



**FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1**

Monday 12 November 2001 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp *EL-9600*, Texas Instruments *TI-85*.

*A correct answer with **no** indication of the method used will usually receive **no** marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

1. Let $S = \{1, 2, 3, 4\}$ and let $A = S \times S$. Define the relation R on A by:

$$(a, b) R (x, y) \text{ if and only if } a + b = x + y.$$

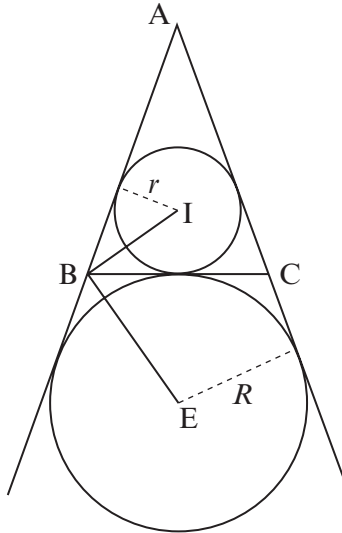
Show that R is an equivalence relation and find the partition it creates on A .

2. Determine whether the series $\sum_{k=1}^{\infty} \frac{k}{e^k}$ converges or diverges. Note the test you use.
3. Find the order of a group G generated by two elements x and y , subject only to the following relations $x^3 = y^2 = (xy)^2 = 1$. List all subgroups of G .
4. Draw a graph given by the following adjacency matrix.

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Determine how many graphs with the same number of edges are possible on this set of vertices.

5. The following diagram shows an isosceles triangle ABC , and 2 circles. The circle whose centre is I and radius is r is inscribed in $\triangle ABC$. The circle whose centre is E and radius is R is the escribed circle, *ie* it is outside $\triangle ABC$, and the lines (BC) , (AB) and (AC) are tangents to this circle.



- (a) Show that angle IBE is a right angle.
- (b) Find BC in terms of r and R .
6. Find the solution to the recurrence relation

$$a_n = 7a_{n-1} - 6a_{n-2}, \text{ with } a_0 = -1 \text{ and } a_1 = 4.$$

7. Use a binary search tree to find 43 on the following list

10, 15, 20, 28, 37, 39, 43, 58, 67, 77, 81, 99.

Show all steps.

8. A computer repair shop replaces corrupt hard disks at a rate of 4 per week. Assuming that such repairs occur at random, find the probability that
- (a) exactly 7 hard disks are replaced in one week;
- (b) in a 3-week period, at least 7 disks are replaced in two of these weeks.

9. In a triangle ABC , $AB = 8$, $AC = 10$, and the median to the side $[BC]$ has length 8. Find the area of the triangle.
10. Estimate $e^{0.2}$ correct to 3 decimal places, using the Taylor approximation

$$f(a + x) = f(a) + x f'(a) + \dots + \frac{x^n}{n!} f^{(n)}(a) + \frac{x^{n+1}}{(n+1)!} f^{(n+1)}(c)$$
