

香港考試及評核局  
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY  
香港中學文憑考試  
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

練習卷  
PRACTICE PAPER

數學                      必修部分                      試卷一  
MATHEMATICS    COMPULSORY PART    PAPER 1

評卷參考  
MARKING SCHEME

本評卷參考乃香港考試及評核局專為本科練習卷而編寫，供教師和學生參考之用。學生不應將評卷參考視為標準答案，硬背死記，活剝生吞。這種學習態度，既無助學生改善學習，學懂應對及解難，亦有違考試着重理解能力與運用技巧之旨。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for teachers' and students' reference. This marking scheme should NOT be regarded as a set of model answers. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, will not help students to improve their learning nor develop their abilities in addressing and solving problems.



**Hong Kong Diploma of Secondary Education Examination  
Mathematics Compulsory Part Paper 1**

**General Marking Instructions**

1. This marking scheme has been updated, with revisions made after the scrutiny of actual samples of student performance in the practice papers. Teachers are strongly advised to conduct their own internal standardisation procedures before applying the marking schemes. After standardisation, teachers should adhere to the marking scheme to ensure a uniform standard of marking within the school.
2. It is very important that all teachers should adhere as closely as possible to the marking scheme. In many cases, however, students will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Teachers should be patient in marking alternative solutions not specified in the marking scheme.
3. In the marking scheme, marks are classified into the following three categories:
 

‘M’ marks	awarded for correct methods being used;
‘A’ marks	awarded for the accuracy of the answers;
Marks without ‘M’ or ‘A’	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).
4. For the convenience of teachers, the marking scheme was written as detailed as possible. However, it is still likely that students would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, teachers should exercise their discretion in marking students’ work. In general, marks for a certain step should be awarded if students’ solution indicated that the relevant concept/technique had been used.
5. Use of notation different from those in the marking scheme should not be penalized.
6. In marking students’ work, the benefit of doubt should be given in the students’ favour.
7. Marks may be deducted for wrong units ( $u$ ) or poor presentation ( $pp$ ).
  - a. The symbol  $(u-1)$  should be used to denote 1 mark deducted for  $u$ . At most deduct **1 mark** for  $u$  in each of Section A(1) and Section A(2). Do not deduct any marks for  $u$  in Section B.
  - b. The symbol  $(pp-1)$  should be used to denote 1 mark deducted for  $pp$ . At most deduct **1 mark** for  $pp$  in each of Section A(1) and Section A(2). Do not deduct any marks for  $pp$  in Section B.
  - c. At most deduct 1 mark in each of Section A(1) and Section A(2).
  - d. In any case, do not deduct any marks in those steps where students could not score any marks.
8. In the marking scheme, ‘r.t.’ stands for ‘accepting answers which can be rounded off to’ and ‘f.t.’ stands for ‘follow through’. Steps which can be skipped are **shaded** whereas alternative answers are enclosed with **rectangles**. All fractional answers must be simplified.

Solution	Marks	Remarks
<p>1. <math>\frac{(m^5 n^{-2})^6}{m^4 n^{-3}}</math></p> $= \frac{m^{30} n^{-12}}{m^4 n^{-3}}$ $= \frac{m^{30-4}}{n^{12-3}}$ $= \frac{m^{26}}{n^9}$	<p>1M</p> <p>1M</p> <p>1A</p> <p>------(3)</p>	<p>for <math>(ab)^p = a^p b^p</math> or <math>(a^p)^q =</math></p> <p>for <math>\frac{a^p}{a^q} = a^{p-q}</math> or <math>\frac{a^p}{a^q} = \frac{1}{a^{q-p}}</math></p>
<p>2. <math>\frac{5+b}{1-a} = 3b</math></p> $5+b = 3b(1-a)$ $5+b = 3b - 3ab$ $3ab = 2b - 5$ $a = \frac{2b-5}{3b}$	<p>1M</p> <p>1M</p> <p>1A</p>	<p>for <math>3b(1-a)</math></p> <p>for putting <math>a</math> on one side</p> <p>or equivalent</p>
$\frac{5+b}{1-a} = 3b$ $5+b = 3b(1-a)$ $a = 1 - \frac{5+b}{3b}$ $a = \frac{3b - (5+b)}{3b}$ $a = \frac{2b-5}{3b}$	<p>1M</p> <p>1M</p> <p>1A</p> <p>------(3)</p>	<p>for <math>3b(1-a)</math></p> <p>for putting <math>a</math> on one side</p> <p>or equivalent</p>
<p>3. (a) <math>9x^2 - 42xy + 49y^2</math></p> $= (3x - 7y)^2$ <p>(b) <math>9x^2 - 42xy + 49y^2 - 6x + 14y</math></p> $= (3x - 7y)^2 - 6x + 14y$ $= (3x - 7y)^2 - 2(3x - 7y)$ $= (3x - 7y)(3x - 7y - 2)$	<p>1A</p> <p>1M</p> <p>1A</p> <p>------(3)</p>	<p>or equivalent</p> <p>for using (a)</p> <p>or equivalent</p>

Solution	Marks	Remarks
<p>4. Let \$ x be the marked price of the chair.</p> $x(1 - 20\%) = 360(1 + 30\%)$ $x = \frac{360(1.3)}{0.8}$ $x = 585$ <p>Thus, the marked price of the chair is \$585 .</p>	<p>1M+1M+1A</p> <p>1A</p>	<p>pp-1 for undefined symbol</p> <p>1M for <math>x(1 - 20\%)</math> + 1M for <math>360(1 + 30\%)</math></p> <p>u-1 for missing unit</p>
<p>The marked price of the chair</p> $= \frac{360(1 + 30\%)}{1 - 20\%}$ $= \$585$	<p>1M+1M+1A</p> <p>1A</p>	<p>1M for <math>360(1 + 30\%)</math> + 1M for dividing by <math>(1 - 20\%)</math> u-1 for missing unit</p>
-----(4)		
<p>5. Let x litres and y litres be the capacities of a bottle and a cup respectively.</p> $\begin{cases} \frac{x}{y} = \frac{4}{3} \\ 7x + 9y = 11 \end{cases}$ <p>So, we have <math>7x + 9\left(\frac{3x}{4}\right) = 11</math> .</p> <p>Solving, we have <math>x = \frac{4}{5}</math> .</p> <p>Thus, the capacity of a bottle is <math>\frac{4}{5}</math> litre.</p>	<p>1A+1A</p> <p>1M</p> <p>1A</p>	<p>pp-1 for undefined symbol</p> <p>for getting a linear equation in x or y only</p> <p>0.8</p> <p>u-1 for missing unit</p>
<p>Let x litres be the capacity of a bottle.</p> $7x + 9\left(\frac{3x}{4}\right) = 11$ <p>Solving, we have <math>x = \frac{4}{5}</math> .</p> <p>Thus, the capacity of a bottle is <math>\frac{4}{5}</math> litre.</p>	<p>1A+1M+1A</p> <p>1A</p>	<p>pp-1 for undefined symbol</p> <p>1A for <math>y = \frac{3x}{4}</math> + 1M for <math>7x + 9y = 11</math></p> <p>0.8</p> <p>u-1 for missing unit</p>
-----(4)		

Solution	Marks	Remarks
<p>6. (a) <math>\angle AOC</math>  <math>= 337^\circ - 157^\circ</math>  <math>= 180^\circ</math>            Thus, <math>A</math>, <math>O</math> and <math>C</math> are collinear.</p>	<p>1M  1A</p>	<p>for considering <math>\angle AOC</math>  f.t.</p>
<p>(b) Note that <math>BO \perp AC</math>.            The area of <math>\triangle ABC</math>  <math>= \frac{1}{2}(13+15)(14)</math>  <math>= 196</math></p>	<p>1M 1A ------(4)</p>	
<p>7. Note that <math>\angle BCD = 90^\circ</math>.            Also note that <math>\angle CBD = 180^\circ - 90^\circ - 36^\circ = 54^\circ</math>.            Further note that <math>\angle BAC = \angle BDC = 36^\circ</math>.            Since <math>AB = AC</math>, we have <math>\angle ACB = \angle ABC</math>.</p>	<p>1A  1M</p>	
<p>So, we have <math>\angle ABC = \frac{180^\circ - 36^\circ}{2}</math>.            Therefore, we have <math>\angle ABC = 72^\circ</math>.</p>	<p>1M</p>	
<p><math>\angle ABD</math>  <math>= \angle ABC - \angle CBD</math>  <math>= 72^\circ - 54^\circ</math>  <math>= 18^\circ</math></p>	<p>1A</p>	<p>u-1 for missing unit</p>
<p>Note that <math>\angle BAC = \angle BDC = 36^\circ</math>.            Since <math>AB = AC</math>, we have <math>\angle ACB = \angle ABC</math>.            So, we have <math>\angle ACB = \frac{180^\circ - 36^\circ}{2}</math>.            Therefore, we have <math>\angle ACB = 72^\circ</math>.            Also note that <math>\angle BCD = 90^\circ</math>.</p>	<p>1M  1M  1A</p>	
<p><math>\angle ACD</math>  <math>= 90^\circ - 72^\circ</math>  <math>= 18^\circ</math></p> <p><math>\angle ABD</math>  <math>= \angle ACD</math>  <math>= 18^\circ</math></p>	<p>1A</p>	<p>u-1 for missing unit</p>
	<p>------(4)</p>	

Solution	Marks	Remarks
<p>8. (a) The coordinates of <math>A'</math>  <math>= (3, 4)</math></p> <p>The coordinates of <math>B'</math>  <math>= (5, -2)</math></p> <p>(b) Let <math>(x, y)</math> be the coordinates of <math>P</math>.  <math>\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-(-2))^2}</math>  <math>x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 10x + 25 + y^2 + 4y + 4</math>  <math>4x - 12y - 4 = 0</math>            Thus, the required equation is <math>x - 3y - 1 = 0</math>.</p>	<p>1A</p> <p>1A</p> <p>1M+1A</p> <p>1A</p>	<p>pp-1 for missing '('</p> <p>pp-1 for missing '(' or ')'</p> <p>or equivalent</p>
<p>The coordinates of the mid-point of <math>A'B'</math>  <math>= \left( \frac{3+5}{2}, \frac{4+(-2)}{2} \right)</math>  <math>= (4, 1)</math></p> <p>The slope of <math>A'B'</math>  <math>= \frac{4 - (-2)}{3 - 5}</math>  <math>= -3</math></p> <p>So, the required equation is <math>y - 1 = \frac{1}{3}(x - 4)</math>.</p> <p>Thus, the required equation is <math>x - 3y - 1 = 0</math>.</p>	<p>1M</p> <p>1A</p> <p>1A</p>	<p>or equivalent</p>
------(5)		
<p>9. (a) The least possible value of the inter-quartile range of the distribution  <math>= 5 - 5</math> or <math>2 - 2</math>  <math>= 0</math></p> <p>The greatest possible value of the inter-quartile range of the distribution  <math>= 5 - 2</math>  <math>= 3</math></p> <p>(b) Since <math>r = 9</math> and the median of the distribution is 3,            we have <math>9 + 8 &gt; 12 + s</math>.            Therefore, we have <math>s &lt; 5</math>.            So, we have <math>s = 1, 2, 3</math> or <math>4</math>.            Thus, there are 4 possible values of <math>s</math>.</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>either one</p> <p>f.t.</p>
------(5)		

Solution	Marks	Remarks
<p>10. (a) Note that when <math>f(x)</math> is divided by <math>x-1</math>, the remainder is 4.</p> $f(x)$ $= (x-1)(6x^2 + 17x - 2) + 4$ $= 6x^3 + 11x^2 - 19x + 6$ $f(-3)$ $= 6(-3)^3 + 11(-3)^2 - 19(-3) + 6$ $= 0$	<p>1M</p> <p>1M</p>  <p>1A</p> <p>------(3)</p>	<p>can be absorbed</p> <p>for <math>(x-1)(6x^2 + 17x - 2)</math></p>
<p>(b) <math>f(x)</math></p> $= (x+3)(6x^2 - 7x + 2)$ $= (x+3)(2x-1)(3x-2)$	<p>1M+1A</p> <p>1A</p> <p>------(3)</p>	<p>1M for <math>(x+3)(ax^2 + bx + c)</math></p>
<p>11. (a) Let <math>C = a + bx^2</math>, where <math>a</math> and <math>b</math> are non-zero constants.</p> <p>So, we have <math>a + (20^2)b = 42</math> and <math>a + (120^2)b = 112</math>.</p> <p>Solving, we have <math>a = 40</math> and <math>b = \frac{1}{200}</math>.</p> <p>The required cost</p> $= 40 + \frac{1}{200}(50^2)$ $= \$ 52.5$	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>------(4)</p>	<p>for either substitution</p> <p>for both correct</p> <p>u-1 for missing unit</p>
<p>(b) <math>40 + \frac{1}{200}x^2 = 58</math></p> $x^2 = 3600$ $x = 60$ <p>Thus, the required length is 60 cm.</p>	<p>1M</p> <p>1A</p> <p>------(2)</p>	<p>u-1 for having unit</p>

Solution	Marks	Remarks
12. (a) The required duration $= 63 - 32$ $= 31$ minutes	1M 1A -----(2)	u-1 for missing unit
(b) Suppose Ada and Billy meet at a place which is at a distance of $x$ km from town $P$ . $\frac{x}{78} = \frac{12}{120}$ $x = 7.8$ Thus, Ada and Billy meet at a place which is at a distance of 7.8 km from town $P$ .	1M+1A 1A -----(3)	pp-1 for undefined symbol 1M for ratio 78 : 120 u-1 for missing unit
(c) The average speed of Ada $= \frac{12}{2}$ $= 6$ km/h The average speed of Billy $= \frac{16-2}{2}$ $= 7$ km/h Note that $7 > 6$ . Thus, Billy runs faster.	1M  1A -----(2)	either one  f.t.
During the period, Ada runs 12 km while Billy runs 14 km. Note that $14 > 12$ . So, the average speed of Billy is higher than that of Ada. Thus, Billy runs faster.	1M 1A -----(2)	f.t.



Solution	Marks	Remarks
<p>13. (a) Let <math>n</math> be the number of students in the group.</p> $\frac{6}{n} = \frac{3}{20}$ $n = 40$ $\frac{k}{40} = \frac{6}{40} + \frac{11}{40} + \frac{5}{40} + \frac{10}{40}$ $k = 8$	<p>1M</p> <p>1M</p> <p>1A</p> <p>----- (3)</p>	<p>pp-1 for undefined</p>
<p>(b) (i) The required angle</p> $= \frac{5}{40}(360^\circ)$ $= 45^\circ$ <p>(ii) Let <math>m</math> be the number of new students.            Assume that the angle of the sector representing that the most favourite fruit is orange will be doubled.</p> $\frac{5+m}{40+m} = \frac{(45)(2)}{360}$ $20 + 4m = 40 + m$ $3m = 20$ <p>Since 20 is not a multiple of 3, the angle of the sector representing that the most favourite fruit is orange will not be doubled.</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>----- (4)</p>	<p>u-1 for missing unit</p> <p>pp-1 for undefined symbol</p> <p>for considering <math>\frac{5+m}{n+m}</math></p> <p>f.t.</p>

Solution	Marks	Remarks
14. (a) $\triangle BCD \sim \triangle OAD$	2A ------(2)	
(b) (i) Let $(0, h)$ be the coordinates of $C$ .	1M	
By (a), we have $\left(\frac{CD}{AD}\right)^2 = \frac{16}{45}$ .	1M	for using similarity
$\left(\frac{12-h}{\sqrt{6^2+12^2}}\right)^2 = \frac{16}{45}$	1M	for either $AD$ or $CD$
$h^2 - 24h + 80 = 0$		
$h = 4$ or $h = 20$ (rejected)	1A	
Thus, the coordinates of $C$ are $(0, 4)$ .		pp-1 for missing '(' or ')'
(ii) Note that $AC$ is a diameter of the circle $OABC$ .	1M	
So, the coordinates of the centre of the circle are $(3, 2)$ .	1M	-----  either one -----
Also, the radius of the circle is $\sqrt{(3-0)^2 + (4-2)^2} = \sqrt{13}$ .		
Thus, the equation of the circle $OABC$ is $(x-3)^2 + (y-2)^2 = 13$ .	1A	$x^2 + y^2 - 6x - 4y = 0$
<p>Suppose that the equation of the circle <math>OABC</math> is <math>x^2 + y^2 + k_1x + k_2y + k_3 = 0</math>, where <math>k_1, k_2</math> and <math>k_3</math> are constants.</p> $\begin{cases} 0^2 + 0^2 + k_1(0) + k_2(0) + k_3 = 0 \\ 6^2 + 0^2 + k_1(6) + k_2(0) + k_3 = 0 \\ 0^2 + 4^2 + k_1(0) + k_2(4) + k_3 = 0 \end{cases}$ <p>Solving, we have <math>k_1 = -6</math>, <math>k_2 = -4</math> and <math>k_3 = 0</math>.</p> <p>Thus, the equation of the circle <math>OABC</math> is <math>x^2 + y^2 - 6x - 4y = 0</math>.</p>	1M  1M  1A	for solving system of equations
	------(7)	



Solution	Marks	Remarks
16. (a) The required probability $= \frac{C_4^{18}}{C_4^{30}}$ $= \frac{68}{609}$	1M  1A	for numerator or denominator  r.t. 0.112
The required probability $= \left(\frac{18}{30}\right)\left(\frac{17}{29}\right)\left(\frac{16}{28}\right)\left(\frac{15}{27}\right)$ $= \frac{68}{609}$	1M  1A	for $\left(\frac{r}{n}\right)\left(\frac{r-1}{n-1}\right)\left(\frac{r-2}{n-2}\right)\left(\frac{r-3}{n-3}\right)$ , $r < n$  r.t. 0.112
-----(2)		
(b) The required probability $= 1 - \frac{68}{609} - \frac{C_4^{12}}{C_4^{30}}$ $= \frac{530}{609}$	1M  1A	for $1 - (a) - p_1$  r.t. 0.870
The required probability $= \frac{C_1^{18}C_3^{12} + C_2^{18}C_2^{12} + C_3^{18}C_1^{12}}{C_4^{30}}$ $= \frac{530}{609}$	1M  1A	for considering 3 cases  r.t. 0.870
The required probability $= 1 - \frac{68}{609} - \left(\frac{12}{30}\right)\left(\frac{11}{29}\right)\left(\frac{10}{28}\right)\left(\frac{9}{27}\right)$ $= \frac{530}{609}$	1M  1A	for $1 - (a) - p_2$  r.t. 0.870
The required probability $= 4\left(\frac{18}{30}\right)\left(\frac{12}{29}\right)\left(\frac{11}{28}\right)\left(\frac{10}{27}\right) + 6\left(\frac{18}{30}\right)\left(\frac{17}{29}\right)\left(\frac{12}{28}\right)\left(\frac{11}{27}\right) + 4\left(\frac{18}{30}\right)\left(\frac{17}{29}\right)\left(\frac{16}{28}\right)\left(\frac{12}{27}\right)$ $= \frac{530}{609}$	1M  1A	for considering 14 cases  r.t. 0.870
-----(2)		

Solution	Marks	Remarks
<p>17. (a) <math>\frac{1}{1+2i}</math></p> $= \left( \frac{1}{1+2i} \right) \left( \frac{1-2i}{1-2i} \right)$ $= \frac{1}{5} - \frac{2}{5}i$	<p>1M</p> <p>1A</p> <p>------(2)</p>	
<p>(b) (i) Note that <math>\frac{10}{1+2i} = 2-4i</math> and <math>\frac{10}{1-2i} = 2+4i</math>.</p> <p>The sum of roots</p> $= \frac{10}{1+2i} + \frac{10}{1-2i}$ $= (2-4i) + (2+4i)$ $= 4$ <p>The product of roots</p> $= \left( \frac{10}{1+2i} \right) \left( \frac{10}{1-2i} \right)$ $= 20$ <p>Thus, we have <math>p = -4</math> and <math>q = 20</math>.</p>	<p>1M</p> <p>1A</p> <p>1A</p>	<p>-----</p> <p>either</p> <p>-----</p> <p>either</p> <p>-----</p> <p>for both correct</p>
<p>(ii) When the equation <math>x^2 - 4x + 20 = r</math> has real roots, we have <math>\Delta \geq 0</math>.</p> <p>So, we have <math>(-4)^2 - 4(1)(20-r) \geq 0</math>.</p> <p>Thus, we have <math>r \geq 16</math>.</p>	<p>1M</p> <p>1A</p> <p>------(5)</p>	

Solution	Marks	Remarks
<p>18. (a) By cosine formula,  <math>AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos \angle ACB</math>  <math>AB^2 = 20^2 + 12^2 - 2(20)(12)\cos 60^\circ</math>  <math>AB = 4\sqrt{19}</math> cm</p> <p>(b) By sine formula,  <math>\frac{\sin \angle BAC}{BC} = \frac{\sin \angle ACB}{AB}</math>  <math>\frac{\sin \angle BAC}{12} = \frac{\sin 60^\circ}{4\sqrt{19}}</math>  <math>\angle BAC \approx 36.58677555^\circ</math>            Let <math>Q</math> be the foot of the perpendicular from <math>C</math> to <math>AB</math>.  <math>\sin \angle BAC = \frac{CQ}{AC}</math>  <math>CQ \approx 20 \sin 36.58677555^\circ</math>  <math>CQ \approx 11.92079121</math> cm            Since <math>\triangle ABC \cong \triangle ABD</math>, the required angle is <math>\angle CQD</math>.  <math>\sin \frac{\angle CQD}{2} = \frac{\frac{1}{2}CD}{CQ}</math>  <math>\sin \frac{\angle CQD}{2} \approx 0.587209345</math>  <math>\angle CQD \approx 71.91844786^\circ</math>  <math>\angle CQD \approx 71.9^\circ</math>            Thus, the angle between the plane <math>ABC</math> and the plane <math>ABD</math> is <math>71.9^\circ</math>.</p>	<p>1M</p> <p>1A</p> <p>-----(2)</p> <p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>r.t. 17.4 cm  <math>AB \approx 17.43559577</math> cm</p> <p>for identifying the angle</p> <p>r.t. <math>71.9^\circ</math></p>
<p>By sine formula,  <math>\frac{\sin \angle ABC}{AC} = \frac{\sin \angle ACB}{AB}</math>  <math>\frac{\sin \angle ABC}{20} = \frac{\sin 60^\circ}{4\sqrt{19}}</math>  <math>\angle ABC \approx 83.41322445^\circ</math>            Let <math>Q</math> be the foot of the perpendicular from <math>C</math> to <math>AB</math>.  <math>\sin \angle ABC = \frac{CQ}{BC}</math>  <math>CQ \approx 12 \sin 83.41322445^\circ</math>  <math>CQ \approx 11.92079121</math> cm            Since <math>\triangle ABC \cong \triangle ABD</math>, the required angle is <math>\angle CQD</math>.  <math>\sin \frac{\angle CQD}{2} = \frac{\frac{1}{2}CD}{CQ}</math>  <math>\sin \frac{\angle CQD}{2} \approx 0.587209345</math>  <math>\angle CQD \approx 71.91844786^\circ</math>  <math>\angle CQD \approx 71.9^\circ</math>            Thus, the angle between the plane <math>ABC</math> and the plane <math>ABD</math> is <math>71.9^\circ</math>.</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>for identifying the angle</p> <p>r.t. <math>71.9^\circ</math></p>

Solution	Marks	Remarks
<p>By sine formula,  <math display="block">\frac{\sin \angle BAC}{BC} = \frac{\sin \angle ACB}{AB}</math> <math display="block">\frac{\sin \angle BAC}{12} = \frac{\sin 60^\circ}{4\sqrt{19}}</math> <math display="block">\angle BAC \approx 36.58677555^\circ</math> <p>Let <math>Q</math> be the foot of the perpendicular from <math>C</math> to <math>AB</math>.  <math display="block">\sin \angle BAC = \frac{CQ}{AC}</math> <math display="block">CQ \approx 20 \sin 36.58677555^\circ</math> <math display="block">CQ \approx 11.92079121 \text{ cm}</math> <p>By symmetry, we have <math>DQ = CQ</math>.  <math display="block">DQ \approx 11.92079121 \text{ cm}</math> <p>Since <math>\triangle ABC \cong \triangle ABD</math>, the required angle is <math>\angle CQD</math>.  <math display="block">CD^2 = CQ^2 + DQ^2 - 2(CQ)(DQ) \cos \angle CQD</math> <math display="block">14^2 \approx 11.92079121^2 + 11.92079121^2 - 2(11.92079121)(11.92079121) \cos \angle CQD</math> <math display="block">\angle CQD \approx 71.91844786^\circ</math> <math display="block">\angle CQD \approx 71.9^\circ</math> <p>Thus, the angle between the plane <math>ABC</math> and the plane <math>ABD</math> is <math>71.9^\circ</math>.</p> </p></p></p></p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p></p> <p>for identifying the angle</p> <p>r.t. <math>71.9^\circ</math></p>
<p>The area of <math>\triangle ABC</math>  <math display="block">= \frac{1}{2}(AC)(BC) \sin \angle ACB</math> <math display="block">= \frac{1}{2}(20)(12) \sin 60^\circ</math> <math display="block">= 60\sqrt{3} \text{ cm}^2</math> <p>Let <math>Q</math> be the foot of the perpendicular from <math>C</math> to <math>AB</math>.  <math display="block">\frac{1}{2}(AB)(CQ) = 60\sqrt{3}</math> <math display="block">\frac{1}{2}(4\sqrt{19})(CQ) = 60\sqrt{3}</math> <math display="block">CQ \approx 11.92079121 \text{ cm}</math> <p>Since <math>\triangle ABC \cong \triangle ABD</math>, the required angle is <math>\angle CQD</math>.  <math display="block">\sin \frac{\angle CQD}{2} = \frac{\frac{1}{2}CD}{CQ}</math> <math display="block">\sin \frac{\angle CQD}{2} \approx 0.587209345</math> <math display="block">\angle CQD \approx 71.91844786^\circ</math> <math display="block">\angle CQD \approx 71.9^\circ</math> <p>Thus, the angle between the plane <math>ABC</math> and the plane <math>ABD</math> is <math>71.9^\circ</math>.</p> </p></p></p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p></p> <p>for identifying the angle</p> <p>r.t. <math>71.9^\circ</math></p>
<p>(c) Let <math>Q</math> be the foot of the perpendicular from <math>C</math> to <math>AB</math>.  <p>Note that <math>\sin \frac{\angle CPD}{2} = \frac{\frac{1}{2}CD}{CP}</math>.</p> <p>Since <math>CP \geq CQ</math>, we have <math>\angle CPD \leq \angle CQD</math>.</p> <p>Thus, <math>\angle CPD</math> increases as <math>P</math> moves from <math>A</math> to <math>Q</math> and decreases as <math>P</math> moves from <math>Q</math> to <math>B</math>.</p> </p>	<p>------(4)</p> <p>1M</p> <p>1A</p> <p>------(2)</p>	<p></p> <p>f.t.</p>

Solution	Marks	Remarks
<p>19. (a) <math>4000000(1-r\%)^3 = 1048576</math>  <math>(1-r\%)^3 = \frac{1048576}{4000000}</math>  <math>1-r\% = 0.64</math>  <math>r = 36</math></p>	<p>1M  1A ------(2)</p>	
<p>(b) (i) Let <math>n</math> be the number of years needed for the total revenue made by the firm to exceed \$ 9000000 .  <math>2000000 + 2000000(1-20\%) + \dots + 2000000(1-20\%)^{n-1} &gt; 9000000</math>  <math>\frac{2000000(1-(0.8)^n)}{1-0.8} &gt; 9000000</math>  <math>(0.8)^n &lt; 0.1</math>  <math>n \log 0.8 &lt; \log 0.1</math>  <math>n &gt; \frac{\log 0.1}{\log 0.8}</math>  <math>n &gt; 10.31885116</math>  Thus, the least number of years needed is 11 .</p>	<p>1M for left side 1M for sum of geometric sequence  1M for solving inequality  1A</p>	
<p>(ii) The total revenue made by the firm  <math>&lt; 2000000 + 2000000(1-20\%) + 2000000(1-20\%)^2 + \dots</math>  <math>= \frac{2000000}{1-0.8}</math>  <math>= 10000000</math>  Thus, the total revenue made by the firm will not exceed \$ 10000000 .</p>	<p>1M 1A f.t.</p>	
<p>(iii) The total revenue made by the firm minus the total amount of investment in the first <math>m</math> years  <math>= \frac{2000000(1-(0.8)^m)}{1-0.8} - \frac{4000000(1-(0.64)^m)}{1-0.64}</math>  <math>= 10000000 \left( (1-(0.8)^m) - \frac{10}{9}(1-(0.64)^m) \right)</math>  <math>= 10000000 \left( (1-(0.8)^m) - \frac{10}{9}(1-(0.8)^{2m}) \right)</math>  <math>= \frac{10000000}{9} (10((0.8)^m)^2 - 9(0.8)^m - 1)</math>  <math>= \frac{10000000}{9} (10(0.8)^m + 1)((0.8)^m - 1)</math>  Note that <math>(0.8)^m &gt; 0</math> and <math>(0.8)^m &lt; 1</math> for any positive integer <math>m</math> .  Therefore, we have <math>10(0.8)^m + 1 &gt; 0</math> and <math>(0.8)^m - 1 &lt; 0</math> .  So, we have <math>\frac{2000000(1-(0.8)^m)}{1-0.8} - \frac{4000000(1-(0.64)^m)}{1-0.64} &lt; 0</math> .  Thus, the claim is disagreed.</p>	<p>1M  1M for quadratic expression  1M for either one  1A f.t. ------(10)</p>	



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PRACTICE PAPER

MATHEMATICS COMPULSORY PART PAPER 2

Question No.	Key	Question No.	Key
1.	A	31.	D
2.	C	32.	B
3.	A	33.	C
4.	D	34.	D
5.	D	35.	A
6.	C	36.	B
7.	B	37.	A
8.	D	38.	C
9.	A	39.	A
10.	B	40.	C
11.	D	41.	B
12.	A	42.	A
13.	A	43.	B
14.	B	44.	D
15.	C	45.	C
16.	D		
17.	C		
18.	A		
19.	D		
20.	C		
21.	C		
22.	B		
23.	C		
24.	D		
25.	B		
26.	D		
27.	B		
28.	A		
29.	B		
30.	C		