INTERNATIONAL BACCALAUREATE
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N98/510/H(1)M

MARKSCHEME

November 1998

MATHEMATICS

Higher Level

Paper 1

12 pages

Paper 1 Markscheme

Instructions to Examiners

1 **Method of Marking**

(a) All marking must be done using a **red** pen.

(b) In this paper, the maximum mark is awarded for a **correct answer**, irrespective of the method used. Thus, if the correct answer appears in the answer box, award the maximum mark and move onto the next question; in this case there is no need to check the method.

(c) If an **answer is wrong**, then marks should be awarded for the method according to the markscheme. (A correct answer incorrectly transferred to the answer box is awarded the maximum mark.)

2 **Abbreviations**

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

C Marks awarded for **Correct** answers (irrespective of working shown)

R Marks awarded for clear **Reasoning**

3 **Follow Through (ft) Marks**

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, follow through (ft) marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) **penalise the error when it first occurs;**

(ii) **accept the incorrect answer as the appropriate value or quantity to be used in all subsequent working;**

(iii) award M marks for a correct method and A(ft) **marks if the subsequent working contains no further errors.**

Follow through procedures may be applied repeatedly throughout the same problem.

The errors made by a candidate may be: arithmetical errors; errors in algebraic manipulation; errors in geometrical representation; use of an incorrect formula; errors in conceptual understanding.

The following illustrates a use of the **follow through procedure.**

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	**M1**	Amount earned = \$ 600 × 1.02	✓	**M1**
= \$ 612	**A1**	= \$602	×	**A0**
\$ (306 × 1.02) + (306 × 1.04)	**M1**	Amount = 301 × 1.02 + 301 × 1.04	✓	**M1**
= \$ 630.36	**A1**	= \$ 620.06	✓	**A1(ft)**

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by '**(d)**' (to indicate that these marks have been awarded at **the discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by '**(d)**' (to indicate that the marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7 , 1,7 ; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

(a) In the case when the accuracy of the answer is **specified in the question** (for example: "find the size of angle *A* to the nearest degree") the maximum mark is awarded **only if** the correct answer is given to the accuracy required.

(b) When the accuracy is **not** specified in the question, then the general rule applies:

> *Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.*

In this case, the candidate is **penalised once only IN THE PAPER** for giving a correct answer to the wrong degree of accuracy. Hence, on the **first** occasion in the paper when a correct answer is given to the wrong degree of accuracy maximum marks are **not** awarded, but on **all subsequent occasions** when correct answers are given to the wrong degree of accuracy then maximum marks **are** awarded.

1. (a) let $y=\dfrac{1}{x+1} \Leftrightarrow x+1=\dfrac{1}{y} \Leftrightarrow x=\dfrac{1}{y}-1$, thus $f^{-1}(x)=\dfrac{1}{x}-1$ *(M1)(A1)*

   (b) $(f \circ g)(x)=f\left(\sqrt{x^2-1}\right)=\dfrac{1}{\sqrt{x^2-1}+1}$ *(M1)(A1)*

   Answers: (a) $f^{-1}(x)=\dfrac{1}{x}-1$ *(C2)*

   (b) $(f \circ g)(x)=\dfrac{1}{\sqrt{x^2-1}+1}$ *(C2)*

2. $4^x = 8^y \Leftrightarrow 2^{2x} = 2^{3y} \Leftrightarrow 2x=3y$, and *(M1)*

   $x+2y=5 \Leftrightarrow 2x+4y=10$

   $\Leftrightarrow 3y+4y=10 \Leftrightarrow y=\dfrac{10}{7}$, and since $x=\dfrac{3y}{2} \Leftrightarrow x=\dfrac{15}{7}$ *(M1)(A2)*

   Answer: $x=\dfrac{15}{7}, y=\dfrac{10}{7}$ *(C4)*

3. (a) $P(-2)=(-2)^3-3(-2)^2+4(-2)+c=-28+c$ *(M1)(A1)*

   (b) $-28+c=-23$ therefore $c=5$ *(M1)(A1)*

   Answers: (a) $-28+c$ *(C2)*

   (b) $c=5$ *(C2)*

4. $\dbinom{5}{3}k^3=\dbinom{5}{4}k^4 \Rightarrow 10k^3=5k^4$ *(M1)(A1)*

   $\Rightarrow 5k^3(2-k)=0$ *(A1)*

   $\Rightarrow k=2$ *(A1)*

   Answer: $k=2$ *(C4)*

5. (a) The values seem to repeat every 6 units. The period is 6. *(R1)(A1)*

   (b) $f(41) = f(36+5) = f(5) = 4$ *(M1)(A1)*

   Answers: (a) The period is 6. *(C2)*
   (b) $f(41) = 4$ *(C2)*

6. (a) $k + \frac{1}{4} + \frac{1}{4} + 3k = 1 \Rightarrow 4k = \frac{1}{2} \Rightarrow k = \frac{1}{8}$ *(M1)(A1)*

   (b) $E(X) = 1 \times \frac{1}{4} + 2 \times \frac{1}{4} + 3 \times \frac{3}{8} = \frac{15}{8}$ *(M1)(A1)*

   Answers: (a) $k = \frac{1}{8}$ *(C2)*

   (b) $E(X) = \frac{15}{8}$ *(C*

7. $\vec{a} \perp \vec{b} \Leftrightarrow (-2 \times 1) + (p \times 3)(p+4) - 1(2p-5) = 0$ *(M1)*

   $\Leftrightarrow 3p^2 + 10p + 3 = 0 \Leftrightarrow (3p+1)(p+3) = 0$ *(M1)*

   $\Leftrightarrow p = -\frac{1}{3},\ \text{or } p = -3$ *(A2)*

   Answer: $p = -\frac{1}{3},\ \text{or } p = -3$ *(C4)*

8. In triangle ABL, it is clear that $\hat{L} = 20^\circ$. Using the law of sines we have *(A1)*

   $\frac{AB}{\sin 20^\circ} = \frac{BL}{\sin 50^\circ} \Leftrightarrow \frac{50}{\sin 20^\circ} = \frac{BL}{\sin 50^\circ}$ *(M1)*

   $\Rightarrow BL = \frac{50 \sin 50^\circ}{\sin 20^\circ} = 111.99 \text{ km} = 112 \text{ km}$ *(M1)(A*

   Answer: $BL = 112$ km *(C4)*

9. (a) For $f(x)$ to be defined, $6x^2 - 5x - 6$ must be larger than zero. *(R1)*

$6x^2 - 5x - 6 > 0 \Leftrightarrow (3x+2)(2x-3) > 0$ *(M1)*

$\Leftrightarrow \left\{ x \in \mathbb{R} \,\middle|\, x < \frac{-2}{3} \text{ or } x > \frac{3}{2} \right\}$ *(A1)*

(b) $\mathbb{R}$ *(A1)*

Answers: (a) $\left\{ x \in \mathbb{R} \,\middle|\, x < \frac{-2}{3} \text{ or } x > \frac{3}{2} \right\}$ *(C3)*

(b) $\mathbb{R}$ *(C1)*

10. $z^2 = -5 + 12\mathrm{i} \Leftrightarrow (a^2 - b^2) + (2ab)\mathrm{i} = -5 + 12\mathrm{i}$ *(M1)*

$\Leftrightarrow a^2 - b^2 = -5$, and $2ab = 12$, or $ab = 6$

$\Leftrightarrow a^2 - \frac{36}{a^2} = -5 \Leftrightarrow a^4 + 5a^2 - 36 = 0$ *(M1)*

$\Leftrightarrow (a^2 + 9)(a^2 - 4) = 0 \Rightarrow a = \pm 2$ and $b = \pm 3$

($a^2 + 9 = 0$ has no real solution)

The ordered pairs are then $(2, 3)$ and $(-2, -3)$ *(A2)*

Answer: $(2, 3)$ and $(-2, -3)$ *(C4)*

11. (a) For a quadratic equation to have real roots, the discriminant $b^2 - 4ac$ must be larger than or equal to zero.

$4(m+2)^2 - 4m(m+2) \geq 0 \Leftrightarrow 4(m+2)(m+2-m) \geq 0 \Rightarrow m+2 \geq 0 \Rightarrow m \geq -2$ *(M1)(A1)*

(b) For the quadratic equation to have two roots of opposite sign their product must be negative.

$\frac{m+2}{m} < 0 \Rightarrow -2 < m < 0$ *(M1)(A1)*

Answers: (a) $m+2 \geq 0 \Rightarrow m \geq -2$ *(C2)*

(b) $-2 < m < 0$ *(C2)*

12. (a) $p(A|B)=\dfrac{p(A\cap B)}{p(B)} \Rightarrow p(B)=\dfrac{\frac{1}{5}}{\frac{3}{10}}=\dfrac{2}{3}$ *(M1)(A1)*

(b) $p(B|A)=\dfrac{p(A\cap B)}{p(A)} \Rightarrow p(A)=\dfrac{\frac{1}{5}}{\frac{1}{2}}=\dfrac{2}{5}$ *(A1)*

(c) $p(A\cup B)=p(A)+p(B)-p(A\cap B)=\dfrac{2}{5}+\dfrac{2}{3}-\dfrac{1}{5}=\dfrac{13}{15}$ *(A1)*

Answers: (a) $p(B)=\dfrac{2}{3}$ *(C2)*

(b) $p(A)=\dfrac{2}{5}$ *(C1)*

(c) $p(A\cup B)=\dfrac{13}{15}$ *(C1)*

13. $(3\cos x+5)^2=(4\sin x)^2$ *(M1)*

$9\cos^2 x+30\cos x+25=16\sin^2 x$

$25\cos^2 x+30\cos x+9=0$ *(M1)*

$(5\cos x+3)^2=0$

$\cos x=-\dfrac{3}{5}$, hence $\tan x=-\dfrac{4}{3}$ *(M1)(A1)*

Answer: $\tan x=-\dfrac{4}{3}$ *(C4)*

14. $\dfrac{1}{2}\begin{pmatrix} b & -5 & 4\\ -1 & 1 & 0\\ 1 & -3 & 2\end{pmatrix}\begin{pmatrix} 1 & -1 & -2\\ 1 & 1 & -2\\ 1 & 2 & a\end{pmatrix}=\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ *(M1)*

$\Leftrightarrow \dfrac{1}{2}\begin{pmatrix} b-1 & 3-b & 4a-2b+10\\ 0 & 2 & 0\\ 0 & 0 & 2a+4\end{pmatrix}=\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ *(A1)*

$\Leftrightarrow b-1=2, 3-b=0, 4a-2b+10=0, 2a+4=2$

$\Leftrightarrow a=-1, b=3$ *(M1)(A1)*

Answer: $a=-1, b=3$ *(C4)*

15. For a plane to be perpendicular to two intersecting planes, it must be perpendicular to their line of intersection. Hence, if we find the cross product of normals to the two given planes, we can use it as a normal to the new plane. *(R1)*

$(6\vec{i} - 2\vec{j} + 3\vec{k}) \times (\vec{i} - 3\vec{j}) = 9\vec{i} + 3\vec{j} - 16\vec{k}$ *(A1)*

Therefore the equation of the plane is of the form $9x + 3y - 16z = d$. *(R1)*
Since the plane contains the point $(2, 2, 3), d = -24$, and the equation is $9x + 3y - 16z = -24$. (Other forms of the equation are acceptable.) *(A1)*

Answer: $9x + 3y - 16z = -24$ *(C4)*

16. The largest parallelogram is a rhombus, its area is $4 \times 4 \times \sin 60° = 8\sqrt{3}$, *(R1)*
the second parallelogram is a rectangle with one side 2 and the other $2\sqrt{3}, \Rightarrow$ Area $= 4\sqrt{3}$, *(A1)*
the third is a rhombus again with side 2, its area is then $2\sqrt{3}$, and so on.

The sum is then an infinite geometric series with a common ratio of $\frac{1}{2}$. *(M1)*

Therefore the sum is $\dfrac{8\sqrt{3}}{1 - \frac{1}{2}} = 16\sqrt{3}$. *(A1)*

Answer: $\dfrac{8\sqrt{3}}{1 - \frac{1}{2}} = 16\sqrt{3}$ *(C4)*

17. $\int_1^k \frac{x}{\sqrt{3}} \sqrt{x^2 - 1}\, dx = 1 \Rightarrow \left[\frac{1}{3\sqrt{3}} (x^2 - 1)^{\frac{3}{2}} \right]_1^k = 1$ *(M1)(A1)*

$\Rightarrow (k^2 - 1)^{\frac{3}{2}} = 3\sqrt{3} \Rightarrow (k^2 - 1)^3 = 27 \Rightarrow k^2 - 1 = 3$ *(A1)*

$\Rightarrow k = 2$ (since $k \geq 1$). *(A1)*

Answer: $k = 2$ (since $k \geq 1$). *(C4)*

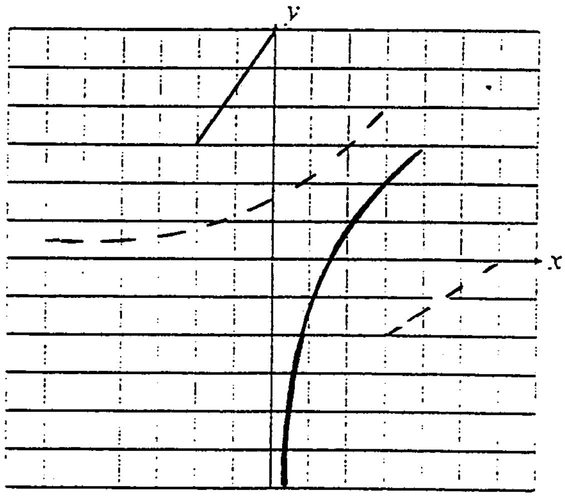
18. For the three vectors to be coplanar, then a vector perpendicular to two of them should be perpendicular to the third, hence, the cross product of two of them multiplied (scalar product) by the third should be zero. *(R1)*

$\vec{w}\cdot\vec{u}\times\vec{v}=0 \Rightarrow \left(\vec{i}+(2-t)\vec{j}+(t+1)\vec{k}\right)\cdot(-12\vec{i}+9\vec{j}+2\vec{k})=0$ *(M1)(A1)*

$\Rightarrow 8-7t=0 \Rightarrow t=\frac{8}{7}$ *(A1)*

Answer: $t=\frac{8}{7}$ *(C4)*

19. (a)

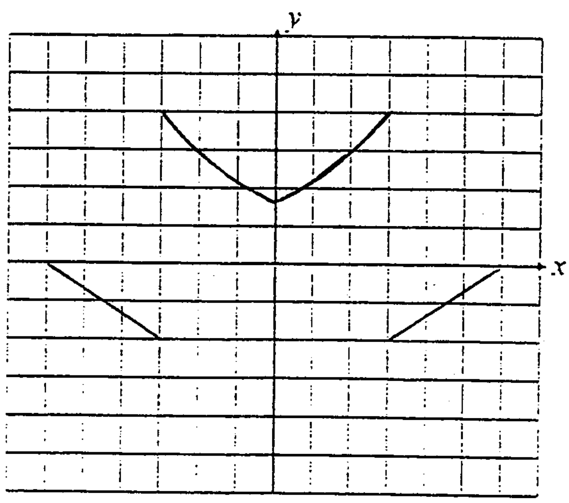


(Deduct 1 mark for each error)

(C2)

The dotted line is the original graph.

(b)



(Deduct 1 mark for each error)

(C2)

20. (a) $f(x) = \dfrac{1}{1+e^{x^2}}$ *(A1)*

(b) Since $1+e^{x^2}$ has a minimum value of 2, $f(x)$ will have a maximum value of $\dfrac{1}{2}$. *(R2)*

Therefore $k \le \dfrac{1}{2}$ *(A1)*

Answers: (a) $f(x) = \dfrac{1}{1+e^{x^2}}$ *(C1)*

(b) $0 < k \le \dfrac{1}{2}$ *(C3)*

1. (a) $(\vec{i}+\vec{j})\times(\vec{i}-2\vec{k}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 1 & 0 & -2 \end{vmatrix}$ *(M1)*

$= \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}\vec{k}$ *(M1)*

$= -2\vec{i} + 2\vec{j} - \vec{k}$ *(A1)*

OR $(\vec{i}+\vec{j})\times(\vec{i}-2\vec{k}) = \vec{i}\times\vec{i} - 2\vec{i}\times\vec{k} + \vec{j}\times\vec{i} - 2\vec{j}\times\vec{k}$ *(M1)*

$= 0 - 2(-\vec{j}) + (-\vec{k}) - 2\vec{i}$ *(M1)*

$= -2\vec{i} + 2\vec{j} - \vec{k}$ *(A1)*

(b) (i) $\vec{n}_1 = 6\vec{i} + 3\vec{j} - 2\vec{k}$ and $\vec{n}_2 = -2\vec{i} + 2\vec{j} - \vec{k}$ *(A2)*

(ii) If θ is the required (acute) angle, then

$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$ *(M1)*

$= \frac{|-12+6+2|}{(7)(3)}$

$= \frac{4}{21}$ *(A1)*

Therefore, the required angle is 79.0° *(A1)*

(c) The equation of L_2 is $\frac{x+30}{2} = \frac{y+39}{1} = \frac{z-\frac{2}{3}}{-\frac{2}{3}}$ *(M1)*

Thus, the direction of L_2 is $\vec{l}_2 = 2\vec{i} + \vec{j} - \frac{2}{3}\vec{k} = \frac{1}{3}\vec{n}_1$ *(A2)*

Therefore L_2 is normal to plane P_1 *(AG)*

Question 1 continued

(d) (i) L_2 has parametric equations

$x = 2u - 30$, $y = u - 39$, $z = \frac{2}{3}(1-u)$, where u is a parameter

L_2 meets P_1 when $6(2u-30) + 3(u-39) - 2\left(\frac{2}{3}(1-u)\right) = 12$ *(M1)*

$\Rightarrow u = 19$ *(A1)*

Thus, the required coordinates are $(8, -20, -12)$ *(A1)*

(ii) The point found in part (i) lies on L_1 since when $t = 1$, $\vec{r} = 8\vec{i} - 20\vec{j} - 12\vec{k}$ *(M1)*

This point lies on P_2 since when $\mu = 6$ and $\lambda = -20$,

$\vec{r} = 22\vec{i} - 20(\vec{i}+\vec{j}) + 6(\vec{i} - 2\vec{k}) = 8\vec{i} - 20\vec{j} - 12\vec{k}$ *(M1)*

Thus, the two lines and two planes have the point $(8, -20, -12)$ in common. *(AG)*

2. (i) The successive distance through which the ball falls form a geometric sequence with first term 81 and the common ratio $\frac{2}{3}$.

(a) The maximum height of the ball between the fifth and the sixth bounce is

$(81)\left(\frac{2}{3}\right)^5 = \frac{32}{3}$ metre. *(M2)(A1)*

(b) The total distance traveled by the ball from the time it is dropped until it strikes the ground the sixth time is

$$\sum_{n=0}^{5} 81\left(\frac{2}{3}\right)^n + \sum_{n=0}^{4} 81\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^n$$

$$= \frac{81\left(1-\left(\frac{2}{3}\right)^6\right)}{1-\frac{2}{3}} + \frac{54\left(1-\left(\frac{2}{3}\right)^5\right)}{1-\frac{2}{3}}$$

$$= \frac{665}{3} + \frac{422}{3} = \frac{1087}{3} = 362\frac{1}{3} \text{ metres}$$ *(M2)(A2)*

Note: Some candidates may calculate the total distance as follows:

$$\text{Total distance} = 81 + 2\times\left\{54 + 54\left(\frac{2}{3}\right) + 54\left(\frac{2}{3}\right)^2 + 54\left(\frac{2}{3}\right)^3 + 54\left(\frac{2}{3}\right)^4\right\}$$

$$= 81 + 108\left(\frac{1-\left(\frac{2}{3}\right)^5}{1-\frac{2}{3}}\right) = 81 + 324\left(\frac{243-42}{243}\right)$$

$$= 81 + 281\frac{1}{3} = 362\frac{1}{3} \text{ metres}$$ *Award (M2)(A2)*

(c) If the ball continues to bounce indefinitely, then the distance traveled is

$$\sum_{0}^{\infty} 81\left(\frac{2}{3}\right)^n + \sum_{0}^{\infty} 81\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^n$$

$$= \frac{81}{1-\frac{2}{3}} + \frac{54}{1-\frac{2}{3}} = 243 + 162 = 405 \text{ metres}$$ *(M2)(A1)*

Note: Some candidates may also mention distance traveled

$$= 81 + 108\left(1 + \frac{2}{3} + \ldots\right)$$

$$= 81 + 108\left(\frac{1}{1-\frac{1}{3}}\right) = 81 + 324$$

$$= 405 \text{ metres}$$ *Award (M2)(A1)*

(ii) FIRST METHOD

Let the three numbers in arithmetic progression be $x, x+r, x+2r$. Their sum is

$$x + (x+r) + (x+2r) = 3x + 3r = 24$$

Hence $x + r = 8$ or $r = 8 - x$ *(M1)(A1)*

We are also given that $x-1, x+r-2$ and $x+2r$ are in geometric progression. So

$$\frac{x+r-2}{x-1} = \frac{x+2r}{x+r-2}$$

or $(x+r-2)^2 = (x-1)(x+2r)$. *(M1)(A1)*

Substituting $x + r = 8$ and $r = 8 - x$, we get

$$(8-2)^2 = (x-1)\{x + 2(8-x)\}$$

or $(x-1)(16-x) = 36$

or $-x^2 + 17x - 16 = 36$

or $x^2 - 17x + 52 = 0$

or $(x-13)(x-4) = 0$

Hence, $x = 13$ or 4 *(M1)(A1)*

The solutions are obtained by taking $x = 13, r = 8 - 13 = -5$ and $x = 4, r = 4$.

So there are two sets of solutions

viz. 13, 8, 3 and 4, 8, 12 *(R1)(R1)*

SECOND METHOD

Since the three numbers are in arithmetic progression with sum equal to 24, let the numbers be $8 - x,\ 8,\ 8 + x$. *(M1)(A1)*

From these we form the new numbers $7 - x,\ 6,\ 8 + x$ which are in geometric progression.

Hence $(7 - x)(8 + x) = 6^2$ *(M1)(A1)*

We get $x^2 + x - 20 = 0$ i.e. $(x + 5)(x - 4) = 0$

Hence, $x = -5$ or $x = 4$ *(M1)(A1)*

When $x = 4$, the numbers are 4, 8, 12
and when $x = -5$, the numbers are 13, 8, 3 *(M1)(A1)*