INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

FURTHER MATHEMATICS
STANDARD LEVEL
**PAPER 1**

Monday 15 May 2000 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.

- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80.

*A correct answer with **no** indication of the method used will usually receive **no** marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

1.   Construct a graph for the following diagram of a map.
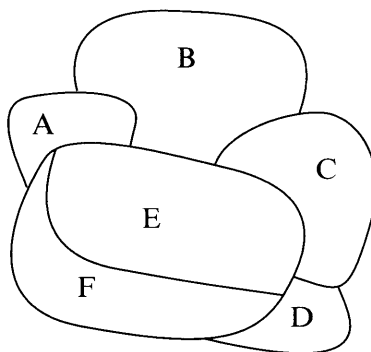
   Use the graph to determine the number of colours required for a proper colouring of the map.

2.   (a)   Define and give an example of a well-ordered set.

   (b)   Prove that the intersection of two well-ordered, non-disjoint sets is well-ordered.

3.   (a)   The inside diameter of a cylindrical tube is a random variable with a mean of 3 inches and a standard deviation of 0.02 inches. The thickness of the tube is also a random variable with a mean of 0.3 inches and a standard deviation of 0.005 inches. Given that these two random variables are independent, calculate the mean and standard deviation of the outside diameter of the tube.

   (b)   If X is a discrete random variable, prove that Var $(X) = E(X^2) - (E(X))^2$.

4.   (a)   Explain what is meant by saying that the series $\displaystyle\sum_{n=1}^{\infty} u_n$ is conditionally convergent.

   (b)   Show that the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \, \frac{1}{\sqrt{n}}$ is conditionally convergent.

5.   Let $(G, \circ)$ be a group with four elements. Prove that $(G, \circ)$ is Abelian.

6.   Two groups of 100 students, from the graduating classes of two high schools, are randomly chosen to take a Mathematics Competency Test. The tests are given on the same day under identical conditions. It is found that 75 students from the first group and 65 from the second achieved a pass grade or better in that test. The test providers claim that 70% of graduating students would achieve a pass grade or better in the test. Test this claim at 5% level of significance.

7.   An ellipse has centre O and major and minor axes of lengths $2a$, $2b$ respectively. The line segments (produced if necessary), joining any point P on the ellipse to the ends of the minor axis, meet the major axis at the points R and Q. Prove that

$$\mathrm{OQ} \times \mathrm{OR} = a^2 \,.$$

8.   Show that two positive integers a and b are congruent modulo m ($m < a$ and $m < b$) if and only if they have the same remainder when divided by m, where m is a positive integer.

9.   Show that the series $S_n = \displaystyle\sum_{k=1}^{n} \frac{(-1)^{k+1}}{\ln(k+1)}$ converges as $n \to \infty$. Hence find the approximate value of S_{10} to four decimal places, and find the maximum error in that estimation.

10.   The sides [AB], [BC], [CD] and [AD] of a quadrilateral ABCD (produced if necessary) are cut by a transversal in the points P, Q, R and S, respectively. Prove that

$$\frac{\mathrm{AP}}{\mathrm{PB}} \times \frac{\mathrm{BQ}}{\mathrm{QC}} \times \frac{\mathrm{CR}}{\mathrm{RD}} \times \frac{\mathrm{DS}}{\mathrm{SA}} = 1 \,.$$