INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MATHEMATICS

Higher Level

Tuesday 4 May 1999 (afternoon)

Paper 1 — 2 hours

A

Candidate name:	Candidate category & number:

This examination paper consists of 20 questions.
The maximum mark for each question is 4.
The maximum mark for this paper is 80.

INSTRUCTIONS TO CANDIDATES

Write your candidate name and number in the boxes above.

Do NOT open this examination paper until instructed to do so.

Answer ALL questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

B

QUESTIONS ANSWERED
ALL

C

EXAMINER	TEAM LEADER
TOTAL /80	TOTAL /80

D

IBCA
TOTAL /80

EXAMINATION MATERIALS

Required:
IB Statistical Tables
Calculator
Ruler and compasses

Allowed:
A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Trigonometrical identities:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$

$$\text{If } \tan\frac{\theta}{2} = t \text{ then } \sin\theta = \frac{2t}{1+t^2} \text{ and } \cos\theta = \frac{1-t^2}{1+t^2}$$

Integration by parts:

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Standard integrals:

$$\int\frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\int\frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + c \quad (|x| < a)$$

Statistics: If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard deviation s are given by

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad s = \sqrt{\frac{\Sigma f_i (x_i - m)^2}{\Sigma f_i}}, \qquad i = 1, 2, \ldots, n$$

Binomial distribution:

$$p_x = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Maximum marks will be given for correct answers. Where an answer is wrong some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary, or on extra sheets of paper provided these are securely fastened to this examination paper.

1. When the function $f(x) = 6x^4 + 11x^3 - 22x^2 + ax + 6$ is divided by $(x + 1)$ the remainder is -20. Find the value of a.

Working:

Answer:

2. A bag contains 2 red balls, 3 blue balls and 4 green balls. A ball is chosen at random from the bag and is not replaced. A second ball is chosen. Find the probability of choosing one green ball and one blue ball in any order.

Working:

Answer:

3. The second term of an arithmetic sequence is 7. The sum of the first four terms of the arithmetic sequence is 12. Find the first term, a, and the common difference, d, of the sequence.

Working:

Answers:

4. Find the coordinates of the point where the line given by the parametric equations $x = 2\lambda + 4$, $y = -\lambda - 2$, $z = 3\lambda + 2$, intersects the plane with equation $2x + 3y - z = 2$.

Working:

Answer:

5. Let $z = x + y\mathrm{i}$. Find the values of x and y if $(1 - \mathrm{i})z = 1 - 3\mathrm{i}$.

Working:

Answers:

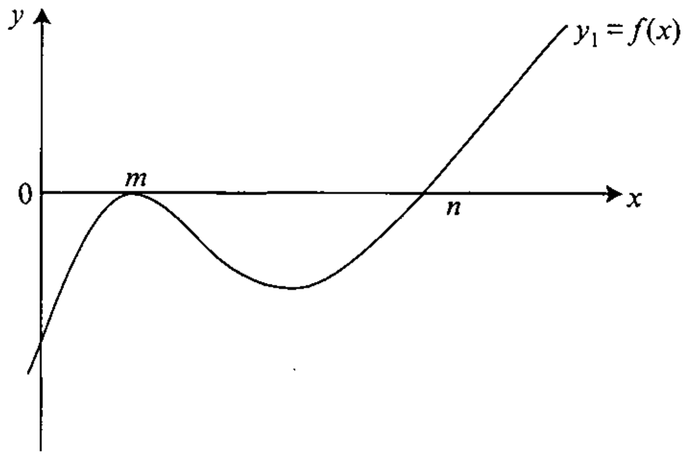
6. Find the value of a for which the following system of equations does not have a unique solution.

$$\begin{aligned} 4x - y + 2z &= 1 \\ 2x + 3y &= -6 \\ x - 2y + az &= \frac{7}{2} \end{aligned}$$

Working:

Answer:

7. The diagram below shows the graph of $y_1 = f(x)$. The x-axis is a tangent to $f(x)$ at $x = m$ and $f(x)$ crosses the x-axis at $x = n$.



On the same diagram sketch the graph of $y_2 = f(x - k)$, where $0 < k < n - m$ and indicate the coordinates of the points of intersection of y_2 with the x-axis.

Working:

8. In a bilingual school there is a class of 21 pupils. In this class, 15 of the pupils speak Spanish as their first language and 12 of these 15 pupils are Argentine. The other 6 pupils in the class speak English as their first language and 3 of these 6 pupils are Argentine.

A pupil is selected at random from the class and is found to be Argentine. Find the probability that the pupil speaks Spanish as his/her first language.

Working:

Answer:

9. If $2x^2 - 3y^2 = 2$, find the two values of $\frac{dy}{dx}$ when $x = 5$.

Working:

Answers:

10. (a) Find a vector perpendicular to the two vectors:

$$\overrightarrow{OP} = \vec{i} - 3\vec{j} + 2\vec{k}$$

$$\overrightarrow{OQ} = -2\vec{i} + \vec{j} - \vec{k}$$

(b) If $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are position vectors for the points P and Q, use your answer to part (a), or otherwise, to find the area of the triangle OPQ.

Working:

Answers:

(a) ________________________

(b) ________________________

11. Differentiate $y = \arccos(1 - 2x^2)$ with respect to x, and simplify your answer.

Working:

Answer:

____

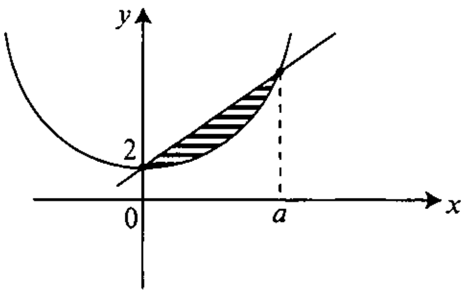
12. Given $f(x) = x^2 + x(2-k) + k^2$, find the range of values of k for which $f(x) > 0$ for all real values of x .

Working:

Answer:

13. The area of the enclosed region shown in the diagram is defined by

$$y \geqslant x^2 + 2\,,\; y \leqslant ax + 2\,, \text{ where } a > 0\,.$$



This region is rotated 360° about the x-axis to form a solid of revolution. Find, in terms of a, the volume of this solid of revolution.

Working:

Answer:

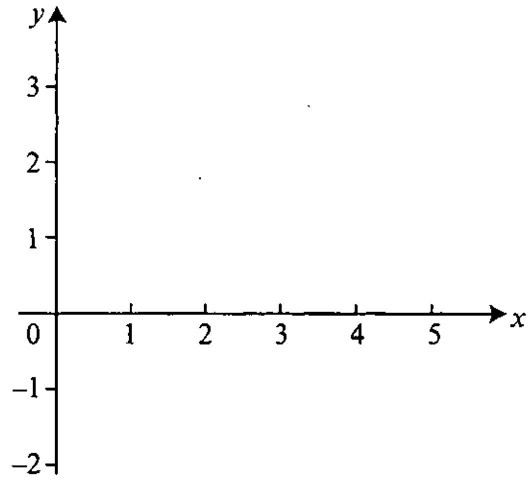
14. Using the substitution $u = \frac{1}{2}x + 1$, or otherwise, find the integral

$$\int x\sqrt{\frac{1}{2}x + 1}\ \mathrm{d}x.$$

Working:

Answer:

15. On the diagram below, draw the locus of the point $P(x, y)$, representing the complex number $z = x + yi$, given that $|z - 4 - 3i| = |z - 2 + i|$.



Working:

16. Given that

$$(1 + x)^5 (1 + ax)^6 \equiv 1 + bx + 10x^2 + \ldots\ldots + a^6x^{11},$$

find the values of a, $b \in \mathbb{Z}^*$.

Working:

Answers:

17. A biased die with four faces is used in a game. A player pays 10 counters to roll the die. The table below shows the possible scores on the die, the probability of each score and the number of counters the player receives in return for each score.

Score	1	2	3	4
Probability	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$
Number of counters player receives	4	5	15	n

Find the value of n in order for the player to get an expected return of 9 counters per roll.

Working:

Answer:

18. A factory has a machine designed to produce 1 kg bags of sugar. It is found that the average weight of sugar in the bags is 1.02 kg. Assuming that the weights of the bags are normally distributed, find the standard deviation if 1.7% of the bags weigh below 1 kg. Give your answer correct to the nearest 0.1 gram.

Working:

Answer:

19. When air is released from an inflated balloon it is found that the rate of decrease of the volume of the balloon is proportional to the volume of the balloon. This can be represented by the differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} = -kv$, where v is the volume, t is the time and k is the constant of proportionality.

(a) If the initial volume of the balloon is v_0, find an expression, in terms of k, for the volume of the balloon at time t.

(b) Find an expression, in terms of k, for the time when the volume is $\frac{v_0}{2}$.

Working:

Answers:

(a) ____________________

(b) ____________________

20. A particle moves along a straight line. When it is a distance s from a fixed point, where $s > 1$, the velocity v is given by $v = \frac{(3s+2)}{(2s-1)}$. Find the acceleration when $s = 2$.

Working:

Answer: