working:*

*Answer:*

6. Given that events  $A$  and  $B$  are independent with  $P(A \cap B) = 0.3$  and  $P(A \cap B') = 0.3$ , find  $P(A \cup B)$ .

*Working:*

*Answer:*

7. Find the sum of the positive terms of the arithmetic sequence 85, 78, 71, . . . .

*Working:*

*Answer:*

8. For the function  $f : x \mapsto \frac{1}{2} \sin 2x + \cos x$ , find the possible values of  $\sin x$  for which  $f'(x) = 0$ .

*Working:*

*Answers:*

9. (a) Describe the transformation of the plane whose matrix is

$$M = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}.$$

- (b) Find the smallest positive integer  $n$  for which  $M^n = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ .

*Working:*

*Answers:*

- (a) \_\_\_\_\_  
\_\_\_\_\_  
(b) \_\_\_\_\_



10. Find the real number  $k$  for which  $1 + ki$ , ( $i = \sqrt{-1}$ ), is a zero of the polynomial  $z^2 + kz + 5$ .

*Working:*

*Answer:*

11. Let  $\alpha$  be the angle between the vectors  $\mathbf{a}$  and  $\mathbf{b}$ , where  $\mathbf{a} = (\cos \theta)\mathbf{i} + (\sin \theta)\mathbf{j}$ ,  
 $\mathbf{b} = (\sin \theta)\mathbf{i} + (\cos \theta)\mathbf{j}$  and  $0 < \theta < \frac{\pi}{4}$ .

Express  $\alpha$  in terms of  $\theta$ .

*Working:*

*Answer:*

12. The coefficient of  $x$  in the expansion of  $\left(x + \frac{1}{ax^2}\right)^7$  is  $\frac{7}{3}$ . Find the possible values of  $a$ .

*Working:*

*Answers:*

13. For what values of  $m$  is the line  $y = mx + 5$  a tangent to the parabola  $y = 4 - x^2$ ?

*Working:*

*Answers:*

14. The tangent to the curve  $y^2 = x^3$  at the point  $P(1, 1)$  meets the  $x$ -axis at  $Q$  and the  $y$ -axis at  $R$ . Find the ratio  $PQ : QR$ .

*Working:*

*Answer:*

15. The sum of an infinite geometric sequence is  $13\frac{1}{2}$ , and the sum of the first three terms is 13. Find the first term.

*Working:*

*Answer:*

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16. In a triangle ABC,  $\widehat{ABC} = 30^\circ$ ,  $AB = 6$  cm and  $AC = 3\sqrt{2}$  cm. Find the possible lengths of [BC].

*Working:*

*Answers:*

17. Solve the differential equation  $xy \frac{dy}{dx} = 1 + y^2$ , given that  $y = 0$  when  $x = 2$ .

*Working:*

*Answer:*

18. If  $z$  is a complex number and  $|z + 16| = 4|z + 1|$ , find the value of  $|z|$ .

*Working:*

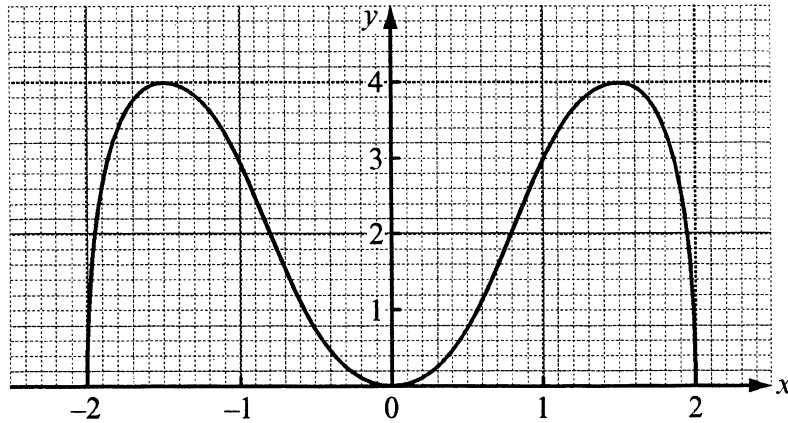
*Answer:*

19. In how many ways can six different coins be divided between two students so that each student receives at least one coin?

*Working:*

*Answer:*

20. The following graph is that part of the graph of  $y = f(x)$  for which  $f(x) \geq 0$ .



Sketch, on the axes provided below, the graph of  $y^2 = f(x)$  for  $-2 \leq x \leq 2$ .

