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General Certificate of Secondary Education

Mathematics 3301
Specification A

Paper 2 Higher Tier

Mark Scheme

2006 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

The following abbreviations are used on the mark scheme:

M	Method marks awarded for a correct method.
A	Accuracy marks awarded when following on from a correct method. It is not necessary always to see the method. This can be implied.
B	Marks awarded independent of method.
M dep	A method mark which is dependent on a previous method mark being awarded.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special Case. Marks awarded for a common misinterpretation which has some mathematical worth.
oe	Or equivalent.
eeoo	Each error or omission.

Paper 2H

1	$5x > 7$, $5x > 10 - 3$	M1	Accept $5x \geq 7$ or $5x \geq 10 - 3$
	$x > \frac{7}{5}$	A1	oe eg Accept $x > 1.4$ or $x > 1\frac{2}{5}$, $\frac{7}{5} < x$ $x = 1.4$ after correct answer seen is incorrect further work so A0. $5x > 13 \Rightarrow x > 2.6$ **SC1**

2	Attempt to add at least 12 correct values from Stem and Leaf	M1	Values used **must** indicate that the candidates understands the stem and leaf notation.
	270	A1	
	(Mean =) 'their 270' ÷ 30	M1dep	Must divide by 30
	9	A1	No follow through

3(a)	Plot (50, 0.4)	B1	1mm tolerance
3(b)	27	B1	
3(c)	Yes, stated or implied, with a reason that uses a valid numerical value	B2	eg if fair expect 15 As in 60 spins Yes because 27 out of 60 is bigger than a $\frac{1}{4}$ B1 for yes, stated or implied, with a reason that is valid but does not use numerical values.
3(d)	$1000 \times (0.3 \text{ to } 0.5)$	M1	
	300 to 500	A1	
3(e)	$0.3 + 0.4$	M1	
	0.7	A1	oe

4	Sporty bar $3.4 \times \frac{100}{10.3}$	M1	oe
	33.(…..)	A1	
	Fruity bar $17.4 \times \frac{62.6}{100}$	M1	oe
	10.9	A1	Any correctly rounded accuracy 10.8924. Accept 11 with working.

5	Total reduction £6 + £1 (= £7)	M1	25% of £4 = £1, 25% of 40% = 10% 75% of 40% = 30%
	Hence Jill plus justification eg £7 is 70% of £10	A1	60% + 10% = 70%, 85% off £10 = £1.50

6	$1.7 \div 250$	M1	ie Digits '17' ÷ digits '25' eg $1700 \div 250$
	0.0068	A1	6.8 accept answer with digits '68' if consistent with units used. eg 0.068 kg/cl
	kg/cm^3	B1	Units consistent with working ie allow g/cm^3 if working and answer supports it NB If M0 awarded all B1 for **any** units of density.

7	$400 - 2 \times 80$	B1	240
	$2\pi r$ or πd or πr = anything except 400	M1	
	$240 \div \pi$	M1dep	$240 \div 2\pi$ (= 38.2)
	76.4, 76.39…..	A1	76 with working or 80 with working SC if 80 misread as total straight length then answer in range 101.8 – 101.9 **SC2**

8(a)	0.007	B1	
8(b)(i)	0.9119215(052)	B1	
8(b)(ii)	0.9, 0.91, 0.912, 9 or 9.1 or 9.12×10^{-1}	B1ft	ft their answer for (b)(i) to 1, 2 or 3sf eg Gradians (b)(i) 0.02221673729 B0 (b)(ii) 0.02, 2×10^{-2}, etc B1 ft
8(c)	0.00805 or 8.05×10^{-3}	B1	

9	Sight of 1.072	B1	7.2% of 2000 = 144
	(2000) × 'their 1.072'^{10}	M1 A1	Their 1.072 must be 1.72 or 1.0072 Calculating at least 5 intermediate values correctly **2144, 2298.37 (368), 2463.85(0496), 2641.25 (.247732), 2831.42 (.417568) M1** All 10 correct A1 3035.28(.279633), 3253.82(.819767), 3488.09 (.09479), 3739.24(.237615) 4008.46(.462723) No penalty for rounding or truncating to nearest pound or 1 decimal place. Truncated values 2144, 2298, 2463, 2640, 2830, 3033, 3251, 3485, 3735, 4004 (4003.82) Rounded values 2144, 2298, 2463, 2640, 2830, 3034, 3252, 3486, 3737, 4006 (4006.06) No penalty for incorrect money notation eg 4008.5 > 2 × 2000
	Yes 4008.(46) or 2.004(2..)	A1ft	ft if only one error made and relevant conclusion drawn. Accept $1.072^{10} > 2$ for 3/4 marks **NB** student who takes 2000 as year 1 gets to 3739 for year 10 and says 'no' 2/4 **SC** 2000×1.072^9 2/4 marks (B1, M1)

10(a)	$4x + 12 = 9x - 18$	M1	Allow one error
	$5x = 30$, $-30 = -5x$	A1ft	ft if M1 awarded and equation is in form $ax = b$ with no further errors
	$x = 6$	A1ft	Follow through only if M1 awarded for fully correct first line and one error made in rearranging so A0 awarded, and their equation of form $ax = b$ is solved correctly
10(b)	Attempt to balance x or y and eliminate by adding or subtracting	M1	eg $15x + 9y = 18$ $15x - 35y = 95$ Followed by an attempt to subtract $44y = -77$ or eg $35x + 21y = 42$ $9x - 21y = 57$ Followed by an attempt to add $44x = 99$ Award M1 for attempt to rearrange one equation and substitute into the other
	Solving resulting equation to find $x = 2.25$ or $y = -1.75$	A1	
	Attempt to eliminate other variable or substitution of found value into an equation	M1	**NB** Could start again eg $11.25 + 3y = 6$, $5x - 5.25 = 6$
	Solving to find other value $y = -1.75$ or $x = 2.25$	A1	

11(a)	$(x + 4)^2$	M1	
	$a = 4$, $b = -21$	A1	Do not award if b given as 21
11(a) **Alt.**	$= x^2 + 2ax + a^2 + b$ **and** $8 = 2a$ or $-5 = a^2 + b$	M1	
	$a = 4$, $b = -21$	A1	
11(b)	$x = \pm\sqrt{21} \pm 4$ or $x = \pm\sqrt{21} - 4$	M1	ft their 'a' if solvable for M1 and A1 T&I M0 unless both answers given.
	$x = 0.58, -8.58$	A1ft	Accept 0.583 $x = \sqrt{21} - 4 = 0.58$ **SC1** Allow SC1 on follow through for positive root only.
11(b) **Alt.**	$x = \dfrac{-8 \pm \sqrt{(8^2 - 4 \times (1) \times -5)}}{2}$	M1	Allow $x = \dfrac{-8 \pm \sqrt{(64 - 20)}}{2}$ as only error for M1
	$x = 0.58, -8.58$	A1	Accept 0.583

12(a)(i)	$10^2 - 5^2$ (= QR^2)	M1	
	(QR =) $\sqrt{75}$	M1dep	
	8.66(0…), $5\sqrt{3}$	A1	Accept 8.7 for 3/3 8.6 **only** implies M2
12(a)(ii)	Sight of cosine	M1	
	(Angle QPR =) $\cos^{-1}(5 \div 10)$ oe	M1dep	Alternative ratios using (a)(i) must be $\sin^{-1}$('(a)(i)' ÷ 10) M2 $\tan^{-1}$('(a)(i)' ÷ 5) M2 sine rule: sin$x$ = (anything × sin90 ÷ 10) M2
	60°	A1	No ft 60 seen with no working full marks.
12(b)	Area ADX = their (a)(i) × 2.5 oe	M1	= 21.65(0…) if exact value or 8.66 used = 21.75 if 8.7 used
	Area AXB = (any angle) ÷ 360 × $\pi \times 10^2$	M1	Accept 100π ÷ 3, 4, 5, 6, 8, 9, 10, 15, 20
	= 26.2, 26.18, 26.1799(3..)	A1ft	ft their angle
	Shaded area 50 – ('their 21.65' + 'their 26.18')	M1dep	Dependent on both Ms
	2.16 to 2.183 (if 8.66 used) 2.06 to 2.083 (if 8.7 used)	A1	Allow 2.2 with working using 8.66 Allow 2.1 with working using 8.7 NB accuracy is as stated, which allows for a range of values of π from 3.14 to 3.142 and final answer to 2sf or 3sf accuracy.

13	(Area =) $\frac{1}{2}x(x+1+x+2)$	M1	oe $(x+1)+\frac{1}{2}\times x\times(1)$
	$2x^2+3x-20=0$	A1	oe eg $x^2+1.5x-10=0$
	$(2x-5)(x+4)=0$	M1dep A1	M1 for an attempt at using an algebraic method such as factorising, formula (allow one error) or completing the square (allow one error) to solve the quadratic eg for $(2x+a)(x+b)$ where $ab=\pm 20$ A1 for a completely correct method
	$x=2.5$	A1	Do not award last A1 if a negative value given as possible answer eg if –4 given 2.5 seen with no or incomplete work **SC2** 2.5 after first M1, A1 give 5/5

14	Angle $ATB=22°$	B1	
	$\frac{BT}{\sin 48}=\frac{50}{\sin 22}$	M1	$\frac{AT}{\sin 110}=\frac{50}{\sin 22}$
	$BT=\frac{50\sin 48}{\sin 22}$	M1	$AT=\frac{50\sin 110}{\sin 22}$
	$BT=99$ or 99.19 or 99.2	A1	$AT=125$ or $125.4(\ldots)$ or better accuracy
	$h+60=$ 'their BT' $\times\sin 70$	M1dep	Dependent on previous use of sine rule. $h+60=$ 'their AT' $\times\sin 48$ oe $\frac{h+60}{\text{their}BT}=70$
	$h=33$, or 33.2 or 33.21	A1	or better accuracy
14 Alt.	$h+60=\tan 48\times(50+x)$	M1	oe x is distance from B to base of cliff
	$h+60=\tan 70\times x$	M1	oe
	$50\tan48=x(\tan70-\tan48)$	M1	oe
	$x=34$ or 33.9 or 33.92	A1	or better accuracy
	$h+60=$ 'their x' $\times\tan 70$	M1dep	Dependent on previous Ms $h=$ 'their x' $\times\tan 70-60$
	$h=33$, or 33.2 or 33.21	A1	or better accuracy

15(a)	$5x(x + 4)$	B1	
15(b)	$(x + 7)(x - 7)$	B1	
15(c)	M1 for expanding and collecting to general quad form, allow one error but expansions must have x^2 term, x term and constant term. Allow misuse of minus.	M1	eg $9x^2 + 24x + 16 - 4x^2 + 4x + 1$ Difference of two squares $((3x + 4) - (2x + 1)) \times ((3x + 4) + (2x + 1))$
	$5x^2 + 20x + 15$	A1	A1 for either $(x + 3)$ or $(5x + 5)$ if difference of 2 squares used.
	$5(x + 3)(x + 1)$	A1	Accept $(x + 3)(5x + 5)$ or $(5x + 15)(x + 1)$

16	$y(3x - 4) = xy + 2$	M1	$y \times 3x - 4 = xy + 2$ is M0 unless recovered
	$3xy - 4y = xy + 2$	A1	
	$2xy = 4y + 2$	M1dep	$3xy - xy = 4y + 2$ Allow one 'sign' error
	$x = \frac{2y+1}{y}$	A1	oe Do not award if $x =$ not written SC $x = \frac{3}{y}$ B2
16 Alt.	$y(3x - 4) = xy + 2$	M1	$y \times 3x - 4 = xy + 2$ is M0 unless recovered
	$3x - 4 = x + \frac{2}{y}$	A1	$3x - 4 = \frac{xy}{y} + \frac{2}{y}$
	$2x = 4 + \frac{2}{y}$	M1dep	$3x - x = 4 + \frac{2}{y}$ Allow one 'sign' error
	$x = 2 + \frac{1}{y}$	A1	oe Deduct mark if $x =$ not written SC $x = \frac{3}{y}$ B2

17	Attempt to find gradient of perpendicular line	M1	Must be negative reciprocal of their gradient for AB
	(Gradient =) $-\frac{2}{3}$	A1	oe eg –0.66, –0.67
	Use of midpoint (3, 1)	M1	Must be **used** either on the diagram with an attempt at a perpendicular or in $y = mx + c$ to find c.
	$y = -\frac{2}{3}x + 3$	A1ft	ft their gradient if first M1 awarded Accept equivalents eg $3y + 2x = 9$

18	*ABD* = 66 (Alt segment)	B1	or angles in triangle if *ADB* found first
	DCB = 104 (opposite in cyclic)	B1	In all alternatives, for first 3 B marks do not award B1 the first time no reason or wrong reason given, otherwise accept angles identified in answer or on diagram. **NB** Mark 'positively' ie, ignore wrong values or reasons unless totally contradictory.
	DBC = 38 (isosceles) *CBA* = 104	B1	
	CBA + *BAD* = 180 (interior)	B1	In all alternatives, reason must be given for final B1 Accept 'allied' or 'angles between parallel lines'. Dependent on correct angles.
18 Alt. 1	*ADB* = 38 (Alt segment)	B1	
	DCB = 104 (opposite in cyclic)	B1	
	CBD = 38 (isosceles)	B1	
	CBD = *ADB* (alternate)	B1	Use of 'Z angles' is not acceptable Dependent on correct angles
18 Alt. 2	*ADB* = 38 (Alt segment)	B1	
	DCB = 104 (opposite in cyclic)	B1	
	BDC = 38 (isosceles) *ADC* = 76	B1	
	BDC + *BCD* = 180 (interior)	B1	Dependent on correct angles
18 Alt. 3	*ADB* = 38 and *ABD* = 66 (Alt segment)	B1	
	DCB = 104 (opposite in cyclic)	B1	
	CBD = *CBD* = 38 (isosceles)	B1	
	DCB = CBA and CDA and BAD = (isosceles trapezium)	B1	

19(a)	$y = \cos x + 1$	B1	$y = 1 + \cos x$
19(b)	$y = 2\cos x$	B1	
19(c)	$y = \cos 2x$	B1	
19(d)	$y = \cos(90 - x)$, $y = \cos(x + 270)$ $y = \cos(x - 90)$ or $y = \sin x$	B1	

20(a)	Upper limit wattage = 2550	B1	Allow $2549.\dot{9}$
	400 000 ÷ 'Their upper limit'	M1	Attempt at an upper limit must be made
	156 lights	A1ft	ft their upper limit if 'truncated' eg 400000 ÷ 2549 = 156.92 is 1/3 if answer 157, 2/3 if answer 156. **NB** Check answer is from correct work. **SC** 400000 ÷ 2450 = 163 lights B2
20(b)	Upper limit I 25.5 And lower limit W 2450	B1	Allow $25.4\dot{9}$
	(R =) $2450 \div 25.5^2$	M1	ft their limits for M1 only if limits attempted and their lower W is divided by their upper I squared
	(R =) 3.77, 3.767(….)	A1ft	ft their limits only if the lower limit of 2450 is divided by 'their upper limit of I' squared. Follow through must be given to at least 3sf. eg $2450 \div 25.49^2 = 3.7707..$ is 2/3. **NB** make sure answer is from correct work. **SC** $400000 \div 25.5^2 = 615$ lights B1

21	Radius A = 1.5cm Radius B = 2.5 cm	B1	These must be clearly stated or implied (eg 2.25π, 6.25π) at some stage in solution. **NB** Scale factor such as 0.6, 1.66, $\frac{5}{3}$ implies B1 **NB** Check pie charts for these.
	1980 (population A)	B1	
	$6.25(\pi) \div 2.25(\pi)$	M1	M1 for attempt to 'compare' areas. eg 25:9, 2.777…
	5500 population in B	A1	Allow 5450 – 5550
	1375	A1	
21 Alt.	Radius A = 1.5cm Radius B = 2.5cm	B1	These must be clearly stated or implied (eg 2.25π, 6.25π) at some stage in solution. **NB** Scale factor such as 0.6, 1.66, $\frac{5}{3}$ implies B1 **NB** Check pie charts for these.
	Compares a population to an area for village A	M1	eg $660 \equiv (=) 2.35619449..$ $660 \div \frac{1}{3} \times 2.25\pi$ $1980:2.25\pi$
	Finds a value for person per area or area per person	A1	275 – 285 people per cm^2 $3.5 - 3.6 \times 10^{-3} cm^2$ per person
	Calculates area of quadrant in B and either multiplies or divides by appropriate value	M1	
	1375	A1	