香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

練習卷
PRACTICE PAPER

數學　延伸部分
單元一（微積分與統計）

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)

評卷參考

MARKING SCHEME

本評卷參考乃香港考試及評核局專爲本科練習卷而編寫，供教師和學生參考之用。學生不應將評卷參考視爲標準答案，硬背死記，活剝生吞。這種學習態度，既無助學生改善學習，學懂應對及解難，亦有違考試着重理解能力與運用技巧之旨。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for teachers' and students' reference. This marking scheme should NOT be regarded as a set of model answers. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, will not help students to improve their learning nor develop their abilities in addressing and solving problems.

General Notes for Teachers on Marking

Adherence to marking scheme

1. This marking scheme has been updated, with revisions made after the scrutiny of actual samples of student performanc practice papers. Teachers are strongly advised to conduct their own internal standardisation procedures before applying marking schemes. After standardisation, teachers should adhere to the marking scheme to ensure a uniform standard of marki within the school.

2. It is very important that all teachers should adhere as closely as possible to the marking scheme. In many cases, however, students may have arrived at a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Teachers should be patient in marking alternative solutions not specified in the marking scheme.

Acceptance of alternative answers

3. For the convenience of teachers, the marking scheme was written as detailed as possible. However, it is likely that students would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, teachers should exercise their discretion in marking students’ work. In general, marks for a certain step should be awarded if students’ solution indicate that the relevant concept / technique has been used.

4. In marking students’ work, the benefit of doubt should be given in students’ favour.

5. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

6. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

Defining symbols used in the marking scheme

7. In the marking scheme, marks are classified into the following three categories:

‘M’ marks	–	awarded for applying correct methods
‘A’ marks	–	awarded for the accuracy of the answers
Marks without ‘M’ or ‘A’	–	awarded for correctly completing a proof or arriving at an answer given in the question.

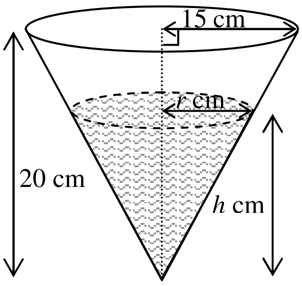
In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Teachers should follow through students’ work in awarding ‘M’ marks.) However, ‘A’ marks for the corresponding answers should NOT be awarded, unless otherwise specified.

8. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles , whereas alternative answers are enclosed by solid rectangles .

Others

9. Marks may be deducted for poor presentation (*pp*), including wrong / no unit. Note the following points:
   (a) At most deduct 1 mark for *pp* in each section.
   (b) In any case, do not deduct any marks for *pp* in those steps where students could not score any marks.

10. (a) Unless otherwise specified in the question, numerical answers not given in exact values or 4 decimal places should not be accepted.
    (b) Answers not accurate up to specified degree of accuracy should not be accepted. For answers with an excess degree of accuracy, deduct 1 mark for *pp*. In any case, do not deduct any marks for excess degree of accuracy in those steps where students could not score any marks.

Solution	Marks	
1. (a) $(2x+1)^3=8x^3+12x^2+6x+1$	1A	
(b) $e^{-ax}=1-ax+\frac{a^2x^2}{2}-\cdots$	1A	
(c) $\frac{(2x+1)^3}{e^{ax}}=(8x^3+12x^2+6x+1)\left(1-ax+\frac{a^2x^2}{2}-\cdots\right)$	1M	
The coefficient of $x^2=12(1)+6(-a)+(1)\frac{a^2}{2}$	1M	
$\therefore\ \frac{a^2}{2}-6a+12=-4$		
$a^2-12a+32=0$		
$a=4$ or 8	1A	
	(5)	
2. (a) $t=y^3+2y^{\frac{-1}{2}}+1$		
$\frac{\mathrm{d}t}{\mathrm{d}y}=3y^2-y^{\frac{-3}{2}}$	1A	
(b) $e^t=x^{x^2+1}$		
$t=(x^2+1)\ln x$	1A	
$\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{x^2+1}{x}+2x\ln x$	1A	
(c) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}t}{\mathrm{d}x}\div\frac{\mathrm{d}t}{\mathrm{d}y}$	1M	
$=\frac{(x^2+1+2x^2\ \ln x)y^{\frac{3}{2}}}{x\left(3y^{\frac{7}{2}}-1\right)}$	1A	OR $\frac{\frac{x^2+1}{x}+2x\ln x}{3y^2-y^{\frac{-3}{2}}}$
	(5)	
3. (a) By similar triangles, we have $\frac{h}{r}=\frac{20}{15}$.	1M	
$h=\frac{4r}{3}$		
$\therefore\ V=\frac{1}{3}\pi r^2\left(\frac{4r}{3}\right)$		
$=\frac{4}{9}\pi r^3$	1A	
$A=\pi r\sqrt{r^2+\left(\frac{4r}{3}\right)^2}$		
$=\frac{5}{3}\pi r^2$	1A	

Solution	Marks	
(b) (i) $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}r}\cdot\dfrac{\mathrm{d}r}{\mathrm{d}t}$	1M	
$=\dfrac{4}{3}\pi r^2\dfrac{\mathrm{d}r}{\mathrm{d}t}$		
$-2\pi=\dfrac{4}{3}\pi(3)^2\dfrac{\mathrm{d}r}{\mathrm{d}t}$		
$\dfrac{\mathrm{d}r}{\mathrm{d}t}=\dfrac{-1}{6}$	1A	Either one
Hence the rate of change of the radius of the water surface is $\dfrac{-1}{6}$ cm/s.		
(ii) $\dfrac{\mathrm{d}A}{\mathrm{d}t}=\dfrac{\mathrm{d}A}{\mathrm{d}r}\cdot\dfrac{\mathrm{d}r}{\mathrm{d}t}$		
$=\dfrac{10}{3}\pi r\dfrac{\mathrm{d}r}{\mathrm{d}t}$		
$=\dfrac{10}{3}\pi(3)\left(\dfrac{-1}{6}\right)$		
$=\dfrac{-5}{3}\pi$	1A	
Hence the rate of change of the area of the wet surface is $\dfrac{-5}{3}\pi$ cm^2/s.		
	(6)	
4. (a) $y=x(2x-1)^{\frac{1}{2}}$		
$\dfrac{\mathrm{d}y}{\mathrm{d}x}=(2x-1)^{\frac{1}{2}}+x\cdot\dfrac{1}{2}(2x-1)^{\frac{-1}{2}}(2)$	1M	For product rule
$=\dfrac{3x-1}{(2x-1)^{\frac{1}{2}}}$	1A	
(b) For tangents parallel to $2x-y=0$, we need $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2$.		
$\dfrac{3x-1}{(2x-1)^{\frac{1}{2}}}=2$	1M	
$9x^2-6x+1=4(2x-1)$		
$9x^2-14x+5=0$		
$x=1$ or $\dfrac{5}{9}$	1A	
For $x=1$, $y=1$ and hence the equation of the tangent is		
$y-1=2(x-1)$		
$2x-y-1=0$	1A	
For $x=\dfrac{5}{9}$, $y=\dfrac{5}{27}$ and hence the equation of the tangent is		
$y-\dfrac{5}{27}=2\left(x-\dfrac{5}{9}\right)$		
$54x-27y-25=0$	1A	
	(6)	

Solution	Marks	
5. (a) $1-\frac{e}{e^x}=e^x-e$		
$(e^x)^2-(e+1)e^x+e=0$	1A	
$e^x=1$ or e $x=0$ or 1	1A	
(b) The area of the region bounded by C_1 and C_2		
$=\int_0^1\left[1-\frac{e}{e^x}-(e^x-e)\right]\mathrm{d}x$	1M	For lower and upper limits
$=\left[x+e\cdot e^{-x}-e^x+ex\right]_0^1$	1M	Accept $\left[e^x-ex-x-e\cdot e^{-x}\right]_0^1$
$=1+1-e+e-e+1$ $=3-e$	1A	
	(5)	
6. (a) $\mathrm{Var}(2\overline{X}+7)=4\mathrm{Var}(\overline{X})$		
$=4\left(\frac{8}{10}\right)$	1M	For $\mathrm{Var}(\overline{X})=\frac{\mathrm{Var}(X)}{n}$
$=3.2$	1A	
(b) A 97% confidence interval for μ		
$=\left(50-2.17\times\frac{\sqrt{8}}{\sqrt{10}},50+2.17\times\frac{\sqrt{8}}{\sqrt{10}}\right)$	1M+1A	1M for $50\pm d$ 1A for 2.17
$=(48.0591,51.9409)$	1A	
	(5)	
7. (a) P(a player is rewarded) $=\frac{1}{2}\cdot\frac{2}{5}+\frac{1}{2}\cdot\frac{1}{5}$		
$=0.3$	1A	
(b) P(both players are rewarded | one player is rewarded) $=\frac{0.3\times0.3}{0.3\times0.3+0.3\times0.7\times2}$	1M	OR $\frac{0.3\times0.3}{1-0.7\times0.7}$
$=\frac{3}{17}$	1A	OR 0.1765
(c) E(no. of players having drawn a blue ball from A) $=60\times\frac{\frac{1}{2}\times\frac{2}{5}}{0.3}$	1M	
$=40$	1A	
	(5)	
8. (a) P(a box contains more than 1 rotten eggs)		
$=1-(0.96)^{30}-C_1^{30}(0.96)^{29}(0.04)$	1M+1M	1M for binomial prob 1M for correct cases
≈0.338820302 ≈0.3388	1A	
(b) (i) P(the 1st box containing more than 1 rotten egg is the 6th box inspected)		
$=(1-0.338820302)^5(0.338820302)$	1M	
≈0.0428	1A	

Solution	Marks
(ii) E(no. of boxes inspected until a box containing more than 1 rotten egg is found)	
$= \dfrac{1}{0.338820302}$	1M
≈ 2.9514	1A
	(7)
9. (a) $\mathrm{P}(A) = \mathrm{P}(A \cap B) + \mathrm{P}(A \cap B')$	
$= 0.12 + k$	1A
$\mathrm{P}(A \mid B') = \dfrac{\mathrm{P}(A \cap B')}{\mathrm{P}(B')}$	
$0.6 = \dfrac{k}{1 - \mathrm{P}(B)}$	
$\mathrm{P}(B) = 1 - \dfrac{5k}{3}$	1A
$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$	
$= (0.12 + k) + \left(1 - \dfrac{5k}{3}\right) - 0.12$	1M
$= 1 - \dfrac{2k}{3}$	1A
(b) If A and B are independent, $\mathrm{P}(A)\mathrm{P}(B) = \mathrm{P}(A \cap B)$.	
$(0.12 + k)\left(1 - \dfrac{5k}{3}\right) = 0.12$	1M
$0.8k - \dfrac{5k^2}{3} = 0$	
$k = 0.48$ or 0 (rejected)	1A
Alternative solution 1	
If A and B are independent, $\mathrm{P}(A) = \mathrm{P}(A \mid B')$.	
$0.12 + k = 0.6$	1M
$k = 0.48$	1A
Alternative solution 2	
If A and B are independent, $\mathrm{P}(A)\mathrm{P}(B') = \mathrm{P}(A \cap B')$.	
$(0.12 + k)\left(\dfrac{5k}{3}\right) = k$	1M
$\dfrac{5k^2}{3} - 0.8k = 0$	
$k = 0.48$ or 0 (rejected)	1A
Alternative solution 3	
If A and B are independent, $\mathrm{P}(A \mid B) = \mathrm{P}(A \mid B')$.	
$\therefore \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = \mathrm{P}(A \mid B')$	
$\dfrac{0.12}{1 - \dfrac{5k}{3}} = 0.6$	1M
$k = 0.48$	1A
	(6)

Solution	Marks	
10. (a) $\frac{dx}{dt}=\frac{61t}{(t+1)^{\frac{5}{2}}}$		
Let $u=t+1$ and hence $du=dt$.		
The amount of alloy produced by A		
$=\int_0^{10}\frac{61t}{(t+1)^{\frac{5}{2}}}\,dt$	1A	
$=\int_1^{11}\frac{61(u-1)}{u^{\frac{5}{2}}}\,du$		
$=\int_1^{11}\left(61u^{\frac{-3}{2}}-61u^{\frac{-5}{2}}\right)du$	1M	
$=\left[-122u^{\frac{-1}{2}}+\frac{122}{3}u^{\frac{-3}{2}}\right]_1^{11}$	1A	For primitive function
Alternative Solution		
$x=\int\frac{61t}{(t+1)^{\frac{5}{2}}}\,dt$		
$=\int\frac{61(u-1)}{u^{\frac{5}{2}}}\,du$	1A	
$=\int\left(61u^{\frac{-3}{2}}-61u^{\frac{-5}{2}}\right)du$	1M	
$=-122u^{\frac{-1}{2}}+\frac{122}{3}u^{\frac{-3}{2}}+C$		
$=-122(t+1)^{\frac{-1}{2}}+\frac{122}{3}(t+1)^{\frac{-3}{2}}+C$	1A	
The amount of alloy produced by A		
$=\left[-122(10+1)^{\frac{-1}{2}}+\frac{122}{3}(10+1)^{\frac{-3}{2}}+C\right]-\left[-122+\frac{122}{3}+C\right]$		
≈ 45.6636	1A	OR $=\frac{244}{3}-\frac{3904}{33\sqrt{11}}$
	(4)	
(b) The amount of alloy produced by B		
$=\int_0^{10}\frac{15\ln(t^2+100)}{16}\,dt$		
$\approx\frac{2}{2}\cdot\frac{15}{16}\{\ln(0+100)+\ln(10^2+100)+2[\ln(2^2+100)$ $+\ln(4^2+100)+\ln(6^2+100)+\ln(8^2+100)]\}$	1M	
≈ 45.6792	1A	
	(2)	

Solution	Marks

(c) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)=\dfrac{\mathrm{d}}{\mathrm{d}t}\dfrac{15\ln(t^2+100)}{16}$

$=\dfrac{15t}{8(t^2+100)}$ 1A

$\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)=\dfrac{15}{8}\cdot\dfrac{(t^2+100)-t(2t)}{(t^2+100)^2}$

$=\dfrac{15(100-t^2)}{8(t^2+100)^2}$ 1A

$\therefore\ \dfrac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)>0\ \text{ for }\ 0<t<10$

Thus, 45.6792 is an over-estimate of the amount of alloy produced by B. 1A

Hence it is uncertain whether machine B is more productive than machine A by the results of (a) and (b). The engineer cannot be agreed with. } 1A

(4)

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11. (a) $\mathrm{P}'(t)=kte^{\frac{a}{20}t}$

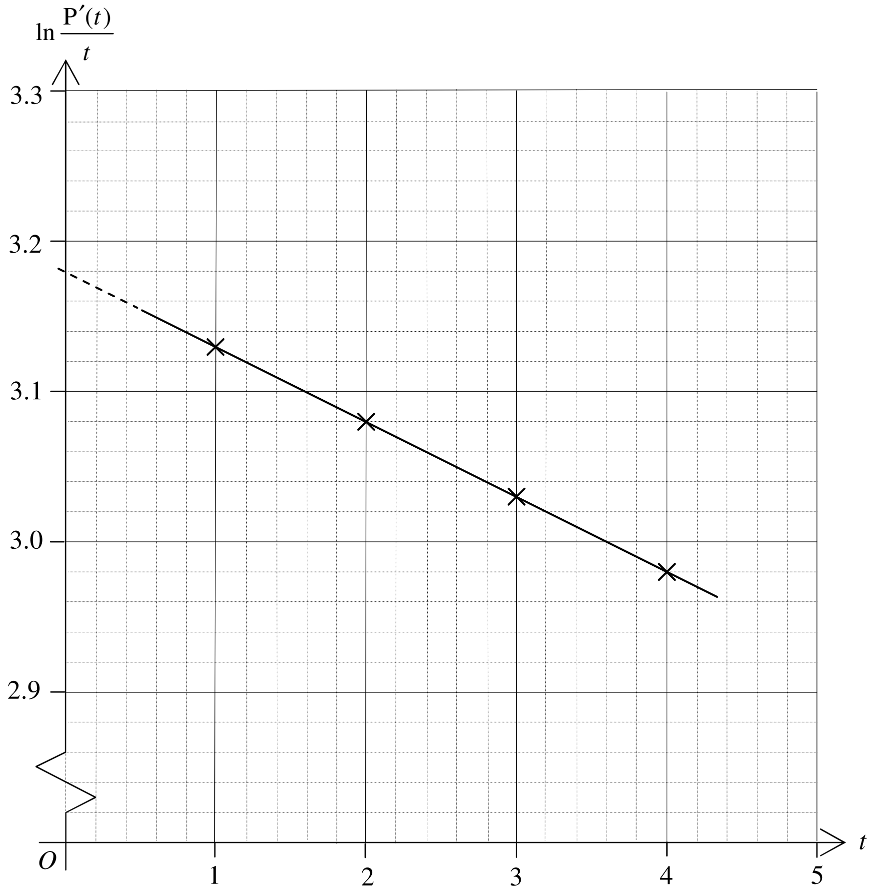
$\ln\dfrac{\mathrm{P}'(t)}{t}=\dfrac{a}{20}t+\ln k$ 1A

(1)

(b)

t	1	2	3	4
$\mathrm{P}'(t)$	22.83	43.43	61.97	78.60
$\ln\dfrac{\mathrm{P}'(t)}{t}$	3.13	3.08	3.03	2.98

1A



1A

Solution | Marks

From the graph, $\frac{a}{20} \approx \frac{2.98-3.13}{4-1}$ 1M

$a \approx -1$ 1A

From the graph, $\ln k \approx 3.18$

Either one

$k \approx 24$ 1A

(5)

(c) (i) $\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{P}'(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(24te^{\frac{-t}{20}}\right)$

$= 24e^{\frac{-t}{20}}\left(1-\frac{t}{20}\right)$ 1A

$\therefore \frac{\mathrm{d}}{\mathrm{d}t}\mathrm{P}'(t) = 0$ when $t = 20$

t	< 20	20	> 20
$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{P}'(t)$	+ve	0	-ve

1M

Alternative Solution

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathrm{P}'(t) = 24e^{\frac{-t}{20}}\left[\frac{-1}{20}\left(1-\frac{t}{20}\right)+\frac{-1}{20}\right]$$

$$= \frac{6}{5}e^{\frac{-t}{20}}\left(\frac{t}{20}-2\right)$$

$\therefore \frac{\mathrm{d}^2}{\mathrm{d}t^2}\mathrm{P}'(t) < 0$ when $t = 20$

1M

Hence the rate of change of the population size is greatest when $t = 20$. 1A

(ii) $\frac{\mathrm{d}}{\mathrm{d}t}\left(te^{\frac{-t}{20}}\right) = e^{\frac{-t}{20}} - \frac{1}{20}te^{\frac{-t}{20}}$ 1A

$24te^{\frac{-t}{20}} = 480e^{\frac{-t}{20}} - 480\frac{\mathrm{d}}{\mathrm{d}t}\left(te^{\frac{-t}{20}}\right)$

$\int 24te^{\frac{-t}{20}}\,\mathrm{d}t = -9600e^{\frac{-t}{20}} - 480te^{\frac{-t}{20}} + C$ 1M

$\mathrm{P}(t) = C - 480te^{\frac{-t}{20}} - 9600e^{\frac{-t}{20}}$ 1A

Since $\mathrm{P}(0) = 30$, we have

$C - 480(0)e^0 - 9600e^0 = 30$ 1M

$C = 9630$

$\therefore \mathrm{P}(t) = 9630 - 480te^{\frac{-t}{20}} - 9600e^{\frac{-t}{20}}$ 1A

(iii) $\lim_{t\to\infty}\mathrm{P}(t) = \lim_{t\to\infty}\left(9630 - 480te^{\frac{-t}{20}} - 9600e^{\frac{-t}{20}}\right)$

$= 9630$ 1A

$\therefore$ the population size after a very long time is estimated to be 9630 thousands.

(9)

Solution	Marks	
12. (a) The estimate of the mean $=\frac{0\times 6+\cdots+7\times 4}{100}$		
$=3.21$	1A	
	(1)	
(b) (i) The sample proportion of school days with less than 4 visits $=\frac{57}{100}$	1A	
(ii) An approximate 95% confidence interval for the proportion		
$=\left(0.57-1.96\sqrt{\frac{0.57\times 0.43}{100}}\ ,\ 0.57+1.96\sqrt{\frac{0.57\times 0.43}{100}}\right)$	1M	
$=(0.4730\ ,\ 0.6670)$	1A	
	(3)	
(c) (i) By (a), $\lambda=3.21$.		
P(*crowded* on a day) $=1-e^{-3.21}\left(1+3.21+\frac{3.21^2}{2!}+\frac{3.21^3}{3!}\right)$	1M	For Poisson probability
≈ 0.399705729		
≈ 0.3997	1A	
(ii) P(*crowded* on alternate days $\mid$ *crowded* on at least 2 days)		
$=\frac{(0.399705729)^3(1-0.399705729)^2+(1-0.399705729)^3(0.399705729)^2}{1-(1-0.399705729)^5-5(1-0.399705729)^4(0.399705729)}$	1M+1M+1M	1M for numerator 1M for denominator 1M for binomial probability
≈ 0.0869	1A	
	(6)	

Solution	Marks	
13. Let X_r minutes and X_e minutes be the waiting times for a customer in the regular and express counter respectively.		
(a) $\text{P}(X_r > 6) = \text{P}\left(Z > \frac{6-6.6}{1.2}\right)$	1M	
$= \text{P}(Z > -0.5)$		
≈ 0.6915	1A	
	(2)	
(b) (i) P(more than 10 from 12 customers with $X_r > 6$)		
$= C^{12}_{11}(0.6915)^{11}(1-0.6915) + (0.6915)^{12}$	1M+1M	
≈ 0.0759	1A	
(ii) Let Y minutes be the average waiting time of the 12 customers		
$Y \sim \text{N}\left(6.6, \frac{1.2^2}{12}\right) = \text{N}(6.6\ ,\ 0.12)$	1A	
$\text{P}(Y > 6) = \text{P}\left(Z > \frac{6-6.6}{\sqrt{0.12}}\right)$		
$\approx \text{P}(Z > -1.73)$		OR $\text{P}(Z > -1.732)$
≈ 0.9582	1A	OR 0.9584
	(5)	
(c) (i) $\text{P}(X_r < k) = 0.2119$		
$\text{P}\left(Z < \frac{k-6.6}{1.2}\right) = 0.2119$	1M	
$\frac{k-6.6}{1.2} = -0.8$		
$k = 5.64$	1A	
$\text{P}(X_e > k) = 0.0359$		
$\text{P}\left(Z > \frac{5.64-\mu}{0.8}\right) = 0.0359$	1M	
$\frac{5.64-\mu}{0.8} = 1.8$		
$\mu = 4.2$	1A	
(ii) $\text{P}(X_r > \mu) = \text{P}\left(Z > \frac{4.2-6.6}{1.2}\right)$		
≈ 0.9772	1A	
P(1 customer pays at regular counter | 2 customers wait more than μ min)		
$\approx \frac{2(0.88)(0.9772)(0.12)(0.5)}{[(0.88)(0.9772)+(0.12)(0.5)]^2}$	1M+1M	1M for numerator 1M for denominator
≈ 0.1219	1A	
	(8)	