M14/5/FURMA/HP2/ENG/TZ0/XX/M

International Baccalaureate®
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MARKSCHEME

May 2014

FURTHER MATHEMATICS

Higher Level

Paper 2

19 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) are often dependent on the preceding ***M*** mark.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.* do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

Follow through ***(FT)*** *marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent** ***A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates may not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x)=2\sin(5x-3)$, the markscheme gives:

$f'(x)=\left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

The method of dealing with accuracy errors on a whole paper basis by means of the Accuracy Penalty ***(AP)*** *no longer applies.*

Instructions to examiners about such numerical issues will be provided on a question by question basis within the framework of mathematical correctness, numerical understanding and contextual appropriateness.

The rubric on the front page of each question paper is given for the guidance of candidates. The markscheme (MS) may contain instructions to examiners in the form of "Accept answers which round to n significant figures (sf)". Where candidates state answers, required by the question, to fewer than n sf, award ***A0****. Some intermediate numerical answers may be required by the MS but not by the question. In these cases only award the mark(s) if the candidate states the answer exactly or to at least 2sf.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation

The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) $\mathrm{E}\left(\frac{X}{n}\right)=\frac{1}{n}\mathrm{E}(X)$ ***M1***

$=\frac{1}{n}\times np=p$ ***A1***

therefore unbiased ***AG***

[2 marks]

(b) $\mathrm{E}\left[\left(\frac{X}{n}\right)^2\right]=\frac{1}{n^2}\left(\mathrm{Var}(X)+[\mathrm{E}(X)]^2\right)$ ***M1A1***

$=\frac{1}{n^2}\left(np(1-p)+n^2p^2\right)$ ***A1***

$\neq p^2$ ***A1***

therefore not unbiased ***AG***

[4 marks]

(c) $\mathrm{E}\left[\left(\frac{X(X-1)}{n(n-1)}\right)\right]=\frac{\mathrm{E}(X^2)-\mathrm{E}(X)}{n(n-1)}$ ***M1***

$=\frac{np(1-p)+n^2p^2-np}{n(n-1)}$ ***A1***

$=\frac{np^2(n-1)}{n(n-1)}$ ***A1***

$=p^2$

therefore unbiased ***AG***

[3 marks]

Total [9 marks]

2. (a) (i) closure: let $a_1 = \text{cis}\left(\frac{n_1\pi}{4}\right)$ and $a_2 = \text{cis}\left(\frac{n_2\pi}{4}\right) \in S$ ***M1***

then $a_1 \times a_2 = \text{cis}\left(\frac{(n_1+n_2)\pi}{4}\right)$ (which $\in S$ because the addition is carried out modulo 8) ***A1***

identity: the identity is 1 (and corresponds to $n = 0$) ***A1***

inverse: the inverse of $\text{cis}\left(\frac{n\pi}{4}\right)$ is $\text{cis}\left(\frac{(8-n)\pi}{4}\right) \in S$ ***A1***

associatively: multiplication of complex numbers is associative ***A1***

the four group axioms are satisfied so S is a group ***AG***

(ii) S is cyclic ***A1***

because $\text{cis}\left(\frac{\pi}{4}\right)$, for example, is a generator ***R1***

[7 marks]

(b) (i)

	1	**3**	**7**	**9**	**11**	**13**	**17**	**19**
1	1	3	7	9	11	13	17	19
3	3	9	1	7	13	19	11	17
7	7	1	9	3	17	11	19	13
9	9	7	3	**1**	**19**	**17**	**13**	**11**
11	11	13	17	**19**	**1**	**3**	**7**	**9**
13	13	19	11	**17**	**3**	**9**	**1**	**7**
17	17	11	19	**13**	**7**	**1**	**9**	**3**
19	19	17	13	**11**	**9**	**7**	**3**	**1**

A3

Note: Award ***A2*** for 1 or 2 errors, ***A1*** for 3 or 4 errors and ***A0*** for more than 4 errors.

(ii)

Element	**Order**
1	1
3	4
7	4
9	2
11	2
13	4
17	4
19	2

A3

Note: Award ***A2*** for 1 error, ***A1*** for 2 errors and ***A0*** for more than 2 errors.

continued ...

Question 2 continued

(iii) they are not isomorphic ***A1***
because $\{S, \times\}$ is cyclic and $\{G, \times_{20}\}$ is not cyclic ***R1***

(iv) a cyclic subgroup is {1, 3, 7, 9} or {1, 9, 13, 17} ***A1***
with generators 3, 7 or 13, 17 ***A1***

(v) the non-cyclic subgroup is {1, 9, 11, 19} ***A1***

[11 marks]

Total [18 marks]

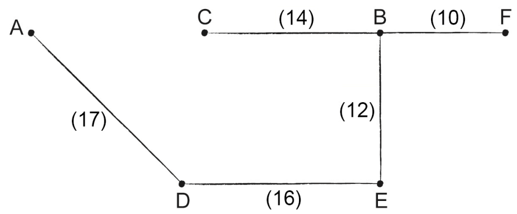
3. (a) (i) using Kruskal's algorithm, the minimum spanning tree is built up as follows

BF ***A1***

BE, BC ***A1***

DE, AD ***A1***

(ii)



A1

[4 marks]

(b) (i) weight of minimum spanning tree $= 69$ ***A1***

Note: This mark may be earned earlier.

upper bound = 138 ***A1***

(ii) starting at A, the cycle is $A \to D \to E \to B \to F \to C \to A$ ***M1A1***

an upper bound is the total weight of this cycle ***(M1)***

$17 + 16 + 12 + 10 + 20 + 19 = 94$ ***A1***

(iii) the minimum spanning tree of the reduced graph is as above with AD removed ***(R1)***

its total weight is $10 + 12 + 14 + 16 = 52$ ***A1***

adding the weights of the two deleted edges of the minimum spanning tree gives ***(M1)***

lower bound $= 52 + 17 + 18 = 87$ ***A1***

[10 marks]

(c) (i) the possible cycles, and their weights, are

$A \to B \to C \to D \to E \to F \to A$ Weight 102 (or 70 exc $A \to B \to C$)

$A \to B \to C \to D \to F \to E \to A$ Weight 107 (or 75 exc $A \to B \to C$)

$A \to B \to C \to E \to D \to F \to A$ Weight 106 (or 74 exc $A \to B \to C$)

$A \to B \to C \to E \to F \to D \to A$ Weight 99 (or 67 exc $A \to B \to C$)

$A \to B \to C \to F \to D \to E \to A$ Weight 110 (or 78 exc $A \to B \to C$)

$A \to B \to C \to F \to E \to D \to A$ Weight 98 (or 66 exc $A \to B \to C$) ***A3***

Note: Award ***A(3 – n)*** for n errors up to $n = 2$, ***A0*** thereafter.

the solution is therefore the cycle $A \to B \to C \to F \to E \to D \to A$ (with weight 98) ***A1***

(ii) no, it has no effect ***A1***

[5 marks]

Total [19 marks]

4. (a) (i) using row reduction, ***M1***

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 2 \\ 3 & 8 & 11 & 8 & 4 \\ 1 & 3 & 4 & 2 & \mu \\ 2 & 5 & 7 & 6 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 2 \\ 0 & 2 & 2 & -4 & -2 \\ 0 & 1 & 1 & -2 & \mu-2 \\ 0 & 1 & 1 & -2 & -1 \end{pmatrix}$$ ***(A2)***

for consistency,

$\mu - 2 = -1$ ***(M1)***

$\mu = 1$ ***A1***

put $z = \alpha, t = \beta$ ***M1***

$y = -1 - \alpha + 2\beta$; $x = 4 - \alpha - 8\beta$ ***A1A1***

(ii) the rank of a matrix is the number of independent rows (or columns) ***A1***

$\text{rank}(\boldsymbol{A}) = 2$ ***A1***

[10 marks]

(b) (i) $\det(\boldsymbol{A}) = 2$ ***(M1)A1***

since $\det(\boldsymbol{A}) \neq 0$, the vectors form a basis ***R1***

(ii) let $\begin{pmatrix} 6 \\ 28 \\ 12 \\ 15 \end{pmatrix} = a\begin{pmatrix} 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} 2 \\ 8 \\ 3 \\ 5 \end{pmatrix} + c\begin{pmatrix} 3 \\ 11 \\ 4 \\ 7 \end{pmatrix} + d\begin{pmatrix} 4 \\ 8 \\ 1 \\ 6 \end{pmatrix}$ ***M1***

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 8 & 11 & 8 \\ 1 & 3 & 4 & 1 \\ 1 & 5 & 7 & 6 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

it follows that

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 8 & 11 & 8 \\ 1 & 3 & 4 & 1 \\ 1 & 5 & 7 & 6 \end{pmatrix}^{-1}\begin{pmatrix} 6 \\ 28 \\ 12 \\ 15 \end{pmatrix}$$

continued ...

Question 4 continued

$$= \begin{pmatrix} 2 \\ 1 \\ 2 \\ -1 \end{pmatrix}$$

therefore

$a = 2$ ***A1***

$b = 1$ ***A1***

$c = 2$ ***A1***

$d = -1$ ***A1***

$$\begin{pmatrix} 6 \\ 28 \\ 12 \\ 15 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 3 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 8 \\ 3 \\ 5 \end{pmatrix} + 2\begin{pmatrix} 3 \\ 11 \\ 4 \\ 7 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \\ 1 \\ 6 \end{pmatrix}$$

[8 marks]

Total [18 marks]

5. (a) integrating factor $= e^{\int \tan x \mathrm{d}x}$ ***M1***

$= \mathrm{e}^{\ln \sec x}$ ***A1***

$= \sec x$ ***A1***

$\sec x \frac{\mathrm{d}y}{\mathrm{d}x} + y \sec x \tan x = 2\cos^3 x$ ***(M1)***

integrating,

$y \sec x = 2\int \cos^3 x \mathrm{d}x$ ***A1***

$= 2\int \cos x (1 - \sin^2 x)\mathrm{d}x$ ***A1***

$= 2\left(\sin x - \frac{\sin^3 x}{3}\right) + C$ ***A1***

Note: Condone the absence of C.

(substituting $x = 0$, $y = 1$)

$1 = C$ ***M1***

the solution is

$y = 2\cos x\left(\sin x - \frac{\sin^3 x}{3}\right) + \cos x$ ***A1***

[9 marks]

(b) (i) differentiating the equation,

$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y\sec^2 x + \tan x \frac{\mathrm{d}y}{\mathrm{d}x} = -8\cos^3 x \sin x$ ***A1A1***

Note: ***A1*** for each side.

substituting for $\frac{\mathrm{d}y}{\mathrm{d}x}$,

$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y\sec^2 x + \tan x\left(2\cos^4 x - y\tan x\right) = -8\cos^3 x \sin x$ ***A1***

$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y(\sec^2 x - \tan^2 x) = -8\cos^3 x \sin x - 2\tan x \cos^4 x$ (or equivalent) ***A1***

$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y = -10\sin x \cos^3 x$ ***AG***

continued ...

Question 5 continued

(ii) differentiating again,

$\frac{d^3 y}{dx^3} + \frac{dy}{dx} = -10\cos^4 x +$ term involving $\sin x$ ***A1***

it follows that

$y(0) = 1,\ y'(0) = 2$ ***A1***

$y''(0) = -1,\ y'''(0) = -12$ ***A1***

attempting to use $y = y(0) + xy'(0) + \frac{x^2}{2} y''(0) + \frac{x^3}{6} y'''(0) + \ldots$ ***(M1)***

$y = 1 + 2x - \frac{x^2}{2} - 2x^3$ ***A1***

[9 marks]

Total [18 marks]

6. (a) consider

$au_{n+1} + bu_n + cu_{n-1} = aA\lambda^{n+1} + aB\mu^{n+1} + bA\lambda^n + bB\mu^n + cA\lambda^{n-1} + cB\mu^{n-1}$ ***M1A1***

$= A\lambda^{n-1}\left(a\lambda^2 + b\lambda + c\right) + B\mu^{n-1}\left(a\mu^2 + b\mu + c\right)$ ***A1***

$= 0$

[3 marks]

(b) (i) to terminate at 0 starting from *n*, the particle must either move to $n+1$ and terminate at 0 starting from there or move to $n-1$ and terminate at 0 starting from there

therefore,

$p_n = 0.4p_{n+1} + 0.6p_{n-1}$ ***M1A2***

leading to $2p_{n+1} - 5p_n + 3p_{n-1} = 0$ ***AG***

(ii) solving the auxiliary equation $2x^2 - 5x + 3 = 0$ ***M1***

$x = 1,\ 1.5$ ***A1***

the general solution is

$p_n = A + B(1.5)^n$ ***A1***

substituting the boundary conditions,

$A + B = 1$

$A + B(1.5)^{10} = 0$ ***M1A1***

solving,

$A = \dfrac{1.5^{10}}{1.5^{10} - 1};\ B = -\dfrac{1}{1.5^{10} - 1}$ ***A1A1***

giving

$p_n = \dfrac{1.5^{10} - 1.5^n}{1.5^{10} - 1}$ ***AG***

[10 marks]

Total [13 marks]

7. (a) (i) $x' = x\sec\alpha\cos(\theta+\alpha)$ ***M1***

$= x\sec\alpha(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$ ***A1***

$= x\cos\theta - x\tan\alpha\sin\theta$ ***A1***

$= x\cos\theta - y\sin\theta$ ***AG***

$y' = x\sec\alpha\sin(\theta+\alpha)$ ***M1***

$= x\sec\alpha(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$ ***A1***

$= x\sin\theta + x\tan\alpha\cos\theta$

$= x\sin\theta + y\cos\theta$ ***A1***

(ii) the matrix $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ represents the rotation ***A1***

[7 marks]

(b) (i) the above relationship can be written in the form

$$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x'\\ y'\end{bmatrix}$$ ***M1***

let $\theta = -\frac{\pi}{4}$

$$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$
$$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$ ***A1***

substituting in the equation of the ellipse,

$$5\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)^2 + 5\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)^2 - 6\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) = 8$$ ***M1***

$$5\left(\frac{x'^2}{2} + \frac{y'^2}{2} - x'y'\right) + 5\left(\frac{x'^2}{2} + \frac{y'^2}{2} + x'y'\right) - 6\left(\frac{x'^2}{2} - \frac{y'^2}{2}\right) = 8$$ ***A1***

leading to $\frac{x'^2}{4} + y'^2 = 1$ ***AG***

Note: Award ***M1A0M1A0*** for using $\theta = \frac{\pi}{4}$ leading to $\frac{y'^2}{4} + x'^2 = 1$.

continued …

Question 7 continued

(ii) in the usual notation, $a = 2, b = 1$ ***(M1)***

the coordinates of the foci of the rotated ellipse are $\left(\sqrt{3}, 0\right)$ and $\left(-\sqrt{3}, 0\right)$ ***A1***

the coordinates of the foci of E are therefore $\left(\frac{\sqrt{3}}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}\right)$ and $\left(\frac{-\sqrt{3}}{\sqrt{2}}, \frac{-\sqrt{3}}{\sqrt{2}}\right)$ ***A1***

[7 marks]

Total [14 marks]

8. (a) (i) using l'Hopital's rule once,

$$\lim_{x\to\infty}\frac{x^n}{e^{\lambda x}}=\lim_{x\to\infty}\frac{nx^{n-1}}{\lambda e^{\lambda x}}$$ ***(A1)(A1)***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

if $n>1$, this still gives $\frac{\infty}{\infty}$ so differentiate again giving

$$\lim_{x\to\infty}\frac{n(n-1)x^{n-2}}{\lambda^2 e^{\lambda x}}$$ ***(A1)***

if $n>2$, this still gives $\frac{\infty}{\infty}$ so differentiate a further $n-2$ times giving ***M1***

$$\lim_{x\to\infty}\frac{n!}{\lambda^n e^{\lambda x}}$$ ***A1***

$=0$ ***AG***

(ii) first prove the result true for $n=0$

$\int_0^\infty e^{-\lambda x}dx=-\frac{1}{\lambda}\left[e^{-\lambda x}\right]_0^\infty=\frac{1}{\lambda}$ as required ***M1A1***

assume the result is true for $n=k$ ***M1***

$$\int_0^\infty x^k e^{-\lambda x}dx=\frac{k!}{\lambda^{k+1}}$$

consider, for $n=k+1$,

$$\int_0^\infty x^{k+1}e^{-\lambda x}dx=-\frac{1}{\lambda}\left[x^{k+1}e^{-\lambda x}\right]_0^\infty+\frac{k+1}{\lambda}\int_0^\infty x^k e^{-\lambda x}dx$$ ***M1A1***

$=(0+)\frac{k+1}{\lambda}\times\frac{k!}{\lambda^{k+1}}$ ***A1***

$=\frac{(k+1)!}{\lambda^{k+2}}$ ***A1***

therefore true for $n=k\Rightarrow$ true for $n=k+1$ and since true for $n=0$, the result is proved by induction ***R1***

Note: Only award the ***R1*** if at least 4 of the previous marks have been awarded.

Note: If a candidate starts at $n=1$, do not award the first 2 marks but follow through thereafter.

[13 marks]

continued …

Question 8 continued

(b) (i) $\mathrm{E}(X)=\dfrac{\lambda^{n+1}}{n!}\displaystyle\int_0^{\infty} x^{n+1}\,\mathrm{e}^{-\lambda x}\,\mathrm{d}x$ ***M1***

$=\dfrac{\lambda^{n+1}}{n!}\times\dfrac{(n+1)!}{\lambda^{n+2}}$ ***A1***

$=\dfrac{(n+1)}{\lambda}$ ***A1***

(ii) the mode satisfies $f'(x)=0$ ***M1***

$f'(x)=\dfrac{\lambda^{n+1}}{n!}\left(nx^{n-1}\mathrm{e}^{-\lambda x}-\lambda x^{n}\mathrm{e}^{-\lambda x}\right)$ ***A1***

mode $=\dfrac{n}{\lambda}$ ***A1***

[6 marks]

(c) (i) $\mathrm{P}(T>t)=\mathrm{P}$(0, 1 or 2 arrivals in [0, t]) ***(M1)***

$=\mathrm{e}^{-8t}+\mathrm{e}^{-8t}\times 8t+\mathrm{e}^{-8t}\times\dfrac{(8t)^2}{2}$ ***A1***

$=\mathrm{e}^{-8t}\left(1+8t+32t^2\right)$ ***AG***

(ii) differentiating,

$-f(t)=-8\mathrm{e}^{-8t}\left(1+8t+32t^2\right)+\mathrm{e}^{-8t}\left(8+64t\right)$ ***A1A1***

Note: Award ***A1*** for LHS, ***A1*** for RHS.

$f(t)=256t^2\mathrm{e}^{-8t}$ ***A1***

(iii) with the previous notation, $n=2$, $\lambda=8$. ***(M1)***

mean $=\dfrac{3}{8}$ ***A1***

mode $=\dfrac{1}{4}$ ***A1***

[8 marks]

Note: Do not follow through if they use a negative probability density function.

Total [27 marks]

9. (a) applying Ceva's theorem to triangle ABC,

$\frac{CD}{DB} \times \frac{AE}{EC} \times \frac{BF}{FA} = 1$ ***M1A1***

applying Menelaus' theorem to triangle ABC with transversal (GFE),

$\frac{BG}{GC} \times \frac{CE}{EA} \times \frac{AF}{FB} = -1$ ***M1A1***

multiplying the two equations, ***M1***

$\frac{CD}{DB} \times \frac{BG}{GC} = -1$ ***A1***

so that $\frac{BD}{DC} = -\frac{BG}{GC}$ ***AG***

[6 marks]

(b) similarly

$\frac{CE}{EA} = -\frac{CH}{HA}$ ***M1A1***

and $\frac{AF}{FB} = -\frac{AI}{IB}$ ***A1***

multiplying the three results,

$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = -\frac{BG}{GC} \times \frac{CH}{HA} \times \frac{AI}{IB}$ ***M1A1***

by Ceva's theorem, as shown previously, the left hand side is equal to 1, therefore so is the right hand side ***R1***

that is $\frac{BG}{GC} \times \frac{CH}{HA} \times \frac{AI}{IB} = -1$ ***A1***

it follows from the converse to Menelaus' theorem that G, H, I are collinear ***R1***

[8 marks]

Total [14 marks]