IBO

IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Tuesday 12 November 2002 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all ten questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

*You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive **no** marks.*

1.   Consider the group $(\mathbb{Z}_{12}, +)$.

   (a)   Find the order of the elements 4 , 5 and 9 .

   (b)   Show that this group is cyclic. Find all possible generators.

2.   Consider κ_n a complete graph with n vertices.

   (a)   Draw κ_5 and find an Eulerian circuit in it.

   (b)   Find the value of n such that κ_n contains an Eulerian path but not an Eulerian circuit. Justify your answer.

3.   Determine whether the following series converges or diverges.

$$\frac{1}{\sqrt{2}} + \frac{3}{2} + \frac{5}{2\sqrt{2}} + \frac{7}{4} + \frac{9}{4\sqrt{2}} + \cdots .$$

4.   Find all the integers x that satisfy the equation $2x^3 - 3x + 1 \equiv 4 \,(\mathrm{mod}\ 6)$.


5.   Eggs are packed in boxes of four. During one day 200 boxes were selected and the number of broken eggs in each box was recorded.

Number of broken eggs	0	1	2	3	4
Number of boxes	73	80	31	14	2

   Test at the 5 % level of significance whether this data follows a binomial distribution with $n = 4$ and $p = 0.24$.

6. The function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3^{\cos x} + \dfrac{1}{6}$.

    (a)   Determine whether the function is injective or surjective, giving your reasons.

    (b)   If the domain of *f* is restricted to $[0, \pi]$ find its inverse function.

7. Consider the triangle ABC. The points M, N and P are on the sides [BC], [CA] and [AB] respectively, such that the lines (AM), (BN) and (CP) are concurrent.

Given that $\dfrac{AP}{AB} = \lambda$, and $\dfrac{CM}{CB} = \mu$, where $\lambda, \mu, \in \mathbb{R}^+$, find $\dfrac{NA}{CN}$.

8. Find a cubic Taylor polynomial approximation for the function $f(x) = \tan x$, about $x = \dfrac{\pi}{4}$.

9. A school newspaper consists of three sections. The number of misprints in each section has a Poisson distribution with parameters 0.9, 1.1 and 1.5 respectively. Misprints occur independently.

    (a)   Find the probability that there will be no misprints in the newspaper.

    (b)   The probability that there are more than *n* misprints in the newspaper is less than 0.5. Find the smallest value of *n*.

Turn over

10. Consider the hyperbola H with equation $b^2x^2 - a^2y^2 = a^2b^2$. The angle between the asymptotes of H is $\frac{\pi}{3}$, as shown in the diagram below.
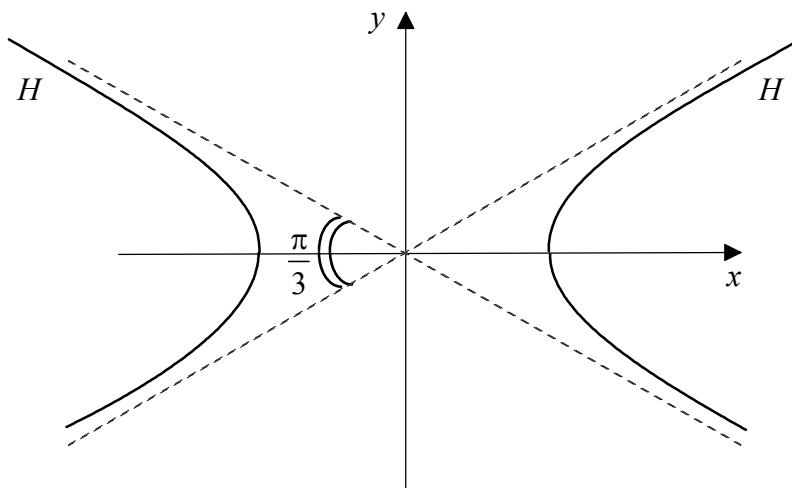
(a) Calculate the eccentricity of H.

(b) Find the equations of the directrices of H, giving your answers in terms of a.