INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
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MARKSCHEME

November 2001

FURTHER MATHEMATICS

Standard Level

Paper 1

9 pages

1. R is reflexive since $x+y=x+y \Rightarrow (x, y)\,R\,(x, y)$ ***(C1)***

R is symmetric since if $x+y=a+b$ then $a+b=x+y$ and hence

$(x, y)\,R\,(a, b) \Rightarrow (a, b)\,R\,(x, y)$ ***(C1)***

R is transitive since if $x+y=a+b$ and $a+b=c+d$, then $x+y=c+d$ and hence ***(M1)***

if $(x, y)\,R\,(a, b)$ and $(a, b)\,R\,(c, d)$ then $(x, y)\,R\,(c, d)$. ***(C1)***

Partition: $\{\{(1,1)\};\{(1,2),(2,1)\},\{(2,2),(1,3),(3,1)\},\{(2,3),(3,2),(1,4),(4,1)\},$

$\{(3,3),(2,4),(4,2)\},\{(3,4),(4,3)\},\{(4,4)\}\}$ ***(C1)***

[5 marks]

2. The series converges by the ratio test. ***(C1)***

$$\lim_{k\to\infty}\frac{a_{k+1}}{a_k}=\lim_{k\to\infty}\frac{k+1}{\mathrm{e}^{k+1}}\times\frac{\mathrm{e}^k}{k}=\lim_{k\to\infty}\frac{k+1}{k}\times\lim_{k\to\infty}\frac{\mathrm{e}^k}{\mathrm{e}^{k+1}}$$ ***(M2)***

$$=\frac{1}{\mathrm{e}}<1$$ ***(A1)***

Thus, $\lim_{k\to\infty}\frac{a_{k+1}}{a_k}<1$ ***(R1)***

[5 marks]

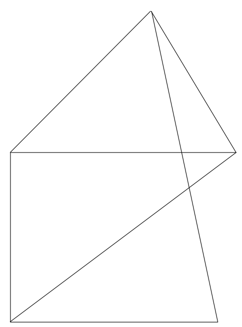
3. Order is 6. ***(C1)***

$\{1\},\{1,x,x^2\},\{1,y\},\{1,xy\},\{1,x^2y\},G$ ***(A4)***

> **Note:** Award ***(A4)*** for all 6 correct, ***(A3)*** for 5, ***(A2)*** for 4, ***(A1)*** for 3.

[5 marks]

4.



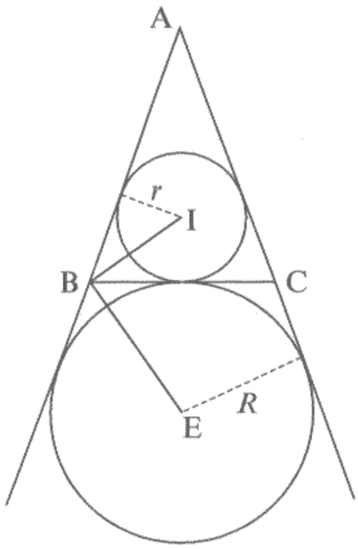
(A2)

$$\binom{\binom{5}{2}}{7}=\binom{10}{7}$$ ***(M2)***

$$=120$$ ***(A1)***

[5 marks]

5.



(a) IB bisects the interior angle, EB bisects the exterior angle, hence the angle is a right angle. ***(R2)***

(b) Since triangle IBE is a right angled triangle then

$\left(\frac{\text{BC}}{2}\right)^2 = r \times R \Rightarrow (\text{BC})^2 = 4rR$ ***[M2]***

$\Rightarrow \text{BC} = 2\sqrt{rR}$ ***[A1]***

[5 marks]

6. The characteristic equation is $r^2 - 7r + 6 = 0$. ***(C1)***

The characteristic roots are 1 and 6. ***(C1)***

The solutions are of the form $a_n = c_1 \times 1^n + c_2 \times 6^n$. ***(R1)***

With the initial conditions we have a system of equations: $c_1 + c_2 = -1$ and $c_1 + 6c_2 = 4$,

which gives $c_1 = -2$ and $c_2 = 1$. ***(M1)***

Hence the solution is $a_n = -2 + 6^n$. ***(A1)***

[5 marks]

7. There are 12 numbers on the list, so we set $i = 1$ and $j = 12$.

Then let $k = \lfloor (1+12)/2 \rfloor = 6$ ***(R1)***

$a(6) = 39 < 43$, so we set $i = 6 + 1 = 7$ ***(M1)***

$k = \lfloor (7+12)/2 \rfloor = 9$ ***(A1)***

$a(9) = 67 > 43$, so set $j = 8$. ***(M1)***

Now, $k = \lfloor (7+8)/2 \rfloor = 7$

$a(7) = 43$, so the algorithm terminates by finding 43. ***(R1)***

[5 marks]

8. (a) Let X be the number of discs replaced.
This is a Poisson distribution $X \sim \text{Po}(4)$ ***(R1)***

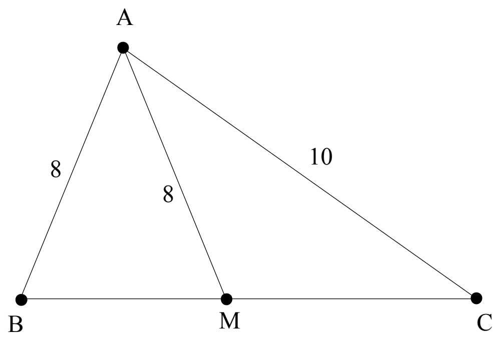
$$\text{P}(X = 7) = 0.0595$$ ***(G1)***

(b) $\text{P}(X \geq 7) = 1 - \text{P}(X \leq 6) = 1 - 0.8893 = 0.1107$ ***(M1)***

Let Y be the number of weeks in which at least 7 discs are replaced.
$Y \sim \text{B}(3, 0.1107)$ *i.e.* Y is binomially distributed ***(R1)***
$\text{P}(Y = 2) \equiv 0.0327$ ***(G1)***

[5 marks]

9.



Using Apollonius' theorem, $2\left(\frac{a}{2}\right)^2 + 2(8)^2 = 8^2 + 10^2$ ***(R1)(M1)***

$\frac{a^2}{2} = 36 \Rightarrow a = 6\sqrt{2}$ ***(A1)***

Then use Heron's formula or any other approach to find the area

Area $= \sqrt{\left(3\sqrt{2}+9\right)\left(9-3\sqrt{2}\right)\left(3\sqrt{2}+1\right)\left(3\sqrt{2}-1\right)} = \sqrt{(81-18)(18-1)} = 3\sqrt{119}$ ***(M1)***

Area $= 32.7$ (3 s.f.) ***(A1)***

[5 marks]

10. We know that $P_n(0.2) = \sum_0^n \frac{0.2^n}{n!}$. The condition means that the remainder must be less than 0.0005 ***(C1)***

$$\left|R_{n+1}(0.2)\right| \leq \text{e}^{0.2}\frac{\left|0.2\right|^{n+1}}{(n+1)!} < 3^{0.2}\frac{\left|0.2\right|^{n+1}}{(n+1)!} < 1.25\frac{1}{5^{n+1}(n+1)!}$$ ***(R1)***

$$\Rightarrow 1.25\frac{1}{5^{n+1}(n+1)!} < 0.0005 \Rightarrow 5^{n+1}(n+1)! > 2500$$ ***(R1)***

The smallest integer that satisfies this inequality is $n = 3$. Thus

$$\text{e}^{0.2} = 1 + 0.2 + \frac{0.2^2}{2!} + \frac{0.2^3}{3!}$$ ***(C1)***

$$\cong 1.221$$ ***(A1)***

Note: Award no marks for a calculator answer of 1.221402758.

[5 marks]