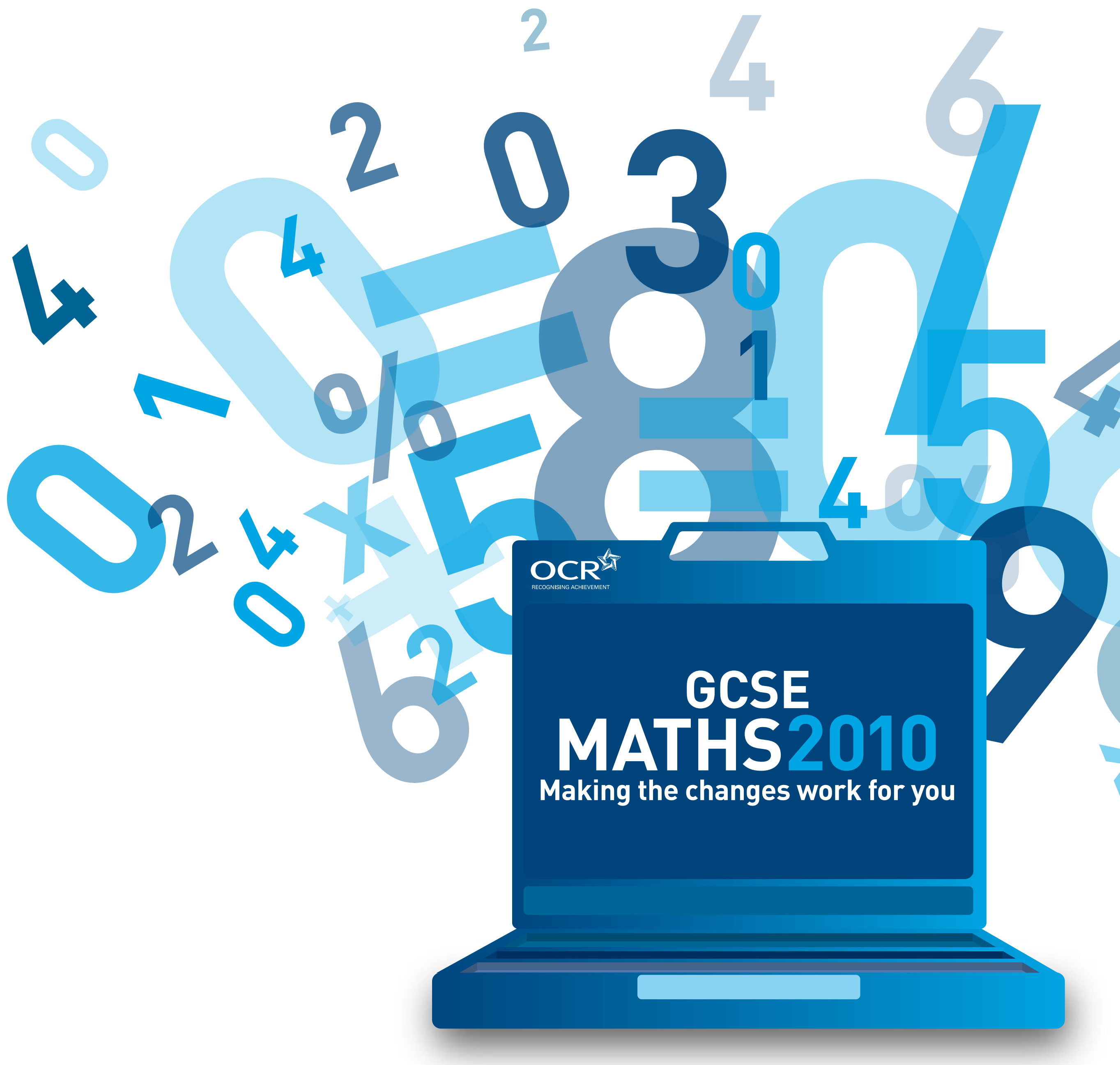
OCR
RECOGNISING ACHIEVEMENT

OCR GCSE in Mathematics A J562

AO3 Guide

This guide is designed to accompany the OCR GCSE Mathematics A specification for teaching from September 2010.



Contents

Contents 2

Section 1: What is the difference between AO2 and AO3? 3

Section 2: Questions in the classroom 5

Section 3: Teaching resources for AO3 7

3.1 Case 1: Area and Perimeter (Foundation) 7
3.2 Case 2: Mean, Mode, Median and Range (Foundation/Higher) 20
3.3 Case 3: Pythagoras' Theorem (Foundation/Higher) 35
3.4 Case 4: Hierarchy of Operations (Foundation) 42
3.5 Case 5: Sequences (Foundation/ Higher) 49
3.6 Case 6: Trigonometry (Higher) 61
3.7 Case 7: Graphs - Gradient, Parallel and Perpendicular (Foundation/Higher) 69
3.8 Case 8: Area and Volume (Higher) 84
3.9 Case 9: Measures (Foundation/Higher) 86
3.10 Case 10: Understand and Use Direct Proportion (Foundation) 90

Section 4: Some worked examples 92

Other forms of Support 95

Published Resources 95
OCR Training 97
e-Communities 98
Interchange 98

Acknowledgments 99

Section 1: What is the difference between AO2 and AO3?

Assessment Objective - AO3

AO3 seeks to assess a learner's ability to:

- Process information
- Pose problems and pursue them
- Conjecture and investigate within mathematics
- Reason concisely
- Evaluate and check methods and results
- Present solution(s) effectively

Educators will need to prepare learners with these skills and examiners will need to devise questions that assess them.

Within specifications, AO3 is defined as the following:

	Assessment Objective	Weighting (%)
AO3	• interpret and analyse problems and generate strategies to solve them	15-25

Note the link to the three strands in Functional Mathematics:
- Represent (strategy)
- Analyse
- Interpret

Not all AO3 questions will be "functional".

A learner who is prepared for AO3 should be able to solve problems that:

- Are set in a context which may be
    - Mathematical or "real world"
    - Likely to be less familiar
- Are susceptible to solution by more than one method
- Have solution(s) that serve a purpose

(The degree of unfamiliarity and extended response will depend on the tier of the examination)

In doing so the learner may be required to:

- Select and apply an appropriate strategy, or strategies, to solve the problem
- Select and use relevant information and reject redundant information
- Interpret their, or a given, solution(s)
- Explain, evaluate and justify their strategy
- Explain their assumptions

AO3 questions in examination papers must provide opportunities for these skills to be assessed.

Questions will be characterised by some of the following:

- A question may be in the form of a problem that has a purpose
  - the purpose may be mathematical or "real world"
- Redundant information may be included
- Questions may be unstructured or have reduced structure but require an extended answer
- A strategy may be given that the candidate is asked to evaluate
- A candidate may be required to construct his or her own strategy
- They may be asked or expected to justify their strategy
- Proof may be required or expected or flaws in a proof might have to be explained and/or corrected
- A solution might have to be presented with the needs of the audience in mind

Assessment Objective - AO2

Within specifications, AO2 is defined as the following:

	Assessment Objective	Weighting (%)
AO2	• select and apply mathematical methods in a range of contexts	25-35

AO2 questions may require problem solving skills and may be characterised by some of the following:

- They will be set in context and will look much like current questions
- There will be no redundant information
- Candidates choose a method (usually standard and reasonably straightforward) within a closed problem
- The solution to a question will follow the direction of the question, so "lead in" sections may be present
- Questions will often require shorter responses than for AO3

Examples of contrasting AO2 and AO3 questions appear throughout Section 3: Teaching resources for AO3.

Section 2: Questions in the classroom

Delivering AO3 in the Classroom

It may not be possible, or desirable, to deliver AO3 in every topic nor in every lesson. It is quite appropriate to have lessons that involve instruction, recall and practice of skills. However, it is important to remember the hierarchy of questions. Further information may be found by following the link http://members.fortunecity.com/rapidrytr/dist-ed/bloom.html

As teachers we tend to ask questions in the **Low**, "knowledge" category most frequently.
In responding to HIGHER ORDER questions learners develop their own thinking skills and understanding.

Low order questions require the learner to recall learned facts and information. (**Knowledge**)

> What is 3 x 15?
>
> What is the shape of the graph for $y = \frac{1}{x^2}$?

The order then progresses through:

Some understanding of the facts (**Comprehension**)

> Which numbers seem to be out of place in this table?

Applying knowledge to find results (**Application**)

> How does knowing the formula for the area of a triangle help you find a formula for the area of a trapezium?
>
> Solve $x^2 - 3x - 6 = 0$ using your graph.

Consideration of results to understand what is happening (**Analysis**)

> What patterns do you see in the results?
>
> Classify these shapes according to their symmetries.

Generalising from known facts – new ideas from old (**Synthesis**)

> All these numbers have something in common. How can you use….to write a formula to….
>
> What would you predict/infer from...?

Making choices based on reasoned judgement (**Evaluation**)

> What evidence would you use to support/refute…?
>
> Under what conditions is ….. true/not true?

Even in a lesson on instruction and recall, appropriate questioning can develop learners' understanding and ability to reason.

Developing AO3 skills is a slow process that depends on giving learners confidence to:

- Try different approaches
- Make mistakes and rectify them in a constructive environment
- Justify decisions and explain consequences
- Express ideas and communicate conclusions in a variety of ways and to different audiences.

This may be done by undertaking a variety of targeted activities throughout the GCSE course, encouraging interaction and exploring understanding through appropriate questioning.

This section sets out seven extended cases, containing ideas for short and longer activities, and three cases with a single activity. In each activity the curriculum may be delivered or revised in a manner that develops learners' competence with the demands of AO3.

It is not intended that this guide should give specific lesson plans. However, the activities are intended to spark ideas about how activities may be developed and used within a series of lessons.

Section 3: Teaching resources for AO3

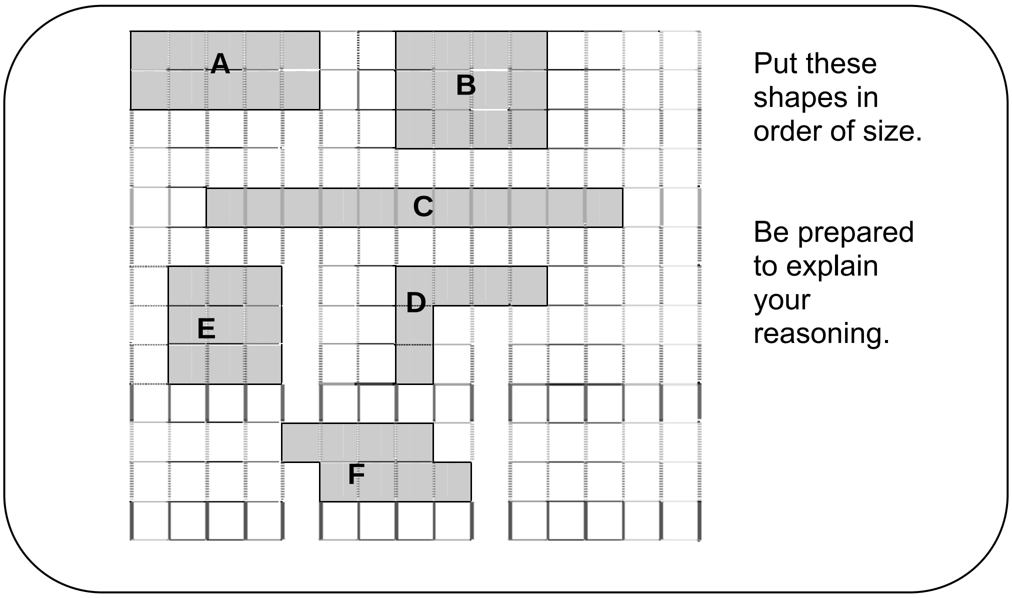
3.1 Case 1: Area and Perimeter (Foundation)

FC9 Area and volume	
9.1 - Perimeter, area (including circles), and volume	Candidates should be able to: a find areas of rectangles, recalling the formula, understanding the connection to counting squares d calculate perimeters and areas of shapes made from triangles and rectangles

It is assumed that candidates will have experience of finding areas of rectangular shapes by counting squares.

Starter

This might be used as a lead in to a lesson on area and perimeter of shapes made from combinations of rectangles.



A 10 squares or perimeter 14 edges
B 12 squares or perimeter 14 edges
C 11 squares or perimeter 24 edges
D 6 squares or perimeter 14 edges
E 9 squares or perimeter 12 edges
F 8 squares or perimeter 14 edges

Some possible questions

- How did you decide what the size of the shape was?
- What is the number of squares that a shape covers called?
- If the shape had covered a lot of squares, do you know any other way to find how many squares it covers?
- Could you have put the shapes in a different order if you did not count squares?
- What is the length around a shape called?
- How do the two orders compare for perimeter and area?
- What do you notice about the perimeters of four of these shapes?
- You have 3 minutes. Find four other shapes that have a perimeter of 14 that you can draw on the grid.
- Do they all have the same area?

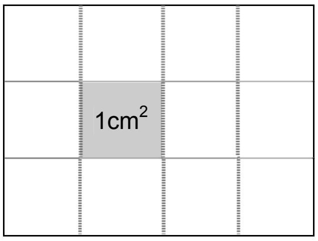
Activity 1

This activity will easily fill a lesson. Questions at the end may be developed to include maximum perimeter and the conditions that apply to this. Also the maximisation of the number of squares that may be removed yet preserve the perimeter. Is the resulting shape unique?

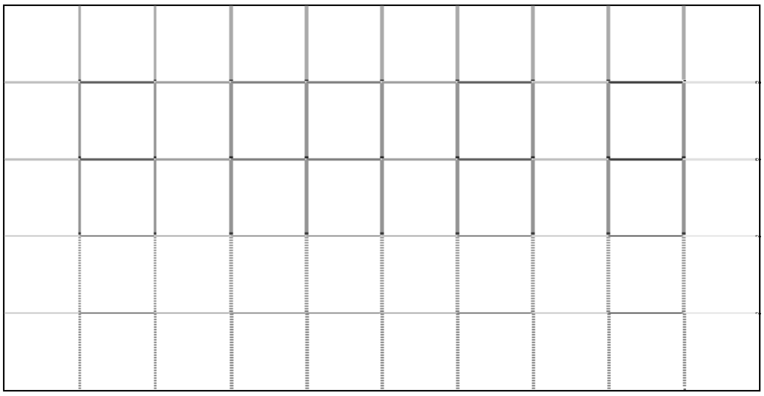
<u>PerImeter Challenge</u>

This rectangle has an area of 12cm^2 (as it covers 12 squares each with sides 1 cm long).

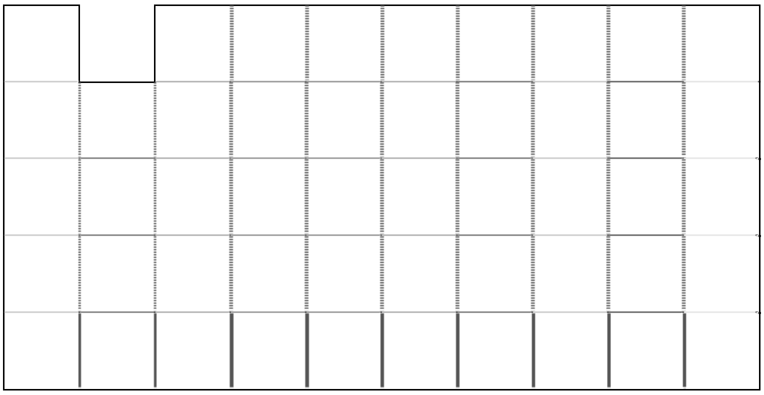
Its perimeter is 3 + 4 + 3 + 4 = 14cm (as this is the distance all around it).



This shape has a perimeter of 30 cm.
It has an area is 50 cm^2.



Now take away a square.



The area is now 49 cm^2 but what is the perimeter?

1 Find how to take away squares but keep the perimeter the same.
2 Find how to take away squares but make the perimeter larger.
3 Can you take away squares and make the perimeter smaller?

Draw diagrams to explain your answers.

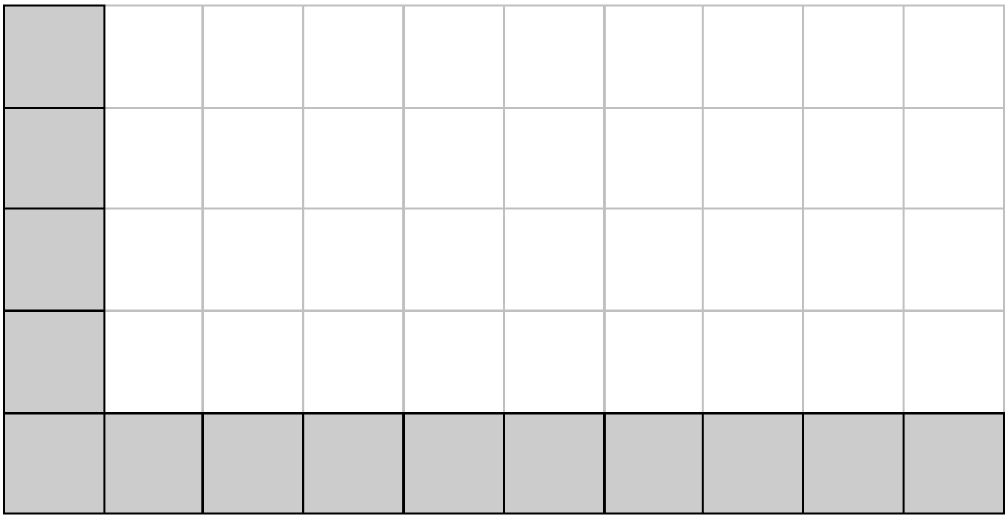
4 Ask one or more questions of your own about taking away squares and how this changes the perimeter and show how to answer them.

Some possible questions

- Where could you take squares away from and not change the perimeter?
- Why did that happen?
- Where could you take squares away from and make the perimeter larger?
- Why did that happen?
- What questions did you try to answer?
- What did you find out?
- How could you check that what you found out was right?
- What is the biggest number of squares you could take away and still have the same perimeter?
- Is there only one way to do this?
- What was the biggest perimeter you found?

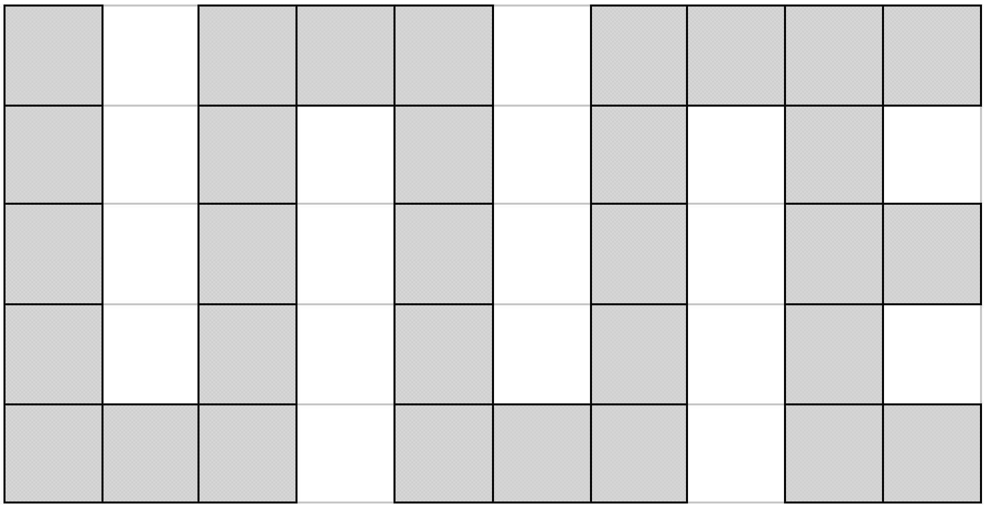
These are possible outcomes from the activity.

1) Maximum number of squares removed but perimeter remains unchanged



This result is not unique but the conditions are that the squares that are removed must come from the corners where the number of edges removed equals the number added (2).

2) Maximum perimeter



This is not unique but occurs when each removed square ADDS two edges to the perimeter. Removed squares cannot join corner to corner as the integrity of the shape is destroyed.

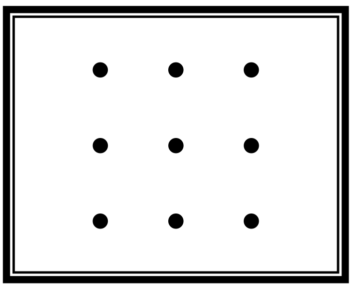
Activity 2

This activity is a short investigation designed to last approximately 30 minutes. It is important that learners discuss their results and the methods used to work out the areas. The technique of surrounding and removing triangles occurs when dealing with obtuse-angled triangles.

Systematic working is essential. Learners should be encouraged to discover the formula $A = \frac{bh}{2}$.

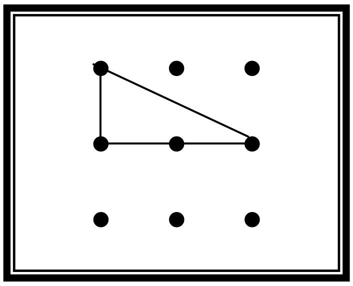
Spotty Triangles

1) Draw as many **different sized** triangles as you can when three dots are joined on a 3 dot by 3 dot grid. The lines can pass through other dots.



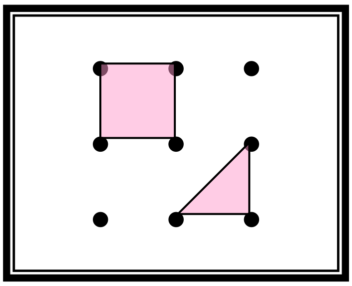
2) Work out the area of each triangle. (All answers are either whole numbers or halves.)

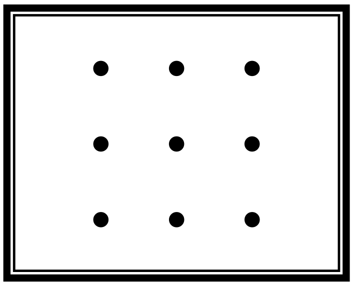
Example: the triangle below has an area of $1cm^2$.

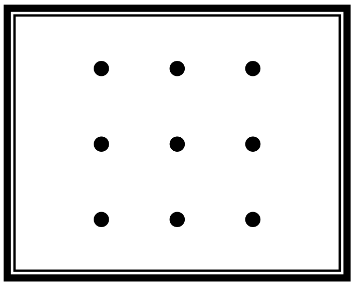


HINT

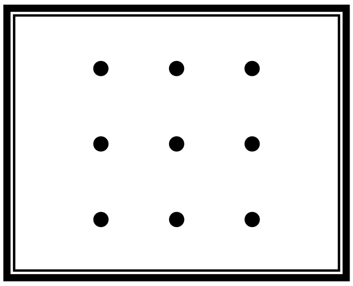
<u>This square is $1cm^2$</u>
<u>This triangle is half the square</u>

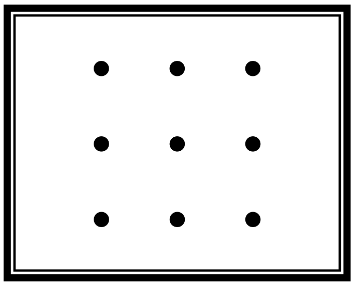






It would be a good idea to provide plenty of these grids.





Some possible questions

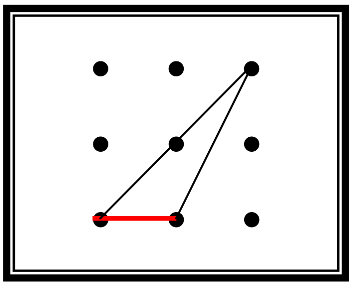
- How many triangles did you find?
- What method did you use to make sure you did not miss out some triangles?
- Which triangles had areas which were easy to work out?
- Which ones were hardest to work out?
- Are you sure about your answers?
- Who can come and show me how to work out the area of this triangle?
- Do you know another method to work out the area of each triangle?

Indicate the base and height of a triangle.

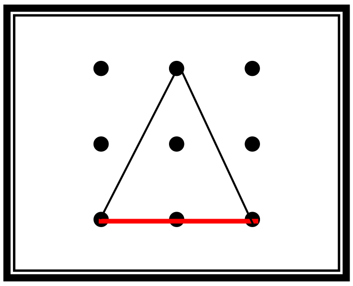
- What is the base and height for each triangle you found?
- What is the connection between that and the area of the triangle?
- Can you write that as a formula?
- How can we check that gives the right answer for the areas of different triangles?

Possible answers to question 1 (on previous page)

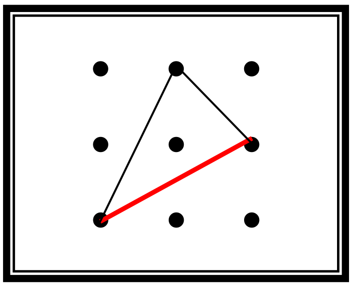
4 triangles based on this line



4 triangles based on this line



1 triangle based on this line

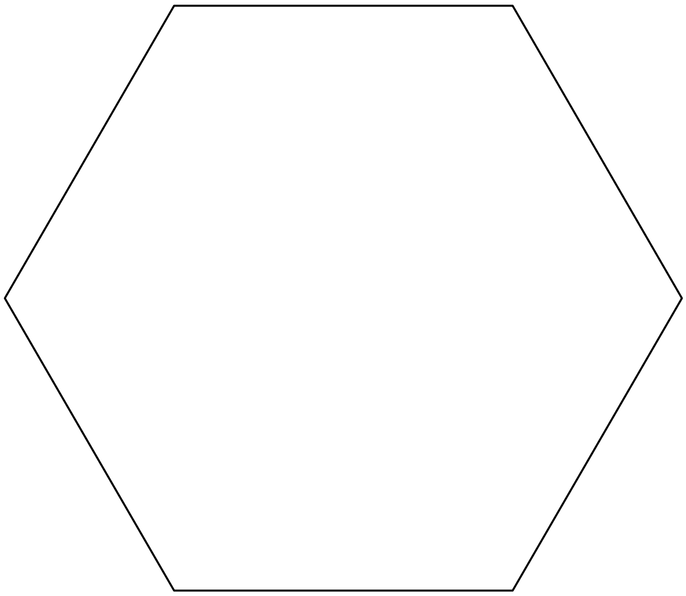


The size of the grid may be extended for additional challenge.

Homework

Find the area of this regular hexagon.

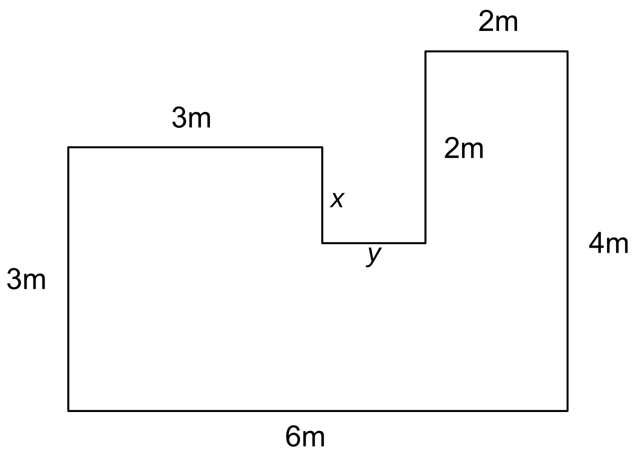
You can make any measurements that you like.



Bring in your results and be ready to explain your methods to the group.

A possible AO2 question (Working space has been deliberately reduced)

1 This is a plan of Annie's back garden.
She is going to make it into a lawn.



Not to scale

(a) **(i)** Work out the length marked *x.*

(a)(i)_________________________ m **[1]**

(ii) Work out the length marked *y.*

(ii) _________________________ m **[1]**

(iii) Annie will put a strip of edging all round her lawn.

How many metres of edging will Annie need?

(iii)_________________________ m **[1]**

Annie is going to sow grass seed to make the lawn.

(b) **(i)** Calculate the area of her lawn.

(b)(i)________________________ m^2 **[2]**

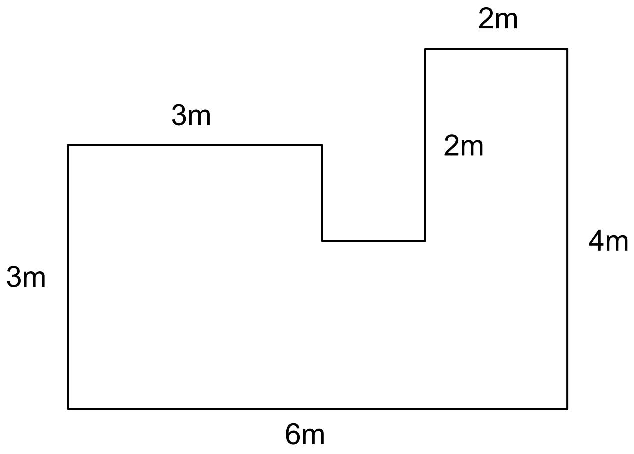
(ii) Each packet of grass seed is enough to cover $4m^2$.
Grass seed costs £3·45 a packet.

What will it cost Annie to buy the grass seed to make her lawn?

(ii) £________________________ **[3]**

A possible AO3 question

2 This is a plan of Annie's back garden.
She is going to make it into a lawn.



Not to scale

(a) Annie will put a strip of edging all round her lawn.
Edging costs £1·35 per metre.

Work out how many metres of edging Annie needs.

(a) ______________________m **[3]**

(b) Annie is going to sow grass seed to make the lawn.
Each packet of grass seed is enough to cover $4m^2$.
Grass seed costs £3·45 a packet.

What will it cost Annie to buy the grass seed to make her lawn?

(b) £__________________________ **[5]**

AO2 mark scheme

Q	Answer	Mark	Notes
1(a)(i)	1	**1**	
(ii)	1	**1**	
(iii)	22	**1FT**	FT their *x* and *y*
(b)(i)	19 www	**2FT**	**If 0** full FT from their *x* and *y* **B1** for 9 or 2 or 8 or 3 or 12 or 4 or 6 OR FT their *x* and *y*
(ii)	5 www 17·25	**2FT** **1FT**	**M1** their 19 ÷ 4 Their 5 x 3·45 correct

AO3 mark scheme

Q	Answer	Mark	Notes
2(a)	1 and 1 22	**2** **1FT**	**B1** for 1 or 2 seen clearly as missing lengths Correct sum of 20 + their lengths
(b)	19 (£)17·25 www	**2FT** **3FT**	**B1** for splitting into rectangles or surrounding and splitting or for two correct areas. FT their missing lengths from (a). **M1** for their 19 ÷ 4 **M1** for rounding UP

3.2 Case 2: Mean, Mode, Median and Range (Foundation/Higher)

FA13 General data handling	
13.3 - Processing	Candidates should be able to: a draw and produce pie charts for categorical data, and diagrams for continuous data, frequency diagrams (bar charts, frequency polygons and fixed interval histograms) and stem and leaf diagrams b calculate mean, range and median of small data sets with discrete then continuous data c identify the modal class for grouped data d find the median for large data sets and calculate an estimate of the mean for large data sets with grouped data

Starter

This starter is designed to lead into a lesson that explores the use of averages and ranges to compare distributions.

- Write down five numbers that have a mean of 5
- Write down five numbers with a mean of 5 and a range of 6
- Write down five numbers with a mean of 5 and a range of 1
- What do all the answers have in common?
- Write down five integers with a mean of 5 and a range of 1

Possible responses

5, 5, 5, 5, 5

2, 5, 5, 5, 8

4.5, 5, 5, 5, 5.5

They have a total of 25

Cannot be done

Possible questions

- How did you work out your answers?
- What information does the range give you?
- What can you say about two sets of numbers that have the same mean but different ranges?
- Why can you not answer the last question?
- Write some related questions like this and try them on others in the group.

Activity 1

This activity would be suitable for a more able Foundation group or a Higher group.

This table shows the world ATP Tennis rankings for Men's Singles events.

Rank	Name & Nationality	Points	Position Moved	Tournaments Played
1	Perrier, Gilles (SUI)	11,255	0	19
2	Fernandez, Miguel (ESP)	8,945	0	18
3	Smith, George (GBR)	8,390	0	19
4	Vasic, Novak (SRB)	7,740	0	22
5	Cano, Juan (ARG)	6,555	0	22
6	Williams, Andy (USA)	4,830	0	21
7	Dubois, Adrien (FRA)	4,100	0	25
8	Ivanov, Nikolay (RUS)	3,710	0	25
9	Pesquera, Fernando (ESP)	3,400	0	23
10	Simon, Claude (FRA)	3,285	0	27
11	Johansson, Robin (SWE)	3,125	0	26
12	Gonzalez, Raúl (CHI)	2,700	0	17
13	Horvat, Marin (CRO)	2,495	2	22
14	Blanc, Etienne (FRA)	2,285	-1	22
15	Robredo, Luis (ESP)	2,090	2	27
16	Novák, Radek (CZE)	2,055	0	23
17	Hernadez, Felipe (ARG)	1,865	-3	19
18	Wilhelm, Thomas (GER)	1,780	0	16
19	Alonso, Federico (ESP)	1,770	0	25
20	Marek, Tomas (CZE)	1,740	1	28
21	Martinez, Carlos (ESP)	1,675	-1	22
22	Federer, Stanislas (SUI)	1,650	1	21
23	Hart, Lewis (AUS)	1,555	3	19
24	Bach, Philipp (GER)	1,460	-2	27
25	Lee, Samuel (USA)	1,460	0	26
26	Brown, James (USA)	1,415	-2	21
27	Peters, Mike (AUS)	1,286	0	29
28	De La Torre, Juan (ARG)	1,225	3	25
29	Alvarez, Nicolas (ESP)	1,225	0	24
30	Novak, Ivo (CRO)	1,220	0	23
31	Lukovic, Viktor (SRB)	1,170	-3	28
32	Pavlov, Mikhail (RUS)	1,165	17	31
33	Duval, Julien (FRA)	1,154	2	27
34	Renard, Gérard (FRA)	1,132	-2	28
35	Lacroix, Henri (FRA)	1,100	-1	28
36	Meier, Andreas (GER)	1,071	4	27
37	Petrović, Ivan (CRO)	1,070	8	25
38	Becker, Bruno (GER)	1,064	0	26
39	Caballero, Luis (ESP)	1,045	-2	26
40	Petrov, Igor (RUS)	1,040	1	32
41	Lopez, Francisco (ESP)	1,005	-8	24
42	Green, John (USA)	987	1	19
43	Lang, Dieter (AUT)	985	-4	27
44	Cohen, Dudi (ISR)	978	0	27
45	Corona, Pablo (URU)	883	9	29
46	Janic, Janko (SRB)	880	14	27
47	Blanco, Miguel (ARG)	876	4	25
48	Preston, Mark (USA)	865	2	26
49	Collet, Jacques (FRA)	863	-7	25
50	Rossi, Adolfo (ITA)	850	5	30
51	Garcia, Manuel (ESP)	849	1	25
52	Gutierrez, Guillermo (ARG)	835	-5	29
53	Fernandes, Cristiano (BRA)	833	3	25
54	Volkov, Evgeny (RUS)	831	3	27
55	Caballero, Miguel (ARG)	821	3	30
56	Jung, Daniel (AUT)	820	5	29
57	Aubry, Jean (FRA)	810	-4	21
58	Mancini, Simone (ITA)	806	6	27
59	Bogdanov, Dmitry (RUS)	785	6	28
60	Gonzalez, Daniel (ARG)	769	8	25
61	Hoffmann, Simon (GER)	767	10	29
62	Manzano, Carlos (ESP)	759	7	24
63	Horák, Jan (CZE)	747	-1	24
64	Wells, Peter (AUS)	742	2	29
65	Roberts, Pater (USA)	730	2	23
66	Lombardi, Fabio (ITA)	720	8	28
67	Klein, Mischa (GER)	720	-19	27
68	Camacho, Gabriel (ESP)	715	7	34
69	Campos, Daniel (ARG)	710	-6	27
70	Wong, Yen-Hsun (TPE)	706	6	29
71	Leblanc, Marc (FRA)	705	6	28
72	Remy, Florent (FRA)	691	6	33
73	Svoboda, Ivo (CZE)	690	6	25
74	Pekár, Karol (SVK)	685	7	24
75	Russo, Francesco (ITA)	682	5	30
76	Perez, Oscar (ESP)	663	-6	34
77	Pereira, Frederico (POR)	659	6	29
78	Schmitz, Adolf (GER)	637	-42	29
79	Keller, Georg (GER)	636	6	21
80	Danielis, Janis (LAT)	630	18	26
81	Masulka, Olivier (BEL)	623	3	23
82	Babić, Mario (CRO)	617	-9	18
83	Gilardi, Marco (SUI)	616	17	23
84	Silva, Marcos (BRA)	615	2	24
85	Marshall, Richard (USA)	609	-3	25
86	Smirnov, Marat (RUS)	600	-27	22
87	Constantinou, Marcos (CYP)	595	3	22
88	Bykau, Christophe (BEL)	588	5	33
89	Maslov, Steve (BEL)	586	5	28
90	De Luca, Aurelio (ITA)	577	15	31
91	Enriquez, Jaime (ESP)	576	21	33
92	Gorbunov, Kristof (BEL)	575	4	21
93	Schmid, Markus (GER)	574	4	31
94	Phillips, Wayne (USA)	571	-7	26
95	Martin, Kevin (USA)	555	6	31
96	Russell, Liam (USA)	555	-5	25
97	Fuentes, Ignacio (ARG)	554	7	23
98	Ballesteros, Oscar (CHI)	551	-3	26
99	Wolf, Philipp (GER)	545	0	31
100	White, John (USA)	544	3	24

Use the information to comment on these statements.

1) “The bottom 20 players change places in the rankings much more than the top 20 do.”

2) “The average number of points a top ten player scores in a tournament is ten times what a bottom ten player scores.”

What could you find out from these statistics?
Make up a question or statement of your own and test it.
(You could even find some data of your own).

Possible answers

1) Total size of top 20 moves = 9 places (ignoring + and –)

Mean move = 9 ÷ 20 = 0·45 places

Mode move = 0 (In this case the mode is probably the most representative)

Median move = 0

Range = 3 – 0 = 3 so the players are very consistent

Total of bottom 20 moves = 149 places

Mean move = 149 ÷ 20 = 7·45 places

Places moved	Tally	Frequency
0	|	1
2	|	1
3	~~||||~~	5
4	||	2
5	|||	3
6	|	1
7	||	2
9	|	1
15	|	1
17	|	1
21	|	1
27	|	1

Mode move = 3 places

Median move = 5 places

Range = 27 – 0 = 27 (The data is biased by inclusion of four “big movers”. A fairer result might be obtained if the final four were left out.)

On any measure the bottom 20 players change places much more than the top 20 players.

2) The number of tournaments played by the players is not the same so it is advisable to work out the mean number of points scored per tournament first.

Alternatively this could be treated as a weighted mean.

Points	Played	Mean
11,255	19	592·4
8,945	18	496·9
8,390	19	441·6
7,740	22	351·8
6,555	22	298·0
4,830	21	230·0
4,100	25	164·0
3,710	25	148·4
3,400	23	147·8
3,285	27	121·7
	Total	2992·6
	Mean	299·3

Points	Played	Mean
576	33	17·5
575	21	27·4
574	31	18·5
571	26	22·0
555	31	17·9
555	25	22·2
554	23	24·1
551	26	21·2
545	31	17·6
544	24	22·7
	Total	210·9
	Mean	21·.1

Ratio 14·2

The ratio of points is closer to 14 times than ten times.

Activity 2

This is designed as a whole class activity based around the simple “Drop the ruler to measure reaction time” task.

It is essential that discussions take place before the activity to understand why it is being done. (For our purposes to generate a large amount of data which can be pooled to compare male and female, right and left, using different averages and the purpose of the range as a measure of dispersion.)

By setting that data in a context that is deliberately competitive learners often take ownership of the data and engage strongly with the process and outcomes.

Learners work in pairs and it is also useful to have two classes working on the activity so that different groups may be compared.

Discuss how to show the data and, when this has been done, how it will be processed.

There is also a suggestion for use linking with ICT.

Who has the fastest reactions?

Measure your reaction times by catching a ruler.

1. Work in pairs.
2. One person holds the ruler with the ZERO line level with the other’s fingers.
3. Drop the ruler WITHOUT WARNING.
4. Other person catches the ruler.
5. Measure how far the ruler has dropped, in cm, to the nearest mm.
6. Do this TEN times and then swap.
7. Repeat this until both people have dropped the ruler 20 times.

Name	
Drop	Distance in cm
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
Total	cm

Name	
Drop	Distance in cm
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
Total	cm

Data may be pooled and males/females/groups compared between classes.

It is essential that results are discussed and appropriate statements made, using the processed data. This is an ideal vehicle for display work where the evidence may be weighed and discussed.

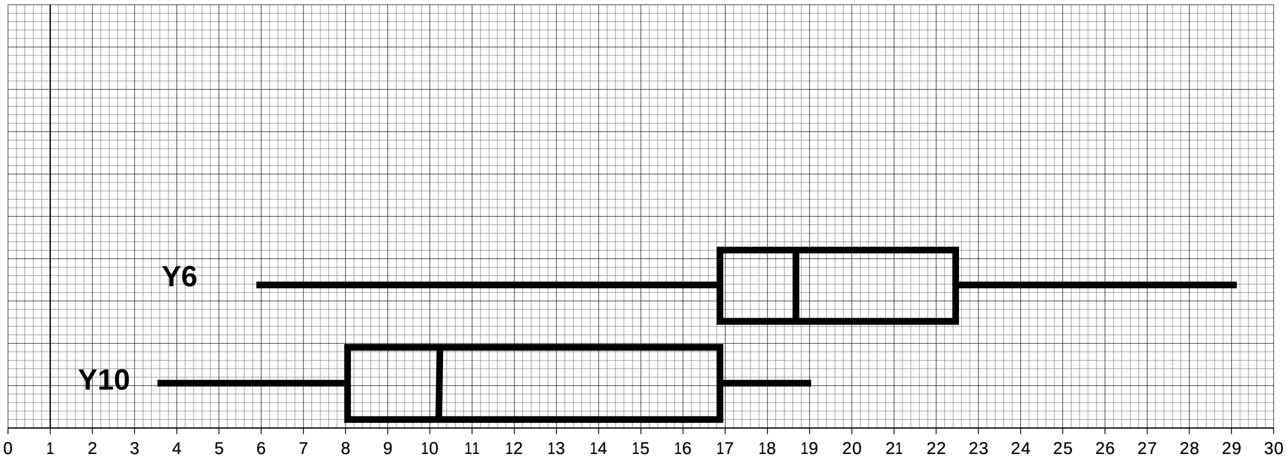
Extension activity

HA14 General data handling	
14.3 - Processing	Candidates should be able to: e **draw and produce cumulative frequency tables and diagrams, box plots and histograms for grouped continuous data** f **find the quartiles and interquartile range for large data sets**

The activity may be extended to Higher tier by analysis using box and whisker plots.

A contrived comparison might be:

I asked two groups to measure their reaction times by catching a ruler.
These are the results for both groups.



Reaction measured in cm

Plot your own reaction times on the graph. How do you compare?

Why could your box and whisker plot be misleading?

A better comparison would be between different groups undertaking the activity at the same time and plotting their results.

Discussion of statements made, drawing on the evidence of the data and graphs is essential to correct misconceptions.

This is also a useful vehicle for grouping data, calculating estimates of means and comparing with means from raw data at both Foundation tier and Higher tier.

Reaction Times and ICT

AIM: “Which person has the fastest reactions?”

Processing the data

Log on and open a spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2												
3												
4												
5												
6												
7												
8												
9												

Put your name and the drop numbers in column A and enter your data into column B.

	A	B	C	D	E	F
1	Name	Brian Jones				
2						
3	Drop	Distance in cm				
4	1	20				
5	2	12.6				
6	3	15				
7	4	15.6				
8	5	23.1				
9	6	2.8				
10	7	29				
11	8	13.5				
12	9	9.3				

You can get the spreadsheet to work out the **mean drop** by putting the formula =AVERAGE(B4:B23) into a cell at the bottom of your list. [Remember to start with =]

	A	B	C	D	E	F	G	H	I
23									
24									
25		Mean is	=AVERAGE(B4:B23)						
26									
27									
28									
29									

The **first cell** reference in the formula is where your data **starts** and the *second* is where it *finishes*.

Now draw a graph of your data

Remember to sort your data first.

Drop	Distance in cm
1	20
2	12.6
3	15
4	15.6
5	23.1
6	2.8
7	29
8	13.5
9	9.3

Once the data is sorted, your spreadsheet will look like this.

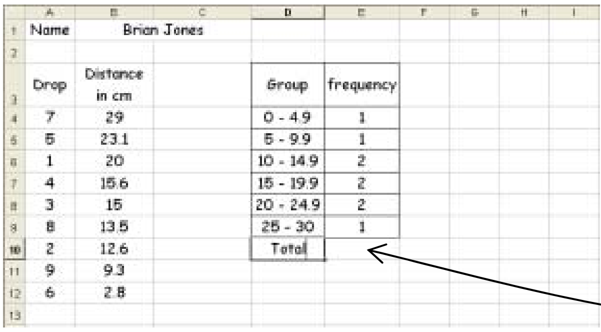
Name		Brian Jones
Drop	Distance in cm	
7	29	
5	23.1	
1	20	
4	15.6	
3	15	
8	13.5	
2	12.6	
9	9.3	
6	2.8	

Now, create a table so that you can group the drops into about **six equal sized groups**.

These could be: 0 - 4·9; 5 - 9·9; ………; 25 - 30 (I know the last one is a bit bigger!!)

Name	Brian Jones		
Drop	Distance in cm	Group	frequency
7	29	0 - 4.9	
5	23.1	5 - 9.9	
1	20	10 - 14.9	
4	15.6	15 - 19.9	
3	15	20 - 24.9	
8	13.5	25 - 30	
2	12.6		
9	9.3		
6	2.8		

Count how many drops are in each group and enter this in the frequency column.

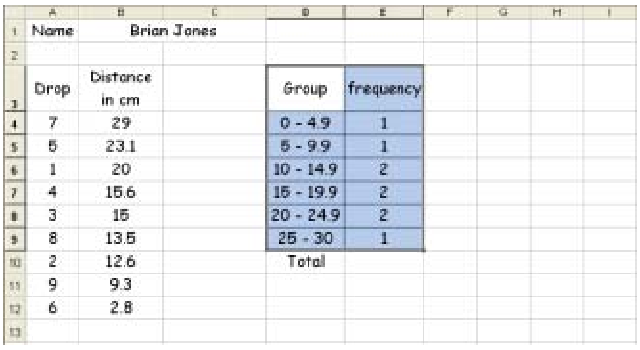


	A	B	C	D	E
1	Name	Brian Jones			
2					
3	Drop	Distance in cm		Group	frequency
4	7	29		0 - 4.9	1
5	5	23.1		5 - 9.9	1
6	1	20		10 - 14.9	2
7	4	15.6		15 - 19.9	2
8	3	15		20 - 24.9	2
9	8	13.5		25 - 30	1
10	2	12.6		Total	
11	9	9.3			
12	6	2.8			
13					

You can complete the total by entering the formula =SUM(E4:E9) in the cell under the frequency column of your table.

Remember the first cell is where your data starts and the second is where it ends.

Highlight the table but not the total row.



	A	B	C	D	E
1	Name	Brian Jones			
2					
3	Drop	Distance in cm		Group	frequency
4	7	29		0 - 4.9	1
5	5	23.1		5 - 9.9	1
6	1	20		10 - 14.9	2
7	4	15.6		15 - 19.9	2
8	3	15		20 - 24.9	2
9	8	13.5		25 - 30	1
10	2	12.6		Total	
11	9	9.3			
12	6	2.8			
13					

Now select the graph type you want and draw the graph.

Print out your results and save.

1. Use a word processing program and write up the experiment that you did.
   (Aim: what you were trying to find out; method: how you did it, results: the spreadsheet; findings)

2. Describe what your results say about your reaction times.

3. Compare your graph with some others near to you.

4. Get a copy of one other person’s results graph.

5. How did you do your reaction times compare with the other person?

Homework

Women's: WTA World Rankings			
	Player		Pts
1	S Lang	Ger	9810
2	N Smith	US	8558
3	V Green	US	6865
4	M Vazquez	Esp	6620
5	L Popova	Rus	6235
6	S Vinogradova	Rus	5960
7	V Ivanova	Rus	5300
8	N Elmer	Swi	4810
9	V Mendes	Bra	4553
10	F Rossi	Ita	3420
11	A Camacho	Chi	3270
12	R Bognanova	Rus	3120
13	F Rodriguez	Arg	3040
14	M Remy	Fr	3025
15	R Cibulkova	Svk	2527
16	A Blanc	Fr	2511
17	S Vickers	Aus	2508
18	V Dubois	Fr	2193
19	N Chang	Chn	2132
20	P Klein	Ger	2127
Leading British Players:			
56	A Roberts		1075
103	K Peters		591
104	E Chambers		587
146	M Short		412
186	G Brown		300

Compare the points gained by the leading British Women tennis players with those of the WTA 20 leading players.

Some possible answers

Mean British $\frac{2965}{5}$ = 593 points each Range = 1075 – 300 = 775 points

Mean WTA best $\frac{88584}{20}$ = 4 429·2 points each Range = 9810 – 2127 = 7 683 points

WTA mean nearly 5·5 times greater than British.

Best British player distorts mean which would be 472·5 without her.

A possible AO2 question (Foundation)

1 These are the weekly wages, in pounds, paid to 11 workers.

275 160 842 275 420 359 315 275 740 280 195

(a) Work out the mean wage.

(a) £_______________________ **[3]**

(b) James says the average wage is £280.

Which average has James worked out?

(b) _________________________ **[1]**

A possible AO3 question (Foundation)

2 These are the weekly wages, in pounds, paid to 11 workers.

275 160 842 275 420 359 315 275 740 280 195

James says the average wage is £280.
Jane says the average wage is £376.

Show how they can both be correct.

[5]

AO2 mark scheme

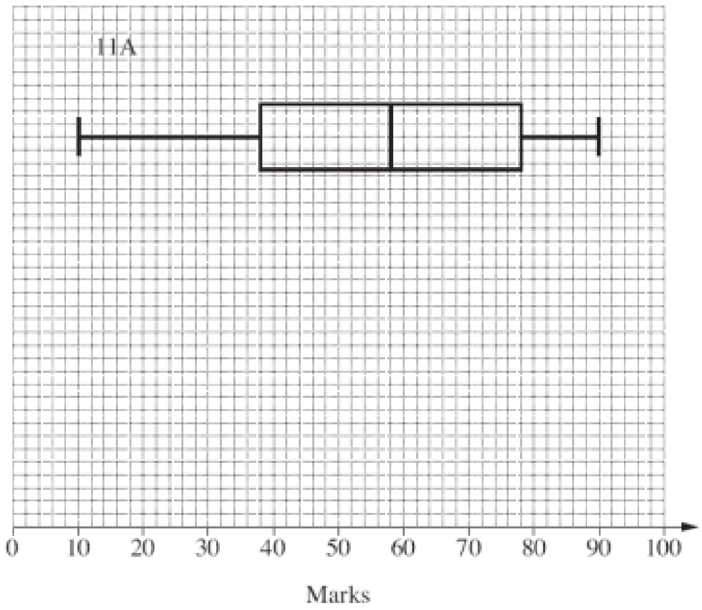
Q	Answer	Mark	Notes
1(a)	(£) 376	**3**	**M1** for attempt to add values implied by 4136 **M1 dep** for their 4136 ÷ 11
(b)	Median	**1**	

AO3 mark scheme

Q	Answer	Mark	Notes
2	Mean and median calculated	**5**	**M1** for attempt to add values implied by 4136 **M1 dep** for their 4136 ÷ 11 **A1** for 376 seen AND **M2** for all values listed in order and median indicated or stated or **M1** for at least 10 values **listed in order**

A possible AO1 and AO2 question (Higher)

3 This box plot summarises the distribution of marks scored in a mathematics examination by class 11A.



Class 11B took the same examination.

(a) Here is some information about the marks for class 11B.

Lowest score	12	Highest score	98
Median	52		
Lower quartile	34	Interquartile range	30

On the grid above draw the box plot for class 11B. **[2]**

(b) Make one comparison of the marks for class 11A and class 11B.

____

____

____ **[1]**

A possible AO3 question (Higher)

4 Two Year 11 classes both took the same test.

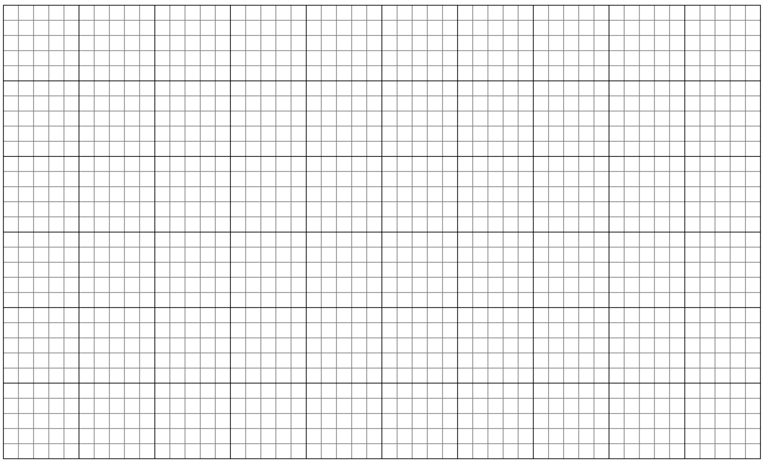
These tables summarise results for the two classes.

11A	
Lowest mark	10
Highest mark	90
Median	58
Interquartile range	40
Lower quartile	38

11B	
Lowest mark	12
Highest mark	98
Median	52
Interquartile range	30
Lower quartile	34

The class with the better marks will receive a prize.

Which class should be given the prize?



[6]

AO1 and AO2 mark scheme

Q	Answer	Mark	Notes
3(a)	Box plot whisker 12 to 98 Box from 34 to 64 Median at 52	**1** **1** **1**	
(b)	Any correct comparison that interprets median or IQR	**1**	eg 11A are better oe (on average) 11B are more consistent oe (IQR smaller) 11B's average was lower than 11A's On average 11A had a higher median score

AO3 mark scheme

Q	Answer	Mark	Notes
4	Construct two box plots <u>11A</u> Box plot whisker 10 to 90 Box from 38 to 78 Median at 58 <u>11B</u> Box plot whisker 12 to 98 Box from 34 to 64 Median at 52	**4**	**3** one correct box plot **2** for 4 out of these 6 elements correct **1** for 2 out of these 6 correct
	One correct comparison that interprets median and IQR	**1**	Eg 11A are better oe (on average) 11B are more consistent oe (IQR smaller) 11B's average was lower than 11A's On average 11A had a higher median score
	One correct comparison that refers to another aspect of the data	**1**	Eg At least one person in 11B scored more than the highest mark in 11A

3.3 Case 3: Pythagoras' Theorem (Foundation/Higher)

FA12	**Pythagoras' theorem in 2D**
12.1 - Use Pythagoras' theorem	Candidates should be able to: a understand, recall and use Pythagoras' theorem to solve simple cases in 2D

Starter 1

I'm sure you can't add the areas of two squares together and get the area of a third square.
Not if they all have to be whole numbers.

Do you agree?

Show how you decide.

Possible responses

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, ……

9 + 25 = 36

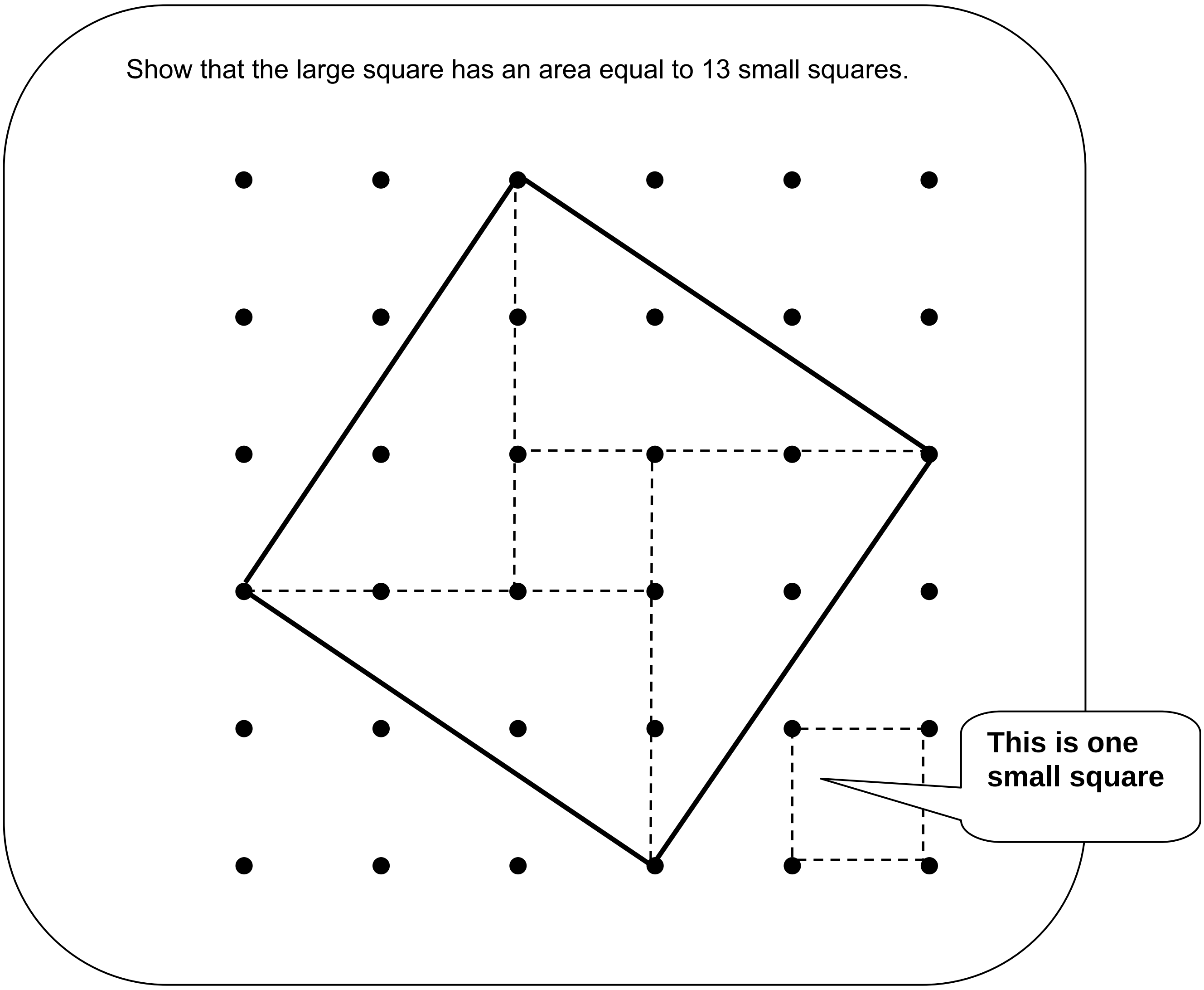
36 + 64 = 100

25 + 144 = 169

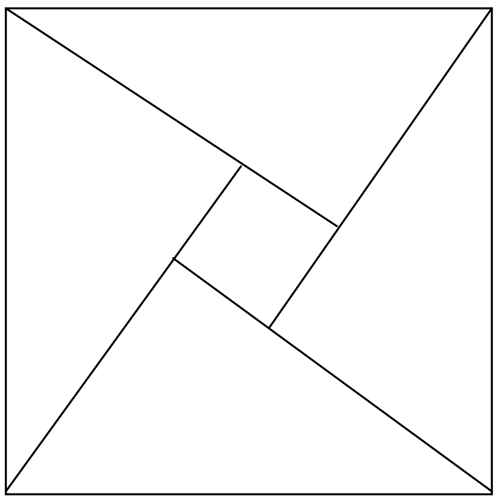
I don't agree, there are lots of examples

Starter 2 leading to Activity 1

The starter is designed to allow create confidence in finding areas by dissection.
The "13 units^2" case must be discussed before moving to the following activity.



Expected response



Each triangle has area 3 × 2 ÷ 2 = 3 units^2

Four triangles 3 × 4 = 12 units^2

Central square = 1 unit^2

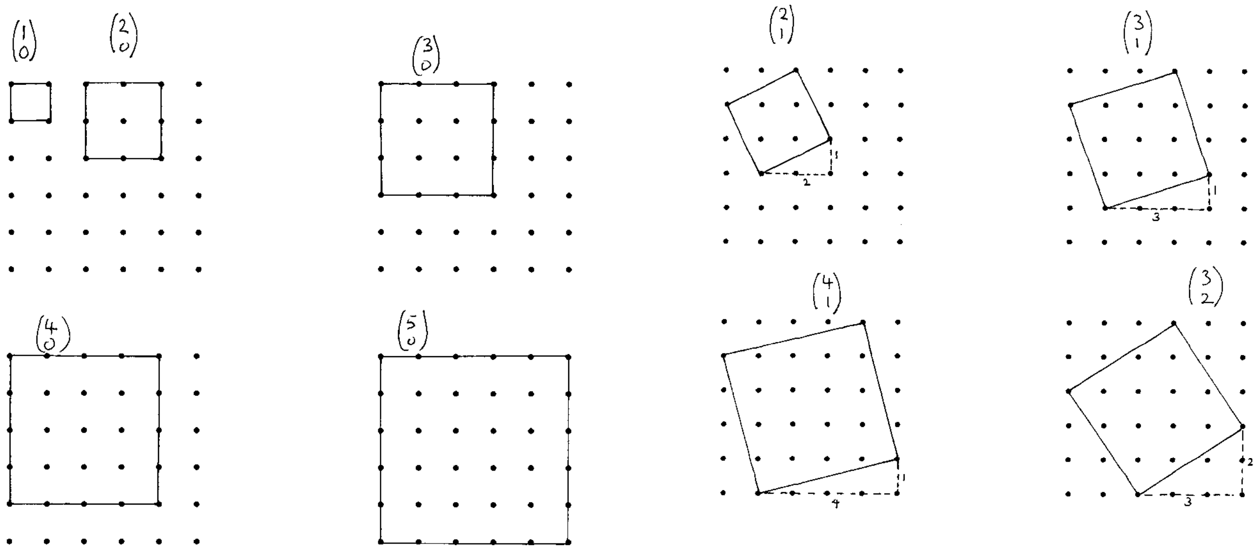
Total area = 13 units^2

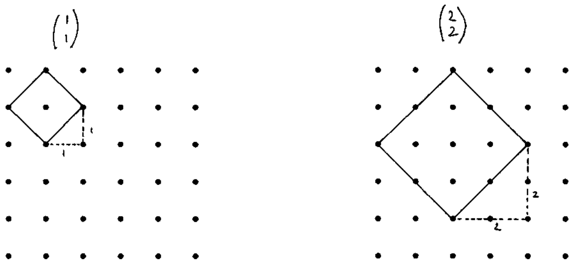
Activity 1

Draw some other squares on dotty paper by joining four dots.

Find their areas.

Possible responses





Each square can be defined by a horizontal and vertical move (a vector, if you wish).

The vectors are written above the squares and shown in some cases with dashed lines.

Some possible questions

- How can you describe what dots to join so that each new side of a square can be drawn?
- What is the link between the moves you make and a right-angled triangle?
- How can you record the results so that you can see any patterns?
- What is the link between the two sides of the triangle and the area of the square?
- Could you write this as a formula?
- How can we work out how long the sides of the squares are?
- Measure the side of the square with an area of 13 units^2. How long is it? Does this agree with the length you calculated?

The activity discovers Pythagoras but can also be used to consider using square roots to find lengths of sides of squares and the inverse.

Learners often find it difficult to reconcile their measured lengths with the calculated ones. This can be a fruitful discussion.

Activity 2

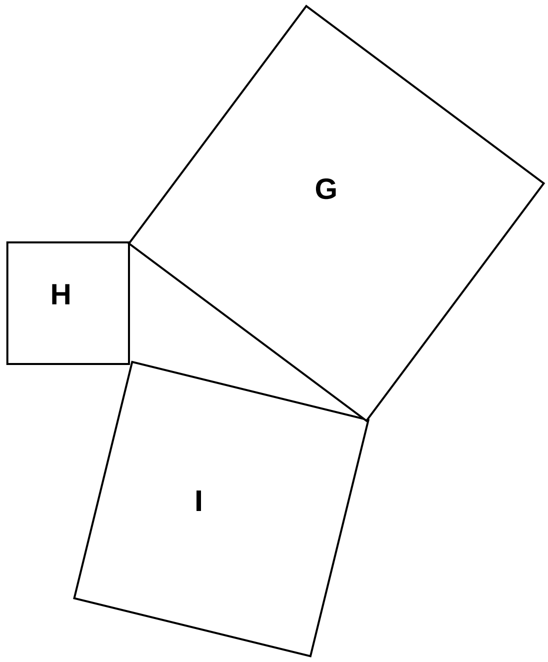
This is designed to be undertaken by pairs or small groups.

Draw squares with sides from 1cm to 20cm.

On each square write the area.

Arrange sets of three squares that join corner to corner and draw the triangles that result.

Note the sizes of the squares used.



Group the triangles according to their properties.

What connections can you find between the areas of the squares used?

Acute-angled triangles: **H** + **I** > **G**

Right-angled triangles: **H** + **I** = **G**

Obtuse-angled triangles: **H** + **I** < **G**

Right-angled triangles are formed between squares with these areas (and sides):

1	9 + 16 = 25	(3, 4, 5)
2	36 + 64 = 100	(6, 8, 10)
3	25 + 144 = 169	(5, 12, 13)
4	81 + 144 = 225	(9, 12, 15)
5	64 + 225 = 289	(8, 15, 17)
6	144 + 256 = 400	(12, 16, 20)

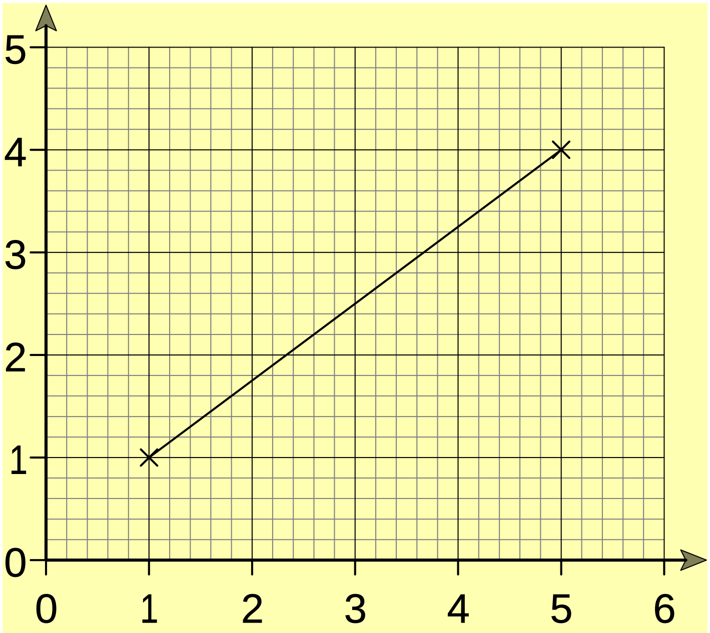
Learners should be encouraged to note the link between triangles 1, 2, 4 and 6.

This may be done through questioning.

- Have you found any other links between these sets of numbers?
- If you found the “3, 4, 5 triangle” was it possible to immediately find another that used different squares?

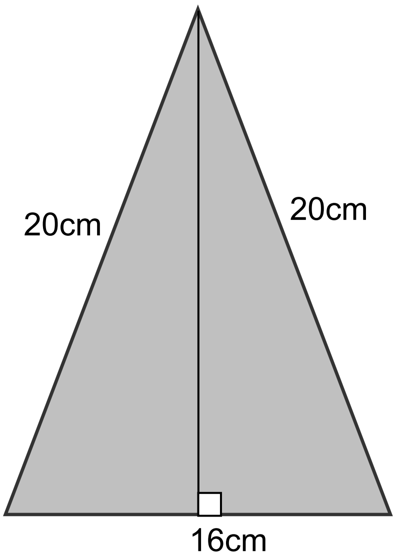
Homework

1 **Calculate** the length of this line drawn on a grid.
The division from (0, 0) to (1, 0) is 1·6cm long.



Not to scale

2 Calculate the area of this triangle.
Give your answer to a sensible degree of accuracy.



Not to scale

Answers

1 5 × 1·6 = 8cm

2 Height = $\sqrt{20^2 - 8^2}$

= 18·3303…cm

Area = 8 × 18·3303… (Premature rounding at this stage would give a final answer of 146cm^2 so should be avoided.)

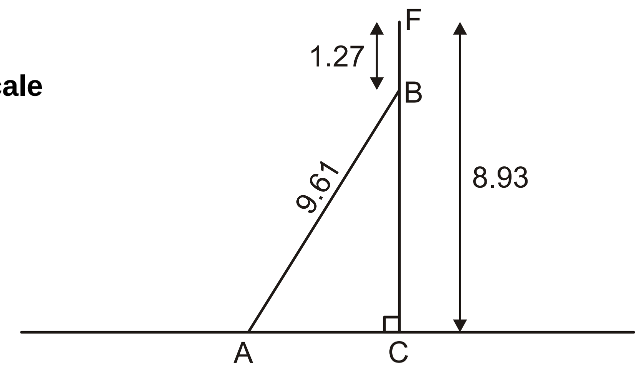
Area = 146·64

Area = 147cm^2

A possible AO2 question

1 The diagram shows a vertical flag pole FC standing on horizontal ground.
The pole is 8·93 m tall and is held by cables.
Cable AB is 9·61 m long and B is 1·27 m from the top of the pole.

Not to scale

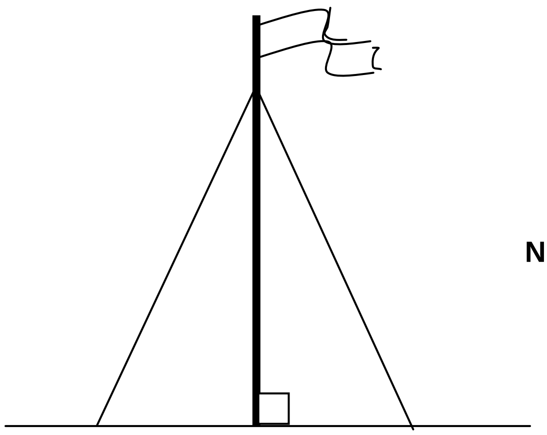


Calculate the distance AC.
Give your answer to a sensible degree of accuracy.

_____________________ m **[5]**

A possible AO3 question

2 The diagram shows a vertical flagpole standing on horizontal ground.
The pole is 8·93 m tall and is secured to the ground by four equal length cables.
Each cable is fastened to the pole 1·27 m from the top and 5·8 m away from the base of the pole.
Each fastening uses 45 cm of cable.



Not to scale

Can the flagpole be secured to the ground with four 10m cables? **[6]**

AO2 mark scheme

Q	Answer	Mark	Notes
1	(BC =) 7·66 $9{\cdot}61^2 = 7{\cdot}66^2 + AC^2$ AC = $\sqrt{9 \cdot 61^2 - 7 \cdot 66^2}$ 5·8031…. 5·8 or 5·80	**1** **1** **1** **1** **1**	

AO3 mark scheme

Q	Answer	Mark	Notes
2	7·66 $\text{length}^2 = 7{\cdot}66^2 + 5{\cdot}8^2$ length = ($\sqrt{7 \cdot 66^2 + 5 \cdot 8^2}$ =) 9·608… 9·608... + 2 × 0·45 10·508 No because total exceeds 10m **OR** Choice of scale Scale flagpole and horizontal Scale length of one cable Actual length of cable Length + 2 × 0·45 = 0·9 No because 9·61 + 0·9 exceeds 10m	**1** **1FT** **1 FT** **1 FT** **1 FT** **1 FT** **OR** **1** **1** **1** **1** **1** **1**	From top 10.508 implies previous mark Must be "correct" statement for their length AND both fastenings Scale 1cm to between 0·5 m and 1m inclusive Accurate to ± 2 mm using their scale Must be "correct" statement for their length AND both fastenings

3.4 Case 4: Hierarchy of Operations (Foundation)

FA3 Hierarchy of operations	
3.1 - Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations	Candidates should be able to: a use brackets and the hierarchy of operations

Starter 1

Look at these calculations.
They are all correct.

a. $12 + 3 - 5 = 10$ ✓

b. $12 + (3 - 5) = 10$ ✓

c. $15 - 3 \times 5 = 0$ ✓

d. $(15 - 3) \times 5 = 60$ ✓

e. $10 \div 2 + 3 = 8$ ✓

f. $10 \div (2 + 3) = 2$ ✓

When does using bracket make a difference?

Make some calculations of your own to show brackets making and not making a difference.

B
O
D
M
A
S

Some possible questions

1 What do the letters of BODMAS stand for?

2 Why do we use this rule?

3 Why did brackets not make a difference to the answer to questions a and b?

4 Why did the brackets change the answers to questions c and e?

5 What questions have you found where brackets made a difference?

6 What questions have you found where brackets did not make a difference?

7 How did you decide what calculations to use? (What was your strategy?)

8 Why do we use brackets in calculations?

9 How do you know your answers are correct? (Did you check your answers? How did you/can you check your answers?)

10 Did you use a calculator?

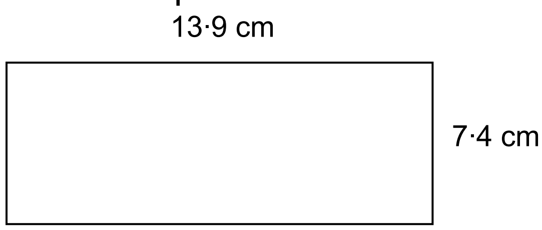
11 Was it always helpful? Why?

Plenary 1

(The second part of this would be an ideal homework to allow research. Discussion could take place after the short activity as to the sort of operations that could be considered, mathematical formulae, working out costs, people's pay over a week if they work at an hourly rate …..)

The perimeter of a rectangle can be worked out by adding the length of the longer side to the length of the shorter side and doubling the answer.

Work out the perimeter of this rectangle.



Show how this calculation can be written as a **single** calculation with one = sign.

Work with a partner to write down some other examples of calculations done in stages and write them as a single calculation.
Don't forget to check!!

B
O
D
M
A
S

Some possible questions

1 How did you check that your answer for the perimeter was right?

2 Where did you put brackets in your calculation?

3 Why did you need brackets?

4 Is there a connection between brackets and the memory key on your calculator key pad?

5 What other examples did you find?

6 How did you find them?

7 How did you check the answers were right?

8 Could you have checked in any other ways?

Activity (The old ones are the best)

Four Fours

The challenge

- ✓ Write calculations that give the answers 0 to 20.
- ✓ In each calculation you must use **four** number 4s. (No more, no fewer)
- ✓ You can use any operation signs you like. (+, –, ×, ÷, √, ^{2}, ^{3}, ! and Σ)
- ✓ It is possible to find more than one sum for some numbers, but you only need to find ONE for EACH.

These may help
4! = 4 × 3 × 2 × 1 = 24
Σ4 = 4 + 3 + 2 + 1 = 10

You are welcome to have a go at numbers over 20.

Answer	Calculation (Remember BODMAS)
0	
1	
2	
3	(4 + 4 + 4) ÷ 4 [= 12 ÷ 4 = 3]
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	4 × 4 + 4 – 4 [= 16 + 0 = 16]
17	
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	44 – 4 × 4 [= 44 – 16 = 28]
29	
30	

It is always more fun to be creative about solutions rather than just work through an exercise, and this generates useful display work too! Try a variation using the date or digits suggested by the group.

A solution

Answer	Calculation (Remember BODMAS)
0	4 + 4 – 4 – 4
1	4 × 4 ÷ (4 × 4)
2	4 ÷ 4 + 4 ÷ 4
3	(4 + 4 + 4) ÷ 4 [= 12 ÷ 4 = 3]
4	√(4 + 4 + 4 + 4)
5	√4 + √4 + (4 ÷ 4)
6	4 × 4 ÷ 4 + √4
7	4 + 4 – (4 ÷ 4)
8	4 × √4 + 4 – 4
9	4 + 4 + (4 ÷ 4)
10	4 × √4 + 4 – √4
11	4^2 – 4 – 4 ÷ 4
12	4 × 4 – (√4 + √4)
13	4^2 – 4 + 4 ÷ 4
14	4^2 – (4 + 4) ÷ 4
15	4 × 4 – (4 ÷ 4)
16	4 × 4 + 4 – 4 [= 16 + 0 = 16]
17	4 × 4 + (4 ÷ 4)
18	4^2 + (4 + 4) ÷ 4
19	4! – 4 + 4 ÷ 4
20	4! – 4 + 4 – 4
21	4^2 + 4 + 4 ÷ 4
22	4! – 4 + 4 ÷ √4
23	4! + 4 ÷ 4 – √4
24	4^2 + 4 × (4 ÷ √4)
25	4! + √4 – 4 ÷ 4
26	4! + √4 + √4 - √4
27	$((4 + 4 + 4) \div 4)^3$ *This is a bit of a "fudge" but is easier than some*
28	44 – 4 × 4 [= 44 – 16 = 28]
29	4! + 4 + 4 ÷ 4
30	4! + Σ4 – 4^2 ÷ 4

Some possible questions

1 What strategies did you use to find further answers from one you had already worked out?

2 Where in the table did solutions seem to "pair up"?

3 How did knowing √4 = 2 help you?

4 How did knowing 4! = 24 help you?

5 How did you make use of your calculator?

6 Would you be able to use just 1, 1, 1, 1 to calculate numbers from 1 to 20? Why?

Homework

Work out the answer to this calculation.

$$12 + 4 \times 6 \div 2 - 1$$

What other answers are possible if brackets are placed in different positions in the calculation?
(You cannot change the numbers used, the symbols or their positions.)

Do brackets always make a difference to the answers?

How many different answers could you get?
Show stages in your working.
What methods did you use to find different answers?
What other questions did you ask yourself that helped you find more solutions?

Create your own calculation and show how the answer can be changed through the use of brackets.
Bring your work in to the next lesson and be prepared to present some of it to the group.
(Remember to check for errors!)

Some possible answers

$12 + 4 \times 6 \div 2 - 1 = 23$

$(12 + 4) \times 6 \div 2 - 1 = 47$

$(12 + 4 \times 6) \div 2 - 1 = 17$

$12 + 4 \times (6 \div 2 - 1) = 20$

$(12 + 4) \times (6 \div 2 - 1) = 32$

$12 + 4 \times 6 \div (2 - 1) = 36$ *$(12 + 4 \times 6) \div (2 - 1) = 36$*

$((12 + 4) \times 6) \div (2 - 1) = 96$

A possible AO1 question

1 Work out.

(a)

(i) (6 + 9) ÷ 3

(a)(i) ________________ **[1]**

(ii) 8 × 2 + 3 × 5

(ii) ___________________ **[2]**

(iii) 7 + 4 × 4

(iii) ____________________ **[1]**

(b) Put one pair of brackets into each calculation to make the answer correct.

(i) 10 + 5 × 4 – 2 = 58

(b)(i) 10 + 5 × 4 – 2 = 58 **[1]**

(ii) 10 + 5 × 4 – 2 = 20

(ii) 10 + 5 × 4 – 2 = 20 **[1]**

A possible AO3 question

2 Brian writes a calculation using the digits 1, 2, 3, 4 and 5.
He can only use each number once in each calculation.
He is allowed to use all four operations and brackets.

What is the largest possible answer to Brian's calculation?
Support your answer with working. **[3]**

AO1 mark scheme

Q	Answer	Mark	Notes
1(a)(i)	5	**1**	
(ii)	31	**2**	**B1** for 15 or 16 seen
(iii)	23	**1**	
(b)(i)	(10 + 5) × 4 – 2	**1**	
(ii)	10 + 5 × (4 – 2)	**1**	

AO3 mark scheme

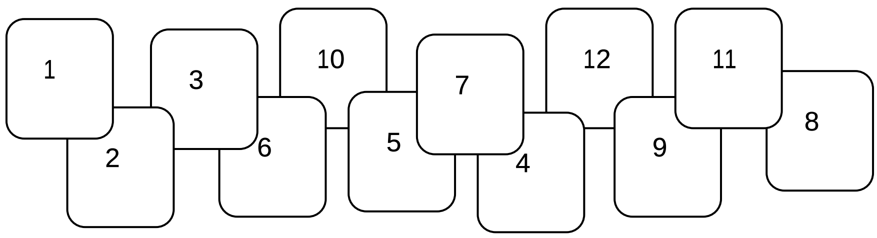
Q	Answer	Mark	Notes
2	180 www	**3**	**M1** for correct calculation with all 5 numbers and no subtractions or divisions And **M1** for second calculation with value greater than 120

3.5 Case 5: Sequences (Foundation/ Higher)

FA7/HA7 Sequences and formulae	
7.2 - Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence	Candidates should be able to: a generate terms of a sequence using term-to-term and position-to-term definitions of the sequence b generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers
7.3 - Use linear expressions to describe the *n*th term of an arithmetic sequence	Candidates should be able to: a use linear expressions to describe the *n*th term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated

Starter 1

Here are 12 cards numbered from 1 to 12.



Make as many sequences of numbers as you can.

- You must use each card once only in a sequence
- A sequence must have at least four terms

There are many possibilities such as:

1	1, 2, 3, 4, 5,……	Add 1	$t_n = n$	counting numbers
2	2, 4, 6, 8, 10, 12	Add 2	$t_n = 2n$	even numbers
3	1, 3, 5, 7,…	Add 2	$t_n = 2n - 1$	odd numbers
4	1, 2, 3, 5, 8	Add previous two terms		Fibonacci numbers
5	1, 3, 4, 7, 11	Add previous two terms		Fibonacci numbers
6	12, 10, …..	Subtract 2	$t_n = 14 - 2n$	
7	3, 6, 9, 12	Add 3	$t_n = 3n$	multiples of 3

Possible questions

- What is a sequence?
- What is a term?
- What is a rule?
- How can a sequence be described?
- Why do the numbers generated in a National Lottery draw not form a sequence?
- Why does the formula for the *n*th term give more information than the term-to-term rule?
- Give me an example of when knowing a sequence might be useful.
- Tell me …… sequences you know and their rules. (Term-to-term or *n*th, depending on level.)

Starter 2

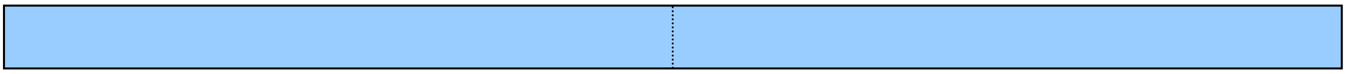
Each learner, and the teacher, has a strip cut from the greatest length of an A3 sheet.

They also have a table (see next page) and a copy on the whiteboard.

"This sheet is not folded. How many regions do you see?" [To establish meaning of the term "regions".]

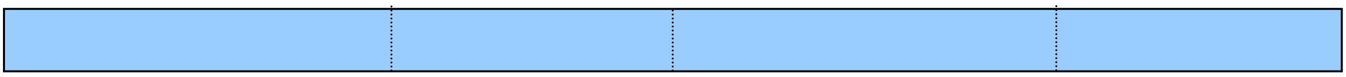
Fill in 0 and 1 on the table.

"Fold the strip across its length and open out." [Demonstrate]
"How many folds and how many regions do you see?"



Fill in 1 and 2 on the table

"Fold the paper again." [Demonstrate] "If we fold the folded paper again how many folds and how many regions do you think we will see when we open it out?" "Why do you think that?" [To predict rules]



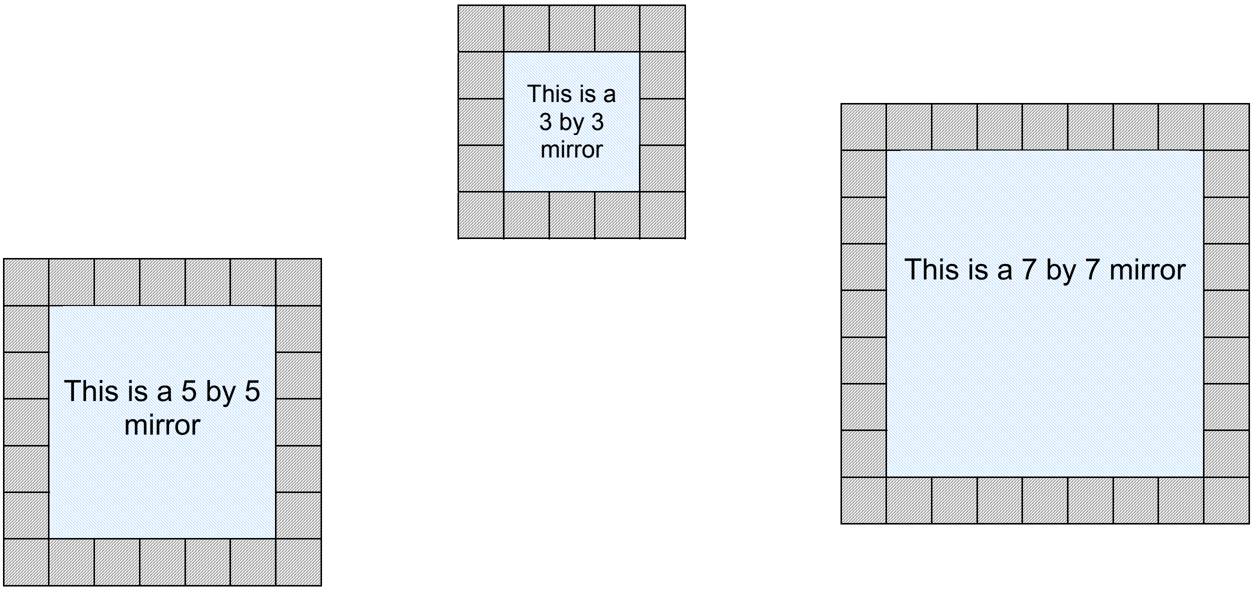
Fill in 3 and 4 on the table

"Is the result what you expected?" [Two terms are not enough to predict a sequence.]
"Why?"
"What is happening?"
"Can you predict the numbers of folds and regions for further foldings?"
"Why is it necessary to predict these numbers?" [Impossible to fold paper more than …. times]
"What are the *n*th term formulae for these sequences?"

Number of times strip has been folded	Number of folds	Number of regions
0	0	1
1	1	2
2	3	4
3	7	8
4	15	16
5		
6		

Activity 1 (Foundation or “easy” Higher)

Mirrors



Sasha uses tiles to make borders for square mirrors.

The pictures show three different sized mirrors, each with a one centimetre border of tiles around.

She only uses 1 by 1 tiles.

1 Investigate the total number of 1 by 1 tiles that are needed to make borders for different sized square mirrors.

Draw some different sized mirrors and try to predict how many tiles will be needed for each mirror.

A “table of results” could be useful.

Try to be systematic.

Find a formula for T, the total number of tiles.

2 Why does your formula work for these mirrors?

3 Investigate square mirrors surrounded by wider borders and try to predict the number of tiles needed for each width of border.

4 Link all your formulas together.

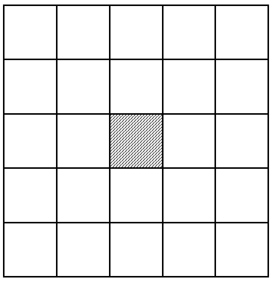
5 Why does your formula work for all square mirrors?

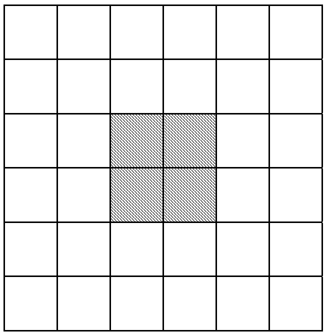
This section could be used where learners require more support.

Developing Mirrors

What if the border was two layers thick?
Draw some (at least four) mirrors but be systematic.

Eg





Put the results in a table.

Number of tiles for a two layer border

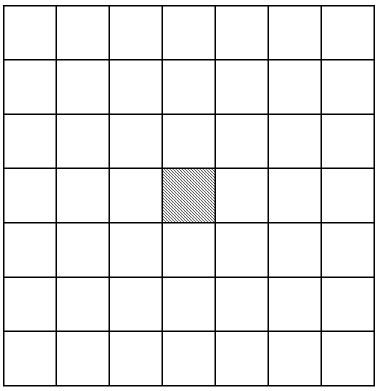
Size of mirror	Number of tiles
1 by 1	24
2 by 2	

Can you work out a formula for the number of tiles for any sized mirror (*n* by *n*)?

- Try it on a size of mirror you have not drawn yet.
- Does it give the right number of tiles?

Now what if the border is THREE tiles thick?

Start with some drawing. (Remember to be systematic).



Results in a table, formula, test.

- ☐ **Why** does the formula work? (Think about how the tiles fit around the mirror. What happens at the corners?)
- ☐ Could you work out the formula for a border that is FOUR tiles thick?
- ☐ Test it, does it work?
- ☐ What if the border was *T* tiles thick? Can you write a formula?
- ☐ Test it, does it work?
- ☐ **Why** does it work? (HINT, think about square numbers 1, 4, 9, 16, ….. x^2)

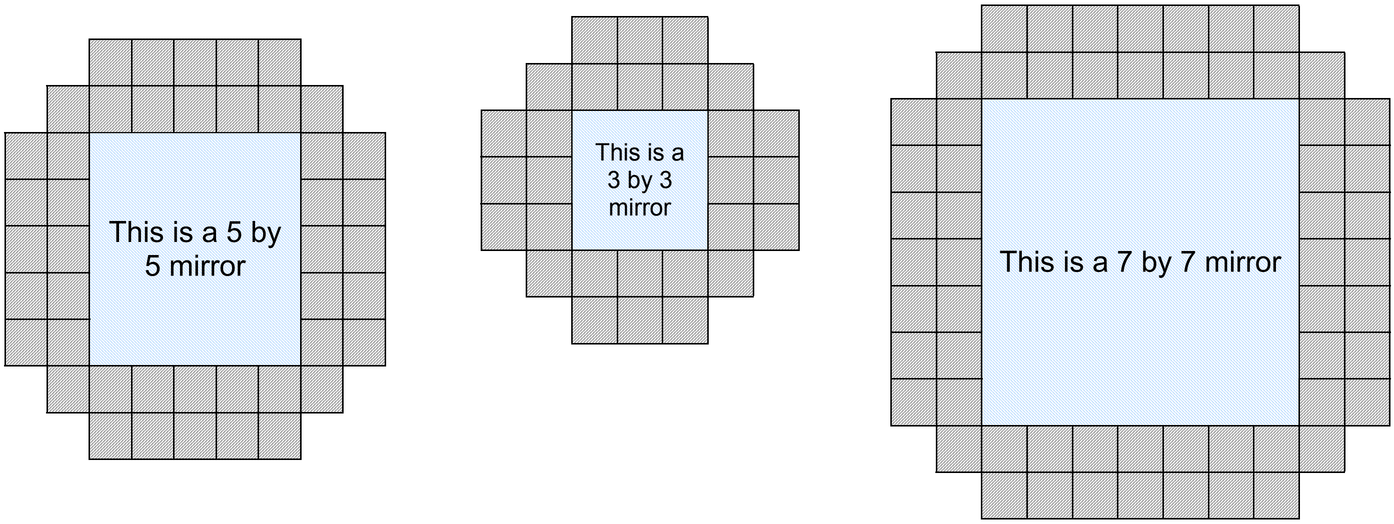
Some Foundation answers

Border 1 deep:	$n = 4x + 4$	OR $T_x = 4x + 4$
Border 2 deep:	$n = 8x + 16$	OR $T_x = 8x + 16$
Border 3 deep:	$n = 12x + 36$	OR $T_x = 12x + 36$
Border 4 deep:	$n = 16x + 64$	OR $T_x = 16x + 64$
Border 5 deep:	$n = 20x + 100$	OR $T_x = 20x + 100$
Border 6 deep:	$n = 24x + 144$	OR $T_x = 24x + 144$
Border *d* deep:	$n = 4\mathrm{d}x + 4d^2$	OR $T_x = 4d(x + d)$ OR $T_x = 4dx + 4d^2$

This activity would be followed up by discussion of the results and how the formulae could be used. Learners could be asked to write questions (with the answers) that could be solved using their formulae.

Activity 1 (Higher)

Mirrors



Sasha uses tiles to make borders for square mirrors.

The pictures shows three different sized mirrors, each with a two centimetre border of tiles around.

She only uses 1 by 1 tiles.

1 Investigate the total number of 1 by 1 tiles that are needed to make borders for different sized square mirrors.

Draw some different sized mirrors and try to predict how many tiles will be needed for each mirror.
A "table of results" could be useful.
Try to be systematic.

Find a formula for T, the total number of tiles.

2 Why does your formula work for these mirrors?

3 Investigate square mirrors surrounded by wider borders and try to predict the number of tiles needed for each width of border.

4 Link all your formulas together.

5 Why does your formula work for all square mirrors?

Some Higher answers

Square Mirror
(side = n and tapering design)

	1 row design	$T = 4n + 4$
	2 row design	$T = 8n + 4$,
	3 row design	$T = 12n – 4$
	4 row design	$T = 16n – 20$
	5 row design	$T = 20n – 44$

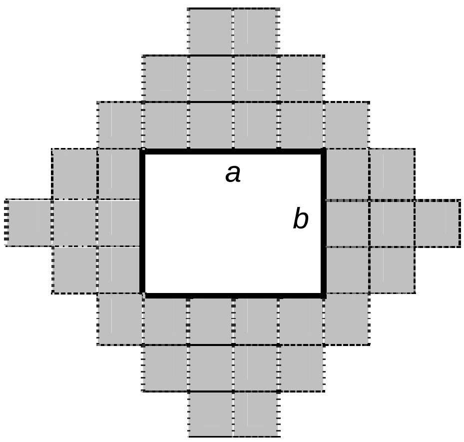
Square Mirror
(side = n, r = number of rows)

	2 by 2	$-4r^2 + 20r – 4$
	3 by 3	$-4r^2 + 24r – 4$
	4 by 4	$-4r^2 + 28r – 4$
	5 by 5	$-4r^2 + 32r – 4$
	6 by 6	$-4r^2 + 36r – 4$
	7 by 7	$-4r^2 + 40r – 4$
	n by n	$-4r^2 + 4r(n + 3) – 4$

If number of rows = r, this leads to:

$$T_r = (4n + 12)r – 4(r^2+1), \qquad \text{where } r \le \frac{(n+3)}{2}$$

Ziggurat arrangement around rectangular mirror



Given that a is horizontal and b is vertical

$$\text{Total} = ½\,(a^2 + 6a + b^2 + 6b + 8 + g)$$

If both a and b are odd, $g = 2$

If either a or b is odd, $g = 1$

If neither a nor b are odd, $g = 0$

Activity 2 (Higher)

Consider these statements.

Use algebra to support your conclusions.

1 For sets of three consecutive numbers, the sum of the numbers is always a multiple of 3.

2 For sets of three consecutive numbers, the product of the first and last is always one less than the square of the middle number.

3 For sets of three numbers that have a common difference, the product of the first and last numbers always differs from the square of the middle number by a square number.

This could easily be extended to sets of 5, 7, ….*d* consecutive numbers.

A key issue is to discuss the difference between "testing", "showing", and "proving".

Learners should be encouraged to present their proofs to the group and face questioning by the group.

Some possible answers

1 $n + n + 1 + n + 2 = 3n + 3 = 3(n + 1)$ [or $n - 1 + n + n + 1 = 3n$]

3 is a factor of $3(n + 1)$ and so 3 is a factor of the sum.

2 $n(n + 2) = n^2 + 2n$ [or $(n - 1)(n + 1) = n^2 - 1$]
$(n + 1)^2 = n^2 + 2n + 1$ [or n^2]

The product of the first and last numbers is clearly one less than the square of the middle number.

3 $n(n + 2a) = n^2 + 2an$ [or $(n - a)(n + a) = n^2 - a^2$]
$(n + a)^2 = n^2 + 2an + a^2$ [or n^2]

The square of the middle number is clearly "the square of the difference between the numbers" more than the product of first and last numbers in any triple.

A possible AO1 question (Foundation/Higher)

1(a) Here are the first four terms of a sequence.

90 85 80 75

(i) Find an expression for the nth term of this sequence.

(a)(i) ______________________ **[2]**

(ii) Use your expression to find the term that has a value of 0.

(ii) ______________________ **[2]**

(b) Here are the first four terms of another sequence.

1 4 9 16

The nth term of this sequence is n^2.

Write down an expression for the nth term of the following sequence.

3 6 11 18 ......

(b) ______________________ **[1]**

A possible AO3 question (Foundation/Higher)

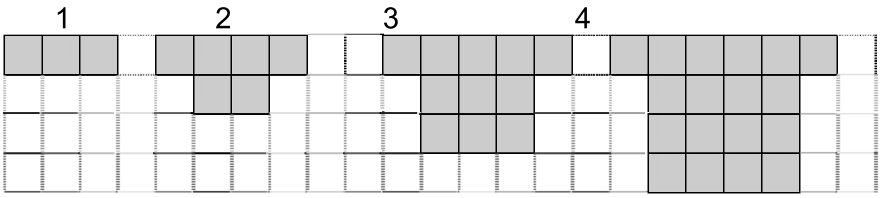
2(a) Here are the first four terms of a sequence and its *n*th term.

90 85 80 75 …… $5(19 - n)$

Show how Jayne can find the position in the sequence of the term that has a value of 0.

[2]

(b) Jayne creates these patterns by shading squares.



Show how Jayne can work out the number of squares in a pattern in **any** position in the sequence.

[3]

AO1 mark scheme

Q	Answer	Mark	Notes
1(a)(i)	(T_n =) 95 – 5n oe	**2**	Also 5(19 – *n*) **B1** for –5*n*
(ii)	(T_n =) 19	**2**	**B1** for setting *their* 95 – 5*n* = 0 and one rearrangement or 5*n* = 95 seen
(b)	$n^2 + 2$	**1**	

AO3 mark scheme

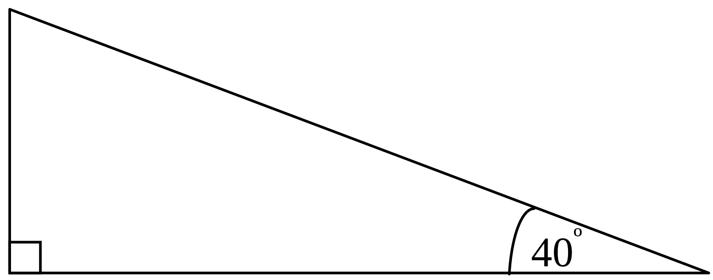
Q	Answer	Mark	Notes
2(a)	5(19 – *n*) = 0 and 19 – *n* = 0 oe *n* = 19 **OR** <u>T&I</u> One substitution correct value of T_n below 95 *n* = 19	**1** **1** **OR** **1** **1**	If expanded, 95 – 5*n* = 0 and 5*n* = 95 **If M0 SC1** for evidence of counting back earns
(b)	Shape is a square + 2 unit squares Square side = *n*, number of squares n^2 (T_n) = n^2 + 2 **OR** 1 2 3 (4 …) 3 6 11 (18…) (squares) 1 4 9 16 The term is two more than the position squared	**1** **1** **1** **OR** **1** **1** **1**	This may be in the form of a statement Attempt to link size of square to term number

3.6 Case 6: Trigonometry (Higher)

HA12 Core trigonometry		
12.1 - Solve 2D problems	Candidates should be able to:	
	a	**understand, recall and use trigonometrical relationships in right-angled triangles, and use these to solve problems, including those involving bearings**

Starter 1

Draw, as accurately as you can, some different triangles that all have these angles.



What common connections can you find between the **lengths of the sides** of your triangles?

This activity depends on careful drawing. Poor precision can ruin such an activity and this should be discussed at the start. Larger triangles help!

The first task learners will have is to decide how they should draw the triangles and what equipment they will need.

Learners should be encouraged to work in groups and divide up the work so that they end up with a collection of triangles having different length sides.

The next challenge will be to decide what the connections could be. Adding and subtracting will prove fruitless as will multiplying. Hence a proportion is required. These ideas could be brought out in discussion either with individuals or the whole group as the activity progresses.

Final discussion should centre around the need to name sides to be able to refer to them and the proportions discovered.

Hopefully, for 40° this will lead to (opposite ÷ adjacent) = 0·839…

(opposite ÷ hypotenuse) = 0·642…

(adjacent ÷ hypotenuse) = 0·766…

There will be all the inverse functions as well and these can be discussed.

The next question is what use this knowledge may be put to and what it means to say that $\sin 40^{\circ} = 0{\cdot}642\ldots$

Here a copy of the triangle can be displayed and lengths may be given for **one** of the sides. Can the learners use the facts they have discovered to work out the other sides?

Discuss how this is done.

This establishes all the basic trigonometric techniques.

It is also appropriate to give two sides as 10 and 8·4 and ask what may be said about the angles of the triangle.

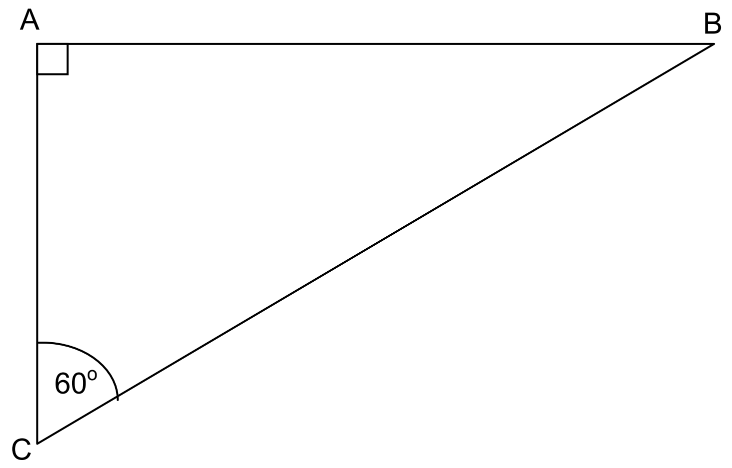
At this point it is useful to have a copy of trig tables (see next page), before referring to the calculator.

Degrees	Sin	Cos	Tan
0	0.00000	1.00000	0.00000
1	0.01745	0.99985	0.01746
2	0.03490	0.99939	0.03492
3	0.05234	0.99863	0.05241
4	0.06976	0.99756	0.06993
5	0.08716	0.99619	0.08749
6	0.10453	0.99452	0.10510
7	0.12187	0.99255	0.12278
8	0.13917	0.99027	0.14054
9	0.15643	0.98769	0.15838
10	0.17365	0.98481	0.17633
11	0.19081	0.98163	0.19438
12	0.20791	0.97815	0.21256
13	0.22495	0.97437	0.23087
14	0.24192	0.97030	0.24933
15	0.25882	0.96593	0.26795
16	0.27564	0.96126	0.28675
17	0.29237	0.95630	0.30573
18	0.30902	0.95106	0.32492
19	0.32557	0.94552	0.34433
20	0.34202	0.93969	0.36397
21	0.35837	0.93358	0.38386
22	0.37461	0.92718	0.40403
23	0.39073	0.92050	0.42447
24	0.40674	0.91355	0.44523
25	0.42262	0.90631	0.46631
26	0.43837	0.89879	0.48773
27	0.45399	0.89101	0.50953
28	0.46947	0.88295	0.53171
29	0.48481	0.87462	0.55431
30	0.50000	0.86603	0.57735
31	0.51504	0.85717	0.60086
32	0.52992	0.84805	0.62487
33	0.54464	0.83867	0.64941
34	0.55919	0.82904	0.67451
35	0.57358	0.81915	0.70021
36	0.58779	0.80902	0.72654
37	0.60182	0.79864	0.75355
38	0.61566	0.78801	0.78129
39	0.62932	0.77715	0.80978
40	0.64279	0.76604	0.83910
41	0.65606	0.75471	0.86929
42	0.66913	0.74314	0.90040
43	0.68200	0.73135	0.93252
44	0.69466	0.71934	0.96569
45	0.70711	0.70711	1.00000

Degrees	Sin	Cos	Tan
46	0.71934	0.69466	1.03553
47	0.73135	0.68200	1.07237
48	0.74314	0.66913	1.11061
49	0.75471	0.65606	1.15037
50	0.76604	0.64279	1.19175
51	0.77715	0.62932	1.23490
52	0.78801	0.61566	1.27994
53	0.79864	0.60182	1.32704
54	0.80902	0.58779	1.37638
55	0.81915	0.57358	1.42815
56	0.82904	0.55919	1.48256
57	0.83867	0.54464	1.53986
58	0.84805	0.52992	1.60033
59	0.85717	0.51504	1.66428
60	0.86603	0.50000	1.73205
61	0.87462	0.48481	1.80405
62	0.88295	0.46947	1.88073
63	0.89101	0.45399	1.96261
64	0.89879	0.43837	2.05030
65	0.90631	0.42262	2.14451
66	0.91355	0.40674	2.24604
67	0.92050	0.39073	2.35585
68	0.92718	0.37461	2.47509
69	0.93358	0.35837	2.60509
70	0.93969	0.34202	2.74748
71	0.94552	0.32557	2.90421
72	0.95106	0.30902	3.07768
73	0.95630	0.29237	3.27085
74	0.96126	0.27564	3.48741
75	0.96593	0.25882	3.73205
76	0.97030	0.24192	4.01078
77	0.97437	0.22495	4.33148
78	0.97815	0.20791	4.70463
79	0.98163	0.19081	5.14455
80	0.98481	0.17365	5.67128
81	0.98769	0.15643	6.31375
82	0.99027	0.13917	7.11537
83	0.99255	0.12187	8.14435
84	0.99452	0.10453	9.51436
85	0.99619	0.08716	11.43005
86	0.99756	0.06976	14.30067
87	0.99863	0.05234	19.08114
88	0.99939	0.03490	28.63625
89	0.99985	0.01745	57.28996
90	1.00000	0.00000	

Starter 2

What could be the lengths of the sides of this triangle?



Can the whole class give different answers and still be right?

The intention of this activity is to reinforce understanding that trigonometry gives the relationship between sides but does not tell us how long the sides are.

There are an infinite number of triangles that have the relationship $\frac{AC}{BC} = 0{\cdot}5$.

Learners need to be able to articulate this fact (and/or its alternatives).

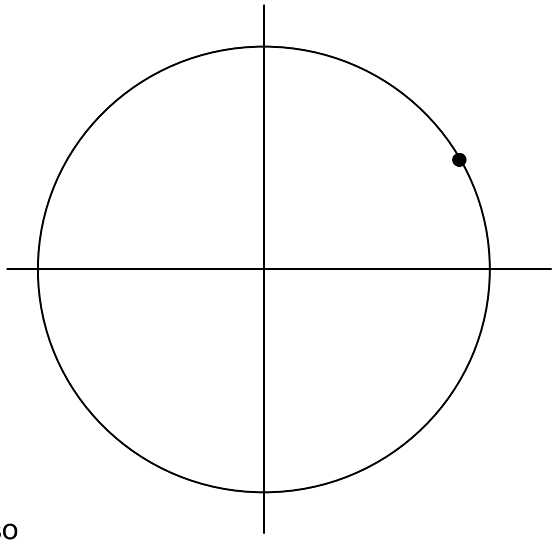
Activity

A point moves anticlockwise around a circle, starting at the point (0, 1).

The centre of the circle is (0, 0).

Calculate the height of the point (above the *x*-axis) at some special positions on the circle without using trigonometry.

Show how your answers link to trigonometric values for these angles.



This task is based on 'Where is the Dot?' published on the NRICH website: http://nrich.maths.org/5615. An animated version and students' solutions are also available on NRICH.

Discussion of what special angles are needed (45°, 30° and 60°) and the methods used (linked to accuracy of checking by drawing) is valuable.

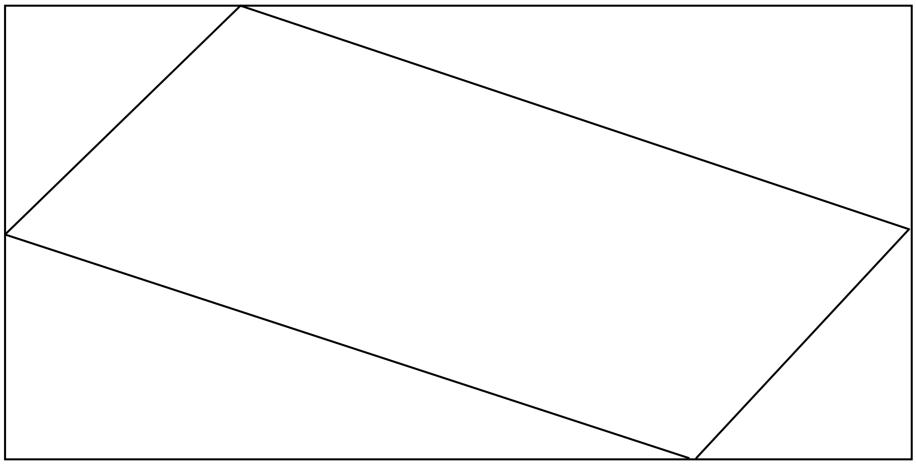
Homework

The sides of the large rectangle are in the ratio 2 : 1

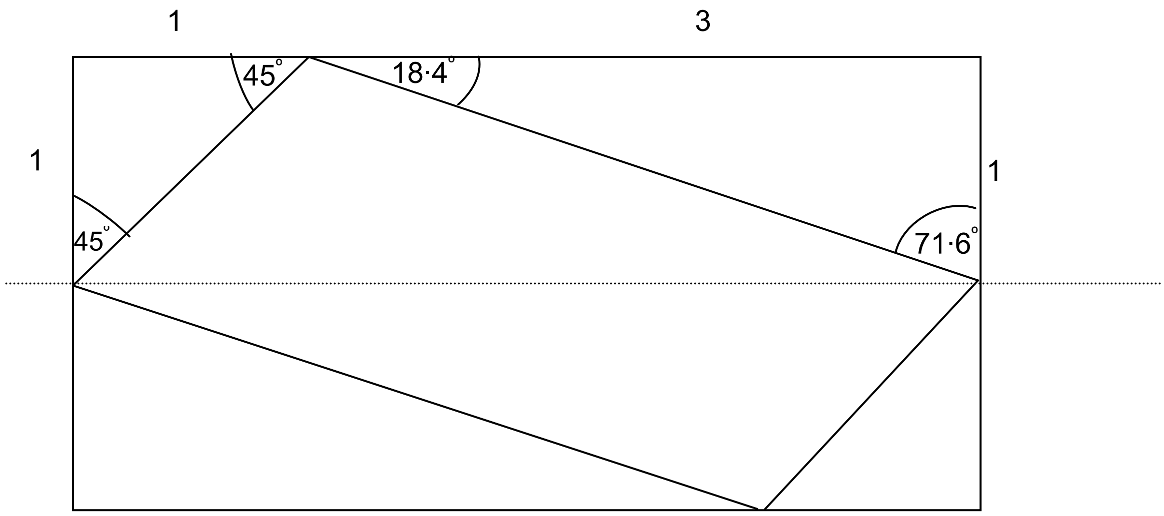
The middle points of the shorter sides are joined to points that divide the longer sides in the ratio 1 : 3 as shown.

Calculate the angles of the quadrilateral drawn within the larger rectangle, making your methods clear.

What is the mathematical name of this shape?



Solution



Angles of rectangle are 90°.

Smaller triangles are isosceles because ratio of sides forming right angle is 1 : 1 (midpoint of shorter side) hence angles are 45°.

For larger triangle sides are in ratio 3 : 1. Hence $\tan^{-1}(1 \div 3) = 18{\cdot}4^\circ$.

Angle within quadrilateral in contact with longer side = 180 – (45 + 18·4) = 116·6°.

Angle within quadrilateral in contact with shorter side = 180 – (45 + 71·6) = 63·4°.

Check 2 × (116·6 + 63·4) = 360°.

(Angles within quadrilateral could be calculated by using alternate angles with, for example, dotted line.)

The shape is a parallelogram. Learners should justify their answer.

A possible AO2 question

1

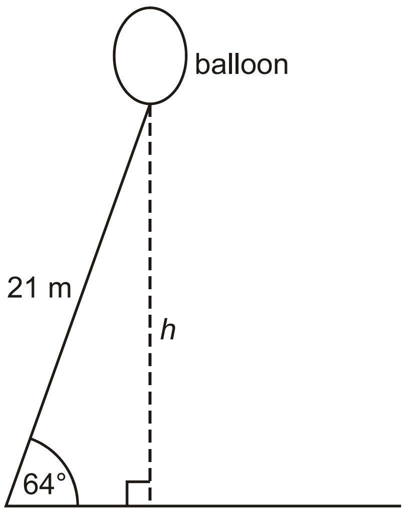


Diagram A

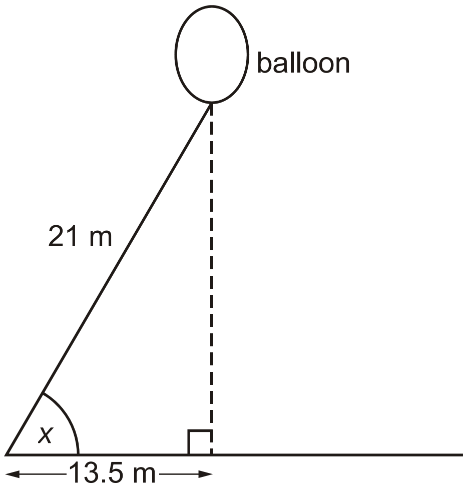


Diagram B

The diagrams show two positions of an advertising balloon.
The balloon is fixed to the ground by a straight cable of length 21 metres.

(a) Calculate the vertical height, *h*, of the balloon above the ground in **Diagram A.**

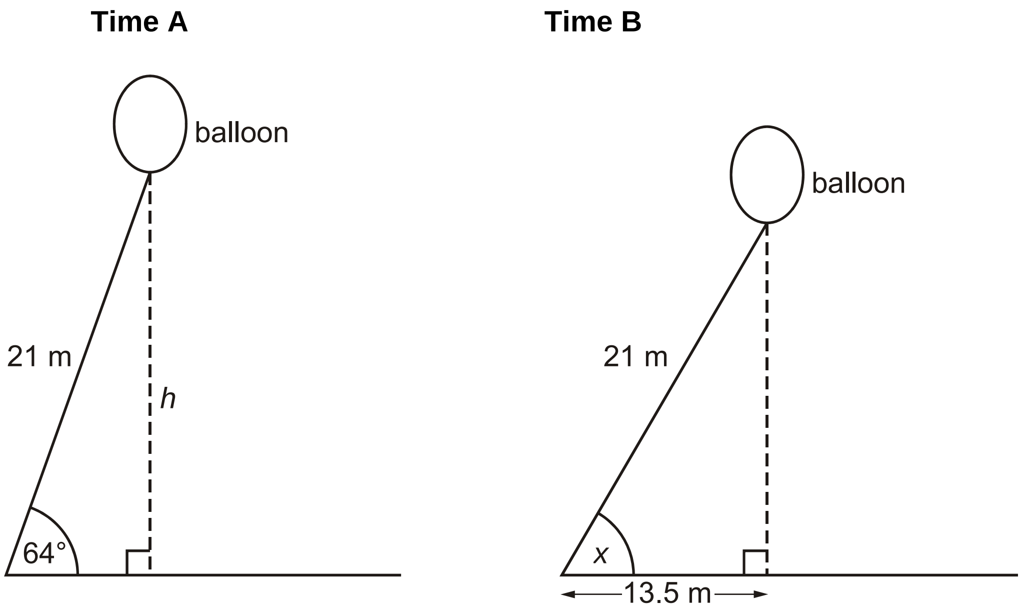
(a)__________________________m **[3]**

(b) Calculate the angle, *x*, which the cable makes with the horizontal ground for the balloon in **Diagram B**.
Give your answer to an appropriate degree of accuracy.

(b)__________________________ **[4]**

A possible AO3 question

2 An advertising balloon is tethered to the ground by a cable 21m long.
It is blown by a strong wind.



How much nearer to the ground is the balloon at **Time B** than at **Time A**?

[8]

AO2 mark scheme

Q	Answer	Mark	Notes
1(a)	18·8 to 19 (m) www	**3**	**M2** 21sin 64 **or M1** sin 64 = $\frac{h}{21}$
(b)	50° www	**4**	**B3** 49 to < 50 **or M2** $\cos^{-1}\left(\frac{13\cdot5}{21}\right)$ **or M1** cos = $\frac{13\cdot5}{21}$ **or SC1** for 1 or 2 after trig seen

AO3 mark scheme

Q	Answer	Mark	Notes
2	18·8 to 19(m) www	**3**	**M2** 21sin 64 **or M1** sin 64 = $\frac{h}{21}$
	16·1 to 16·2(m) www	**2**	**M1** for 24·96 (from addition)
	Their height A – height B	**1**	2·9 to 2·6(m)
	Correct statement	**1**	Units, if used, should be correct with values given eg 2·9m or 290cm
	Clear annotation or organisation of working	**1**	Stages identified with brief headings

3.7 Case 7: Graphs - Gradient, Parallel and Perpendicular (Foundation/Higher)

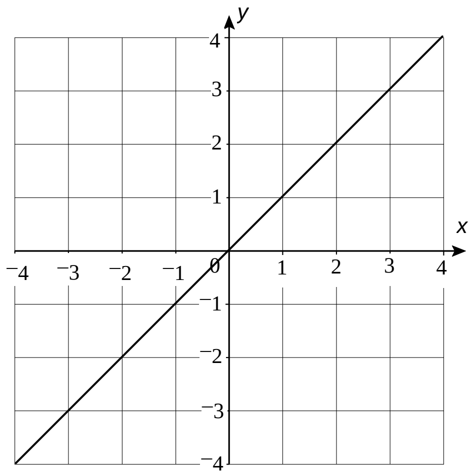
FB6 Functions and graphs	
6.3 - Recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients	Candidates should be able to: a recognise (when values are given for *m* and *c*) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane b find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for *m* and *c*); investigate the gradients of parallel lines c plot graphs of functions in which *y* is given explicitly in terms of *x*, or implicitly, where no table or axes are given

HB6 Functions and graphs	
6.4 - Straight-line graphs and the equation $y = mx + c$	Candidates should be able to: a **understand that the form $y = mx + c$ represents a straight line and that *m* is the gradient of the line and *c* is the value of the *y*-intercept** b **explore the gradients of parallel lines and lines perpendicular to each other**

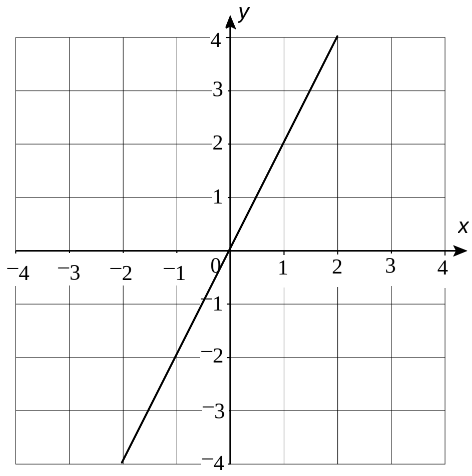
Starter 1

What are the equations of each of these lines?

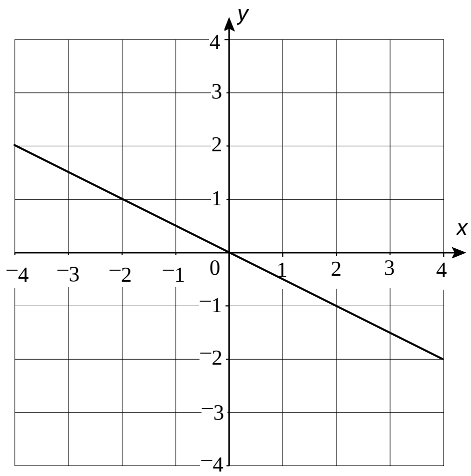
A



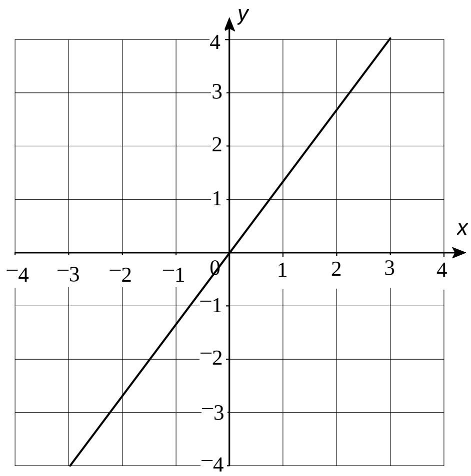
B



C



D



Explain how you know.

Answers

A $y = x$

B $y = 2x$

C $y = -\frac{1}{2}x$

D $y = \frac{4}{3}x$

In all cases equivalent versions are accepted.

Here, the question at the end of the task is the key. Learners should be encouraged to articulate this.

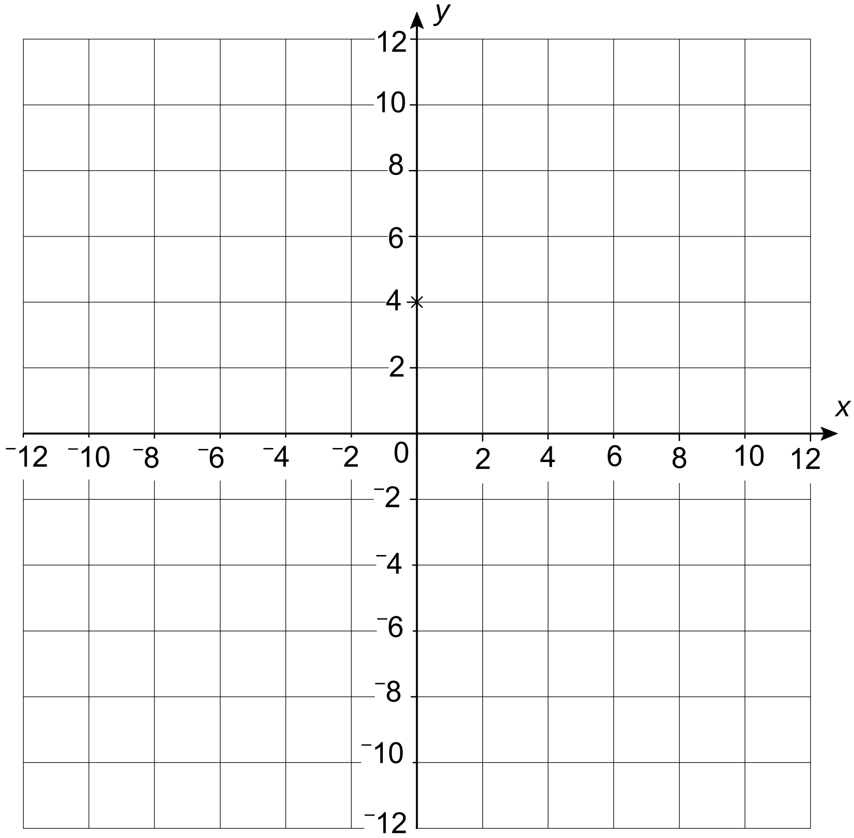
Evidence might be based on relationships between coordinates within pairs or related to gradient.

Further questions could be asked…

- Is the equation for **C**, $2y + x = 0$?
- How do you know?
- Is the equation for **C**, $y = 0{\cdot}5x$?
- Is the equation for **D**, $y = 1{\cdot}3x$?
- Why not?
- What is the advantage of using fractions rather than decimals to express the gradient (equation)?
- Would an extended version of the line on graph **A** pass through the point (123, 124)?
- How do you know?
- How close to the point would it get?
- How could you find out?
- What does it mean if a line has a negative gradient?

Starter 2

Draw some straight lines through the point (0, 4)



Work out the equation of each of the lines you have drawn.
Be prepared to explain your methods to the group.

Learners need copies of the grid.

Timing depends on the ability of the group.

The activity allows learners to work at their level but it is useful to set a challenge, 4 lines, 6 lines, 10 lines…

It is important that methods are explained. Learners may use tables of values or work out gradients.

Simple cases will be horizontal and vertical (y = 2 and $x = 0$). The latter case is the only equation that does not involve the constant 2.

All other cases are expressible in the form $y = mx + 2$.

This could be brought out in questioning:

- What do all but one of your equations have in common?
- Why is this?
- How can you use “this fact” to begin to write down equations of other lines passing through (0, 2)?
- What is true of equations passing through the point (0, 3)?
- What is the equation of a line passing through (0, 3) that does not have + 3 in it?
- What point does $y = x + 4$ pass through on the y-axis?

Activity

This section lends itself to use of ICT. Graphing software allows learners to explore challenges through the electronic medium. It saves a lot of time working out tables of values and lets the learner focus on ideas relating to how graphs behave.

It is, however, vital that learners write up (or verbalise) solutions to challenges and explain the mathematics that lies behind each solution.

Graphing software challenge

Note: To enter a power eg x^2 type $y = x$ and then hold down Alt and 2 together.

To type $\frac{2}{x}$ type $y = 2/x$

Look at these equations and decide:

1 Which ones, when drawn as a graph, will produce a straight line?

2 Write down what you think the gradient of each **straight line** will be.

Equation	Straight line?		Gradient
	Yes	No	
$y = x^2 + 3x$			
$y = 5x – 2$			
$y = 3 – 4x$			
$y = \frac{2}{x} + 7$			
$y = 1{\cdot}4x + 2$			
$x + 3y = 10$			
$y = \frac{2x}{3} - 5$			
$x^2 + 3y = 12$			
$2y = 10 + 2x$			

Now draw the graphs of each equation using your graphing software.
Were you right each time?

Use the printouts of the straight lines to work out the gradient for each line.
Show how the answer for each gradient relates to the equation of the line.

Graph Tasks

Use the graph package to draw these graphs and answer the questions.
Print the graphs and write your conclusions using a word processing program.

<u>Task 1</u> [Raising lines]

1 Draw $y = x + 1$ Where does this cross the *y*-axis?
2 Draw $y = x + 2$ Where does this cross the *y*-axis?
3 Draw $y = x + 3$ Where does this cross the *y*-axis?
4 Draw $y = x + 4$ Where does this cross the *y*-axis?
5 Draw $y = x + 5$ Where does this cross the *y*-axis?
6 Where do you think $y = x + 10$ will cross the *y*-axis?
7 WHY did you decide the line would cross at this point?
8 Draw it and check whether you were right.
9 What can you say about all the lines you have drawn?

<u>Task 2</u> [The “coefficient” of *x*]

1 Draw $y = 2x + 1$ Where does this cross the *y*-axis?
2 Draw $y = 3x + 1$ Where does this cross the *y*-axis?
3 Draw $y = 5x + 1$ Where does this cross the *y*-axis?
4 Draw $y = 1 – x$ Where does this cross the *y*-axis?
5 Draw $y = 1 –2x$ Where does this cross the *y*-axis?
6 Draw $y = 1$ Where does this cross the *y*-axis?
7 What do all these lines have in common?
8 How does your answer to (7) relate to the EQUATION of the line?
9 Does changing the number that MULTIPLIES “*x*”, [the “coefficient” of *x*], change where the graph crosses the *y*-axis?
10 What does changing the coefficient do to the graphs?
- As positive coefficients of “*x*” get bigger, the graph gets _______________
- As positive coefficients of “*x*” get smaller, the graph gets _______________
- When the coefficient of “*x*” is negative the line slopes _______________

<u>Task 3</u>

1 Draw six **different** straight lines that all pass through **4 on the *y*-axis**.
2 What do all the EQUATIONS of the lines you have drawn have in common?

<u>Task 4</u>

Draw all of these graphs.
You should be able to put them into four, different groups according to their shape.

- Draw each set of graphs on your graphing software.
- Write two sentences. One to describe **the shape** of the graphs in the group and one to say **what links the equations** for the group.

$y = x$ $y = x^2$ $y = 5 – x$ $y = \frac{1}{x}$ $y = 4x^3$ $y = 7x – 5$ $y = 19 – 2x$

$y = 3 + \frac{2}{x}$ $y = (1 + x)^2$ $y = 4 – x^3$ $y = 3x$ $y = 4 + x^2$ $y = \frac{5}{x} + 1$ $y = (x – 3)^3$

$y = \frac{1}{2}x$ $y = 5x + 8$ $y = 2 – x^3$ $y = x + x^2$

Although structured, the activity still allows the learner to explore changing *m* and *c* in $y = mx + c$ and the effect of such changes.

Matching cards

1 $y = 2x$	x **1** **2** **3** **4** **G** y 2 4 6 8
2 $y = x + 2$	x **3** **5** **7** **9** **N** y 5 7 9 11
3 $y = \frac{1}{2}x$	x **3** **4** **5** **6** **I** y 1·5 2 2·5 3
4 $x + y = 10$	x **⁻1** **0** **1** **2** **T** y 11 10 9 8
5 $y = 2x + 1$	x **⁻4** **⁻3** **⁻2** **⁻1** **N** y ⁻7 ⁻5 ⁻3 ⁻1
6 $y = 3x$	x **0** **2** **4** **6** **E** y 0 6 12 18
7 $y = 6 - x$	x **3** **4** **5** **6** **S** y 3 2 1 0
8 $y = x^2$	x **⁻4** **⁻2** **0** **2** **E** y 16 4 0 4
9 $2x = y + 3$	x **⁻1** **0** **1** **2** **R** y ⁻5 ⁻3 ⁻1 1
10 $2y = 2x$	x **⁻5** **⁻4** **⁻3** **⁻2** **P** y ⁻5 ⁻4 ⁻3 ⁻2

The equations should be moved around so they do not match the corresponding table and learners should try and match equation number to letter. They could be invited to state the gradient of each line.

A less structured approach could be as follows:

1 Draw six parallel lines using the graphing package.

- What do the equations of each line have in common?

2 Draw a line at 90^o (**perpendicula**r) to $y = x$.

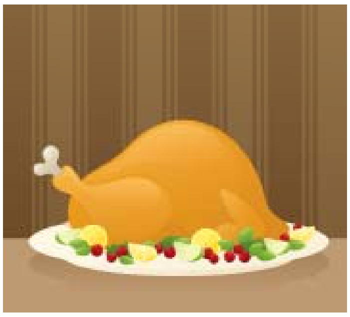
- What is the equation of the new line?
- How is this connected to the equation $y = x$?

3 Draw a line parallel to $y = x$ and another **perpendicular** to it.

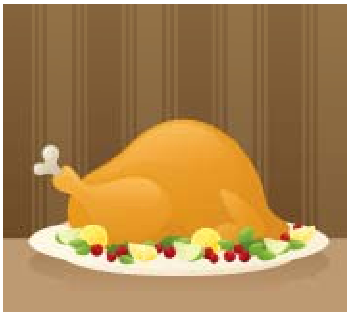
- Explain how you decided what equations you needed to enter to draw these lines.

Activities of this type may be generated to explore graphs but also need to be supported with the usual table creation and point plotting activities.

Homework task



Turkey Dinner



Use the information sheet to help you produce:

- Graphs showing how long to thaw and then cook a frozen turkey for
- Examples showing how to use the graphs
- The equations of the graphs and explanations of what the coefficients you have used mean, in relation to the information on the sheet

COOKING & THAWING INFORMATION SHEET

The following information gives recommendations for thawing and cooking a whole turkey:

WEIGHT	THAWING TIME	COOKING TIME
2·25kg/5lb	20 hours	2 hours
4·5kg/10lb	22-24 hours	3 hours
6·75kg/15lb	24-28 hours	4 hours
9kg/20lb	40-48 hours	5 hours
11·25kg/25lb	Over 48 hours	6 hours

RECOMMENDED THAWING

- Thaw the turkey in a cool room (below 60°F).
- After thawing store in the refrigerator at a temperature of no more than 5°C.

COOKING GUIDELINES

- As a guide, allow 18 minutes per pound (this is 450g).
- To check that your turkey is fully cooked, pierce the thickest part of the thigh with a skewer. If the juices run clear, it is ready, if they run pink, continue cooking.

Note: To convert temperatures in degrees Fahrenheit (°F) to degrees Celsius (°C) use the following formula.

$$C = \frac{5(F - 32)}{9}$$

FC7 Real life and non-linear functions	
7.1 - Functions from real life	Candidates should be able to: a discuss and interpret graphs modelling real situations

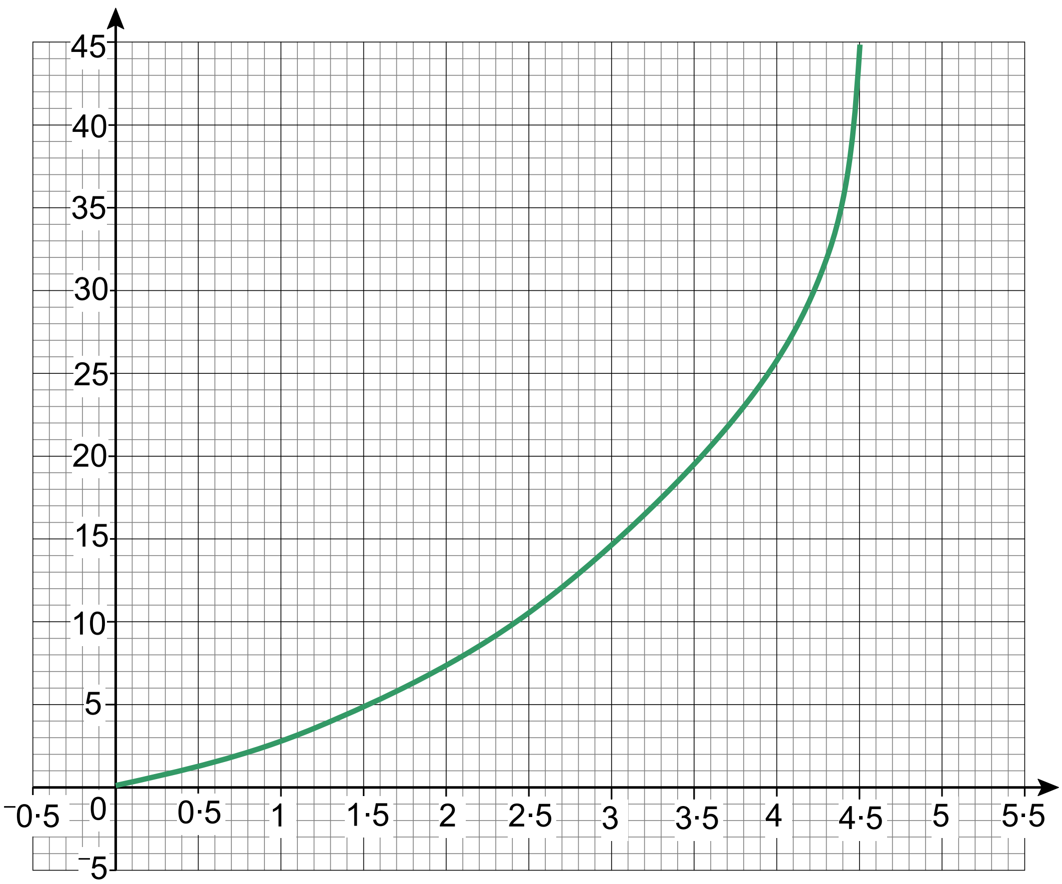
The following graphs represent two different ways of adding water to a cylindrical container.

Describe the two ways, making use of readings from the graphs.

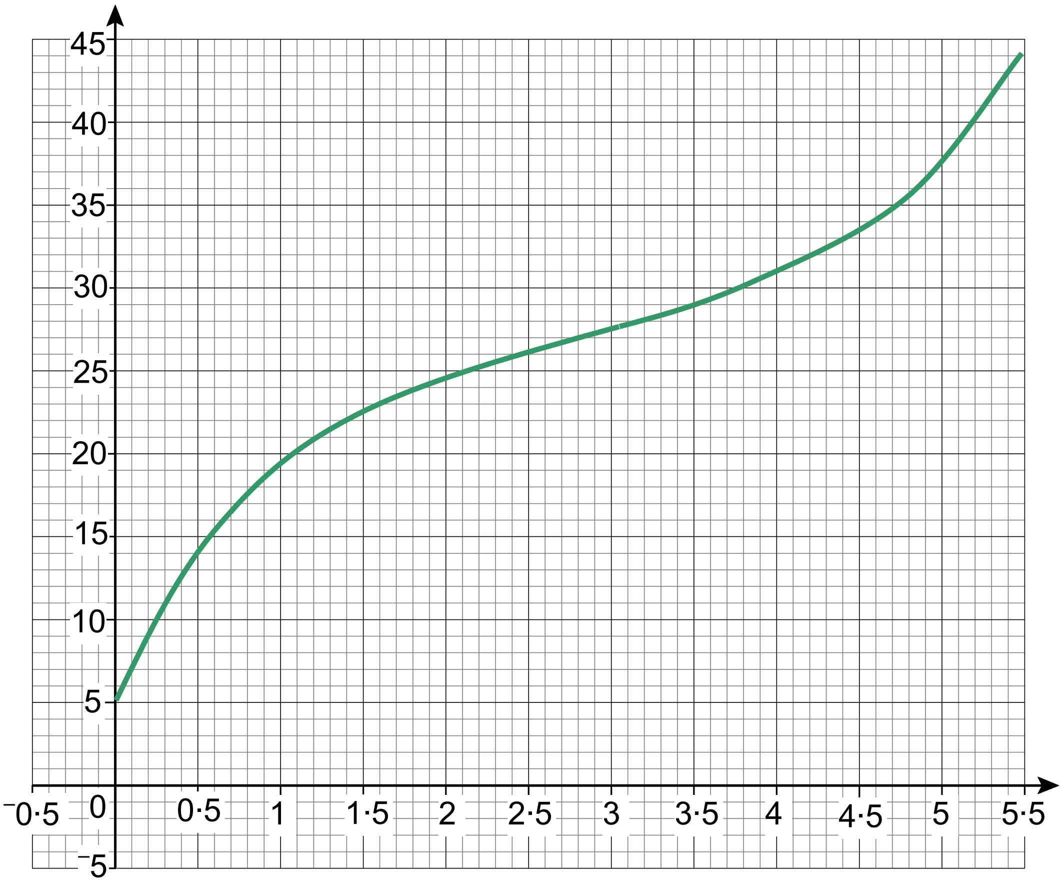
Add any labels needed to make your descriptions clear.

You should describe any similarities and differences between the two ways of adding water.

A



B



Axes should be labelled with titles.

A possible FULL point might be indicated.

In graph **B** the container already contains some fluid but in graph **A** the container is empty.

In graph **A** fluid is added slowly at the beginning but the rate at which this is added is steadily increased until the fluid in the container reaches a height of 45______

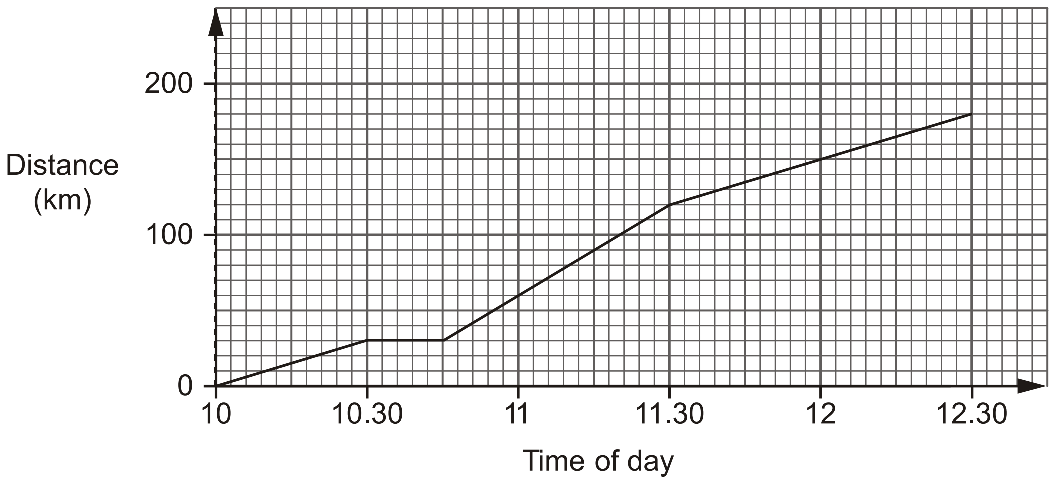
In graph **A** it takes 3·75______ to half fill the container.

Further statements should be made about graph **B**.

The containers might not be cylinders. What shapes might they be if the fluid is, in fact, added steadily?

A possible AO2 question (Foundation/Higher)

1 Hassan is driving along a motorway.
The distance-time graph shows part of his journey.



(a) Between which times is Hassan driving fastest?

Between ________ and ________ **[1]**

(b) Calculate Hassan's speed between 10 am and 10.30 am.

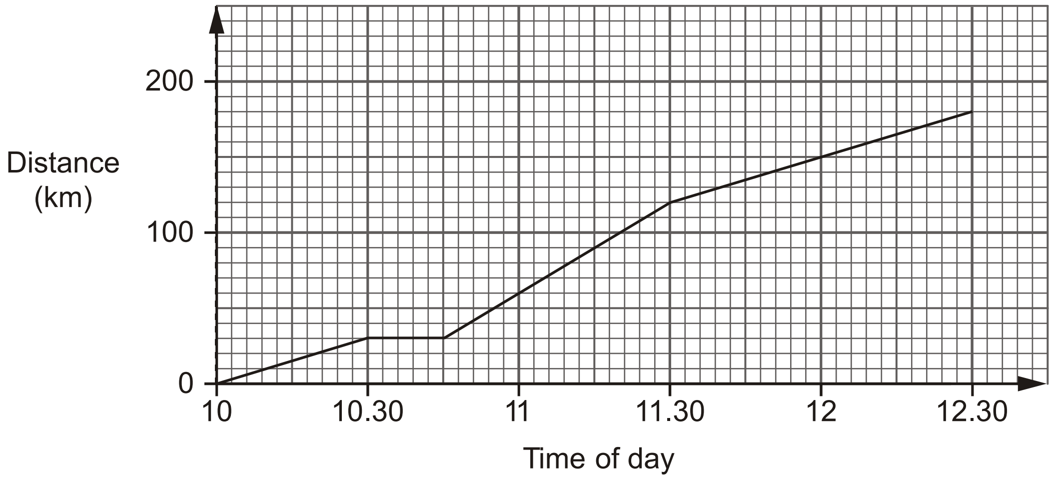
....

....

__________________km/h **[2]**

A possible AO3 question

2 One morning Hassan drove to Birmingham along a motorway.
The distance-time graph shows his journey.



Hassan attends a meeting from 12.30 until 19.00.
He hopes to be home by 20.45.
He knows that the motorway will be much less busy after 19.00.

Given that 5 miles is approximately 8 kilometres and that the speed limit on English motorways is 70 mph, is Hassan likely to be home by 20.45?

[4]

AO2 mark scheme

Q	Answer	Mark	Notes
1(a)	10 45 to 11 30	**1**	
(b)	60	**2**	**M1** for $\frac{30}{\textit{histime}}$ soi by 1 or 100 (30 ÷ 30 minutes OR 30 ÷ 0·3)

AO3 mark scheme

Q	Answer	Mark	Notes
2	112·5 miles (accept 112 to 113)	**1**	Total distance home
	1·6……	**1**	Time home in hours
	1 hour 36 minutes (accept 37)	**1**	Time home in hours and minutes
	Should be able to get home within the 1 hour 45 minutes	**1**	Do not allow for any confusion with 1·6… hours
	OR	**OR**	
	1 hour 45 minutes = 1·75 hours	**1**	
	102·9 km/h	**1**	Average speed home km/h
	64·3 mph	**1**	Average speed home mph
	If the roads are clear he should be able to raise his speed to this	**1**	

3.8 Case 8: Area and Volume (Higher)

HC10 Area and volume	
10.1 - Perimeter, area (including circles), and volume	Candidates should be able to: i **calculate volumes of objects made from cubes, cuboids, pyramids, prisms and spheres** i **calculate the lengths of arcs and the areas of sectors of circles**

In a design competition, entrants were given the following brief:

Design a mug or cup to hold between 350ml and 450ml of hot drink.
The mug or cup must keep the fluid warm for as long as possible.
It should be stable and around 20 of them could stand on a shelf that is 0·85 m long and 14cm wide.
It is known that reducing the surface area of a warm fluid reduces the heat loss and that this is particularly true of the area between a fluid and the open air.

Design a mug and show how you have considered the conditions of the competition.

This is a good activity to challenge more able learners in the application of mensuration.

Learners will need to use formulae for area of circle, surface area of cylinder, volume of cylinder and may need to use formulae for volume of a cone, surface area of a cone and techniques for similar solids and frustums. Learners may choose to use cuboids.

Most mugs take the shape of a cylinder or inverted, truncated cone. This does not need to be the case and learners may move away from this.

It could be useful to devise a “points scheme” to reward good designs.

The task could be undertaken as a homework or classroom activity. Initial discussion could focus on the conditions posed by the task but to identify these and leave the interpretation to the learners.

Candidates may attempt to minimise the total surface area or the area of the exposed surface of the fluid.

They may spend considerable time getting the dimensions correct (with consideration given to the length of the shelf).

There should be space so that the fluid does not FILL the cup to the rim.

They should consider internal and external dimensions.

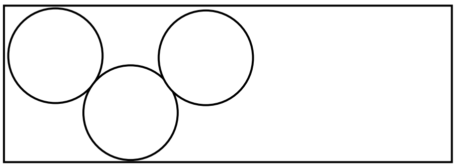
Stability should give the mug a wide base (defined by the dimensions of the shelf) and not too great a height.

A possible solution is a truncated cone but the diameter of the rim should be sufficient to drink from.

These would be useful points to discuss in the light of the solutions presented by the learners.

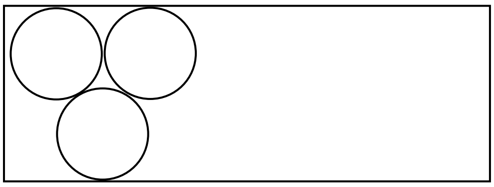
It is worth noting that there is no “right answer” BUT the solutions should be annotated and assumptions identified, correct units used and reasonable explanations given.

Mugs may fit on the shelf like this.



Ceramic mugs are around 2·5mm thick.

Mugs will fit like this to get all the mugs on the shelf, with a small space (≈ 2·5cm) at each end and internal diameter of roughly 7cm.



The resulting height of a **cylinder**, holding 400ml of fluid is 10·4cm.

Supporting working and variations are expected.

3.9 Case 9: Measures (Foundation/Higher)

Extract data from tables, apply standard calculation techniques, approximate and derive unknown facts from those already known (use of imperial units).

FB2 Number	
2.1 - Add, subtract, multiply and divide any number	Candidates should be able to: a recall all positive integer complements to 100 b recall all multiplication facts to 10 × 10, and use them to derive quickly the corresponding division facts c develop a range of strategies for mental calculation; derive unknown facts from those they know d add and subtract mentally numbers with up to two decimal places e multiply and divide numbers with no more than one decimal place, using place value adjustments factorisation and the commutative, associative, and distributive laws, where possible f use standard column procedures for addition and subtraction of integers and decimals understanding where to position the decimal point g perform a calculation involving division by a decimal (up to two decimal places) by transforming it to a calculation involving division by an integer

HB2 Number	
2.2 - Approximate to a specified or appropriate degree of accuracy	Candidates should be able to: a round to the nearest integer, to any number of decimal places and to one significant figure b estimate answers to problems involving decimals c estimate and check answers to problems d use a variety of checking procedures, including working the problem backwards, and considering whether a result is of the right order of magnitude e **round to a given number of significant figures** f **select, and use, an appropriate degree of accuracy in solving a problem** g **develop a range of strategies for mental calculation** h **derive unknown facts from those they already know**

This is a short activity that learners enjoy because of the stories that are associated with it. It is largely Foundation tier but still requires the extraction of information and application to some simple problems.

Learners need the information sheet (see next page) and a set of questions.

The activity may be undertaken in pairs with discussion of methods, answers and units following.

The questions can easily be adapted to suit the situation.

Some "Old Fashioned English" or Imperial measurements of length

Our English measurements of length came from convenient things people could use in their everyday world to estimate how long things were… their feet or their arms, for instance.

Here are some of them:

Measurement	Where it came from	Connection
1 inch	The width of a man's thumb	
1 foot	The length of a man's foot	12 inches make 1 foot
1 hand	The width of a mans hand across the knuckles	4 inches make 1 hand
1 yard	From a man's breast bone to the tip of his fingers OR a step	3 feet make 1 yard
1 fathom	A man's height OR $\frac{1}{1000}$ of a nautical mile (Used to measure the depth of water)	6 feet make 1 fathom
1 chain	A standard measuring chain used by surveyors up to a few years ago OR the length of a cricket pitch	22 yards make 1 chain
1 furlong	The length of a furrow in a medieval field	10 chains make 1 furlong
1 mile	1000 double strides, left foot to left foot	1760 yards make 1 mile

Use the table to work out the answers to these questions.
Write down the calculation you have to do.

1 How many inches in 1 yard?

2 How many feet in a mile?

3 How many hands in a yard?

4 How deep is 4 fathoms in feet?

5 Jules Verne wrote "20 000 leagues under the sea" (A league is 3 miles). What would the title have been in miles?

6 How many chains in a mile?

7 How many inches in 1·5 yards?

8 What is 1 foot 7 inches in inches only?

9 How many feet and how many inches does 66 inches make?

10 How long would a medieval field have been in feet?

11 If a submarine dived to 50 fathoms, how many yards would it be below the surface?

12 I drive 14·5 miles to school each day.

   a How many miles do I drive in one day coming to work and going home?

   b I work three days a week. How many miles do I drive so that I can work in this college in a week?

   c How many furlongs do I drive?

13 How many feet in a mile?

14 How many miles, yards, … are there in 10 000 feet?

15 Make up two questions of your own related to these and show how you answer them.

Extension

My car does 46 miles per gallon of diesel.
A litre of diesel costs approximately £1·05.
There are 4·5 litres in 1 gallon.

How much does it cost me to drive to and from home and school for a week?

Answers

1 How many inches in 1 yard? $3 \times 12 = 36$

2 How many feet in a mile? $3 \times 1760 = 5280$

3 How many hands in a yard? $3 \times 3 = 9$

4 How deep is 4 fathoms in feet? $4 \times 6 = 24$

5 Jules Verne wrote “20 000 leagues under the sea” (A league is 3 miles).
What would the title have been in miles? $3 \times 20\,000 = 60\,000$

6 How many chains in a mile? $1760 \div 22 = 80$

7 How many inches in 1·5 yards? $36 \times 1.5 = 54$

8 What is 1 foot 7 inches in inches only? $12 + 7 = 19$

9 How many feet and how many inches does 66 inches make? $66 \div 12 = 5{\cdot}5$ = 5 feet 6 inches

10 How long would a medieval field have been in feet? $10 \times 22 = 220$

11 If a submarine dived to 50 fathoms, how many yards would it be below the surface?
50×6 [= 300 feet] $\div 3 = 100$ yards

12 I drive 14·5 miles to school each day.

a How many miles do I drive a day coming to work and going home? $14{\cdot}5 \times 2 = 29$

b I work three days a week. How many miles do I drive so that I can work in this college in a week? $29 \times 3 = 87$

c How many furlongs do I drive? 87×352 [= 30 624] $\div 10 = 3062{\cdot}4$

13 200 chains make how many miles? $200 \times 22 = 4400$;
$4400 \div 1760 = 2{\cdot}5$ miles

14 How many miles and yards and feet are there in 10 000 feet? $10\,000 \div 3 = 3333$ yards 1 foot
$3333 \div 1760 = (1{\cdot}89375)$
1 mile 1573 yards 1 foot

3.10 Case 10: Understand and Use Direct Proportion (Foundation)

FC4 Social arithmetic	
4.1 - Apply problem solving skills	Candidates should be able to: a analyse real life problems using mathematical skills b apply mathematical skills when solving real life problems c communicate findings from solutions to real life problems d interpret solutions to real life problems
4.3 - Understand and use direct and indirect proportion	Candidates should be able to: a solve word problems about proportion, including using informal strategies and the unitary method of solution

This is a short starter that may be used to explore a learner's understanding of the situation.

Figures may be adapted to suit the situation.

Responses could be given using whiteboards.

True or False?

Here is some information about products sold by Sainsco.

Use the information to decide whether the information label in the central column is true or false.

Information	Customer Label	T/F	
250ml bottle costs 50p	100ml costs 10p		
10 pack of crisps. Total cost 95p	1 packet of crisps costs 10p		
Milkshake!! Special offer 300ml costs 90p	10ml costs 3p		
Chocolate bars on promotion. 200g costs £1	25p buys 50g of chocolate		
Toilet rolls. 200 sheets each roll. 6 rolls for £1·20	100 sheets costs 20p		

Answers F, F, T, T, F

Plenary

(Figures may be adapted at will.)

Decide which is better value for money.
Show your evidence in each case.

(1000ml = 1 litre, 1000g = 1 kilogram, 100cm = 1 metre)

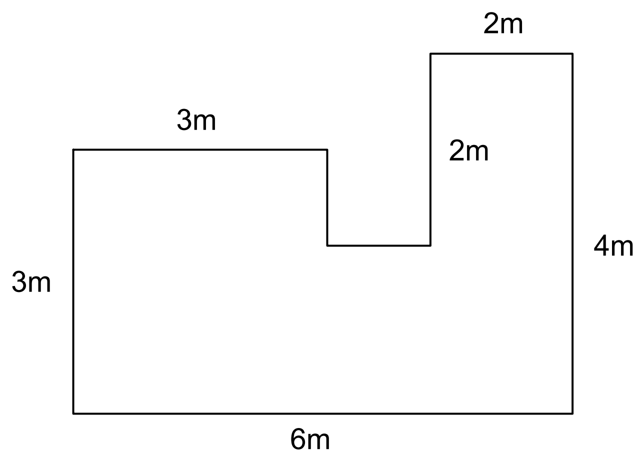
A	**B**	Working
Fabric Softener 700 ml £1·20	Fabric Softener 1 litre £1·60	
Chocolate Drops 50 g 45p	Chocolate Drops 150 g £1·45	
Phone calls 4 minutes 5p	Phone calls 1 hour 65p	
Ball of string 30 m £1·20	Packet of string 2 m 15p	

Answers B, A, B, A

Section 4: Some worked examples

Example 1 (from Section 3.1 - Case 1: Area and Perimeter)

2 This is a plan of Annie's back garden.
She is going to make it into a lawn.



Not to scale

(a) Annie will put a strip of edging all round her lawn.
Edging costs £1·35 per metre.

Work out how many metres of edging Annie needs.

Annotation, not an essay, clarifies the purpose of calculations.

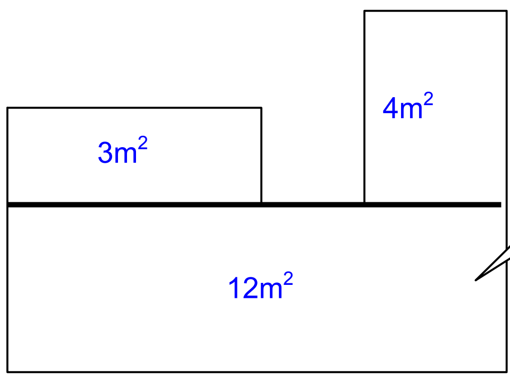
Missing lengths 6 – 3 – 2 = 1m and 4 – 2 = 2·3 – 2 = 1m

Total length = 3 + 3 + 1 + 1 + 2 + 2 + 4 + 6 = 22m

(a) 22m **[3]**

(b) Annie is going to sow grass seed to make the lawn.
Each packet of grass seed is enough to cover $4m^2$.
Grass seed costs £3·45 a packet.

What will it cost Annie to buy the grass seed to make her lawn?



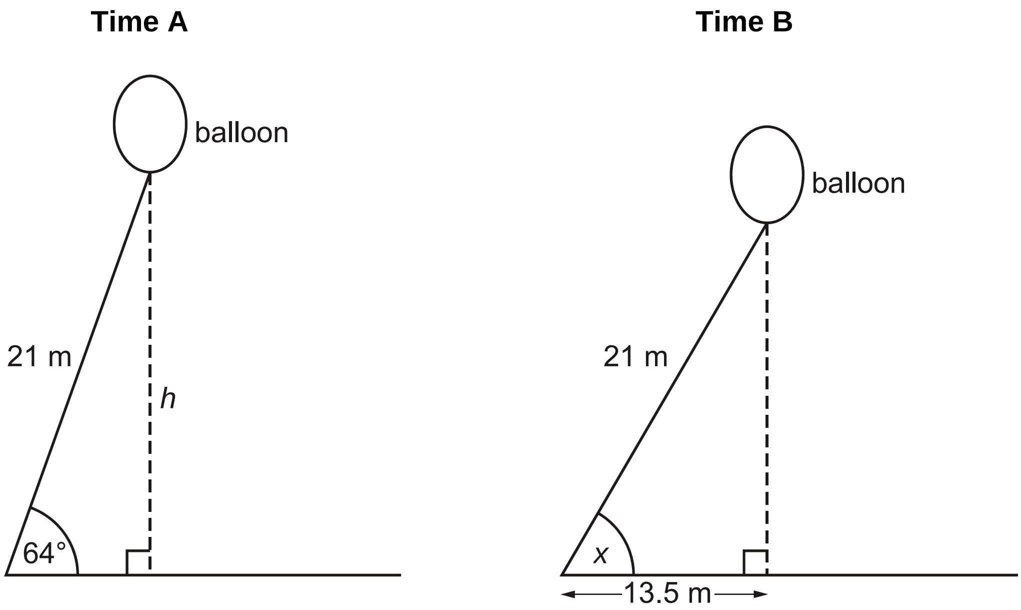
Working on the diagram will be marked.
The dividing line (others are possible) make the method used clear.

$3 + 4 + 12 = 19m^2$
Number of packets = 19 ÷ 4 = 4·75
Must buy 5 packets
Cost = 5 x £3·45
= £17·25

(b) £ 17.25 **[4]**

Example 2 (from Section 3.6 - Case 6: Trigonometry)

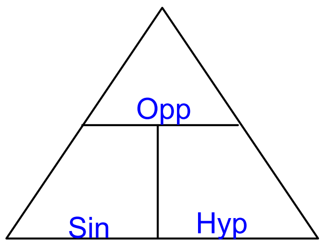
2 An advertising balloon is tethered to the ground by a cable 21m long.
It is blown by a strong wind.



How much nearer to the ground is the balloon at **Time B** than at **Time A**?

Height A $21 \times \sin 64$
$h = 18{\cdot}87$

3 www



Height B $\sqrt{623 \times 25}$

Added not subtracted

24·964

M1

It isn't lower, it's higher.

Organised and annotated **1 mark**

No difference attempted but correct statement **1 mark**

6 marks out of 8

[8]

Example 3

(a) Here are the first four terms of a sequence and its *n*th term.

90 85 80 75 …… 5(19 – *n*)

Show how Jayne can find the position in the sequence of the term that has a value of 0.

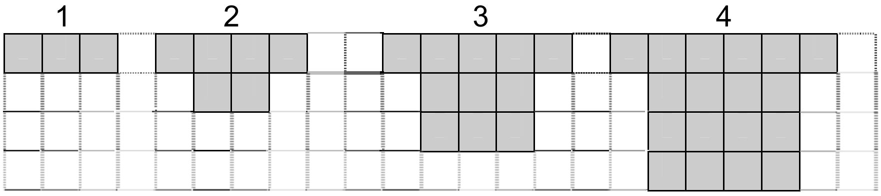
[2]

75 – 5 = 70 – 5 = 65 – 5 = 60 – 5 = 55 – 5 = 50 – 5 = 45 …. – 5 = 0
It's in position 20

Wrong answer following poor counting method

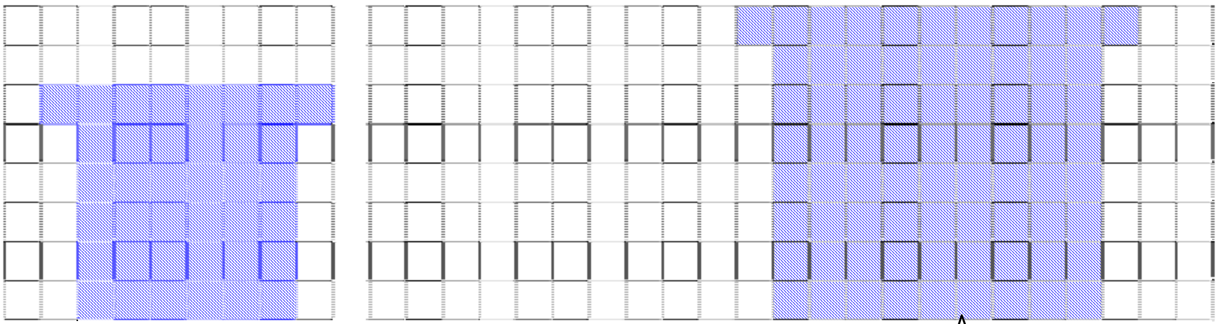
Despite the horrible notation and layout, this is an acceptable method **1 mark**

(b) Jayne creates these patterns by shading squares.



Show how Jayne can work out the number of squares in a pattern in **any** position in the sequence.

[3]



All she has to do is draw a square and add two squares. Then she counts up. Simple!

The diagrams show understanding of the rule that may be applied, as does the comment.
However, there is no attempt to link the understanding to the position number and UNIT is not mentioned.
1 mark

Other forms of Support

In order to help you implement the new GCSE Mathematics A Specification effectively, OCR offers a comprehensive package of support. This includes:

Published Resources

OCR offers centres a wealth of quality published support with a fantastic choice of 'Official Publisher Partner' and 'Approved Publication' resources, all endorsed by OCR for use with OCR specifications.

Publisher partners

OCR works in close collaboration with three Publisher Partners; Hodder Education, Heinemann and Oxford University Press (OUP) to ensure centres have access to:

- Better published support, available when you need it, tailored to OCR specifications
- Quality resources produced in consultation with OCR subject teams, which are linked to OCR's teacher support materials
- More resources for specifications with lower candidate entries
- Materials that are subject to a thorough quality assurance process to achieve endorsement

Oxford University Press is the publisher partner for OCR GCSE Mathematics Specification A.

OCR RECOGNISING ACHIEVEMENT
OXFORD UNIVERSITY PRESS
Official Publisher Partnership

OUP is producing the following resources for OCR GCSE Mathematics A for first teaching in September 2010, which will be available in Summer 2010.

Oxford GCSE Maths for OCR Foundation Student Book
Jayne Kranat and Mike Heylings
ISBN: 9780199139293
Published: 04 March 2010

Oxford GCSE Maths for OCR Higher Student Book
Jayne Kranat, Mike Heylings, Clare Plass and Marguerite Appleton
ISBN: 9780199139286
Published: 04 March 2010

Oxford GCSE Maths for OCR Foundation Teacher' s Guide
Neil Tully
ISBN: 9780199127283
Published: May 2010 (estimated)

Oxford GCSE Maths for OCR Higher Teacher' s Guide
Neil Tully

ISBN: 9780199127290
Published: May 2010 (estimated)

Oxford GCSE Maths for OCR Foundation Practice Book
David Rayner
ISBN: 9780199139309
Published: May 2010 (estimated)

Oxford GCSE Maths for OCR Higher Practice Book
David Rayner
ISBN: 9780199139316
Published: May 2010 (estimated)

Oxford GCSE Maths for OCR Interactive Ox Box CD-ROM
Various Authors
ISBN: 9780199139323
Published: June 2010 (estimated)

Oxford GCSE Maths for OCR Assessment Ox Box CD-ROM
Katie Wood and James Nicholson
ISBN: 9780199127306
Published: June 2010 (estimated)

Approved publications

OCR still endorses other publisher materials, which undergo a thorough quality assurance process to achieve endorsement. By offering a choice of endorsed materials, centres can be assured of quality support for all OCR qualifications.

Hodder is producing the following endorsed resources for OCR GCSE Mathematics A for first teaching in September 2010, which will be available in Summer 2010.

OCR GCSE Mathematics A - Higher Student's Book
ISBN: 9781444112795
Published: April 2010

OCR GCSE Mathematics A - Foundation Student's Book
ISBN: 9781444112801
Published: June 2010

OCR GCSE Mathematics A - Higher Teacher and Assessment Packs with CD-ROM
ISBN: 9781444115314
Published: June 2010

OCR GCSE Mathematics A - Foundation Teacher and Assessment Packs with CD-ROM
ISBN: 9781444115321
Published: July 2010

OCR GCSE Mathematics A – Higher Homework Book
ISBN: 9781444112818
Published: August 2010

OCR GCSE Mathematics A – Foundation Homework Book
ISBN: 9781444112825
Published: August 2010

Endorsement

OCR endorses a range of publisher materials to provide quality support for centres delivering its qualifications. You can be confident that materials branded with OCR's "Official Publishing Partner" or "Approved publication" logos have undergone a thorough quality assurance process to achieve endorsement. All responsibility for the content of the publisher's materials rests with the publisher.

These endorsements do not mean that the materials are the only suitable resources available or necessary to achieve an OCR qualification. Any resource lists which are produced by OCR shall include a range of appropriate texts.

OCR Training

A full range of training events provide valuable support, for the delivery and assessment of OCR qualifications:

Get Ready…

An overview of new OCR specifications

Get Started…

For teachers preparing to deliver or already delivering OCR specifications

Get Ahead…

For teachers wanting to improve delivery and assessment of a current OCR specification

Lead the way…

To encourage creativity and innovation

View up-to-date event details and make online bookings at www.ocreventbooker.org.uk or view our new training e-books at www.ocr.org.uk/training. If you are unable to find what you are looking for contact us by e-mail training@ocr.org.uk or telephone 02476 496398.

e-Communities

Over 70 e-Communities offer you a fast, dynamic communication channel to make contact with other subject specialists. Our online mailing list covers a wide range of subjects and enables you to share knowledge and views via email.

Visit https://community.ocr.org.uk, choose your community and join the discussion!

Interchange

OCR Interchange has been developed to help you to carry out day to day administration functions online, quickly and easily. The site allows you to register and enter candidates online. In addition, you can gain immediate free access to candidate information at your convenience. Sign up at https://interchange.ocr.org.uk

Acknowledgments

Page 77 Turkey image © www.istockphoto.com

Page 78 Cooking and Thawing Times © www.ukturkeys.co.uk. Permission granted by National Farmers' Union (NFU) www.nfuonline.com.

www.ocr.org.uk

OCR customer contact centre

Vocational qualifications
Telephone 024 76 851509
Facsimile 024 76 851633
Email vocational.qualifications@ocr.org.uk

General qualifications
Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk