

MARKSCHEME

November 2000

MATHEMATICS

Higher Level

Paper 1

1. $\begin{vmatrix} k-4 & 3 \\ -2 & k+1 \end{vmatrix} = 0$
 $\Rightarrow (k-4)(k+1) + 6 = 0$ (M1)
 $\Rightarrow k^2 - 3k + 2 = 0$ (M1)
 $\Rightarrow (k-2)(k-1) = 0$
 $\Rightarrow k = 2$ or $k = 1$ (A1) (C3)

[3 marks]

2. $(f \circ g): x \mapsto x^3 + 1$ (M1)
 $(f \circ g)^{-1}: x \mapsto (x-1)^{1/3}$ (M1)(A1) (C3)

[3 marks]

3. $f(x) = x^2 \ln x$
 $f'(x) = 2x \ln x + x^2 \left(\frac{1}{x}\right)$ (M1)(M1)
 $= 2x \ln x + x$ (A1) (C3)
 $f': x \mapsto 2x \ln x + x$

[3 marks]

4. (a) Required percentage = 25 % (A1) (C1)
 (b) Required percentage = 75 % (A1) (C1)
 (c) Mean height of the male students is ≈ 172 cm ± 1 cm (A1) (C1)

[3 marks]

5. $x \sin(x^2) = 0$ when $x^2 = 0 (+k\pi, k \in \mathbb{Z})$, i.e. $x = 0 (+\sqrt{k\pi})$ (A1)
 The required area = $\int_0^{\sqrt{\pi}} x \sin(x^2) dx$ (M1)
 $= 1$ (A1) (C3)

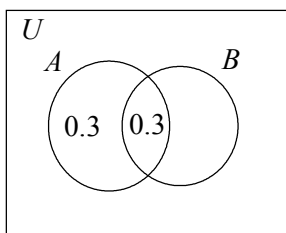
OR

$$\begin{aligned} \text{Area} &= \int_0^{\sqrt{\pi}} x \sin(x^2) dx \\ &= -\frac{1}{2} [\cos(x^2)]_0^{\sqrt{\pi}} \quad (M1) \\ &= -\frac{1}{2} (-1 - 1) \\ &= 1 \quad (A1) \quad (C3) \end{aligned}$$

[3 marks]

6. **Method 1:** (Venn diagram)

(M1)



$$P(A \cap B) = P(A)P(B)$$

(M1)

$$0.3 = 0.6 \times P(B)$$

$$P(B) = 0.5$$

$$\text{Therefore, } P(A \cup B) = 0.8$$

(A1)

(C3)

Method 2: $P(A \cap B') = P(A) - P(A \cap B)$

$$0.3 = P(A) - 0.3$$

$$P(A) = 0.6$$

(A1)

$$P(A \cap B) = P(A)P(B) \text{ since } A, B \text{ are independent}$$

$$0.3 = 0.6 \times P(B)$$

$$P(B) = 0.5$$

(A1)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.6 + 0.5 - 0.3$$

$$= 0.8$$

(A1)

(C3)

[3 marks]

7. Arithmetic progression: 85, 78, 71, ...

$$u_1 = 85, d = -7$$

$$u_n = u_1 + (n-1)d = 85 - 7(n-1) = 92 - 7n$$

(M1)

Thus, $u_n > 0$ provided $n \leq 13$.

$$\text{The required sum} = S_{13} = \frac{13}{2}(u_1 + u_{13}) = \frac{13}{2}(85 + 1).$$

(M1)

$$= 559$$

(A1)

(C3)

[3 marks]

8. $f(x) = \frac{1}{2} \sin 2x + \cos x$
 $f'(x) = \cos 2x - \sin x$ (M1)
 $= 1 - 2 \sin^2 x - \sin x$
 $= (1 + \sin x)(1 - 2 \sin x)$ (M1)
 $= 0$ when $\sin x = -1$ or $\frac{1}{2}$ (A1) (C3)

[3 marks]

9. (a) $M = \begin{pmatrix} \cos \frac{\pi}{3} & -\sin \frac{\pi}{3} \\ \sin \frac{\pi}{3} & \cos \frac{\pi}{3} \end{pmatrix}$ (M1)
 M represents a rotation about the origin through $\frac{\pi}{3}$ or 60° . (A1) (C2)
- (b) The smallest value of n is 6. (A1) (C1)

[3 marks]

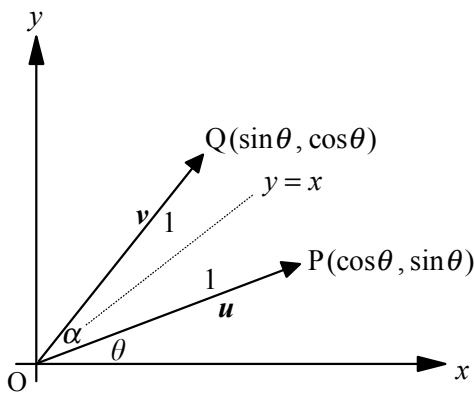
10. $(1+ki)^2 + k(1+ki) + 5 = 0$ (M1)
 $1 + 2ki - k^2 + k + k^2i + 5 = 0$
 $(6+k-k^2) + ki(2+k) = 0$
 Thus, $k(2+k) = 0$ and $6+k-k^2 = 0$ (M1)
 This gives $k = -2$ (A1) (C3)

[3 marks]

11. **Method 1:** Let the angle be α , then $\cos\alpha = \frac{a \cdot b}{|a||b|}$ (M1)
 $= \frac{2 \sin\theta \cos\theta}{(1)(1)}$
 $= \sin 2\theta$ (M1)
 $= \cos\left(\frac{\pi}{2} - 2\theta\right)$

$\alpha = \frac{\pi}{2} - 2\theta$ or $\alpha = \arccos(\sin 2\theta)$ (A1) (C3)

Method 2:



Q is the image of P under a reflection in $y = x$ (M1)

$\theta + \frac{\alpha}{2} = \frac{\pi}{4}$ (A1)

$\alpha = \frac{\pi}{2} - 2\theta$ (A1) (C3)

[3 marks]

12. **Method 1:** $T_{r+1} = \binom{7}{r} x^{7-r} \left(\frac{1}{ax^2}\right)^r = \binom{7}{r} \left(\frac{1}{a}\right)^r x^{7-3r}$ (M1)

$7 - 3r = 1$
 $r = 2$ (A1)

Now, $\binom{7}{2} \frac{1}{a^2} = \frac{7}{3}$

$\Rightarrow a^2 = 9$

$\Rightarrow a = \pm 3$. (A1) (C3)

Method 2: $\left(x + \frac{1}{ax^2}\right)^7 = x^7 \left(1 + \frac{1}{ax^3}\right)^7$ (M1)

Coefficient of $x = \binom{7}{2} \left(\frac{1}{a}\right)^2$ (A1)

Thus, $\frac{21}{a^2} = \frac{7}{3}$ which leads to $a = \pm 3$ (A1) (C3)

[3 marks]

- 13. Method 1:** $y = 4 - x^2$
 $\frac{dy}{dx} = -2x = m$ when $x = -\frac{m}{2}$ (M1)
 Thus, $\left(-\frac{m}{2}, 4 - \frac{m^2}{4}\right)$ lies on $y = mx + 5$. (R1)
 Then, $4 - \frac{m^2}{4} = -\frac{m^2}{2} + 5$, so $m^2 = 4$
 $m = \pm 2$. (A1) (C3)
- Method 2:** For intersection: $mx + 5 = 4 - x^2$ or $x^2 + mx + 1 = 0$. (M1)
 For tangency: discriminant = 0 (M1)
 Thus, $m^2 - 4 = 0$
 $m = \pm 2$ (A1) (C3)
- [3 marks]**

- 14.** $y^2 = x^3$ so $2y \frac{dy}{dx} = 3x^2$.
 At P(1, 1), $\frac{dy}{dx} = \frac{3}{2}$. (M1)
 The tangent is $3x - 2y = 1$, giving Q = $\left(\frac{1}{3}, 0\right)$ and R = $\left(0, -\frac{1}{2}\right)$. (A1)
 Therefore, PQ : QR = $\frac{2}{3} : \frac{1}{3}$ or $1 : \frac{1}{2}$
 $= 2 : 1$. (A1) (C3)
- [3 marks]**

- 15.** $\frac{u_1}{1-r} = \frac{27}{2}$ and $u_1 + u_1 r + u_1 r^2 = 13$ (M1)
 $\frac{27}{2}(1-r)(1+r+r^2) = 13$ (M1)
 $1-r^3 = \frac{26}{27}$ giving $r = \frac{1}{3}$
 Therefore, $u_1 = 9$. (A1) (C3)
- [3 marks]**

16. **Note:** Award full marks for exact answers or answers given to three significant figures.

Method 1:

Using the sine rule: $\frac{\sin C}{6} = \frac{\sin 30^\circ}{3\sqrt{2}}$

$$\sin C = \frac{1}{\sqrt{2}}$$

$$C = 45^\circ, 135^\circ. \quad (M1)$$

Again, $\frac{3\sqrt{2}}{\sin 30^\circ} = \frac{BC}{\sin 105^\circ}$ or $\frac{BC}{\sin 15^\circ}$

Thus, $BC = 6\sqrt{2} \sin 105^\circ$ or $6\sqrt{2} \sin 15^\circ$
 $BC = 8.20 \text{ cm}$ or $BC = 2.20 \text{ cm}.$

(A1)(A1) (C3)

Method 2:

Using the cosine rule: $AC^2 = 6^2 + BC^2 - 2(6)(BC)\cos 30^\circ$

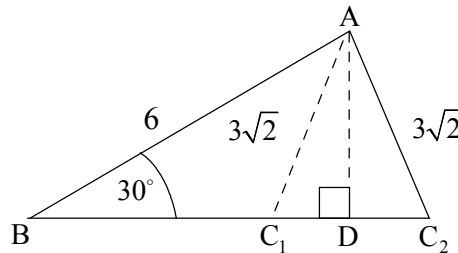
$$18 = 36 + BC^2 - 6\sqrt{3}BC \quad (M1)$$

Therefore, $BC^2 - (6\sqrt{3})BC + 18 = 0$

Therefore, $(BC - 3\sqrt{3})^2 = 27 - 18 = 9$

Therefore, $BC = 3\sqrt{3} \pm 3$, i.e. $BC = 8.20 \text{ cm}$ or $BC = 2.20 \text{ cm}.$ (A1)(A1) (C3)

Method 3:



In $\triangle ABD$, $AD = 3 \text{ cm},$ (A1)

and $BD = \sqrt{27} = 3\sqrt{3} \text{ cm}.$

In $\triangle AC_1D$, $C_1D = 3$ (A1)

Also, $C_2D = 3.$

Therefore $BC = (3\sqrt{3} \pm 3) \text{ cm},$ i.e. $BC = 8.20 \text{ cm}$ or $BC = 2.20 \text{ cm}.$ (A1) (C3)

Note: If only one answer is given, award a maximum of (M1)(A1).

[3 marks]

17. $xy \frac{dy}{dx} = 1 + y^2 \Rightarrow \int \frac{y}{1+y^2} dy = \int \frac{1}{x} dx$ (M1)

$\frac{1}{2} \ln(1+y^2) = \ln x + \ln c$ (M1)

$1+y^2 = kx^2 \quad (k = c^2)$

$y = 0$ when $x = 2$, and so $1 = 4k$

Thus, $1+y^2 = \frac{1}{4}x^2$ or $x^2 - 4y^2 = 4$. (A1) (C3)

[3 marks]

18. Let $z = x + iy$, $x, y \in \mathbb{R}$.

Then, $|z+16|^2 = 16|z+1|^2$

$\Rightarrow (x+16)^2 + y^2 = 16\{(x+1)^2 + y^2\}$ (M1)

$\Rightarrow x^2 + 32x + 256 + y^2 = 16x^2 + 32x + 16 + 16y^2$

$\Rightarrow 15x^2 + 15y^2 = 240$

$\Rightarrow x^2 + y^2 = 16$ (A1)

Therefore, $|z| = 4$. (A1) (C3)

[3 marks]

19. The first student can receive x coins in $\binom{6}{x}$ ways, $1 \leq x \leq 5$. (M1)

[The second student then receives the rest.]

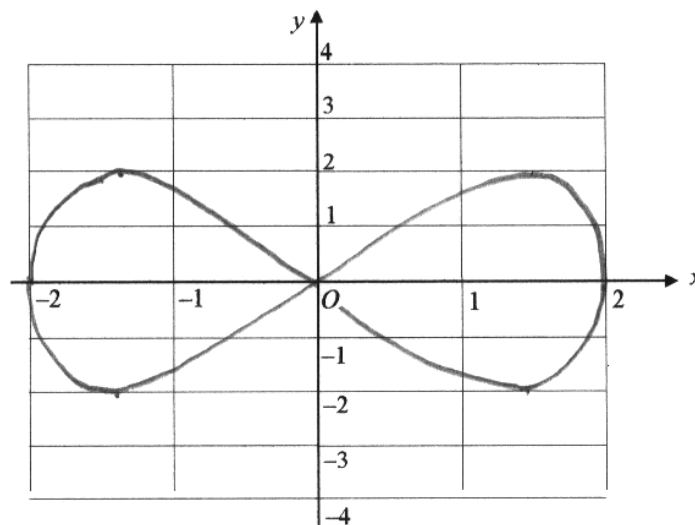
Therefore, the number of ways = $\binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5}$ (A1)

$= 2^6 - 2$

$= 62$. (A1) (C3)

[3 marks]

20.



(A1)(A1)(A1) (C3)

Note: Award (A1) for maxima and minima, (A1) for symmetry, (A1) for zeros.

[3 marks]