Problems with explanations:

1. How many 5 digit numbers can be created if the following terms apply: the leftmost digit is even, the second is odd, the third is a non even prime and the fourth and fifth are two random digits not used before in the number?
a) 2520
b) 3150
c) 3360
d) 6000
e) 7500
2. A drawer holds 4 red hats and 4 blue hats. What is the probability of getting exactly three red hats or exactly three blue hats when taking out 4 hats randomly out of the drawer and returning each hat before taking out the next one?
a) $1 / 8$
b) $1 / 4$
c) $1 / 2$
d) $3 / 8$
e) $7 / 12$
3. What is the probability for a family with three ehirdren to have a boy and two girls (assuming the probability of having a boy or a girl is equa)?
a) $1 / 8$
b) $1 / 4$
c) $1 / 2$
d) $3 / 8$
e) $5 / 8$
4. In how many ways ca you sig 8 people on a bench if 3 of them must sit together?
a) 720
b) 2,160
c) 2,400

40,320
$\underbrace{8}$
Dw many ways can you sit 7 people on a bench if Suzan won't sit on the middle seat or on nd?
a) 720
b) 1,720
c) 2,880
d) 5,040
e) 10,080
6. In a jar there are 15 white balls, 25 red balls, 10 blue balls and 20 green balls. How many balls must be taken out in order to make sure we took out 8 of the same color?
a) 8
b) 23
c) 29
d) 32
e) 53
7. In a jar there are 21 white balls, 24 green balls and 32 blue balls. How many balls must be taken out in order to make sure we have 23 balls of the same color?
a) 23
b) 46
c) 57
d) 66
e) 67
8. How many four-digit numbers that do not contain the digits 3 or 6 are there?
a) 2401
b) 3584
c) 4096
d) 5040
e) 7200
9. How many five-digit numbers are there, if the two leftmost digits are even, the other digits are odd and the digit 4 cannot appear more than once in the number?

10. What is the probability of getting a sum of 12 when rolling 3 dice simultaneously?
a) $10 / 216$
b) $12 / 216$
c) $21 / 216$
d) $23 / 216$
e) $25 / 216$
11. How many diagonals does a polygon with 21 sides have, if one of its vertices does not connect to any diagonal?
a) 21
b) 170
c) 340
d) 357
e) 420
12. How many diagonals does a polygon with 18 sides have if three of its vertices do not send any diagonal?
a) 90
b) 126
c) 210
d) 264
e) 306
13. What is the probability of getting a sum of 8 or 14 when rolling 3 dice simultaneously?
a) $1 / 6$
b) $1 / 4$
c) $1 / 2$
d) $21 / 216$
e) $32 / 216$
14. The telephone company wants to add an area code composed of 2 letters to every phone number. In order to do so, the company chose a special sign language containing 124 different signs. If the company used 122 of the signs fully and two remained unused, how many additional area codes canbe ceated the company uses all 124 signs?
15. How many 8 -letter words can be created using computer language ( $0 / 1$ only)?
a) 16
b) 64
c) 128
d) 256
e) 512
16. Ruth wants to choose 4 books to take with her on a camping trip. If Ruth has a total of 11 books to choose from, how many different book quartets are possible?
a) 28
b) 44
c) 110
d) 210
e) 330
17. A computer game has five difficulty levels. In each level you can choose among four different scenarios except for the first level, where you can choose among three scenarios only. How many different games are possible?
a) 18
b) 19
c) 20
d) 21
e) None of the above
18. In a department store prize box, $40 \%$ the noves give the winner a dreamy vacation; the other notes are blank. What is the approx mate probability that 3 out of 5 people that draw the notes one after the other, and immediately return their note into the box get a dreamy vacation?
a) 0.12
b) 0.23
c) 0.35
d) 0.45
e) 0.68

## Explanations:

1. The best answer is A.

The first aigit has 4 options ( $2,4,6,8$ and not 0 ), the second has 5 options ( $1,3,5,7,9$ ) the third has 3 options ( $3,5,7$ and not 2 ), the fourth has 7 options (10-3 used before) and the fifth has 6 options ( $10-4$ used before). The total is $4 * 5 * 3 * 7 * 6=2520$.
2. The best answer is C.

Getting three red out of 4 that are taken out has 4 options (4!/(3!*1!)) each option has a probability of $(1 / 2)^{4}$ since drawing a red or blue has a $50 \%$ chance. $4^{*} 1 / 16=1 / 4$ to get three red hats. The same goes for three blue hats so $1 / 4+1 / 4=1 / 2$.
3. The best answer is D.

There are three different arrangements of a boy and two girls:(boy, girl, girl), (girl, boy, girl), (girl, girl, boy). Each has a probability of $(1 / 2)^{3}$. The total is $3 *(1 / 2)^{3}=3 / 8$.

## 4. The best answer is D.

Treat the three that sit together as one person for the time being. Now, you have only 6 people ( 5 and the three that act as one) on 6 places: $6!=720$. Now, you have to remember that the three tha sit together can also change places among themselves: $3!=6$. So, The total number of possibilities is $6!* 3!=4320$.
5. The best answer is C.

First, check Suzan: she has 4 seats left (7 minus the one in the middle and the two ends), After Suzan sits down, the rest still have 6 places for 6 people or 6! Options to sit The totab is Suzan and the rest: $4 * 6!=2880$.
6. The best answer is C.

The worst case is that we take out seven balls of each color modilld no have 8 of the same color. The next ball we take out will become the eighth ball of gome golor and our mission is accomplished.
Since we have 4 different colors: $4 * 7$ (of each) $+1=29$ balls total.
Of course you could take out 8 of the same color mmediately however we need to make sure it happens, and we need to consider the worst-
7. The best answer is D .

The worst case would be to take out 21 white barls, 22 green and 22 blue balls and still not having 23 of the same color. Takeonemore ball out and you get 23 of either the green or the blue balls. Notice that you cannot get 23 white balls since there are only 21, however, you must consider them since they might be taken out also.
The total is: $21+22+22+$
8. The best answer is $B$.


The first digit has possibilities ( $10-0,3$ and 6 ). The other three digits have 8 possibilities each. $7 * 8 * 8 * 8=3584$.
9. The best answe is C.

No considering the fact that 4 cannot appear more than once, we have a total of -2500 . Now we deduct the possibilities where 4 does appear more than once (in this case it dan appear only twice on the two leftmost even digits). In order to do so, we put 4 in the first and second leftmost digits. The rest of the digits are odd: $5 * 5 * 5=125.2500-125=2375$.

1a. The best answer is E .
Start checking from the smaller or bigger numbers on the dice. We will check from bigger numbers working downwards: start with 6 , it has the following options: $(6,5,1),(6,4,2),(6,3,3)$. Now pass on to 5: $(5,5,2)$, $(5,4,3)$. Now 4: $(4,4,4)$. And that's it, these are all number
combinations that are possible, if you go on to 3 , you will notice that you need to use 4,5 or 6 , that you have already considered (the same goes for 2 and 1 ). Now analyze every option: $6,5,1$ has 6 options $(6,5,1),(6,1,5),(5,1,6),(5,6,1),(1,6,5),(1,5,6)$. So do $(6,4,2)$ and $(5,4,3)$. Options $(6,3,3)$ and $(5,5,2)$ have 3 options each: $(5,5,2),(5,2,5)$ and $(2,5,5)$. The same goes for $(6,3,3)$. The last option $(4,4,4)$ has only one option. The total is $3 * 6+2 * 3+1=18+6+1=25$ out of $216\left(6^{3}\right)$ options.
11. The best answer is B.

We have 20 vertices linking to 17 others each: that is $17 * 20=340$. We divide that by 2 since every diagonal connects two vertices. 340/2=170. The vertex that does not connectoany diagonal is just not counted.
12. The best answer is A.

We have 15 Vertices that send diagonals to 12 each (not to itself and not to the two adjacent vertices). $15^{*} 12=180$. Divide it by 2 since any diagonal links 2 vertices $=90$. The three vertices that do not send a diagonal also do not receive any since the same diagonalis sent and received. Thus they are not counted.
13. The best answer is A.

The options for a sum of 14 : $(6,4,4)$ has 3 options, $(6,5,3)$ has 6 optons, $(6,6,2)$ has 3 options, $(5,5,4)$ has 3 options. We have 15 options to get 14 .
The options for a sum of $8:(6,1,1)$ has 3 options, $(5,2,1)$ has 6 pptions, $(4,3,1)$ has 6 options, $(4,2,2)$ has 3 options, $(3,3,2)$ has 3 options. We-have 21 options to get 8 .
Total: $21+15=36 / 216=1 / 6$.
14. The best answer is C.

The phone company already created $122 * 122$ atea codes, now it can create $124 * 124$. $124^{2}-122^{2}=(124+122)(124-122)=246 * 2=492$ additional codes.
There are other ways to solve this question. However this way is usually the fastest.
15. The best answer is D.

Every letter must be chosen fron 0 or 1 only. This means we have two options for every word and $2^{8}=256$ wores total.
16. The best answer Is E.

Choosing 4 out of 11 oooks is: $11!/(4!* 7!)=330$ possibilities.
17 The best answer is .
On fourlevels there are 4 scenarios = 16 different games. The first level has 3 different
scenarios. The total is 19 scenarios.
18. The best answer is $B$.

The chance of winning is 0.4 and it stays that way for all people since they return their note. The number of different options to choose 3 winners out of 5 is $5!/(3!* 2!)=10$. Each option has a chance of $0.4^{*} 0.4^{*} 0.4^{*} 0.6 * 0.6=0.02304 * 10=0.2304$.


