GCSE

185/09

MATHEMATICS
HIGHER TIER
PAPER 1

P.M. WEDNESDAY, 9 November 2011

2 hours

CALCULATORS ARE NOT TO BE USED FOR THIS PAPER

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	5	
2	6	
3	10	
4	8	
5	7	
6	4	
7	7	
8	5	
9	10	
10	7	
11	5	
12	4	
13	7	
14	11	
15	4	
TOTAL MARK		

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Write your name, centre number and candidate number in the spaces at the top of this page.

Answer **all** the questions in the spaces provided.

Take π as 3·14.

INFORMATION FOR CANDIDATES

You should give details of your method of solution when appropriate.

Unless stated, diagrams are not drawn to scale.

Scale drawing solutions will not be acceptable where you are asked to calculate.

The number of marks is given in brackets at the end of each question or part-question.

Formula List

Volume of prism = area of cross-section × length
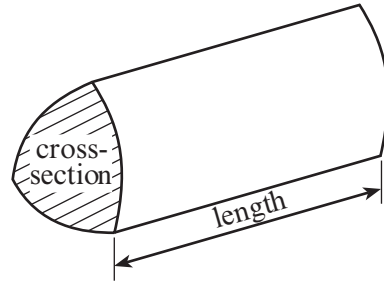
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
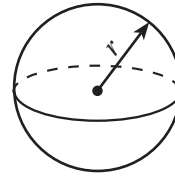
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$
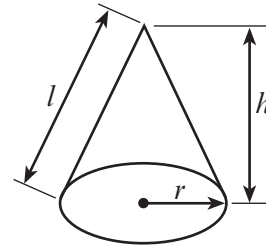
In any triangle ABC

    Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

    Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

    Area of triangle $= \frac{1}{2}ab\sin C$
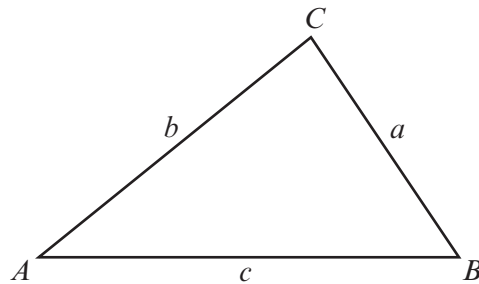
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$

where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Standard Deviation

Standard deviation for a set of numbers

$x_1, x_2, \ldots, x_n,$ having a mean of $\bar{x}$ is given by

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \text{ or } s = \sqrt{\frac{\sum x^2}{n} - \left\{\frac{\sum x}{n}\right\}^2}$$

1. Tomas has one spin of the circular spinner shown below.
Two of the lines shown on the diagram are diameters of the circle.
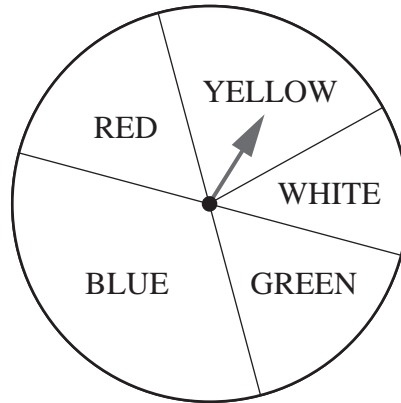
Diagram not drawn to scale

(a) The table below shows the probabilities of Tomas obtaining YELLOW, WHITE and GREEN with one spin of the spinner.
Complete the table.

Colour	YELLOW	WHITE	GREEN	RED	BLUE
Probability	0·2	0·12	0·18		

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[3]

(b) Find the probability of obtaining either WHITE or GREEN on the spinner.

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[2]

2. The scatter diagram shows the height, in cm, and the weight, in kg, for each of 20 members of a sports club.
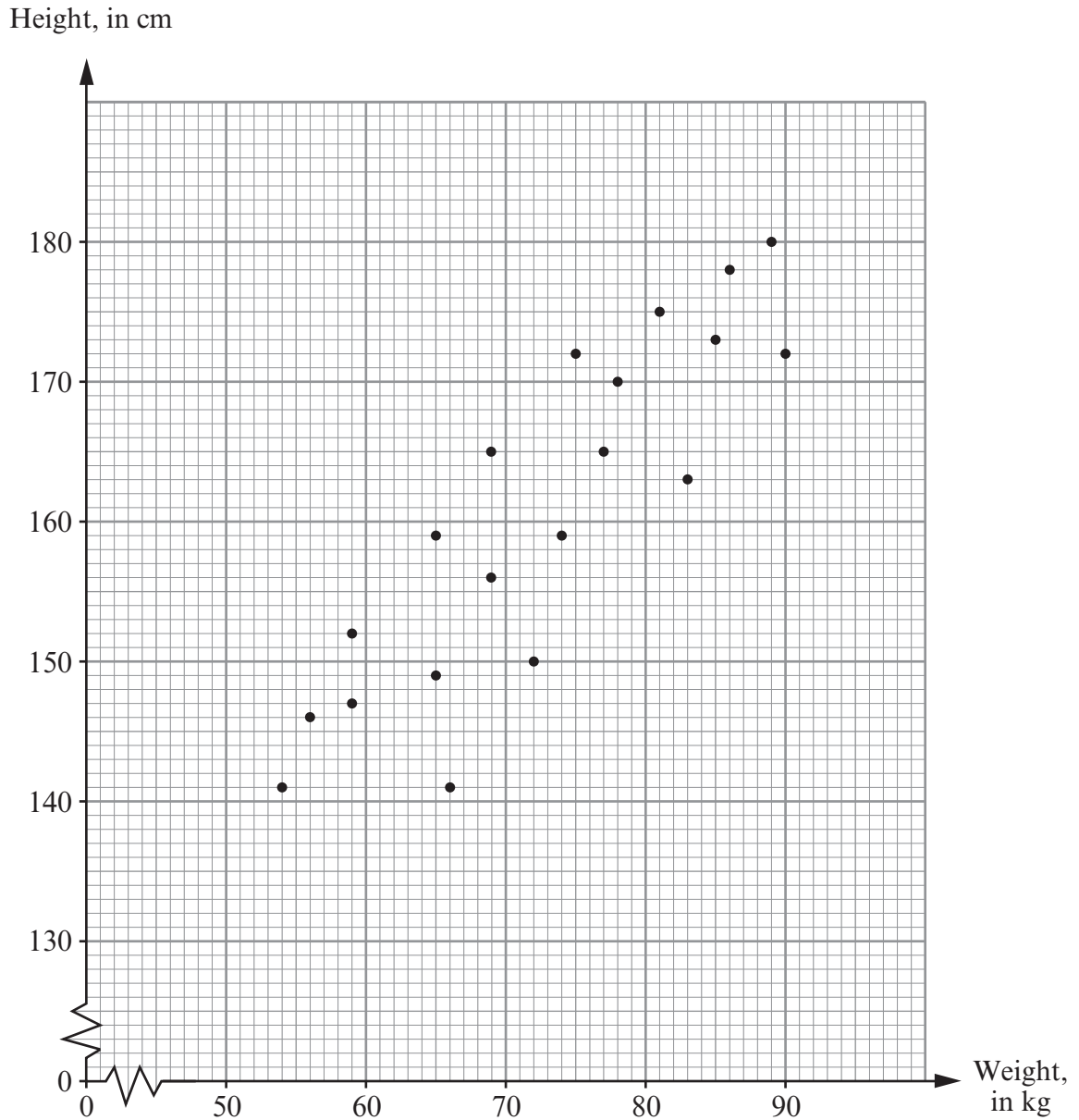
(a) Write down the height and weight of the **heaviest** of the 20 members of the sports club.

Weight .... kg

Height .... cm

[2]

(b) Write down the type of correlation shown by the scatter diagram.

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[1]

(c) Draw, by eye, a line of best fit on the scatter diagram.
[1]

(d) Estimate the weight of a person of height 155 cm.

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[1]

(e) Is it possible to estimate the weight of a person with a height of 210 cm from the scatter diagram? You must give a reason for your answer.

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[1]

Turn over.

3. *(a)*
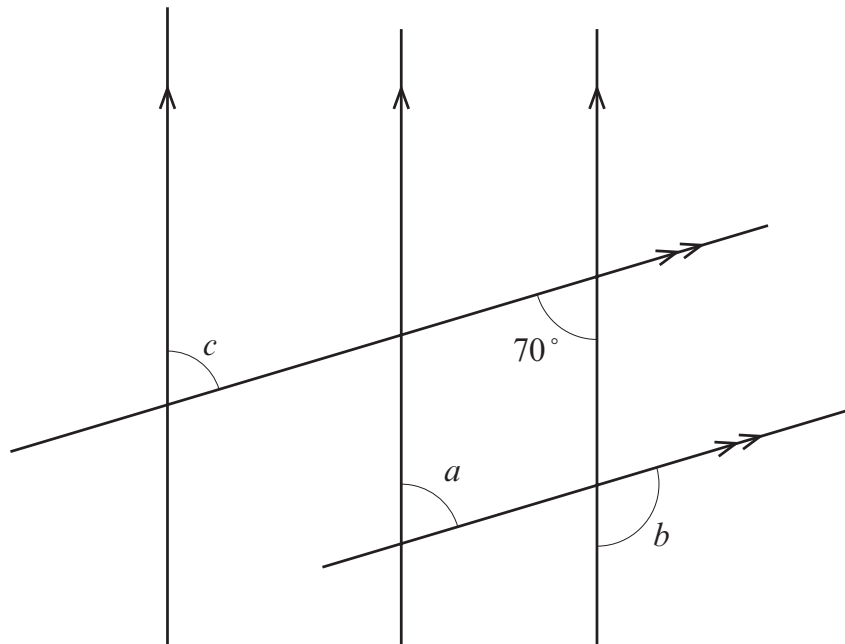
Diagram not drawn to scale

Find the size of each of the angles marked *a, b* and *c*.

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$a =$ .... ° $b =$ .... ° $c =$ .... °

[3]

(b) Calculate the size of the interior angle of a regular 10 sided polygon.

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[3]

(c)
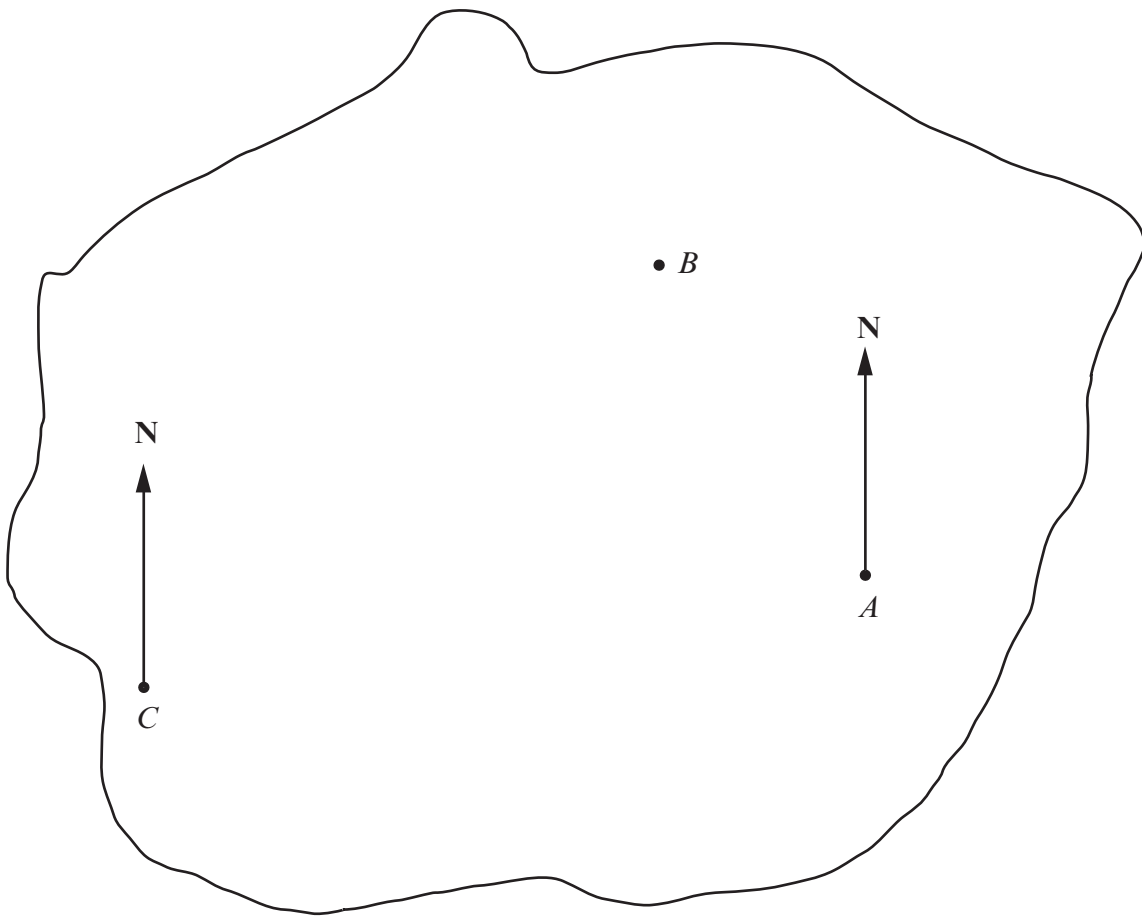
(i)   Write down the bearing of point *B* from point *A*.

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(ii) A point *D* is to be plotted on the above plan.
      The bearing of *D* from *C* is 038°, and the bearing of *D* from *A* is 305°.
      Find and mark the position of *D* on the above plan.

[4]

4. *(a)* Reflect the triangle in the line $x = -1$.

[2]
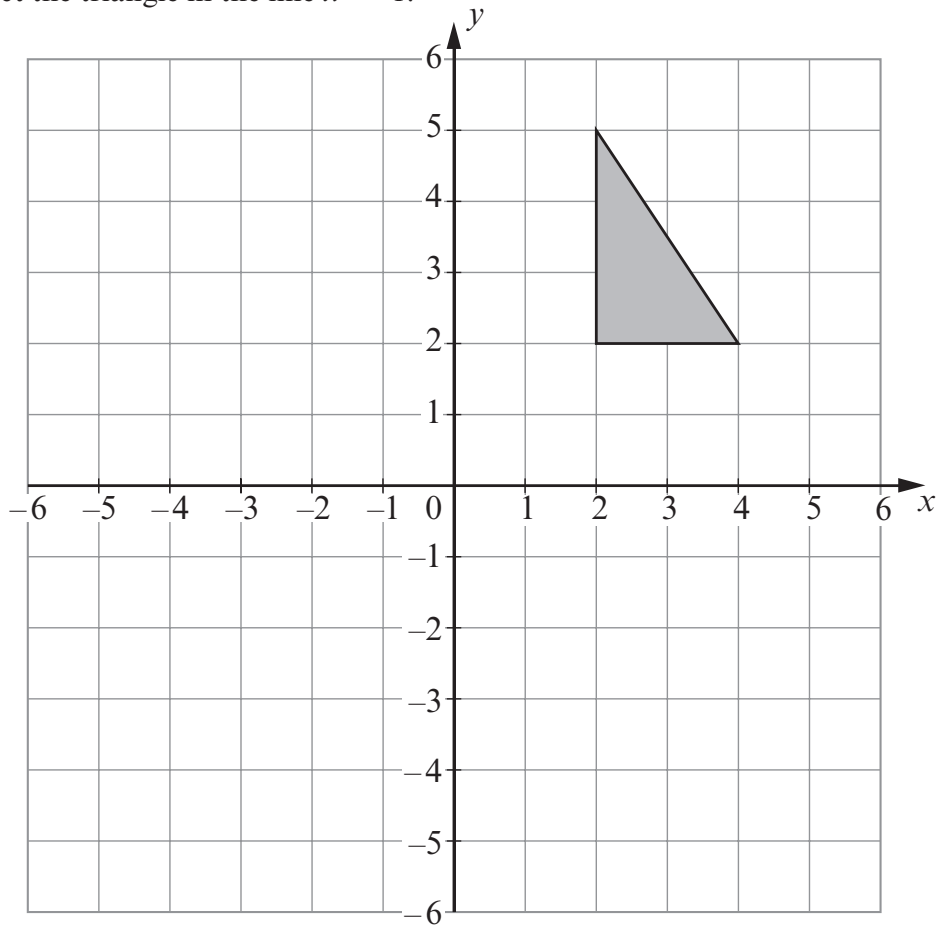
(b) Translate the triangle shown 4 to the left and 2 down.

[1]
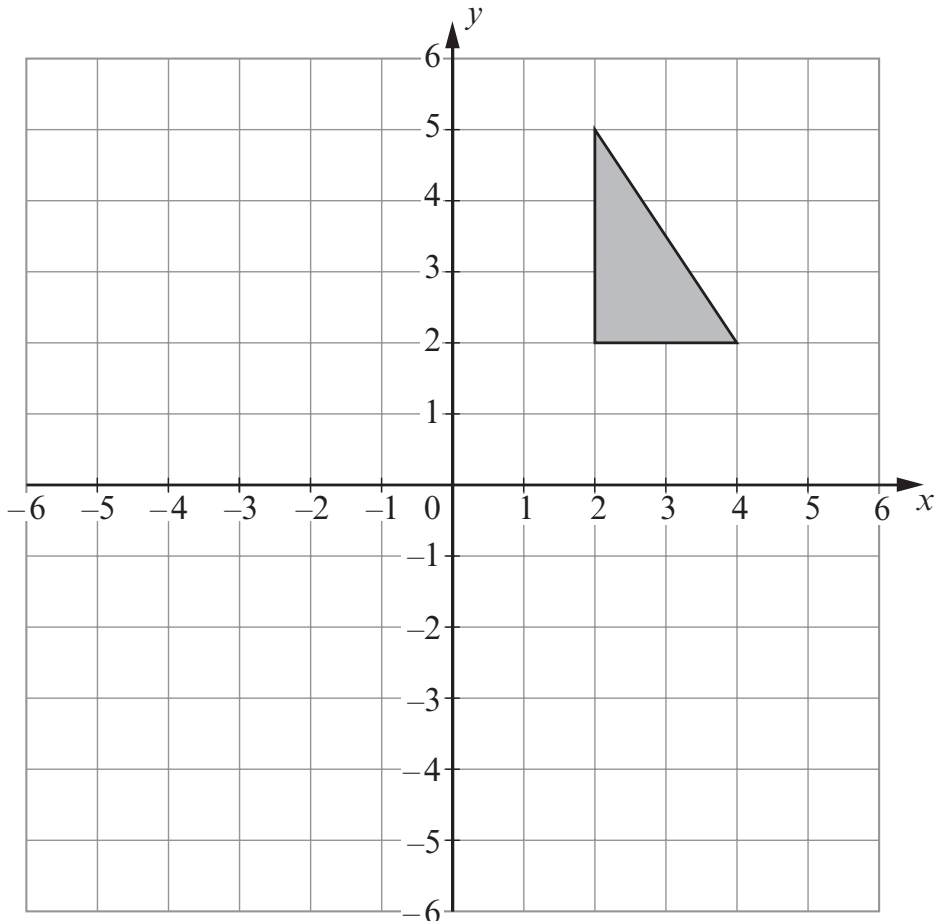
(c) Enlarge the triangle using centre (0, 0) by a scale factor of $\frac{1}{2}$.

[2]
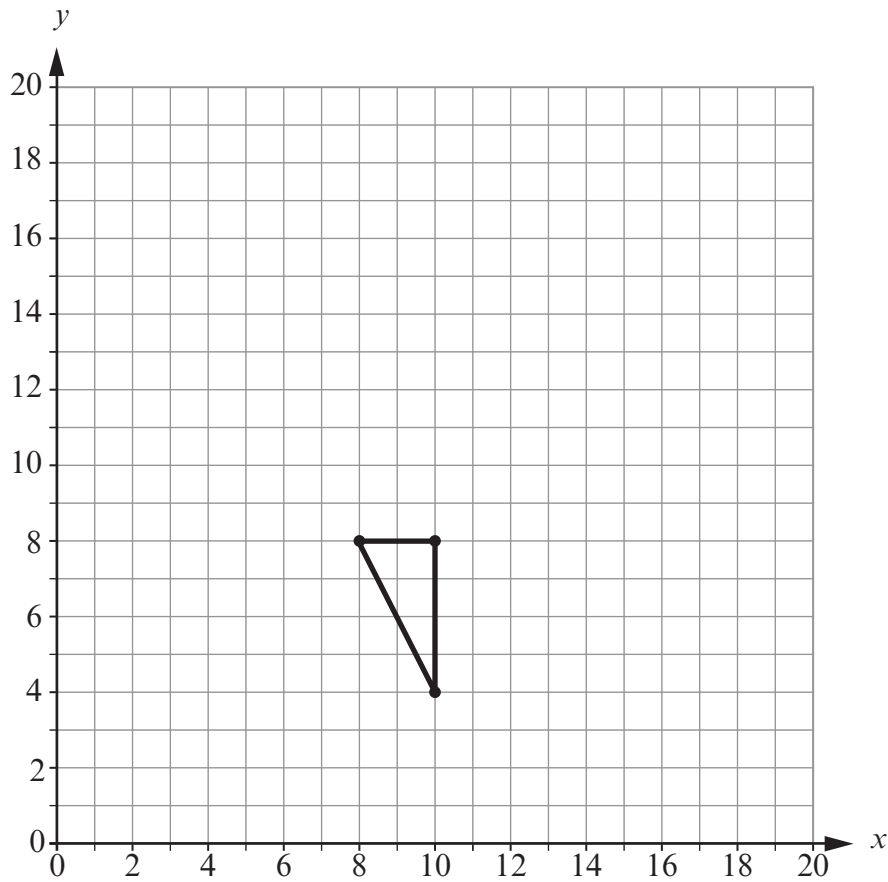
(d) The diagram below shows a triangle *ABC*. Find and **shade** the region inside the triangle which satisfies **both** of the following conditions.

(i) All points in the region are nearer to *AC* than to *AB*.

(ii) All points in the region are less than 5 cm from *A*.

[3]

Turn over.
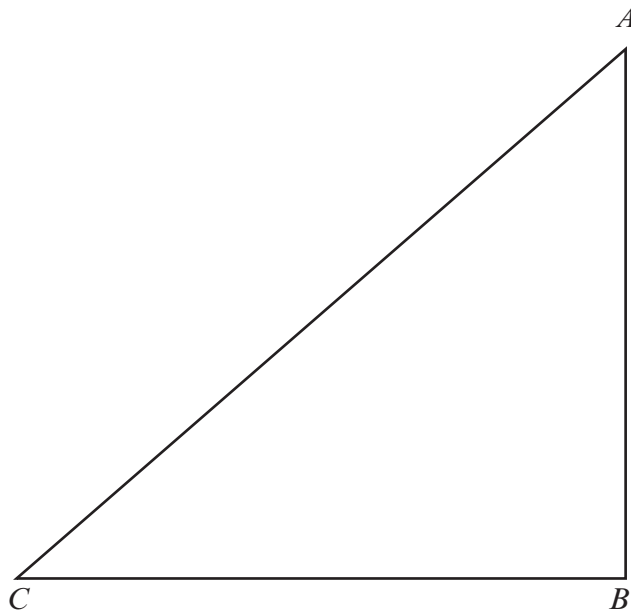
5. *(a)* Cheryl scored 60 marks out of 80 in a test.
Express Cheryl's score as a percentage.

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[2]

(b) Share £300 in the ratio 5 : 7.

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[3]

(c) Calculate 75% of £562.80.

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[2]

6. *(a)* Write down the *n*th term of the sequence 6, 10, 14, 18, 22, ...

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[2]

(b) Solve $3x + 4 = 8 - 7x$.

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[2]

Turn over.

7. *(a)* Explain how you know that 24 is **not** a square number.

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[1]

(b) Express 112 as a product of prime numbers in index form.

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[3]

(c) Evaluate $\frac{1}{5} \div \frac{1}{3}$.

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[1]

(d) Evaluate $2\frac{1}{3} \times 3\frac{3}{7}$. Simplify your answer.

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[2]

8. The length of a table top is 2050 mm, measured to the nearest 10 mm.

(a) Write down the **least** and **greatest** possible values of the length of the table top.

Least value .... mm   Greatest value .... mm

[2]

(b) The width of the table top is 1040 mm, measured to the nearest 10 mm.
Find the least possible perimeter of the table top.

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[3]

Turn over.

9. *(a)* Factorise $x^2 - 2x - 8$.

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[2]

(b) Expand and simplify $(2x + 1)(x - 5)$.

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[2]

(c) Solve $\dfrac{21 - 2x}{5} = 4 - x$.

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[3]

(d) Simplify $6x^7y^2 \times 4x^3y^5$.

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[2]

(e) Simplify $a^5 \times a^{-2}$.

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[1]

Turn over.

10. The heart rate of each of 60 people at a gym was recorded.
The following table shows a grouped frequency distribution of the results.

Heart rate, r (beats per minute)	$80 \leqslant r < 90$	$90 \leqslant r < 100$	$100 \leqslant r < 110$	$110 \leqslant r < 120$	$120 \leqslant r < 130$	$130 \leqslant r < 140$
Number of people	4	14	22	10	6	4

(a) Complete the following cumulative frequency table.

Heart rate (less than)	80	90	100	110	120	130	140
Cumulative frequency	0	4	18				

[1]

(b) On the graph paper below, draw a cumulative frequency diagram to show this information.

[3]

Cumulative frequency
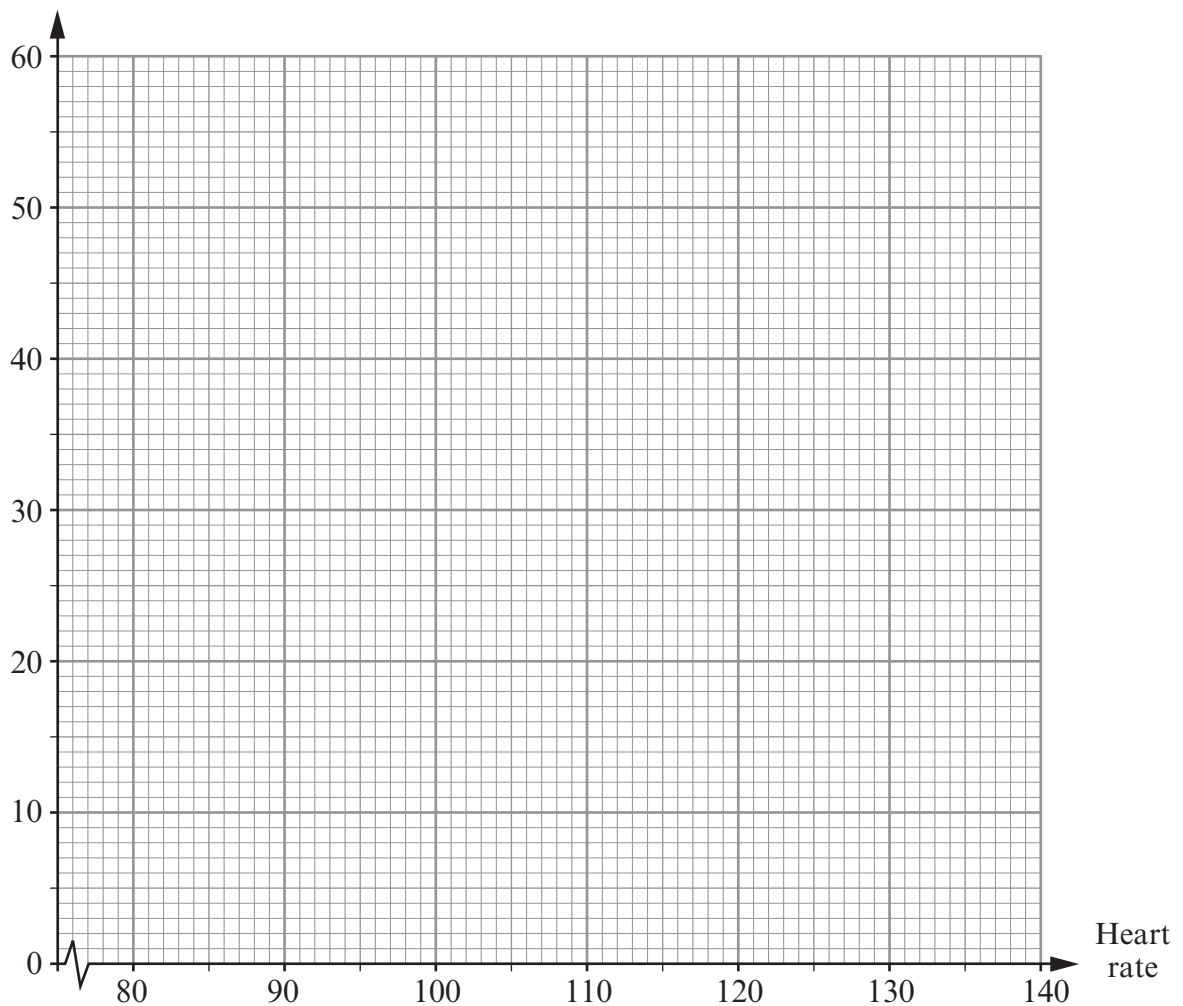
(c) Use your cumulative frequency diagram to find estimates for

(i) the median,

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(ii) the interquartile range.

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[3]

11. *(a)* Factorise the expression $6x^2 - 11x - 10$ and hence solve the equation
$6x^2 - 11x - 10 = 0$.

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[3]

(b) Factorise $4y^2 - 81$.

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[2]

12. At lunchtime, the probability that Molly has a sandwich is 0·7.
Whether or not Molly has a sandwich, the probability that Molly has an ice cream mid-afternoon is 0·2.

(a) Complete the following tree diagram.

Molly has
an ice cream

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Molly has a
sandwich

0·7

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Molly does not
have an ice cream

Molly has
an ice cream

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Molly does not
have a sandwich

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Molly does not
have an ice cream
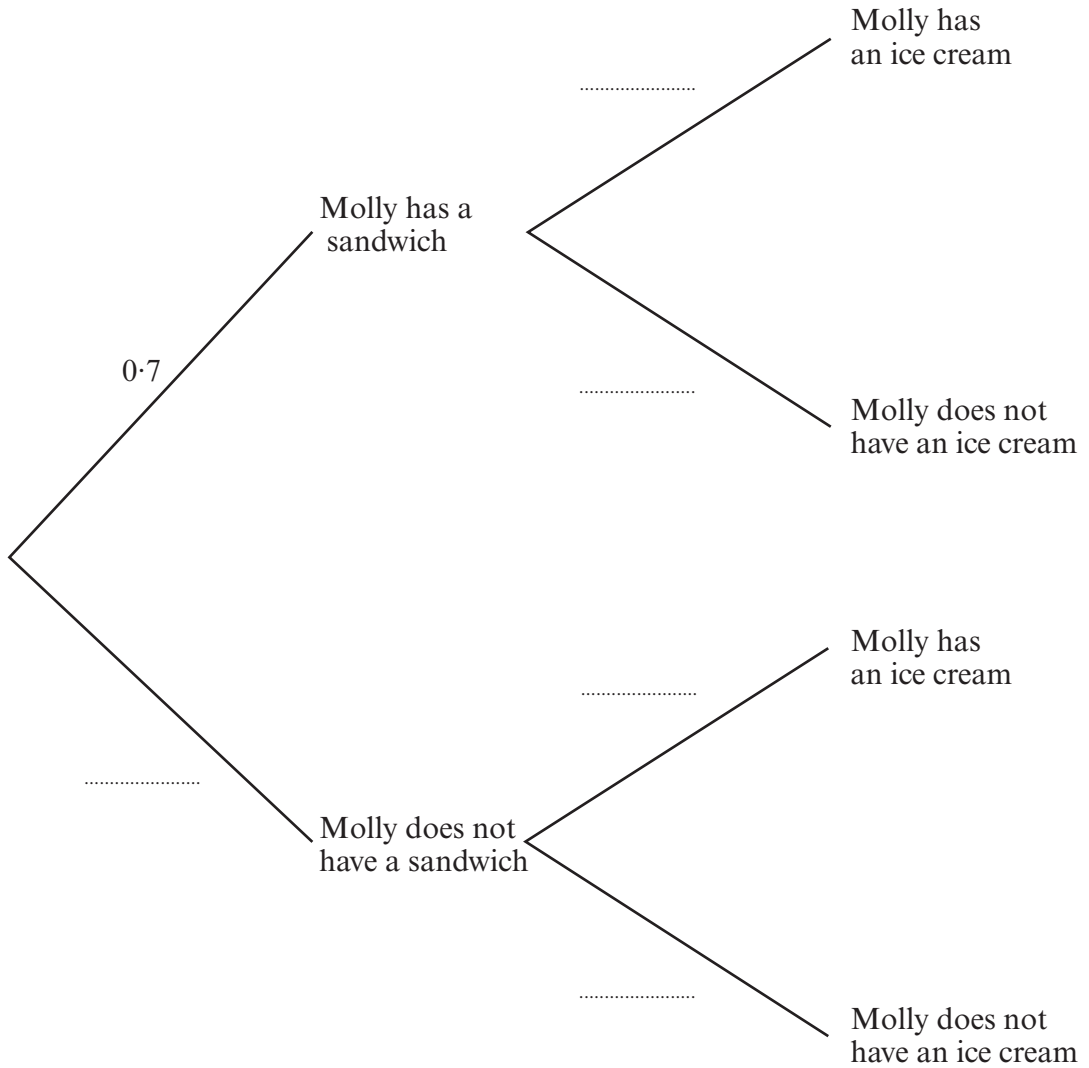
[2]

(b) Calculate the probability that Molly has a sandwich for lunch and has an ice cream mid-afternoon.

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[2]

13. Vectors **OK**, **OL** and **OM** are shown in the diagram below.

Diagram not drawn to scale
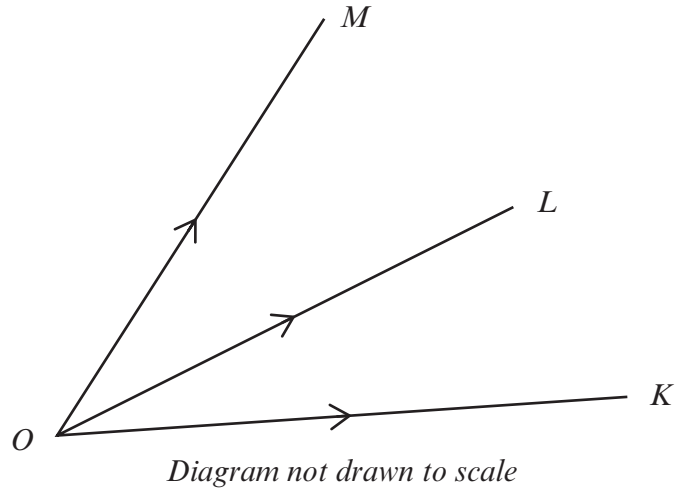
Given that **OK** = 2**a** + **b**, **OL** = 5**a** + 3**b** and **OM** = 14**a** + 9**b**,

(a) express each of the following in terms of **a** and **b** in their simplest form,

(i) **KL**

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(ii) **LM**

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[3]

(b) show that **KM** = *p***KL** where *p* is a constant,

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[2]

(c) explain fully the geometrical implication of your answer in part *(b)*.

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[2]

Turn over.

14. The graph of $y = x^3 - 3x^2 - 13x + 15$, for values of x between $x = -4$ and $x = 6$, has been drawn below.

(a) Use the graph to solve $x^3 - 3x^2 - 13x + 15 = 0$.

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[1]

(b) Using the graph, estimate the gradient of the curve $y = x^3 - 3x^2 - 13x + 15$ when $x = 4$.

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[3]

(c) By drawing an appropriate line on the graph, solve the equation

$$x^3 - 3x^2 - 13x + 5 = 0.$$

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[3]

(d) Use the trapezium rule with 4 strips to estimate the area of the region enclosed by the curve $y = x^3 - 3x^2 - 13x + 15$ and the *x*-axis between $x = -3$ and $x = 1$.

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[4]

Turn over.
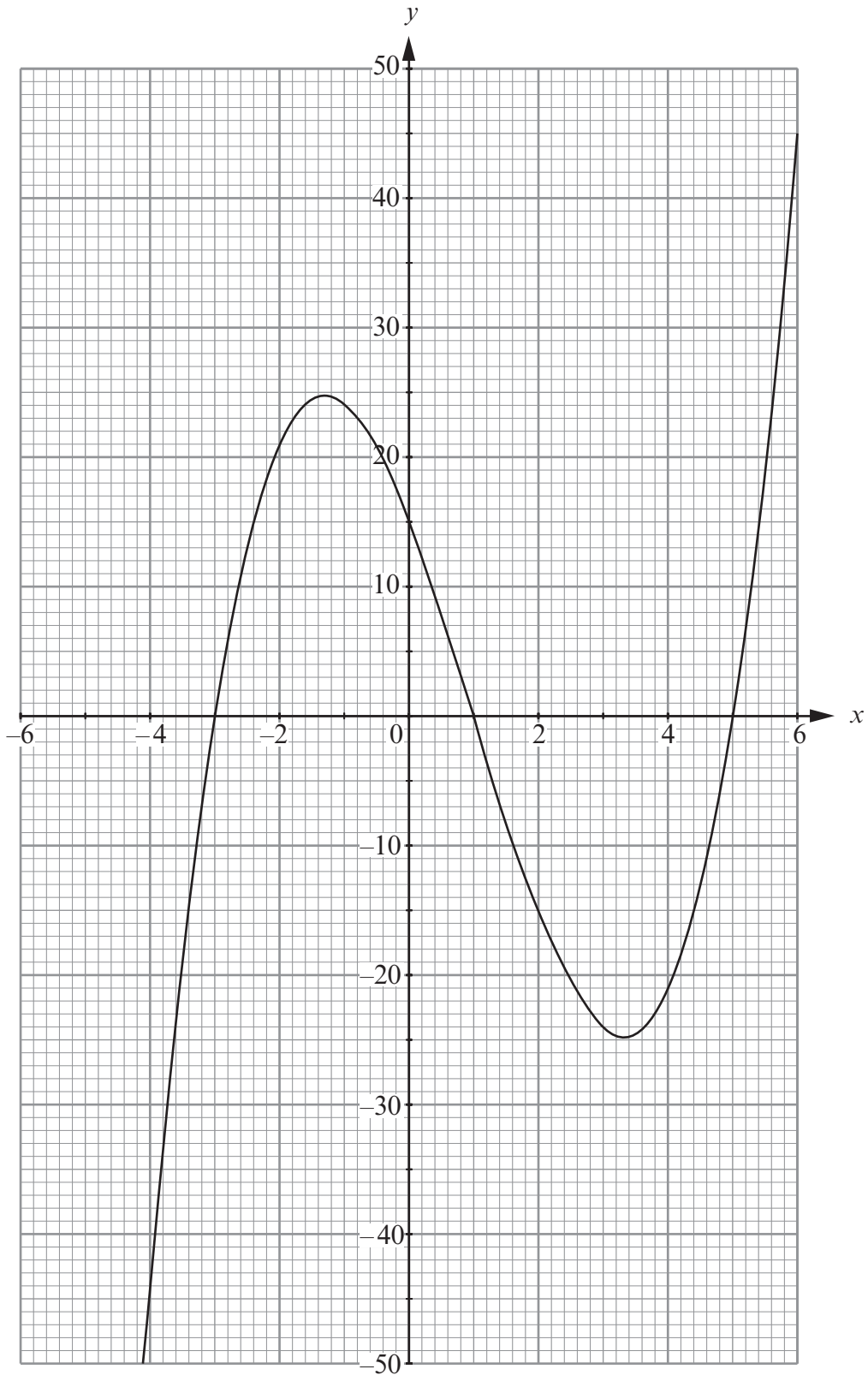
15. *(a)* Express $0.6\dot{5}\dot{2}$ as a fraction.

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[2]

(b) Given that $p = \sqrt{5}$, $q = \sqrt{13}$ and $r = \sqrt{325}$, simplify *pqr*.

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[2]