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FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 5 May 2011 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 11]*

(a) Bottles of iced tea are supposed to contain 500 ml. A random sample of 8 bottles was selected and the volumes measured (in ml) were as follows:

$$497.2,\ 502.0,\ 501.0,\ 498.6,\ 496.3,\ 499.1,\ 500.1,\ 497.7\,.$$

(i) Calculate unbiased estimates of the mean and variance.

(ii) Test at the 5 % significance level the null hypothesis $\mathrm{H}_0 : \mu = 500$ against the alternative hypothesis $\mathrm{H}_1 : \mu < 500$. *[5 marks]*

(b) A random sample of size four is taken from the distribution $\mathrm{N}(60, 36)$. Calculate the probability that the sum of the sample values is less than 250. *[6 marks]*

2. *[Maximum mark: 15]*

(a) (i) Find the range of values of n for which $\int_1^\infty x^n \, \mathrm{d}x$ exists.

(ii) Write down the value of $\int_1^\infty x^n \, \mathrm{d}x$ in terms of n, when it does exist. *[7 marks]*

(b) Find the solution to the differential equation

$$(\cos x - \sin x)\frac{\mathrm{d}y}{\mathrm{d}x} + (\cos x + \sin x)\, y = \cos x + \sin x\,,$$

given that $y = -1$ when $x = \frac{\pi}{2}$. *[8 marks]*

3. *[Maximum mark: 11]*

(a) Prove that the number 14 641 is the fourth power of an integer in any base greater than 6. *[3 marks]*

(b) For $a, b \in \mathbb{Z}$ the relation aRb is defined if and only if $\frac{a}{b} = 2^k$, $k \in \mathbb{Z}$.

(i) Prove that R is an equivalence relation.

(ii) List the equivalence classes of R on the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. *[8 marks]*

4. *[Maximum mark: 11]*

(a) Prove that if $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then $\gcd(a, bc) = 1$. *[5 marks]*

(b) (i) A simple graph has e edges and v vertices, where $v > 2$. Prove that if all the vertices have degree at least k, then $2e \geq kv$.

(ii) **Hence** prove that every planar graph has at least one vertex of degree less than 6. *[6 marks]*

5. *[Maximum mark: 12]*

The rectangle ABCD is inscribed in a circle. Sides [AD] and [AB] have lengths 3 cm and 9 cm respectively. E is a point on side [AB] such that AE is 3 cm. Side [DE] is produced to meet the circumcircle of ABCD at point P. Use Ptolemy's theorem to calculate the length of chord [AP].
