INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MATHEMATICS

Higher Level

Thursday 4 November 1999 (afternoon)

Paper 1 — 2 hours

A

Candidate name:	Candidate category & number:

This examination paper consists of 20 questions.
The maximum mark for each question is 4.
The maximum mark for this paper is 80.

INSTRUCTIONS TO CANDIDATES

Write your candidate name and number in the boxes above.

Do NOT open this examination paper until instructed to do so.

Answer ALL questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

B

QUESTIONS ANSWERED
ALL

C

EXAMINER	TEAM LEADER
TOTAL /80	TOTAL /80

D

IBCA
TOTAL /80

EXAMINATION MATERIALS

Required:
IB Statistical Tables
Calculator
Ruler and compasses

Allowed:
A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Trigonometrical identities:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$

$$\text{If } \tan\frac{\theta}{2} = t \text{ then } \sin\theta = \frac{2t}{1+t^2} \text{ and } \cos\theta = \frac{1-t^2}{1+t^2}$$

Integration by parts:

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Standard integrals:

$$\int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + c \quad (|x| < a)$$

Statistics: If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard deviation s are given by

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i}, \qquad s = \sqrt{\frac{\Sigma f_i (x_i - m)^2}{\Sigma f_i}}, \qquad i = 1, 2, \ldots, n$$

Binomial distribution:

$$p_x = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to this examination paper.

1. An arithmetic sequence has 5 and 13 as its first two terms respectively.

(a) Write down, in terms of n, an expression for the nth term, a_n.

(b) Find the number of terms of the sequence which are less than 400.

Working:

Answers:

(a) ______________________

(b) ______________________

2. Given the matrix $A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$, find the values of the real number k for which $\det(A - kI) = 0$ where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Working:

Answers:

____

3. The vector $\vec{n} = 2\vec{i} - \vec{j} + 3\vec{k}$ is normal to a plane which passes through the point $(2, 1, 2)$.

(a) Find an equation for the plane.

(b) Find a if the point $(a, a-1, a-2)$ lies on the plane.

Working:

Answers:

(a) ______________________________

(b) ______________________________

4. The random variable X is distributed normally with mean 30 and standard deviation 2. Find $p(27 \leqslant X \leqslant 34)$.

Working:

Answer:

5. For which values of the real number x is $|x+k| = |x| + k$, where k is a positive real number?

Working:

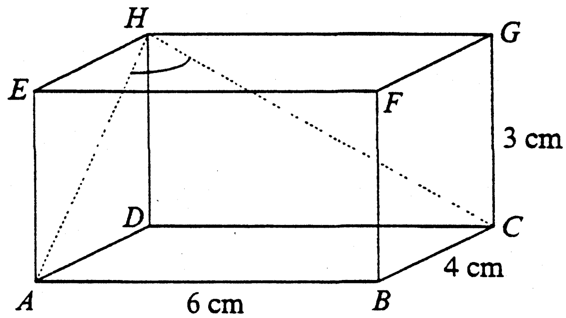
Answer:

6. The area between the graph of $y = e^x$ and the x-axis from $x = 0$ to $x = k$ $(k > 0)$ is rotated through 360° about the x-axis. Find, in terms of k and e, the volume of the solid generated.

Working:

Answer:

7. The rectangle box shown in the diagram has dimensions 6 cm × 4 cm × 3 cm.



Find, correct to the nearest one-tenth of a degree, the size of the angle $A\widehat{H}C$.

Working:

Answer:

8. The roots α and β of the quadratic equation

$$x^2 - kx + (k + 1) = 0$$

are such that $\alpha^2 + \beta^2 = 13$. Find the possible values of the real number k.

Working:

Answers:

9. Express $\dfrac{3x - 4}{x^2 - x}$ in partial fractions.

Working:

Answer:

10. Find the largest domain for the function $f : x \mapsto \dfrac{1}{\sqrt{4-9x^2}}$.

Working:

Answer:

11. Find the real number $k > 1$ for which $\displaystyle\int_1^k \left(1+\frac{1}{x^2}\right) \mathrm{d}x = \frac{3}{2}$.

Working:

Answer:

12. (a) Find the values of a and b given that the matrix $A = \begin{pmatrix} a & -4 & -6 \\ -8 & 5 & 7 \\ -5 & 3 & 4 \end{pmatrix}$ is the inverse of the matrix $B = \begin{pmatrix} 1 & 2 & -2 \\ 3 & b & 1 \\ -1 & 1 & -3 \end{pmatrix}$.

(b) For the values of a and b found in part (a), solve the system of linear equations

$$\begin{aligned} x + 2y - 2z &= 5 \\ 3x + by + z &= 0 \\ -x + y - 3z &= a - 1\,. \end{aligned}$$

Working:

Answers:

(a) ______________________

(b) ______________________

13. The local Football Association consists of ten teams. Team A has a 40 % chance of winning any game against a higher-ranked team, and a 75 % chance of winning any game against a lower-ranked team. If A is currently in fourth position, find the probability that A wins its next game.

Working:

Answer:

14. The polynomial $p(x) = (ax + b)^3$ leaves a remainder of -1 when divided by $(x + 1)$, and a remainder of 27 when divided by $(x - 2)$. Find the values of the real numbers a and b.

Working:

Answers:

15. Find, in terms of the constant a, the equation of the **normal** to the curve defined parametrically by

$$x = a(\theta - \sin\theta)$$
$$y = a(1 - \cos\theta)$$

at the point on the curve where $\theta = \frac{\pi}{2}$.

Working:

Answer:

16. The acceleration, $a(t)$ m s^{-2}, of a fast train during the first 80 seconds of motion is given by

$$a(t) = -\frac{1}{20}t + 2$$

where t is the time in seconds. If the train starts from rest at $t = 0$, find the distance travelled by the train in the first minute.

Working:

Answer:

17. For what values of k is the straight line $y = kx + 1$ a tangent to the circle with centre (5 , 1) and radius 3 ?

Working:

Answers:

18. Calculate the shortest distance from the point $A(0 , 2 , 2)$ to the line

$$\vec{r} = 5\vec{i} + 9\vec{j} + 6\vec{k} + t(\vec{i} + 2\vec{j} + 2\vec{k})$$

where t is a scalar.

Working:

Answer:

19. Solve the differential equation

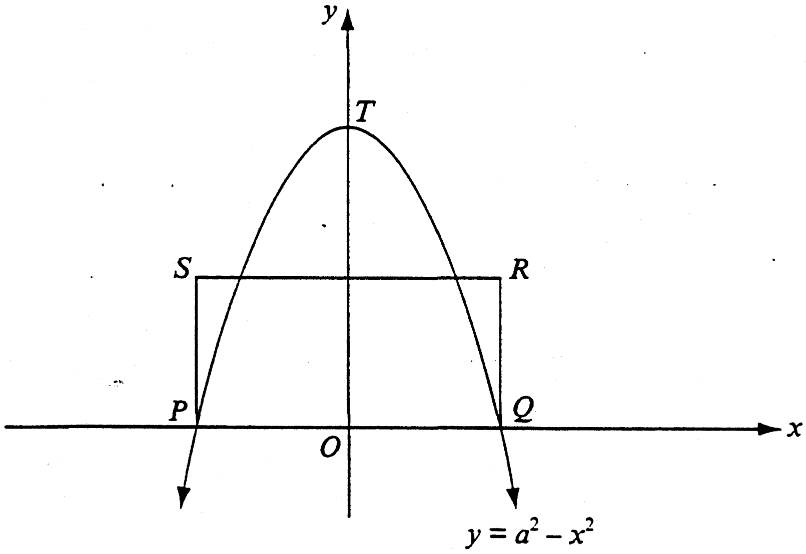
$$\frac{dy}{dx} = y \tan x + 1, \quad 0 \leq x < \frac{\pi}{2},$$

if $y = 1$ when $x = 0$.

Working:

Answer:

20. In the diagram, PTQ is an arc of the parabola $y = a^2 - x^2$, where a is a positive constant, and $PQRS$ is a rectangle. The area of the rectangle $PQRS$ is equal to the area between the arc PTQ of the parabola and the x-axis.



Find, in terms of a, the dimensions of the rectangle.

Working:

Answer: