

### **General Certificate of Secondary Education**

## Mathematics 3301 Specification A

Paper 1 Higher Tier

# Mark Scheme

#### 2006 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

#### Paper 1H

 $2\frac{8}{15}$ 

| -            |                                                                      |      |                                                                                                                                         |
|--------------|----------------------------------------------------------------------|------|-----------------------------------------------------------------------------------------------------------------------------------------|
| 1(a)         | $n^2 + n$ or $n(n+1)$ or $n \times (n+1)$<br>or $(n+1) \times n$     | B2   | B1 for $n^2 \dots$ or brackets omitted                                                                                                  |
| 1(b)         | 42                                                                   | B2   | B1 10 and 11 seen                                                                                                                       |
| 2(a)         | 7x - 14y - 6x + 3y                                                   | M1   | Allow one error                                                                                                                         |
|              | x-11y                                                                | A1   |                                                                                                                                         |
| 2(b)         | 24 - x = 15                                                          | M1   | Allow $24 - x = 3 \times 5$                                                                                                             |
|              | 9                                                                    | A1   |                                                                                                                                         |
| 2(c)(i)      | (y-2)(y-3)                                                           | B2   | B1 $(y \pm 1)(y \pm 6)$ or $(y \pm 2)(y \pm 3)$                                                                                         |
| 2(c)(ii)     | 2 <u>and</u> 3                                                       | B1ft |                                                                                                                                         |
| <b>2(d)</b>  | $\frac{(x+3)}{4}$                                                    | B2   | B1 for each of numerator and denominator                                                                                                |
|              | 4                                                                    |      | If denominator = 4, then numerator must be an algebraic expression to earn B1                                                           |
|              |                                                                      |      | B1 for $\frac{2}{8}(x+3)$ $\frac{2(x+6)}{8}$ $\frac{(x+3)_2}{4(x+3)}$                                                                   |
| 2(e)         | $16m^{12}p^4$                                                        | B2   | - 1 eeoo                                                                                                                                |
| <b>3</b> (a) | $\frac{1}{1.5}$                                                      | M1   | $\frac{1}{\frac{3}{2}}$ and 1 ÷ 1.5 both earn M1                                                                                        |
|              | $\frac{2}{3}$                                                        | A1   | SC1 for $-\frac{2}{3}$                                                                                                                  |
| 3(b)         | 10 (÷) 0.2                                                           | M1   | Both approximations correct                                                                                                             |
|              | 50                                                                   | A1   |                                                                                                                                         |
|              | 7                                                                    | A1   | Allow 7.1                                                                                                                               |
| 3(c)         | 3 <u>and</u> common denominator                                      | M1   | or $\frac{21}{5} - \frac{5}{3}$ 1.66(6)                                                                                                 |
|              | $(3 +) \frac{3}{15} - \frac{10}{15}$<br>Allow one error in numerator | M1   | or $\frac{63}{15} - \frac{25}{15}$ 4.2<br>Allow a total of 1 error in <u>either</u> 1 <sup>st</sup><br><u>or</u> 2 <sup>nd</sup> M mark |

| 4(a) | Reflection        | B1 |                                                                               |
|------|-------------------|----|-------------------------------------------------------------------------------|
|      | y = x             | B1 | oe                                                                            |
| 4(b) | Rotation          | B1 | Allow turn                                                                    |
|      | 90° anticlockwise | B1 | oe $\frac{1}{4}$ turn anticlockwise scores B2<br>$\frac{1}{4}$ turn scores B1 |
|      | (Centre) (1,1)    | B1 |                                                                               |
| 4(c) | Enlargement       | B1 | No combined transformations                                                   |
|      | Scale factor –2   | B1 |                                                                               |
|      | Centre (0,0)      | B1 |                                                                               |

| 5(a) | 0.833(3) <u>or</u> 0.875 and 0.9 | M1 | Allow percentages                                                                 |
|------|----------------------------------|----|-----------------------------------------------------------------------------------|
|      | 0.166(6) <u>or</u> 0.125 and 0.1 |    | or fractions with denominators with prime factors of 2 and/or 5 only terminate oe |
|      | $\frac{5}{6}$                    | A1 | Must see working                                                                  |
| 5(b) | Attempt at 3 ÷ 11                | M1 | Answer attempted to 2dp,<br>(Accept error in 2 <sup>nd</sup> dp )                 |
|      | 0.2727(27)                       | A1 | Minimum of 4dp or recurring notation<br>SC1 sight of digits 27                    |

| 6(a) | (2.5, 1), (7.5, 2), (12.5, 7),<br>(17.5, 9), (22.5, 7), (27.5, 4)<br>joined within 1 small square,<br>straight lines attempted                    | B2 | B1 One error <u>or</u> not joined <u>or</u> joined with curve<br>SC1 for consistent plots at lcb or ucb                                                                                                                                                                                                                                                                                        |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------|----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6(b) | Correct comparison of average<br><u>and</u> spread,<br>or<br>Correct comparison of average or<br>spread <u>and</u> one other valid<br>observation | B2 | <ul> <li>eg Students average time larger oe<br/>Allow eg in general, on average, overall<br/>Spread of student times larger oe<br/>Allow eg larger range, more varied</li> <li>Other valid observations</li> <li>eg More students watch from 15 to 25 h<br/>Same number (7) watch from 10 to 15 h</li> <li>B1 one correct comparison of average/spread<br/>or one valid observation</li> </ul> |

| 7(a) | $7 \times 10^5$                                                 | B2 | B1 35 000 000 ÷ 50 (: 1), or<br>their 700 000 (: 1), or<br>their 700 000 in correct standard form |
|------|-----------------------------------------------------------------|----|---------------------------------------------------------------------------------------------------|
| 7(b) | $3.5\times10^3\div10^6$                                         | M1 | or 3 500 ÷ 1 000 000 or 0.0035                                                                    |
|      | $3.5 \times 10^{-3}$                                            | A1 |                                                                                                   |
|      |                                                                 |    | -                                                                                                 |
| 8    | $1 \rightarrow D$                                               | B1 | $1 \longrightarrow y \ge 2x - 4$                                                                  |
|      | $2 \rightarrow C$                                               | B1 | $2 \longrightarrow y \ge -2x + 4$                                                                 |
|      | $3 \rightarrow E$                                               | B1 | $3 \rightarrow y \leq 2x - 4$                                                                     |
|      | $4 \rightarrow A$                                               | B1 | $4 \longrightarrow y \le -\frac{1}{2}x + 2$                                                       |
|      |                                                                 |    |                                                                                                   |
| 9    | $2s = (u + v)t$ $s/t = \frac{1}{2}(u + v)$                      | M1 | $2s = (u + v)t \qquad M1$                                                                         |
|      | 2                                                               |    | Look out for missing bracket then recovery                                                        |
|      |                                                                 |    | s = ut/2 + vt/2                                                                                   |
|      | 2s (111                                                         | M1 | 2s = ut + vt $s - vt/2 = ut/2$ M1                                                                 |
|      | $\frac{2s}{t} = u + v \qquad s/t - \frac{1}{2}v = \frac{1}{2}u$ |    | (must rearrange for M1)                                                                           |
|      | $\frac{2s}{1} - v = u$                                          | A1 | 2s - vt = ut M1 $2(s - vt/2) = ut$ M1                                                             |
|      | t                                                               |    | $(2s - vt) / t = u$ A1 $\frac{2}{t}(s - vt/2) = u$ A1                                             |
|      |                                                                 |    | Look out for equivalent answers                                                                   |
|      |                                                                 |    | eg $u = (\frac{1}{2}vt - s) \div (-\frac{1}{2}t)$ $u = s \div t \div 0.5 - v$                     |

| 10 | Sight of correct ratio or scale factor ie 20 : 30, 2 : 3, $\frac{2}{3}$ , $1\frac{1}{2}$ | M1 | oe sight of $\frac{3}{5}$ or $\frac{2}{5}$ earns this mark                                                                             |
|----|------------------------------------------------------------------------------------------|----|----------------------------------------------------------------------------------------------------------------------------------------|
|    | $\frac{3}{5} \times 40$                                                                  | M1 | oe eg might work out $\frac{2}{5}$ then subtract                                                                                       |
|    | 24                                                                                       | A1 | Note 2:3 ratio might be scaled up to give<br>ratio of 16:24 (M1, M1)                                                                   |
|    |                                                                                          |    | Must state $h = 24$ for A1                                                                                                             |
|    |                                                                                          |    | Alternatively $h/30 = (40 - h)/20$ M1                                                                                                  |
|    |                                                                                          |    | 20h = 30(40 - h)                                                                                                                       |
|    |                                                                                          |    | 20h = 1200 - 30h M1                                                                                                                    |
|    |                                                                                          |    | 50h = 1200                                                                                                                             |
|    |                                                                                          |    | <i>h</i> = 24 A1                                                                                                                       |
| 11 | Men: 4 5 4 2                                                                             |    | Eully correct 4 marks M2 A1                                                                                                            |
| 11 | Men: 4 5 4 2<br>Women: 6 4 7 3                                                           |    | Fully correct4 marks, M3, A1Slight slip, attempts to round from<br>correct working allow 1 or 2 errors only<br>3 marks, M1, M1, M1, A0 |
|    |                                                                                          |    | eg Men: $3 6 4 2$<br>Women: $6 4 7 3$                                                                                                  |
|    |                                                                                          |    | Correct figures but not rounded allow $\underline{1}$ or $\underline{2}$ errors only $2$ marks, M1, M1, M0, A0                         |
|    |                                                                                          |    | eg Men: 3.6 <u>5.6</u> 4.2 1.8                                                                                                         |
|    |                                                                                          |    | Women: 5.8 <u>3.6</u> 7.0 3.4                                                                                                          |
|    |                                                                                          |    | Correct proportions but no 10% calculation <u>must be correct</u> 1 mark, M1, M0, M0, A0                                               |
|    |                                                                                          |    | eg Men: 36 54 42 18                                                                                                                    |
|    |                                                                                          |    | Women: 58 38 70 34                                                                                                                     |
|    |                                                                                          |    | SC Men <u>or</u> Women <u>fully</u> correct<br>(more than 2 errors on other one) scores B2<br>(with only <u>1</u> error) scores B1     |

| 12    | YZ = ZY                                                                                         | B1    |                                                                                                                                                    |
|-------|-------------------------------------------------------------------------------------------------|-------|----------------------------------------------------------------------------------------------------------------------------------------------------|
|       | Angle <i>MZY</i> = angle <i>NYZ</i>                                                             | B1    | Note Reason necessary                                                                                                                              |
|       | base angles of (Isosceles) $\Delta XYZ$                                                         |       | eg you might see                                                                                                                                   |
|       |                                                                                                 |       | If $XZ = XY$ then angle $XZY =$ angle $XYZ$                                                                                                        |
|       | Angle $MYZ$ = angle $NZY$                                                                       | B1    |                                                                                                                                                    |
|       | Triangles congruent, ASA                                                                        | B1dep | Note Dependent on earning first 3 marks                                                                                                            |
|       |                                                                                                 |       | Must give correct reason for congruence (ASA)                                                                                                      |
|       |                                                                                                 |       | Only allow AAS if complete argument stating 'third angles equal'                                                                                   |
| 13    | Attempt at $y = x - 1$                                                                          | M1    | ' <i>m</i> ' or ' <i>c</i> ' correct $(y = -1 \text{ scores } M0)$                                                                                 |
|       |                                                                                                 |       | Table of values seen with at least one pair<br>correct, with attempt at line, earns M1                                                             |
|       | Correct ruled line                                                                              | A1    |                                                                                                                                                    |
|       | $-2.6 \le x \le -2.5$                                                                           | Alft  | ft their line, <b>two</b> solutions only, tolerance of                                                                                             |
|       | and $1.5 \le x \le 1.6$                                                                         |       | $\pm 0.05$                                                                                                                                         |
| 14(a) | Correct Pythagoras in two<br>appropriate right-angled triangles                                 | M1    | or simply $BH^2 = 12^2 + 3^2 + 4^2$                                                                                                                |
|       | 13                                                                                              | A1    |                                                                                                                                                    |
| 14(b) | $HB = 13, HC = 5 \text{ or } DB = \sqrt{153}$                                                   | M1    | Explanations may not involve any calculations                                                                                                      |
|       | with attempt at trig. Ratio                                                                     |       | eg $BC < BD$ or $HC > HD$ together with<br>some comparison such as $BH$ is common                                                                  |
|       |                                                                                                 |       | (diagrams drawn, to illustrate, are appropriate)                                                                                                   |
|       | Two correct, comparable trig.<br>ratios<br>eg sin $x = \frac{4}{13}$ and sin $y = \frac{5}{13}$ | Al    | For example: <i>BH</i> is common and triangles<br><i>BHD</i> and <i>BHC</i> are right-angled, so y must be<br>bigger because the height is greater |
|       | У                                                                                               | A1    | Good explanation and correct conclusion<br>this earns all 3 marks                                                                                  |

| 15(a)     | Probabilities of 0.7, 0.3, 0.4, 0.6 on '2 <sup>nd</sup> game' column    | B1    | SC1 either 'top half' or 'bottom half' correct                                                                    |
|-----------|-------------------------------------------------------------------------|-------|-------------------------------------------------------------------------------------------------------------------|
|           | Probabilities of 0.4, 0.6, 0.7, 0.3<br>on '3 <sup>rd</sup> game' column | B1    |                                                                                                                   |
| 15(b)     | 0.5 	imes 0.7                                                           | M1    | ft their probabilities (if using values < 1)<br>(not 0.5)                                                         |
|           | $0.5 \times 0.3 \times 0.4$ or $0.5 \times 0.4 \times 0.7$              | M1    | ft as above                                                                                                       |
|           | addition of three <u>valid</u><br>probabilities                         | M1dep | ft their values<br>( <b>note</b> dependent on first two marks)                                                    |
|           | 0.55                                                                    | A1    | If probabilities are all 0.5, and there is a <u>correct</u> <u>attempt</u> at one of the alternatives, award SC 1 |
|           |                                                                         |       | If they work out the prob. that Simon wins,                                                                       |
|           |                                                                         |       | $0.5 \times 0.6$ (M1) ( <u>ft their probs. in all of these</u> )                                                  |
|           |                                                                         |       | $0.5\times0.4\times0.3$ or $0.5\times0.3\times0.6$ (M1)                                                           |
|           |                                                                         |       | 1 - (addition of valid three) (M1dep)                                                                             |
|           |                                                                         |       | 0.55 (A1)                                                                                                         |
| 16(a)(i)  | w <sup>8</sup>                                                          | B1    |                                                                                                                   |
| 16(a)(ii) | w <sup>6</sup>                                                          | B1    |                                                                                                                   |

| 16(a)(ii)  | W <sup>6</sup>  | B1 |                                                                |
|------------|-----------------|----|----------------------------------------------------------------|
| 16(a)(iii) | w <sup>12</sup> | B1 |                                                                |
| 16(b)(i)   | $\frac{x}{y}$   | B1 | Allow $x \div y$ or $x \times y^{-1}$                          |
| 16(b)(ii)  | $x^2$           | B1 | Allow $x \times x$                                             |
| 16(b)(iii) | 9 <i>y</i>      | B1 | Allow $y9  9 \times y  y \times 9  3^2 \times y  y \times 3^2$ |

|              | [                                                                                                                                         |       |                                                                                                                                                          |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 17           | Substitute one pair of data into $t = k \sqrt{m}$ , $t = k / m$ or $t = k / \sqrt{m}$                                                     | M1    | Look for valid alternative methods which will still earn this mark                                                                                       |
|              |                                                                                                                                           |       | eg The first rule might be eliminated by<br>reasoning that the relationship must be an<br>inverse one since as <i>t</i> increases, <i>m</i><br>decreases |
|              | Test one of the rules to reach a conclusion                                                                                               | M1    | ie Test value of k found from first pair on<br>another pair or find a contradictory value<br>of k                                                        |
|              | Test a second rule to reach a conclusion                                                                                                  | M1    | Repeat as above (2-stage process)                                                                                                                        |
|              | Select correct rule (C)                                                                                                                   | A1    | If testing C only, must use all three pairs otherwise max of M1 M1 M0 A0                                                                                 |
| 19(-)        | <b>either</b> $12 + \sqrt{12}\sqrt{3} + \sqrt{12}\sqrt{3} + 3$                                                                            |       |                                                                                                                                                          |
| <b>18(a)</b> | either $12 + \sqrt{12}\sqrt{3} + \sqrt{12}\sqrt{3} + 3$                                                                                   | M1    | Allow $\sqrt{12}\sqrt{12} + \sqrt{12}\sqrt{3} + \sqrt{12}\sqrt{3} + \sqrt{3}\sqrt{3}$                                                                    |
|              |                                                                                                                                           |       | Allow one error (might see $\sqrt{36}$ used here)                                                                                                        |
|              | $\sqrt{12}\sqrt{3} = 2\sqrt{3}\sqrt{3} = 6 \rightarrow \text{sum} = 27$ $\sqrt{12}\sqrt{3} = (\sqrt{36}) = 6 \rightarrow \text{sum} = 27$ | A1    | <u>Clearly</u> shown, must see surds used correctly                                                                                                      |
|              | or $\sqrt{12} = 2\sqrt{3}$<br>and $\sqrt{12} + \sqrt{3} = 3\sqrt{3}$                                                                      | M1    | Expanding $(2\sqrt{3} + \sqrt{3})^2$ , allowing one error, also earns this mark                                                                          |
|              | $(3\sqrt{3})^2 = 9 \times 3 = 27$                                                                                                         | A1    | $2\sqrt{3}\sqrt{3} = 6$ must be seen eventually to earn A1                                                                                               |
| 18(b)(i)     | $(\sqrt{8} + \sqrt{2})^2 = 18$                                                                                                            | M1 A1 | Marked as in Part (a)                                                                                                                                    |
|              | Use of Pythagoras in $\Delta PQS$                                                                                                         | M1    | or Subtraction of $QS^2$ from $PQ^2$ with their                                                                                                          |
|              | ie $(\sqrt{12} + \sqrt{3})^2 - (\sqrt{8} + \sqrt{2})^2$                                                                                   |       | value for $QS^2$                                                                                                                                         |
|              | $PS^2 = 9$                                                                                                                                | A1    | Must be clearly shown ( $PS = 3 \text{ cm}$ , given)                                                                                                     |
| 18(b)(ii)    | $\sqrt{8} + \sqrt{2} + \sqrt{2} = 4\sqrt{2}$                                                                                              | B1    | Could be seen at any stage                                                                                                                               |
|              | Area = $\frac{1}{2} \times (\sqrt{8} + \sqrt{2} + \sqrt{2}) \times 3$                                                                     | M1    | Could be $\frac{1}{2} \times (\sqrt{8} + \sqrt{2}) \times 3 + \frac{1}{2} \times \sqrt{2} \times 3$                                                      |
|              | 6√2                                                                                                                                       | A1    |                                                                                                                                                          |

| 19(a) | $\frac{4}{3} \pi r^{3} = 2 \times \frac{1}{3} \pi r^{2} x$ | M1 | Must include the factor of 2<br>Allow use of <i>h</i> instead of <i>x</i>                                                                                                                                                  |
|-------|------------------------------------------------------------|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|       | Simplified to give $x = 2r$                                | A1 | Alternatively<br>Allow substitution of 2r for height of cone and<br>verification of result<br>ie 2 × Vol cone = $2 \times \frac{1}{3} \times \pi \times r^{2} \times 2r$ M1<br>= $\frac{4}{3} \pi r^{3}$ (must be seen) A1 |
| 19(b) | $(l)^{2} = r^{2} + 4r^{2}$                                 | M1 | $(l)^2 = r^2 + (2r)^2$ is M1 $(l)^2 = r^2 + 2r^2$ is M0                                                                                                                                                                    |
|       | $(l) = \sqrt{5} r$                                         | A1 |                                                                                                                                                                                                                            |
|       | Surface area cone = $\pi \times r \times \sqrt{5} r$       | M1 | Using their <i>l</i> if from an attempt at Pythagoras                                                                                                                                                                      |
|       | 4 : √5                                                     | A1 | Allow $\sqrt{5}$ : 4<br>SC2 for a complete numerical solution                                                                                                                                                              |