INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

# MARKSCHEME

## November 2000

## MATHEMATICS

## Higher Level

## Paper 1

**1.** $\begin{vmatrix} k-4 & 3 \\ -2 & k+1 \end{vmatrix} = 0$

$\Rightarrow$ $(k-4)(k+1)+6=0$ *(M1)*

$\Rightarrow$ $k^2-3k+2=0$ *(M1)*

$\Rightarrow$ $(k-2)(k-1)=0$

$\Rightarrow$ $k=2$ or $k=1$ *(A1)* *(C3)*

***[3 marks]***

**2.** $(f\circ g): x \mapsto x^3+1$ *(M1)*

$(f\circ g)^{-1}: x \mapsto (x-1)^{1/3}$ *(M1)(A1)* *(C3)*

***[3 marks]***

**3.** $f(x) = x^2 \ln x$

$f'(x) = 2x\ln x + x^2\left(\frac{1}{x}\right)$ *(M1)(M1)*

$= 2x\ln x + x$ *(A1)* *(C3)*

$f': x \mapsto 2x\ln x + x$

***[3 marks]***

**4.** (a) Required percentage $= 25\%$ *(A1)* *(C1)*

(b) Required percentage $= 75\%$ *(A1)* *(C1)*

(c) Mean height of the male students is $\approx 172$ cm $\pm 1$ cm *(A1)* *(C1)*

***[3 marks]***

**5.** $x\sin(x^2)=0$ when $x^2 = 0(+k\pi, k\in\mathbb{Z})$, *i.e.* $x=0(+\sqrt{k\pi})$ *(A1)*

The required area $= \int_0^{\sqrt{\pi}} x\sin(x^2)\,\mathrm{d}x$ *(M1)*

$=1$ *(G1)* *(C3)*

**OR**

Area $= \int_0^{\sqrt{\pi}} x\sin(x^2)\,\mathrm{d}x$

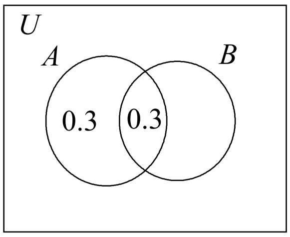
$= -\frac{1}{2}\left[\cos(x^2)\right]_0^{\sqrt{\pi}}$ *(M1)*

$= -\frac{1}{2}(-1-1)$

$=1$ *(A1)* *(C3)*

***[3 marks]***

**6.** **Method 1:** (Venn diagram) *(M1)*




$P(A \cap B) = P(A)P(B)$ *(M1)*

$0.3 = 0.6 \times P(B)$

$P(B) = 0.5$

Therefore, $P(A \cup B) = 0.8$ *(A1)* *(C3)*

**Method 2:**

$P(A \cap B') = P(A) - P(A \cap B)$

$0.3 = P(A) - 0.3$

$P(A) = 0.6$ *(A1)*

$P(A \cap B) = P(A)P(B)$ since $A, B$ are independent

$0.3 = 0.6 \times P(B)$

$P(B) = 0.5$ *(A1)*

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.5 - 0.3$

$= 0.8$ *(A1)* *(C3)*

***[3 marks]***

**7.** Arithmetic progression: 85, 78, 71, ...

$u_1 = 85, d = -7$

$u_n = u_1 + (n-1)d = 85 - 7(n-1) = 92 - 7n$ *(M1)*

Thus, $u_n > 0$ provided $n \le 13$.

The required sum $= S_{13} = \frac{13}{2}(u_1 + u_{13}) = \frac{13}{2}(85+1)$. *(M1)*

$= 559$ *(A1)* *(C3)*

***[3 marks]***

**8.** $f(x) = \frac{1}{2}\sin 2x + \cos x$

$f'(x) = \cos 2x - \sin x$ *(M1)*

$= 1 - 2\sin^2 x - \sin x$

$= (1 + \sin x)(1 - 2\sin x)$ *(M1)*

$= 0$ when $\sin x = -1$ or $\frac{1}{2}$ *(A1)* *(C3)*

***[3 marks]***

**9.** (a) $\boldsymbol{M} = \begin{pmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{pmatrix}$ *(M1)*

***M*** represents a rotation about the origin through $\frac{\pi}{3}$ or $60^\circ$. *(A1)* *(C2)*

(b) The smallest value of $n$ is 6. *(A1)* *(C1)*

***[3 marks]***

**10.** $(1 + k\mathrm{i})^2 + k(1 + k\mathrm{i}) + 5 = 0$ *(M1)*

$1 + 2k\mathrm{i} - k^2 + k + k^2\mathrm{i} + 5 = 0$

$(6 + k - k^2) + k\mathrm{i}(2 + k) = 0$

Thus, $k(2 + k) = 0$ and $6 + k - k^2 = 0$ *(M1)*

This gives $k = -2$ *(A1)* *(C3)*

***[3 marks]***

**11.** **Method 1:** Let the angle be $\alpha$, then $\cos\alpha = \dfrac{\boldsymbol{a}\cdot\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$ *(M1)*

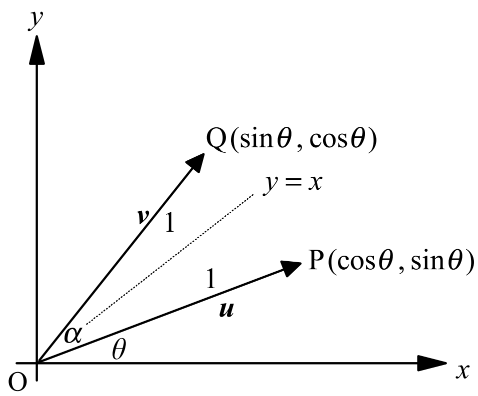
$= \dfrac{2\sin\theta\cos\theta}{(1)(1)}$

$= \sin 2\theta$ *(M1)*

$= \cos\left(\dfrac{\pi}{2} - 2\theta\right)$

$\alpha = \dfrac{\pi}{2} - 2\theta$ or $\alpha = \arccos(\sin 2\theta)$ *(A1)* *(C3)*

**Method 2:**




Q is the image of P under a reflection in $y = x$ *(M1)*

$\theta + \dfrac{\alpha}{2} = \dfrac{\pi}{4}$ *(A1)*

$\alpha = \dfrac{\pi}{2} - 2\theta$ *(A1)* *(C3)*

***[3 marks]***

**12.** **Method 1:** $T_{r+1} = \dbinom{7}{r} x^{7-r}\left(\dfrac{1}{ax^2}\right)^r = \dbinom{7}{r}\left(\dfrac{1}{a}\right)^r x^{7-3r}$ *(M1)*

$7 - 3r = 1$

$r = 2$ *(A1)*

Now, $\dbinom{7}{2}\dfrac{1}{a^2} = \dfrac{7}{3}$

$\Rightarrow \quad a^2 = 9$

$\Rightarrow \quad a = \pm 3.$ *(A1)* *(C3)*

**Method 2:** $\left(x + \dfrac{1}{ax^2}\right)^7 = x^7\left(1 + \dfrac{1}{ax^3}\right)^7$ *(M1)*

Coefficient of $x = \dbinom{7}{2}\left(\dfrac{1}{a}\right)^2$ *(A1)*

Thus, $\dfrac{21}{a^2} = \dfrac{7}{3}$ which leads to $a = \pm 3$ *(A1)* *(C3)*

***[3 marks]***

**13. Method 1:** $y = 4 - x^2$

$\frac{dy}{dx} = -2x = m$ when $x = -\frac{m}{2}$ *(M1)*

Thus, $\left(-\frac{m}{2}, 4 - \frac{m^2}{4}\right)$ lies on $y = mx + 5$. *(R1)*

Then, $4 - \frac{m^2}{4} = -\frac{m^2}{2} + 5$, so $m^2 = 4$

$m = \pm 2$. *(A1)* *(C3)*

**Method 2:** For intersection: $mx + 5 = 4 - x^2$ or $x^2 + mx + 1 = 0$. *(M1)*

For tangency: discriminant $= 0$ *(M1)*

Thus, $m^2 - 4 = 0$

$m = \pm 2$ *(A1)* *(C3)*

***[3 marks]***

**14.** $y^2 = x^3$ so $2y\frac{dy}{dx} = 3x^2$.

At $P(1, 1), \frac{dy}{dx} = \frac{3}{2}$. *(M1)*

The tangent is $3x - 2y = 1$, giving $Q = \left(\frac{1}{3}, 0\right)$ and $R = \left(0, -\frac{1}{2}\right)$. *(A1)*

Therefore, PQ : QR $= \frac{2}{3} : \frac{1}{3}$ or $1 : \frac{1}{2}$

$= 2 : 1$. *(A1)* *(C3)*

***[3 marks]***

**15.** $\frac{u_1}{1-r} = \frac{27}{2}$ and $u_1 + u_1 r + u_1 r^2 = 13$ *(M1)*

$\frac{27}{2}(1-r)(1+r+r^2) = 13$ *(M1)*

$1 - r^3 = \frac{26}{27}$ giving $r = \frac{1}{3}$

Therefore, $u_1 = 9$. *(A1)* *(C3)*

***[3 marks]***

**16.** **Note:** Award full marks for exact answers or answers given to three significant figures.

**Method 1:**

Using the sine rule: $\frac{\sin C}{6} = \frac{\sin 30^\circ}{3\sqrt{2}}$

$\sin C = \frac{1}{\sqrt{2}}$

$C = 45^\circ, 135^\circ$. *(M1)*

Again, $\frac{3\sqrt{2}}{\sin 30^\circ} = \frac{BC}{\sin 105^\circ}$ or $\frac{BC}{\sin 15^\circ}$

Thus, $BC = 6\sqrt{2}\sin 105^\circ$ or $6\sqrt{2}\sin 15^\circ$

$BC = 8.20$ cm or $BC = 2.20$ cm. *(A1)(A1)* *(C3)*

**Method 2:**

Using the cosine rule: $AC^2 = 6^2 + BC^2 - 2(6)(BC)\cos 30°$

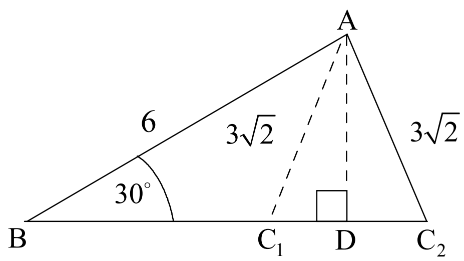
$18 = 36 + BC^2 - 6\sqrt{3}BC$ *(M1)*

Therefore, $BC^2 - (6\sqrt{3})BC + 18 = 0$

Therefore, $(BC - 3\sqrt{3})^2 = 27 - 18 = 9$

Therefore, $BC = 3\sqrt{3} \pm 3$, *i.e.* $BC = 8.20$ cm or $BC = 2.20$ cm. *(A1)(A1)* *(C3)*

**Method 3:**




In $\Delta ABD$, $AD = 3$ cm,
and $BD = \sqrt{27} = 3\sqrt{3}$ cm. *(A1)*

In $\Delta AC_1D$, $C_1D = 3$ *(A1)*

Also, $C_2D = 3$.

Therefore $BC = (3\sqrt{3} \pm 3)$ cm, *i.e.* $BC = 8.20$ cm or $BC = 2.20$ cm. *(A1)* *(C3)*

**Note:** If only one answer is given, award a maximum of ***(M1)(A1)***.

***[3 marks]***

**17.** $xy\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 + y^2 \Rightarrow \int \dfrac{y}{1+y^2}\,\mathrm{d}y = \int \dfrac{1}{x}\,\mathrm{d}x$ ***(M1)***

$\dfrac{1}{2}\ln(1+y^2) = \ln x + \ln c$ ***(M1)***

$1 + y^2 = kx^2 \quad (k = c^2)$

$y = 0$ when $x = 2$, and so $1 = 4k$

Thus, $1 + y^2 = \dfrac{1}{4}x^2$ or $x^2 - 4y^2 = 4$. ***(A1)*** ***(C3)***

***[3 marks]***

**18.** Let $z = x + \mathrm{i}y,\ x, y \in \mathbb{R}$.

Then, $|z+16|^2 = 16|z+1|^2$

$\Rightarrow \quad (x+16)^2 + y^2 = 16\{(x+1)^2 + y^2\}$ ***(M1)***

$\Rightarrow \quad x^2 + 32x + 256 + y^2 = 16x^2 + 32x + 16 + 16y^2$

$\Rightarrow \quad 15x^2 + 15y^2 = 240$

$\Rightarrow \quad x^2 + y^2 = 16$ ***(A1)***

Therefore, $|z| = 4$. ***(A1)*** ***(C3)***

***[3 marks]***

**19.** The first student can receive $x$ coins in $\dbinom{6}{x}$ ways, $1 \le x \le 5$. ***(M1)***

[The second student then receives the rest.]

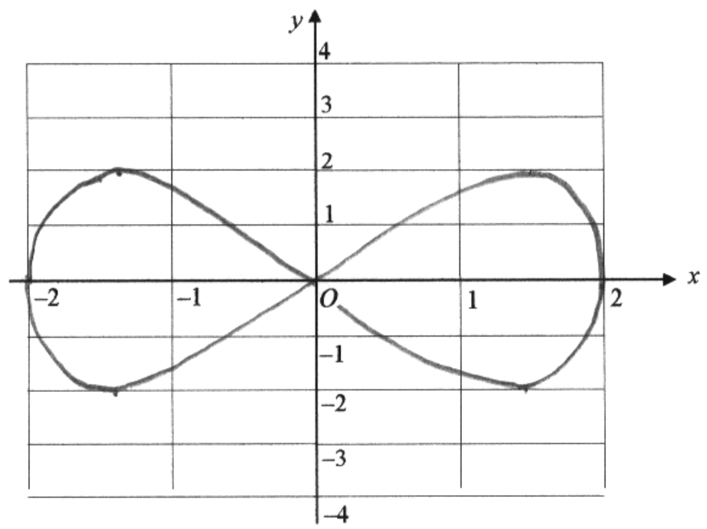
Therefore, the number of ways $= \dbinom{6}{1} + \dbinom{6}{2} + \dbinom{6}{3} + \dbinom{6}{4} + \dbinom{6}{5}$ ***(A1)***

$= 2^6 - 2$

$= 62$. ***(A1)*** ***(C3)***

***[3 marks]***

**20.**




***(A1)(A1)(A1)*** ***(C3)***

| **Note:** Award ***(A1)*** for maxima and minima, ***(A1)*** for symmetry, ***(A1)*** for zeros. |
|---|

***[3 marks]***