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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Secondary Education

Mathematics 4306 and 4307
Specifications A (Linear) and B (Modular)
2009

TEACHER'S GUIDE

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Background Information

1 Introduction

Following a review of the National Curriculum requirements, and the establishment of the National Qualifications Framework, all the unitary awarding bodies revised their GCSE specifications for first examination in 2003. These specifications were revised for examination in 2008 to incorporate necessary changes in moving to two tiers of assessment. The 2008 specifications were revised for examination in 2009 to incorporate necessary changes following the withdrawal of coursework. The new specifications will be used by schools and colleges for two-year courses starting in September 2007.

1.1 Purpose

This Teachers' Guide has been provided to assist teachers in their preparation for the delivery of courses based on the revised AQA GCSE specifications 4306 and 4307. The guide should be read in conjunction with the specification document and the specimen material that accompany them. All documents are available in hard copy. The specifications and specimen assessment materials are also available on the AQA website (www.aqa.org.uk).

1.2 Requirements at GCSE

Tiering

In GCSE Mathematics the scheme of assessment must include question papers targeted at two tiers of grades: A* – D (Higher), and C – G (Foundation).

Candidates should be entered at the tier appropriate to their attainment. In GCSE Mathematics (Modular) each candidate may enter for each individual module at a different tier of entry. However, the final range of grades available to a candidate is determined by the tier of entry for Module 5. Candidates who fail to achieve the mark for the lowest grade available at each tier of Module 5 will be recorded as unclassified (U).

Key Skills

All GCSE specifications must identify, as appropriate, opportunities for generating evidence on which candidates may be assessed in the 'main' Key Skills of *Communication, Application of Number* and *Information and Communication Technology* at the appropriate level(s). Also, where appropriate, they must identify opportunities for developing and generating evidence for addressing the 'wider' Key Skills of *Working with Others, Improving own Learning and Performance* and *Problem Solving*.

Whilst opportunities for delivering Key Skills have been highlighted in the specifications. Section 8 of this Teachers' Guide provides more detail of how they could be delivered and where evidence could be collected.

1.3 Use of ICT

The subject content of all GCSEs must require candidates to make effective use of ICT and provide, where appropriate, assessment opportunities for ICT.

Within this specification candidates will have opportunities to apply and develop their ICT capabilities through the practical use of ICT tools to support their learning. The level to which the use of ICT is developed will depend in part upon the teaching styles and methods used to deliver the subject content, the opportunities available at the centre and the abilities of the candidates.

However, ICT skills are not assessed by any component of this specification.

Suitable opportunities may be found, within each section of the subject content, to use ICT skills to find and develop information and present findings in a variety of appropriate formats. Candidates should be provided with opportunities to support their work by:

- Using the Internet/CD-ROMS/databases/software packages to obtain, select and manipulate information;
- Presenting results from investigations using ICT tools to amend and refine their work and enhance its quality and accuracy;

The National Curriculum for Mathematics identifies opportunities for the use of ICT as candidates learn Mathematics. The references for these opportunities are given below. Each statement is referenced to the corresponding statement in the Foundation or Higher Programme of Study in the National Curriculum. Thus, F2.1a refers to AO2, Foundation Programme of Study, statement 1a.

Specification Reference	**ICT Opportunity**
F2.5f	Candidates could use a spreadsheet to construct formulae to model situations
F2.6d	Candidates could use a spreadsheet to calculate points and draw graphs to explore the effects of varying m and c in the graph of $y = mx + c$
F4.5c	Candidates could use databases to present their findings
H2.6b – H2.6f	Candidates could generate functions from plots of data, (for example, from a science experiment) using simple curve fitting techniques on graphic calculators, or with graphics software
H2.6g	Candidates could use software to explore transformations of graphs
H3.3b – H3.3f	Candidates could use software to explore transformations and their effects on properties of shapes
H4.1c	Candidates could use databases or spreadsheets to present their findings and display their data
H4.5c	Candidates could use databases to present their findings

1.4 Citizenship

Since 2002, students in England have been required to study Citizenship as a national curriculum subject. Each GCSE specification must signpost, where appropriate, opportunities for developing citizenship knowledge, skills and understanding.

Tasks, particularly those for AO4 *Handling Data*, promote the skills of enquiry and communication. They also encourage the skill of participation and responsible action in the educational establishment and/or communication.

Specification Reference	**Aspect of Citizenship**
F3.1e; H3.1d; H3.1f; H3.1j; F4.1a; H4.1a; F4.1b; F4.1e; H4.1d; F4.1i; H4.1f; H4.1g; F4.5a; H4.5a; F4.5b; F4.5c; H4.5c; F4.5d; H4.5d; F4.5e; H4.5e; F4.5f; F4.5g; H4.5g; F4.5h; H4.5h; F4.5j	*Thinking skills,* through helping candidates to engage in social issues that require the use of reasoning, understanding and action through enquiry and evaluation

Specification Reference	**Aspect of Citizenship**
F2.2e; H2.2e; F2.3c; F2.3e; H.23e; F2.3m; H2.3j; F2.3n; H2.3j; H2.3k; F2.3q; H2.3p; F2.4a; F2.4c; F2.5f; H3.4a	*Financial capability,* through developing candidates' understanding of the nature and role of money
F2.2e; H2.2e; F2.3c; F2.3e; H2.3e; F2.3m; H2.3j; F2.3n; H2.3j; H2.3k	*Enterprise and entrepreneurial skills,* through developing candidates' understanding of the importance of these skills for a thriving economy and democracy
F3.4a; F3.4i	*Work related learning,* through helping candidates to appreciate the link between learning and work for a thriving economy and society
H3.3t; F2.6e	*Education for sustainable development,* through developing candidates' skills in, and commitment to, effective participation in the democratic and other decision-making processes that affect the quality, structure and health of environments and society and exploring values that determine people's actions within society, the economy and the environment.

2

Specification at a Glance
Mathematics A

- AQA offers two GCSE Mathematics specifications. Specification A is a traditional linear scheme and is suitable for both pre-16 and post-16 candidates.
- There are two tiers of assessment, Foundation (C-G) and Higher (A*-D).

GCSE Mathematics A (4306)	
Paper 1	
Written Paper (Non-Calculator)	50% of total marks
Foundation Tier	1 hour 30 minutes
Higher Tier	2 hours
Paper 2	
Written Paper (Calculator)	50% of total marks
Foundation Tier	1 hour 30 minutes
Higher Tier	2 hours

Specification at a Glance
Mathematics B (Modular)

AQA offers two GCSE Mathematics specifications. Specification B is modular and is suitable for both pre-16 and post-16 candidates.

- There are two tiers of assessment, Foundation (C – G) and Higher (A* – D).

GCSE Mathematics B (4307)	
Module 1	
Written Paper	18% of total marks
2 x 30 minutes (Both tiers)	
Section A – Calculator Section B – Non-calculator	
Module 3	
Written Paper	27% of total marks
2 x 45 minutes (Both tiers)	
Section A – Calculator Section B – Non-calculator	
Module 5	
Written Paper	55% of total marks
Paper 1 Non-Calculator	
Foundation Tier	1 hour 15 minutes
Higher Tier	1 hour 15 minutes
Paper 2 Calculator	
Foundation Tier	1 hour 15 minutes
Higher Tier	1 hour 15 minutes

Foundation Tier Higher Tier
Modules 1, 3 and 5 are available in both tiers

Scheme of Assessment

3 External Assessment Issues

3.1 National Criteria

Both specifications comply with the following:

The GCSE Subject Criteria for Mathematics;

The GCSE and GCE A/AS Code of Practice;

The GCSE Qualification Specific Criteria;

The Arrangements for the Statutory Regulation of External Qualifications in England, Wales and Northern Ireland: Common Criteria.

3.2 Marks per Grade

The Foundation Tier will have 50% of the assessment targeted at grades G and F and 50% targeted at grades E, D and C.

50%		50%		
G	F	E	D	C

The Higher Tier will have 50% of the assessment targeted at grades D and C and 50% targeted at grades B, A and A*.

50%		50%		
D	C	B	A	A*

Units Mark

One question per tier will have the additional requirement that candidates 'State the units of your answer'.

This will be clearly written in the question.

Note that due to the modular nature if Specification B candidates may meet this requirement more than once. In Specification A it will occur once per pair of papers at each Tier.

Candidates will simply be required to state the units of the answer. A mark will be lost if they do not do this.

This mark is independent of the mathematics required to solve the question.

3.3 Accuracy

The accuracy to which candidates give answers causes problems. In general there is no accuracy requirement unless stated. Answers given to 3 significant figures are always acceptable. The answers to Pythagoras and Trigonometry problems, in particular, could be given to 3 significant figures.

However, there are situations where accuracy is important.

Foundation Tier

Candidates will be required to round off a given value or a calculated answer to a given accuracy.

The required accuracies will be one, two or three decimal places or one significant figure.

Higher Tier

One question per tier will have the additional requirement that candidates 'Give your answer to a suitable/sensible/appropriate degree of accuracy'.

This will be clearly written in the question.

Note that due to the modular nature of Specification B candidates may meet this requirement more than once. In Specification A it will occur once per pair of papers at each Tier.

This requirement assesses candidates' ability to give an answer to an accuracy that is sensible in the context of the problem. For example, questions about astronomical distances do not need answers given to several decimal places.

As a general guide, when this requirement is asked for, candidates should give answers to the same (or fewer) number of significant figures as the numbers used in the question.

3.4 Premature Rounding

Candidates often write intermediate values down, particularly in multi-step questions. These values, taken from a calculator display are sometimes rounded to 2 or 3 significant figures. Further calculation using these values (for example, using inverse sine with a 2 significant figure value) can lead to the final answers being outside the acceptable range of accuracy as determined by the examiner and the subsequent loss of a mark.

Candidates are advised to use the values as shown in the calculator display or to write down intermediate values to at least 4 significant figures. This will then ensure that the final answer will be within the acceptable range of accuracy.

3.5 Specification B – Division of content across modules

For the most part, division of content across the modules is clearly defined in the specification. In the Teachers' Guide, the module in which a particular specification reference is assessed is shown in the third column of the subject content guidance which begins on page 13. In a number of cases, more than one module is given alongside a particular reference. This is generally because a strict assignment of the reference to one module would lead to artificial and undesirable divisions in the subject, which we have sought to avoid. In the interests of clarity, the following notes attempt to give some detail on how and where these references will be assessed.

It should be noted that throughout the question papers, knowledge of the Key Stage 3 curriculum is assumed and, across the Higher tier, knowledge of the Foundation content is assumed.

3.6 Subject Content Guidance

The following two sections give guidance for the interpretation of the subject content for the Foundation and Higher Tiers in Specifications A and B. This guidance is given in the form of notes on the National Curriculum statements.

The first column gives the reference to the appropriate statement in the Foundation or Higher Programme of Study in the National Curriculum. Thus, F2.1a refers to AO2, Foundation Programme of Study, statement 1a. Column two gives the corresponding statement of subject content, column three gives the Specification B module(s) in which the statement will be assessed and column 4 provides guidance in the form of notes on the statement, which are applicable to both specifications.

Foundation Tier Guidance

AO2 Number and algebra

1. Using and applying handling data

Pupils should be taught to:

Foundation Tier	Notes

Problem solving

F2.1a	select and use suitable problem-solving strategies and efficient techniques to solve numerical and algebraic problems	B3, B5	Mini-investigations will not be set but candidates will be expected to make decisions and use the appropriate techniques to solve a problem. Candidates should choose relevant information when some is redundant.
H2.1b	identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches	B3, B5	Candidates will be expected to give reasons for answers or to show working. If a question states, "You **must** show your working", marks will be lost if the instruction is ignored.
F2.1b H2.1c	break down a complex calculation into simpler steps before attempting to solve it	B3, B5	Multi-step problems will be set. At the lowest grades there will normally only be one interim step to be identified. At the higher grade problems may require more than one interim step.

Foundation Tier		Notes	
F2.1c	use algebra to formulate and solve a simple problem – identifying the variable, setting up an equation, solving the equation and interpreting the solution in the context of the problem	B5	Candidates should understand the role of a letter as an unknown, in setting up expressions and solving equations. eg, The angles of a triangle are $2x$, $x + 30$ and $x + 70$. Find the value of x. (Diagram given). eg, Jo is 3 years older than Sam. The sum of their ages is 15. By forming an equation find Jo's age.
F2.1d	make mental estimates of the answers to calculations; use checking procedures, including use of inverse operations; work to stated levels of accuracy	B3	Candidates should be able to round to one significant figure to make mental estimates of calculations. Candidates should be able to round answers to given accuracies of 1, 2 and 3 decimal places and 1 significant figure.

Communicating

F2.1e	interpret and discuss numerical and algebraic information presented in a variety of forms	B3, B5	Candidates will be expected to understand problems in context. eg, Find the cost of 26 tins of polish costing 73p each.
F2.1f	use notation and symbols correctly and consistently within a given problem	B5	Correct use of numerical and algebraic notation is expected. For example $a + a = a2$ will not be accepted
F2.1g	use a range of strategies to create numerical, algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem	B3, B5	Candidates should be able to interpret for example, diagrams, information given in a real-life context, such as a 'Sale' poster and translate this into a mathematical problem.
F2.1h	present and interpret solutions in the context of the original problem	B3, B5	Candidates will be required to give sensible answers to questions. eg, How many 4-seater taxis are needed to carry 14 passengers?
F2.1i	review and justify their choice of mathematical presentation	B3, B5	eg, Candidates should be able to explain patterns in words.

Foundation Tier			Notes

Reasoning

F2.1j	explore, identify, and use pattern and symmetry in algebraic contexts investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems	B5	For example, using simple codes that substitute numbers for letters. See further guidance on counter examples given in section on proof (Appendix C).
F2.1k	show step-by-step deduction in solving a problem	B3, B5	Candidates should always show their working, particularly in multi-step and using and applying questions, where marks are allocated for candidates showing a strategy to solve the problem.
F2.1l	understand the difference between a practical demonstration and a proof	B5	See section on proof (Appendix C).
F2.1m	recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem	B5	Candidates should assume that information given is exact unless the question states or implies otherwise.

2. Numbers and the number system

Pupils should be taught to:

	Foundation Tier		Notes
	Integers		
F2.2a H2.2a	use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use positive numbers and negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition	B3, B5	Candidates could be asked to round a number to the nearest whole number, 10, 100 or 1000. Abbreviations will not be used in examinations. The word 'least' will be used. Candidates will be expected to identify eg, multiples, factors and prime number from lists. There is no requirement that candidates use the prime factor decomposition method to solve HCF and LCM problems. Candidates can write out lists of multiples and factors to identify common factors and multiples.

Foundation Tier	Notes

Powers and roots

F2.2b H2.2b	use the terms square, positive and negative square root, cube and cube root; use index notation for squares, cubes and powers of 10; use index laws for multiplication and division of integer powers; express standard index form both in conventional notation and on a calculator display	B3, B5	Questions will not ask for 'positive square root' but will use the notation $\sqrt{25}$. When a square root is asked for, only the positive value will be required. However, candidates are expected to know that a square root can be negative. If the solution to $x^2 = 25$ is required then both the negative and positive root are expected. Powers of 10 will be used only up to 10^6 Values of simple integer powers eg, 2^4 will be tested. The words cube, square and cube root may be used and should be understood. Candidates may be asked to write 7^5 x 7^2 or $7^5 \div 7^2$ as a single power of 7. Candidates are not expected to know the definition of standard form but should be aware that calculator displays can sometimes show values such as 1.7×10^{-3} or 1.7^{-03} and know how to interpret these.

Fractions

F2.2c	understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator	B3, B5	Candidates may be asked to give a fractional answer in its simplest form. When this requirement is not clearly stated candidates do not have to cancel fractional answers. Candidates will require a basic knowledge of mixed numbers. The term 'common denominator' will not be used.

Foundation Tier			Notes

Decimals

F2.2d H2.2d	use decimal notation and recognise that each terminating decimal is a fraction recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals.	B3, B5	eg, $0.137 = \frac{137}{1000}$ eg, $\frac{1}{7} = 0.142857142857\ldots$ Candidates should know that $0.\dot{3} = \frac{1}{3}$ and $0.\dot{6} = \frac{2}{3}$ and that other fractions give recurring decimals and know how to write eg, $\frac{1}{6} = 0.16666\ldots.$ as $0.1\dot{6}$

Percentages

F2.2e	understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of '; use percentage in real-life situations.	B3, B5	eg, 10% means 10 parts per 100, and 15% of Y means $\frac{15}{100} \times Y$ x% of y will be required. VAT rates will be provided. Percentage problems on non-calculator papers will involve percentages that can be worked out using multiples of 1% and 10%. Some basic knowledge of percentages in every day life eg, commerce and business including rate of inflation, VAT, price index, interest rates and financial capability is required. Note that problems involving simple interest for more than one year and compound interest will not be set but that interest on investments over 1 year could be assessed.

Foundation Tier			Notes

Ratio

F2.2f	use ratio notation, including reduction to its simplest form and its various links to fraction notation	B3	Candidates should be familiar with ratio notation (eg, 2:3) and should know how to reduce to simplest form. Questions asking for the ratio in the form 1:n may be required. Candidates should know that if, say, red balls and blue balls are in the ratio 3:4 then the fraction of red balls is $\frac{3}{7}$

3. Calculations

Pupils should be taught to:

	Foundation Tier		Notes
	Number operations and the relationships between them		
F2.3a H2.3a	add, subtract, multiply and divide integers and then any number; multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers; use inverse operations	B3, B5	The following should be known: table facts up to 10×10; squares up to 15×15; non-calculator methods for adding and subtracting 3 digit numbers; non-calculator methods for multiplying and dividing up to 3 digit numbers by up to 2 digit numbers. Candidates should be able to interpret a remainder from a division problem. Multiplication and division of integers and decimals by powers of 10 will be restricted to 10, 100 and 1000. Questions may involve negative integers eg, Find the reciprocal of $\frac{2}{3}$, 0.8 eg, Express 48 as the product of prime factors

Foundation Tier		Notes	
F2.3b	use brackets and the hierarchy of operations	B3, B5	The BIDMAS or BODMAS convention should be known but the mnemonic need not be known. Candidate could be asked to insert brackets into a calculation to make it true.
F2.3c	calculate a given fraction of a given quantity expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal	B3	For example, for scale drawings and construction of models, down payments, discounts. eg, Work out $\frac{3}{8}$ of 56 (non-calculator question) eg, Find $\frac{2}{7}$ of 5467 (calculator question) Questions involving mixed numbers may be set. eg, Work out $1\frac{2}{5} + \frac{3}{4}$ eg, Work out $3\frac{5}{6} - 2\frac{1}{2}$ eg, Find $\frac{3}{8}$ of 520. (non-calculator question) eg, Find $\frac{2}{7}$ of 520 people. (calculator question with answer given to sensible accuracy)

Foundation Tier		Notes	
F2.3d H2.3d	understand and use unit fractions as multiplicative inverses multiply and divide a fraction by an integer, by a unit fraction and by a general fraction	B3	eg, thinking of multiplication by $\frac{1}{5}$ as division by 5. eg, $4 \times \frac{7}{8}$, $\frac{6}{11} \div 3$ eg, $\frac{3}{4} \times \frac{8}{9}$, $\frac{4}{15} \div \frac{4}{15}$ Multiplication and division problems with mixed numbers will not be assessed.
F2.3e	convert simple fractions of a whole to percentages of the whole and vice versa then understand the multiplicative nature of percentages as operators	B3	The use of multipliers is expected although other methods are acceptable. eg, 32% of £80 = 0.32×80 For example, a 15% increase in value Y, is calculated as $1.15 \times Y$
F2.3f	divide a quantity in a given ratio	B3	eg, Divide £500 in ratio 3:2

Foundation Tier	Notes

Mental methods

F2.3g H2.3g	recall all positive integer complements to 100; recall all multiplication facts to 10 × 10, and use them to derive quickly the corresponding division facts; recall integer squares from 11 × 11 to 15 × 15 and the corresponding square roots, recall the cubes of 2, 3, 4, 5 and 10, and the fraction-to-decimal conversion of familiar simple fractions	B3, B5	For example 37 + 63 = 100 It will be acceptable to give the decimal equivalent of $\frac{1}{3}$ and $\frac{2}{3}$ as 0.33 and 0.66 or 0.67 For example, decimal and percentage equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{8}$ should be known. Mental methods will not be tested in the written papers but a quick recall of basic number facts is expected.
F2.3h	round to the nearest integer and to one significant figure; estimate answers to problems involving decimals	B3	Candidates should be able to round off a calculator display. Money answers should be written as £3.60 not £3.6 Candidates should be able to round to a given number of decimal places and also to the nearest 10, 100 or 1000. When estimating with whole numbers or decimal numbers less than 1, the numbers should be rounded to 1s.f. before the estimation is done.
F2.3i	develop a range of strategies for mental calculation; add and subtract mentally numbers with up to one decimal place; multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments	B3	Knowledge of the terms 'commutative', 'associative' and 'distributive' is not required and the term 'factorise' in the context of number need not be known. eg, $\frac{2000}{0.4} = \frac{20000}{4} = 5000$

Foundation Tier			Notes

Written methods

F2.3j	use standard column procedures for addition and subtraction of integers and decimals	B3	Candidates may use any algorithm in non-calculator papers.
F2.3k	use standard column procedures for multiplication of integers and decimals, understanding where to position the decimal point by considering what happens if they multiply equivalent fractions; solve a problem involving division by a decimal (up to 2 d.p.) by transforming it to a problem involving division by an integer	B3	Candidates may use any algorithm in non-calculator papers. eg, $408 \div 0.17 = 40800 \div 17$ eg, $0.02 \times 0.3 = 0.006$, $0.4^2 = 0.16$
F2.3l	use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer	B3	If asked to calculate $\frac{4}{9} \times \frac{3}{8}$, candidates will not be penalised if they fail to cancel before carrying out the multiplication. They may not gain full marks, however, if they fail to cancel $\frac{12}{72}$
F2.3m	solve simple percentage problems, including increase and decrease	B3	For example, simple interest, VAT, annual rate of inflation, income tax, discounts. Use of a multiplier is expected, although other methods will be accepted. eg, Decrease 76 kg by 12% is calculated as 0.88×76

Foundation Tier		Notes	
F2.3n	solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution	B3	For example, given that m identical items cost £y, then one item costs £$\frac{y}{m}$ and n items cost £ $(n \times \frac{y}{m})$, the number of items that can be bought for £z is $z \times \frac{m}{y}$ eg, 8 pencils cost £2.40. How much do 11 cost? (non-calculator question) eg, 8 pencils cost £2.56. How many can be bought for £4.80? (calculator question)
H2.3n	use π in exact calculations, without a calculator	B3	Candidates should be aware that 6π is an exact answer and $6 \times 3.14159\ldots$ is an estimate. In the non-calculator papers candidates could be asked to give the area of a circle of radius 3cm in terms of π.

Calculator methods

F2.3o	use calculators effectively and efficiently; know how to enter complex calculations and use function keys for reciprocals, squares and powers	B3, B5	eg, Work out $3.2 + 5.4^2$ eg, Work out $3.2 + \frac{1}{5.4}$
F2.3p	enter a range of calculations, including those involving standard index form and measures	B3	For example, time calculations in which fractions of an hour must be entered as fractions or as decimals. Candidates will not be expected to calculate with Standard Form numbers or enter them into a calculator.

Foundation Tier		Notes	
F2.3q H2.3p	understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation	B3	To avoid problems with premature rounding candidates should use the full calculator display. If they wish to write down an intermediate value during a calculation it should be to at least 4 significant figures.

4. Solving numerical problems

Pupils should be taught to:

Foundation Tier		Notes	
F2.4a H2.4a	draw on their knowledge of operations, inverse operations and the relationships between them, and of simple integer powers and their corresponding roots, and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, fractions, percentages and measures and conversion between measures, and compound measures defined within a particular situation	B3, B5	The terms 'commutative', 'associative' and 'distributive' will not be used in the examination. Knowledge of the term 'root' is required. Knowledge of the term 'inverse operation' is required and candidates should know the inverse operations of the four rules: square, square root, cube and cube root. Compound measures may be expressed in the form metres per second, m/s, $m\ s^{-1}$. Candidates would be expected to understand speed and know the relationship between speed, distance and time. Units may be any of those in common usage such as miles per hour or metres per second. The values used in the question will make the required unit clear. Other compound measures that are non-standard would be defined in the question eg, population density is population/km^2. Conversion between measures would involve knowledge of the connection between metric units. Conversions between imperial units will be given.

Foundation Tier		Notes	
F2.4b	select appropriate operations, methods and strategies to solve number problems, including trial and improvement where a more efficient method to find the solution is not obvious	B3	Trial and improvement should not be used when a standard algorithm is expected.
F2.4c H2.4b	estimate answers to problems; use a variety of checking procedures, including working the problem backwards, and considering whether a result is of the right order of magnitude	B3	eg, The heights of 7 men are 150, 151, 148, 133, 138, 142, 140 cm. Give a reason why 143.143 cm is not an appropriate answer for their mean height. For example if the answer to $4x = 2$ is calculated as 2, checking should show that $4 \times 2 = 8 \neq 2$.
F2.4d	give solutions in the context of the problem to an appropriate degree of accuracy, interpreting the solution shown on a calculator display, and recognising limitations on the accuracy of data and measurements	B3	Candidates should be able to round to a given degree of accuracy. This could be 1 s.f., 1, 2 or 3 d.p, to the nearest penny, etc. An understanding that rounded values may be inaccurate by up to half a unit is necessary.

5. Equations, formulae and identities

Pupils should be taught to:

	Foundation Tier		Notes
Use of symbols			
F2.5a H2.5a	distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations defined quantities or variables in formulae, general, unspecified and independent numbers in identities, and in functions they define new expressions or quantities by referring to known quantities.	B5	eg, knowing that $5x + 1 = 16$ is an equation. eg, knowing that $x^2 + 1 = 82$ is an equation. eg, knowing that $V = IR$ is a formula. eg, knowing that $3x + 2x \equiv 5x$, for all values of x is an identity. eg, knowing that $(x + 1)^2 \equiv x^2 + 2x + 1$, for all values of x is an identity. Knowledge of the identity symbol $\equiv$ is required. eg, knowing that $y = 2x$ is a function. eg, knowing that $y = 7 - 2x$ is a function. Candidates will be expected to know the standard convention such as $2x$ for $2 \times x$ and $\frac{1}{2}x$ or $\frac{x}{2}$ Candidates who write $2 \times x$, $x \times 2$, $x \div 2$ or $x/2$ will not be penalised, but $x2$ will not be accepted for $2x$. eg, Write an expression for the cost of x sweets at 12p each.

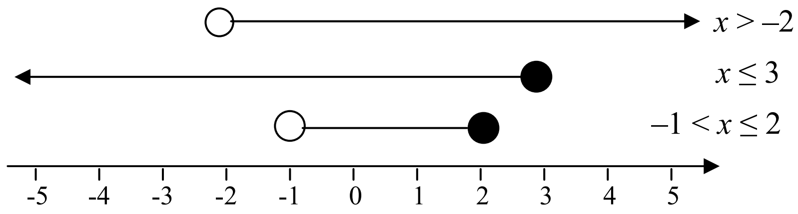
Foundation Tier		Notes	
F2.5b H2.5b	understand that the transformation of algebraic expressions obeys and generalises the rules of generalised arithmetic, expand the product of two linear expressions; manipulate algebraic expressions by collecting like terms by multiplying a single term over a bracket, and by taking out common factors; distinguish in meaning between the words 'equation', 'formula', 'identity' and 'expression'	B5	For example, $x + 5 - 2x - 1 = 4 - x$ Candidates will be expected to know the term 'factorise' in the context of an algebraic simplification. eg, Factorise $2a + 4b$ eg, Simplify $2x + 2 - 2x + 5$ eg, Simplify $2(x - 2) + 5(2x - 3)$ eg, The expression $x^2 + 2x + 1$ has three terms. The equation $3x - 2 = 7$ can be solved, $A = lb$ is a formula. $3n + 2n \equiv 5n$ as it is true for all n. Knowledge of the identity symbol $\equiv$ is required. eg, Expand and simplify $(x + 3)(x - 4)$ Candidates will be expected to know the meaning of 'solve' in relation to linear equations. (eg, Solve the equation $2x - 15 = 3$)

Index notation

F2.5c	use index notation for simple integer powers, and simple instances of index laws substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$	B5	eg, Evaluate 2^5 Evaluate $2x^3$ when $x = -2$, $3x^3 + 4$ when $x = \frac{1}{2}$ or -3 For example, $x^3 \times x^2 = x^5$; $x^8 \div x^2 = x^6$

Foundation Tier			Notes

Inequalities

F2.5d	solve simple linear inequalities in one variable, and represent the solution set on a number line	B5	Candidates should know the difference between >, <, ≤ and ≥ For example Solve $2x + 3 < 7$ Candidates should know the convention of an open circle for a strict inequality and a closed circle for an included boundary. eg, 

Equations

H2.5e	set up simple equations, solve simple equations by using inverse operations or by transforming both sides in the same way	B5	For example, find the angle a in a triangle with angles a, $a + 10$, $a + 20$ For example, $5x = 7$; $11 - 4x = 2$; $3(2x + 1) = 8$, $2(1 - x) = 6(2 + x)$, $4x^2 = 49$, $3 = \frac{12}{x}$

Foundation Tier	Notes

Linear Equations

F2.5e	solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution	B5	eg, Solve $7 - 5x = 20$ eg, Solve $5x + 7 = 2$ eg, Solve $2(x - 4) = 7$ eg, Solve $6x + 7 = x + 13$ eg, Solve $5x + 17 = 3(x + 6)$ eg, Solve $\frac{15 - x}{4} = 2$

Formulae

F2.5f	use formulae from mathematics and other subjects expressed initially in words and then using letters and symbols; substitute numbers into a formula; derive a formula and change its subject	B5	eg, formula for area of a triangle, area of a parallelogram, area of a circle, wage earned = hours worked × hourly rate or wage earned = hours worked × hourly rate plus bonus, volume of a prism. Substitute numbers into a formula; derive a formula (eg, convert temperature between degrees Fahrenheit and degrees Celsius, find the perimeter of a rectangle given its area A and the length l of one side). Formula to be rearranged will need at most two operations to rearrange. Formula where a power of the subject appears will not be required. eg, Rearrange $x + y = 7$ to make x the subject. eg, Rearrange $C = 2\pi r$ to make r the subject eg, Rearrange $y = 2x + 3$ to make x the subject.

Foundation Tier	Notes

Numerical Methods

H2.5m	use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them	B5	For example, Solve $x^3 - x = 900$ Solve $\frac{1}{x} = x^2 - 5$ Answers will be expected to 1 d.p. Candidates will be expected to test the mid-value of the 1 d.p. interval to establish which 1 d.p. value is nearest to the solution.

6. Sequences, functions and graphs

Pupils should be taught to:

Foundation Tier	Notes

Sequences

F2.6a H2.6a	generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); use linear expressions to describe the nth term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated	B5	Candidates should be able to explain how a sequence continues. The nth terms of linear sequences will be required. Candidates will **not** be expected to find the nth term of a non-linear sequence. However, candidates should be familiar with the idea of a non-linear sequence and the fact that the nth term can be generated by an expression of the form $\frac{1}{2}n(n + 1)$, for example. They should also know that the *n*th term of the square number sequence is given by n^2. Candidates should be able to generate simple sequences of eg, odd or even numbers, square integers and sequences derived from diagrams.

Foundation Tier			Notes

Graphs of linear functions

F2.6b	use the conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for m and c) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which y is given explicitly in terms of x or implicitly.	B5	eg, explicitly such as $y = 2x + 3$ or implicitly such as $x + y = 5$. Partially completed tables of values may sometimes be given but candidates should be able to plot the graph of $y = 3x - 1$, say, with no further assistance. Knowledge that m is the gradient and c is the y-intercept will be expected. Candidates will not be expected to find the equation of a given line.
F2.6c	construct linear functions from real-life problems and plot their corresponding graphs; discuss and interpret graphs modelling real situations; understand that the point of intersection of two different lines in the same two variables that simultaneously describe a real situation is the solution to the simultaneous equations represented by the lines; draw line of best fit through a set of linearly related points and find its equation	B5	For example, currency conversion graphs, distance-time graphs, graphs describing trends, graphs of height or weight against age, graphs of quantities that vary against time, such as employment, graphs of costs of units of gas. Candidates should be able to read from graphs. For example, find the cost of a bill for so many units of gas or find the number of units for a given cost. They should understand that the intercept in such a graph represents the fixed charge, for example. Candidates will be expected to find the velocity for sections of a distance time graph and should understand that the steeper the line the faster the speed.

Interpret graphical information

F2.6e	interpret information presented in a range of linear and non-linear graphs	B5	Candidates should be able to interpret graphs showing real-life situations such as the depth of water in containers as they are filled at a steady rate. Candidates may be given non-linear graphs from real life situations to interpret eg, the height of a ball plotted against time.

Foundation Tier			Notes

Gradients

F2.6d	find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for m and c); investigate the gradients of parallel lines	B5	Candidates will not be expected to find the equation of a line when the graph is given but should know that lines of the form $y = 2x + c$, for example, are parallel and lines of the form $y = ax + 1$ all pass through the same point on the y-axis.

Quadratic functions

H2.6e	generate points and plot graphs of simple quadratic functions, then more general quadratic functions find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function	B5	eg, $y = x^2$, $y = 3x^2 + 4$ eg, $y = x^2 - 2x + 1$ If candidates are required to draw a graph then a table may be given in which some y values may have to be calculated. Quadratic graphs are expected to be drawn as a curve. Candidates will be expected to know that the roots of an equation $f(x) = 0$ can be found where the graph of the function intersects the x-axis and that the solution of $f(x) = a$ is found where $y = a$ intersects with $f(x)$.

AO3: Shape, space and measures

1. Using and applying shape, space and measures

Pupils should be taught to:

	Foundation Tier		Notes
	Problem solving		
F3.1a H3.1a	select problem-solving strategies and resources, including ICT tools, to use in geometrical work, and monitor their effectiveness; consider and explain the extent to which the selections they made were appropriate	B5	Mini-investigations will not be set but candidates will be expected to make decisions and use the appropriate techniques to solve a problem drawing on well known facts, such as the sum of angles in a triangle.
F3.1b	select and combine known facts and problem-solving strategies to solve complex problems	B5	Multi-step problems will be set. eg, Find the base angle of an isosceles triangle (apex angle and diagram given).
F3.1c H3.1c	identify what further information is needed to solve a geometrical problem; break complex problems down into a series of tasks; develop and follow alternative lines of enquiry	B5	Redundant information may sometimes be used, for example, the slant height of a parallelogram. Candidates should be able to identify which information given is needed to solve the given problem.

Foundation Tier			Notes

Communicating

F3.1d	interpret, discuss and synthesise geometrical information presented in a variety of forms	B5	Candidates will be expected to interpret information from diagrams such as angles, equal lines marked, parallel lines marked,
F3.1e H3.1d	communicate mathematically with emphasis on a critical examination of the presentation and organisation of results, and on effective use of symbols and geometrical diagrams	B5	Candidates will be expected to use correct mathematical notation and produce a logical solution to a given problem.
F3.1f	use geometrical language appropriately	B5	Use of colloquial terminology may be penalised. For example, words such as 'flip' will not be accepted for reflection.
F3.1g	review and justify their choices of mathematics presentation	B5	Candidates should be able to choose an appropriate method when several methods are possible.

Reasoning

F3.1h	distinguish between practical demonstrations and proofs	B5	Candidates may be required to give specific examples or more general proofs
F3.1i	apply mathematical reasoning, explaining and justifying inferences and deductions	B5	See further guidance given in section on proof (Appendix C).
F3.1j	show step-by-step deduction in solving a geometrical problem	B5	Candidates should be able to explain reasons using words or diagrams
F3.1k	state constraints and give starting points when making deductions	B5	Candidates should realise when an answer is inappropriate.

Foundation Tier		Notes	
F3.1l	recognise the limitations of any assumptions that are made; understand the effects that varying the assumptions may have on the solution	B5	eg, Candidates should assume that lengths and angles given are exact unless the question states otherwise.
F3.1m	identify exceptional cases when solving geometrical problems	B5	

2. Geometrical reasoning

Pupils should be taught to:

Angles

F3.2a	recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex	B5	Candidates should be able to justify an answer with explanations such as 'angles on a straight line', etc. The notations angle ABC, $\angle ABC$ or $A\hat{B}C$ will be used, although questions might sometimes refer to angle B where there is no ambiguity.
F3.2b	distinguish between acute, obtuse, reflex and right angles; estimate the size of an angle in degrees	B5	Candidates should know and understand the terms acute, obtuse, reflex and right angle.

Foundation Tier	Notes

Properties of triangles and other rectilinear shapes

F3.2c H3.2a	distinguish between lines and line segments; use parallel lines, alternate angles and corresponding angles; understand the consequent properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices	B5	Explanations should use the correct terminology. Colloquial terms such as 'Z', 'F' angles are not acceptable as reasons. Acceptable terms for the interior, co-interior or allied See further guidance on proof (Appendix C). *Candidates should know that a straight line drawn between two points or vertices is a line segment.*
F3.2d	use angle properties of equilateral, isosceles and right-angled triangles; understand congruence; explain why the angle sum of a quadrilateral is 360 degrees	B5	Candidates should be able to recognise congruent shapes when rotated, reflected or in different orientations. Angle sum of a quadrilateral is based on the angle sum of a triangle which can be taken as a fact.
F3.2e	use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle	B5	Candidates will not be expected to deduce the formula for area of rectangles, parallelograms and triangles in examinations but will be expected to know them. Questions involving compound shapes made up of rectangles and triangles will be assessed.
F3.2f H3.2c	recall the essential properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties	B5	Questions may be set that test candidates' knowledge of these properties. The properties of a kite should also be known. The formula for the area of a trapezium is given on the formula sheet. Candidates should know the side, angle and diagonal properties of quadrilaterals.

Foundation Tier		Notes	
F3.2g	calculate and use the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons; calculate and use the angles of regular polygons.	B5	Candidates should learn or know how to work out the angle sum of polygons up to a hexagon. Questions involving the interior and exterior angles of regular polygons may be set. Questions could include octagons and decagons
F3.2h	understand, recall and use Pythagoras' theorem	B5	Questions may be set in context, for example, a ladder against a wall, but questions will always include a diagram of a right angled triangle with two sides marked and the third side to be found. Quoting the formula will not gain credit. It must be used with the appropriate numbers, eg, $x^2 = 7^2 + 5^2$, $x^2 = 12^2 - 9^2$ or $x^2 + 9^2 = 12^2$

Properties of circles

F3.2i	recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that inscribed regular polygons can be constructed by equal division of a circle	B5	Questions asking for the angle at the centre of a regular polygon may be set.

3-D shapes

F3.2j	explore the geometry of cuboids (including cubes), and shapes made from cuboids	B5	Candidates should understand Planes of symmetry of a cuboid and be able to find the surface area of a cuboid. Questions could include simple isometric drawing of cuboids (including cubes) and shapes made from cuboids

Foundation Tier		Notes	
F3.2k H3.2i	use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms and cylinders	B5	Isometric drawings should be understood. Dotted lines will indicate hidden edges. Answers in terms of π may be required. (non-calculator question) Questions asking for the surface area of a cylinder will not be set but candidates should have an understanding of the net of a cylinder. The volume of a prism is given on the formula sheet. Formula for the volume of a cylinder is not given. Candidates should know this.

3. Transformations and coordinates

Pupils should be taught to:

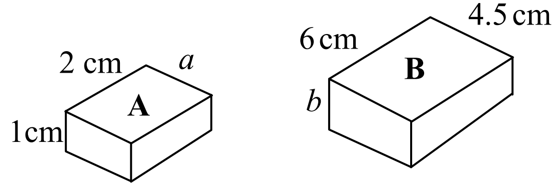
Specifying transformations

F3.3a H3.3a	understand that rotations are specified by a centre and an (anticlockwise) angle; rotate a shape about the origin, or any other point; measure the angle of rotation using right angles, simple fractions of a turn or degrees; understand that reflections are specified by a mirror line, at first using a line parallel to an axis, then a mirror line such as $y = x$ or $y = -x$; understand that translations are specified by a distance and direction(or a vector), and enlargements by a centre and positive scale factor	B5	The direction of rotation will always be given. Column vector notation should be understood. Lines of symmetry will be restricted to $x = a$, $y = a$, $y = x$ and $y = -x$ Scale factors for enlargements can be fractional.

Foundation Tier			Notes

Properties of transformations

F3.3b H3.3b	recognise and visualise rotations, reflections and translations, including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations, recognising that these transformations preserve length and angle, so that any figure is congruent to its image under any of these transformations; distinguish properties that are preserved under particular transformations	B5	When describing transformations, the minimum requirement is Reflection described by a mirror line Translations described by a vector or a clear description such as 3 squares to the right, 5 squares down. Rotations described by centre, direction (unless a half turn) and an amount of turn (as a fraction of a whole or in degrees). Candidates will always be asked to describe a **single** transformation but could be asked to do a combined transformation on a single shape. Questions will be set on line symmetry and order of rotational symmetry, including tessellations.
F3.3c	recognise, visualise and construct enlargements of objects using positive scale factors greater than one, then positive scale factors less than one; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not	B5	Scale factors for enlargements will be restricted to positive integers or simple unit fractions. Enlargements may be drawn on a grid, or on a coordinate grid, where the centre of enlargement will always be at a point of intersection of two grid lines. Questions assessing the properties of similar shapes, including similar triangles, will not be set.

Foundation Tier		Notes	
F3.3d	recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments and apply this to triangles; understand the implications of enlargement for perimeter; use and interpret maps and scale drawings; understand the implications of enlargement for area and for volume; distinguish between formulae for perimeter, area and volume by considering dimensions; understand and use simple examples of the relationship between enlargement and areas and volumes of shapes and solids	B5	Scales will be given as, for example, 1 cm represents 10 km, or 1:100. Area and volume scale factors are not required. Questions involving the effect of enlargement on area and volume will involve a diagram. eg, These boxes are similar.  (a) What is the length a ? (b) What is the length b ? (c) What is the ratio of the volume of box A to box B ? Candidates should be able to distinguish between formula for length, area and volume. eg, Given three formulae, identify one as a length, one as an area and one as a volume. Formula used will be within the experience of candidates such as $2l + 2w$, $\pi r^2 h$ etc.

Foundation Tier			Notes

Coordinates

F3.3e	understand that one coordinate identifies a point on a number line, two coordinates identify a point in a plane and three coordinates identify a point in space, using the terms '1-D', '2-D' and '3-D'; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; find the coordinates of points identified by geometrical information; find the coordinates of the midpoint of the line segment *AB*, given points *A* and *B*, then calculate the length *AB*.	B5	Questions asking for the mid-point will always be accompanied by a grid. The formula $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ need not be known. For example, find the coordinates of the fourth vertex of a parallelogram with vertices at (2, 1) (–7, 3) and (5, 6). eg, identify the coordinates of the vertex of a cuboid on a 3-D grid

Vectors

F3.3f	understand and use vector notation for translations	B5	eg, Candidates could be asked to translate a shape by $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

4. Measures and construction

Pupils should be taught to:

	Foundation Tier		Notes

Measures

F3.4a	interpret scales on a range of measuring instruments, including those for time and mass; know that measurements using real numbers depend on the choice of unit; recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction; convert measurements from one unit to another; know rough metric equivalents of pounds, feet, miles, pints and gallons; make sensible estimates of a range of measures in everyday settings	B5	Conversion between measures would involve knowledge of the connection between metric units. Conversions between imperial units will be given but the rough metric equivalents to common imperial measures should be known. These will be restricted to 8 km $\approx$ 5 miles, 1 litre $\approx$ 1.75 pints, 1 kg $\approx$ 2.2 lbs, 1 gallon $\approx$ 4.5 litres, 1 foot $\approx$ 30 cm. Other conversions will be given in the question eg, Give the upper and lower limits of a length of 11 cm measured to the nearest centimetre. Lower limit = 10.5 cm, Upper limit = 11.5 cm. The notation $10.5 \leq \text{length} < 11.5$ should be understood.
F3.4b	understand angle measure using the associated language	B5	For example, use bearings to specify direction. Bearings will always be given as a 3-figure bearing. The eight points of the compass (N, NE, E, SE, S, SW, W, NW) and their equivalent bearings should be known

Foundation Tier		Notes	
F3.4c H3.4a	understand and use compound measures, including speed and density	B5	For example, what distance is covered travelling at 40mph for 3 hours. Speed may be expressed in the form metres per second, (m/s). Candidates would be expected to understand these, and also units in common usage such as miles per hour (mph) or kilometres per hour (km/h). Calculations involving distance or time will be restricted to $\frac{1}{4}$ hour, $\frac{1}{3}$ hour, $\frac{1}{2}$ hour , $\frac{2}{3}$ hour or a whole number of hours.

Construction

F3.4d	measure and draw lines to the nearest millimetre, and angles to the nearest degree; draw triangles and other 2-D shapes using a ruler and protractor, given information about their side lengths and angles; understand, from their experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; construct cubes, regular tetrahedral, square-based pyramids and other 3-D shapes from given information	B5	Knowledge of SSS, SAS, ASA and RHS terminology will not be required but candidates should be able to recognise when two triangles are congruent. Candidates will be expected to draw a net of a 3-D shape and also to recognise a shape from a given net. eg, Construct a triangle with side of 6 cm, 7 cm and 8 cm When constructing triangles, compasses should be used to measure lengths rather than rulers. Construction arcs need to be seen for full marks to be awarded.
F3.4e	use straight edge and compasses to do standard constructions, including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle	B5	Candidates will be expected to show clear evidence that a straight edge and compasses have been used to do constructions.

Foundation Tier	Notes

Mensuration

F3.4f	find areas of rectangles, recalling the formula, understanding the connection to counting squares and how it extends this approach; recall and use the formulae for the area of a parallelogram and a triangle; find the surface area of simple shapes using the area formulae for triangles and rectangles; calculate perimeters and areas of shapes made from triangles and rectangles	B5	Questions on area and perimeter using compound shapes formed from two or more rectangles may be set. Questions may include the perimeter of simple shapes Questions may include areas of parallelograms and trapezia
F3.4g	find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids	B5	The formula $V = lwh$ should be known, The formula Volume of prism = cross-sectional area × length is given on the formula sheet.
F3.4h	find circumferences of circles and areas enclosed by circles, recalling relevant formulae	B5	Circumference and area formula for a circle should be known. Perimeters and areas of semi-circles or simple fractions of a circle, eg, quarter circles, could be assessed.
F3.4i	convert between area measures, including square centimetres and square metres, and volume measures, including cubic centimetres and cubic metres	B5	eg, A rectangle has sides of 30 cm and 40 cm. Find the area. Give your answer in m^2.

Foundation Tier			Notes

Loci

F3.4j	find loci, both by reasoning and by using ICT to produce shapes and paths	B5	Loci will be restricted to at most two constraints. eg, Find the overlapping area of two transmitters, with ranges of 30km and 40km respectively. Loci problems may be set in practical contexts such as finding the position of a radio transmitter. Constructions expected are the perpendicular bisector of two points, accurate construction of a circle and the angle bisector. Questions involving bearings may also be required.

AO4: Handling data

1. Using and applying handling data

Pupils should be taught to:

Foundation Tier			Notes
Problem solving			
F4.1a	carry out each of the four aspects of the handling data cycle to solve problems: (i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed (ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources (iii) process and represent the data: turn the raw data into usable information that gives insight into the problem (iv) interpret and discuss the data: answer the initial question by drawing conclusions from the data	B1	

Foundation Tier			Notes
F4.1b H4.1b	identify what further information is needed to pursue a particular line of enquiry; select the problem-solving strategies to use in statistical work, and monitor their effectiveness (these strategies should address the scale and manageability of the tasks, and should consider whether the mathematics and approach used are delivering the most appropriate solutions)	B1	
F4.1c	select and organise the appropriate mathematics and resources to use for a task	B1	
F4.1d	review progress while working; check and evaluate solutions	B1	

Communicating

F4.1e	interpret, discuss and synthesise information presented in a variety of forms	B1	Candidates should be familiar with data presented in a variety of forms such as lists of raw data, lists of ordered data, tables, frequency tables, bar charts, pie charts and should be aware of the links between them.
F4.1f	communicate mathematically, including using ICT, making use of diagrams and related explanatory text	B1	Candidates should know and be able to draw a variety of statistical diagrams.
F4.1g	examine critically, and justify, their choices of mathematical presentation of problems involving data	B1	Candidates should be able to recognise when diagrams contain errors or omissions

Reasoning

Foundation Tier		Notes	
F4.1h	apply mathematical reasoning, explaining and justifying inferences and deductions	B1	Candidates should be able to show methods clearly, explaining their answers.
H4.1e	identify exceptional or unexpected cases when solving statistical problems	B1	
F4.1i H4.1f	explore connections in mathematics and look for relationships between variables when analysing data	B1	eg, Describing the relationship from a line of best fit.
F4.1j	recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis	B1	Candidates should know and recognise when answers are inappropriate.

2. Specifying the problem and planning

Pupils should be taught to:

Foundation Tier		Notes	
F4.2a	see that random processes are unpredictable	B1	Candidates should understand that the outcome of a random event cannot be predicted and that the probability of a fair coin landing on heads is 0.5 (or $\frac{1}{2}$), even if the previous six throws have given heads. Probability may be expressed as fractions, decimals or percentages.
F4.2b H4.2b	identify key qquestions that can be addressed by statistical methods	B1	Standard statistical terminology such as average, range, data, etc, should be understood. Candidates should know that questions involving comparison and analysis of data will need to be solved using statistical methods.
F4.2c	discuss how data relate to a problem, identify possible sources of bias and plan to minimise it	B1	Candidates may be asked to criticise survey questions or comment on the results of experimental data (relative frequency).
F4.2d	identify which primary data they need to collect and in what format, including grouped data, considering appropriate equal class intervals	B1	Candidates should be familiar with terms such as raw data, ordered data, discrete data and continuous data.
F4.2e H4.2e	design an experiment or survey; decide what primary and secondary data to use	B1	For example, candidates may be asked to give an appropriate question for a survey or to criticise given questions. Candidates should understand the terms 'primary' and 'secondary' data.

3. Collecting data

Pupils should be taught to:

Foundation Tier		Notes	
F4.3a	design and use data-collection sheets for grouped, discrete and continuous data; collect data using various methods, including observation, controlled experiment, data logging, questionnaires and surveys	B1	Candidates may be asked to design a data collection sheet. Candidates should know, eg, that data logging is when data is collected automatically by machine, eg, the numbers of cars in a car park at any time.
F4.3b	gather data from secondary sources, including printed tables and lists from ICT-based sources	B1	Reading and analysing data from tables, charts and lists will be required.
F4.3c	design and use two-way tables for discrete and grouped data	B1	eg, Design a two-way table to analyse the colour and make of vehicles.

4. Processing and representing data

Pupils should be taught to:

Foundation Tier		Notes	
F4.4a	draw and produce, using paper and ICT, pie charts for categorical data, and diagrams for continuous data, including line graphs for time series, scatter graphs, frequency diagrams and stem-and-leaf diagrams	B1	Includes knowledge and use of pictograms and bar-charts. Includes frequency polygons, histograms with equal class intervals, and frequency diagrams for grouped discrete data. Pie charts should be labelled. If a frequency diagram is required, then it can be an equal interval histogram or a frequency polygon. If a stem-and-leaf diagram is given a key will be provided. If candidates are asked to draw a stem-and-leaf diagram they should give a key.
F4.4b	calculate mean, range and median of small data sets with discrete then continuous data; identify the modal class for grouped data	B1	Includes knowledge and use of the mode. Data in questions will always be given in tabular form with space provided for the addition of an extra column or columns for working.
F4.4c	understand and use the probability scale	B1	Candidates should be familiar with the words impossible, unlikely, evens, likely, certain and their positions on the probability scale. Knowledge that the scale runs from 0 to 1 is expected.
F4.4d	understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency	B1	Questions using coins and dice will be set. Questions using playing cards or involving gambling will not be set. Other situations such as taking counters from bags will also be used. Candidates will be required to know the meaning of 'at random' and this term will be used in questions. Questions may include the addition of simple probabilities

Foundation Tier		Notes	
F4.4e	list all outcomes for single events, and for two successive events, in a systematic way	B1	Candidates should be familiar with sample space diagrams. Lists or sample space diagrams may be given as answers.
F4.4f	identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1.	B1	eg, The probability that a person is left-handed is 0.19. What is the probability that a person is not left-handed?
F4.4g	find the median for large data sets and calculate an estimate of the mean for large data sets with grouped data	B1	Data in questions will always be given in tabular form with space provided for the addition of extra columns for working.
F4.4h	draw lines of best fit by eye, understanding what these represent	B1	Lines of best fit need not go through the mean point but should pass as close to as many data points as possible.
H4.4j	use relevant statistical functions on a calculator or spreadsheet	B1	Candidates may use statistical functions on their calculator. Standard deviation is no longer tested in written examinations but the mean of a discrete or grouped frequency table is. Marks for this are usually awarded for the processes and the sole use of a calculator is not advised as all marks may be lost if the wrong values (for mid-points, for example) are used and there is no evidence of method.

5. Interpreting and discussing results

Pupils should be taught to:

Foundation Tier		Notes	
F4.5a	relate summarised data to the initial questions	B1	
F4.5b	interpret a wide range of graphs and diagrams and draw conclusions	B1	Candidates should be familiar with pictograms, bar charts, bar line graphs, pie charts, stem-and-leaf diagrams, equal width histograms, frequency polygons, two-way tables, scatter graphs and line graphs. Candidates should know that lines joining points, such as on a graph plotting average temperature against month, have no meaning. Candidates will be expected to interpret eg, the median and the range from stem-and-leaf diagrams.
F4.5c	look at data to find patterns and exceptions	B1	For example, identifying a 'rogue' value on a scatter graph.
F4.5d	compare distributions and make inferences, using the shapes of distributions and measures of average and range	B1	Questions comparing distributions will be restricted to comparisons of an average and range.
F4.5e	consider and check results and modify their approach if necessary	B1	
F4.5f H4.5f	appreciate that correlation is a measure of the strength of the association between two variables; distinguish between positive, negative and zero correlation using lines of best fit; appreciate that zero correlation does not necessarily imply 'no relationship' but merely 'no linear relationship'	B1	Candidates will not be expected to make statements about how reliable the correlation is but should be aware that some data can form a perfect linear relationship and that other data may not. They should also be aware that using the line of best fit to predict values beyond the plotted range may not be reliable.
F4.5g	use the vocabulary of probability to interpret results involving uncertainty and prediction	B1	For example, 'there is some evidence from this sample that...'

Foundation Tier		Notes	
F4.5h	compare experimental data and theoretical probabilities	B1	Knowledge of the term 'relative frequency' is required.
F4.5i	understand that if they repeat an experiment, they may - and usually will - get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics	B1	Candidates should know that, for example, throwing a dice will not always give a rectangular distribution but that the more trials are carried out then the better the reliability of the results.
F4.5j	discuss implications of findings in the context of the problem	B1	Candidates may be asked to comment on the implications of their calculations.
F4.5k	interpret social statistics including index numbers; time series and survey data	B1	For example, population growth. For example, the National Census. Candidates should understand the difference between a sample and a census. eg, Candidates should know that if the index number for a base year is 100 and that eg, an index of 120 represents a 20% increase on the base year.

Higher Tier Guidance

AO2 Number and algebra

1. Using and applying number and algebra

Pupils should be taught to:

Higher Tier			Notes
Problem solving			
H2.1a	select and use appropriate and efficient techniques and strategies to solve problems of increasing complexity, involving numerical and algebraic manipulation	B3, B5	Mini-investigations will not be set but candidates will be expected to make decisions and use the appropriate techniques to solve a problem logically. Candidates should choose relevant information when some is redundant.
H2.1b	identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches	B3, B5	Candidates will be expected to give reasons for answers or show working. If a question states, "You **must** show your working", marks will be lost if the instruction is ignored.
H2.1c	break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods	B3, B5	Multi-step problems will be set. Credit can be gained in this type of question for showing a suitable strategy.

Higher Tier		Notes	
H2.1d	make mental estimates of the answers to calculations; present answers to sensible levels of accuracy; understand how errors are compounded in certain calculations	B3, B5	Candidates should be able to round to make mental estimates of calculations. In complex calculations, such as the solution of a trigonometric problem, candidates should not round off intermediate calculations as this could affect the accuracy of the final answer. Any intermediate values should be written to at least 4 significant figures. Candidates should be able to round to any given accuracy. As a general rule, when candidates are asked to give an answer to a suitable degree of accuracy solutions should be given to the same accuracy as the numbers used in the question.

Communicating

H2.1e	discuss their work and explain their reasoning using an increasing range of mathematical language and notation	B3, B5	Candidates will be expected to use correct mathematical notation and derive a solution to a given problem logically
H2.1f	use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem	B3, B5	Candidates should be able to interpret for example, diagrams, information given in a real-life context, such as a 'Sale' poster and translate this into a mathematical problem.
H2.1g	present and interpret solutions in the context of the original problem	B3, B5	Candidates will be required to give sensible answers to questions. eg, How many 4-seater taxis are needed to carry 14 passengers?
H2.1h	use notation and symbols correctly and consistently within a given problem	B5	eg, Candidates should be able to explain patterns in words. Correct use of numerical and algebraic notation is expected. eg, $a + a = a2$ will not be accepted.
H2.1i	examine critically, improve, then justify their choice of mathematical presentation, present a concise, reasoned argument	B3, B5	Questions may be set that can be made easier by standard mathematical techniques such as use of brackets, factorisation and cancelling.

Higher Tier	Notes

Reasoning

H2.1j	explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems	B3, B5	For example, using simple codes that substitute numbers for letters. See further guidance given in section on proof (Appendix C) Candidates may be required to find a counter-example to disprove a statement.
H2.1k	understand the difference between a practical demonstration and a proof	B3, B5	See further guidance given in section on proof (Appendix C)
H21.l	show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning	B3, B5	See further guidance given in section on proof (Appendix C) Candidates should always show working.
H2.1m	recognise the significance of stating constraints and assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem	B3, B5	Candidates should assume that information given is exact unless the question states or implies otherwise.

2. Numbers and the number system

Pupils should be taught to:

Higher Tier			Notes

Integers

H2.2a	use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition	B3	Abbreviations will not be used in examinations. The word 'least' will be used. Candidates could be asked to round a number to any power of 10 up to 10^6. Candidates will be expected to identify eg, multiples, factors and prime numbers from lists. Questions may be set that ask candidates to do prime factor decomposition and find HCF or LCMs. There is no obligation on candidates to use prime factor decomposition to find HCF or LCMs. The writing out of multiples or factors is an acceptable method and may be more efficient.

Higher Tier	Notes

Powers and roots

H2.2b	use the terms square, positive square root, negative square root, cube and cube root; use index notation and index laws for multiplication and division of integer powers; use standard index form, expressed in conventional notation and on a calculator display	B3	Definition of standard index form is $a \times 10^n$ where $1 \le a < 10$ and n is an integer. The term 'standard form' will be used in the examination. Questions will not ask for positive square root but will use the notation $\sqrt{25}$. When a square root is asked for only the positive value will be required. Candidates who give the negative root or both answers will not be penalised, but if the value is to be used in further calculation they may lose marks by using the negative root. If the solution to $x^2 = 25$ is required then both the negative and positive root are expected. Powers of 10 up to 10^6 should be understood. Values of simple integer powers eg, 2^4 will be tested. The words cube, square and cube root may be used and should be understood. Candidates should be aware that calculator displays can sometimes show values such as 1.7×10^{-3} or 1.7^{-3} and know how to interpret these.

Fractions

H2.2c	understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator	B3	Candidates may be asked to give a fractional answer in its simplest form. When this requirement is not clearly stated candidates do not have to cancel fractional answers. Candidates will require knowledge of mixed numbers. The term 'common denominator' will not be used.

Higher Tier	Notes

Decimals

H2.2d	use decimal notation and recognise that each terminating decimal is a fraction; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals	B3	eg, $0.137 = \frac{137}{1000}$ eg, $\frac{1}{7} = 0.142857142857\ldots$ Candidates should know that $0.\dot{3} = \frac{1}{3}$ and $0.\dot{6} = \frac{2}{3}$ and that other fractions give recurring decimals and know how to write eg, $\frac{1}{6} = 0.16666\ldots$ as $0.1\dot{6}$ Candidates could be asked to write a recurring decimal as a rational number and should know a method for converting a recurring decimal to a rational number.

	Higher Tier		Notes

Percentages

F2.2e H2.2e	understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of'; use percentage in real-life situations	B3	For example, 10% means 10 parts per 100, and 15% of Y means $\frac{15}{100} \times Y$ x% of y will be required. VAT rates will be provided. Percentage problems on non calculator papers will involve percentages that can be worked out using multiples of 1% and 10%. Some basic knowledge of percentages in every day life eg, commerce and business including rate of inflation, VAT, price index, interest rates and financial capability is required. Note that problems involving simple interest for more than one year will not be set but that compound interest on investments up to 20 years could be assessed. The use of a percentage multiplier is expected.

Ratio

H2.2f	use ratio notation, including reduction to its simplest form and its various links to fraction notation	B3	For example, in maps and scale drawings, paper sizes and gears. Candidates will be expected to know that a line divided in the ratio 1:2 will be split into $\frac{1}{3}$ and $\frac{2}{3}$ of its length. Candidates should know that if say red balls and blue balls are in the ratio 3:4 then the fraction of red balls is $\frac{3}{7}$ Candidates should be familiar with ratio notation, for example 2:3, and should know how to reduce to simplest form. Questions asking for the ratio in the form 1:n may be required.

3. Calculations

Pupils should be taught to:

	Higher Tier		Notes

Number operations and the relationships between them

H2.3a	multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer, fractional and negative powers; use inverse operations, understanding that the inverse operation of raising a positive number to power n is raising the result of this operation to power $\frac{1}{n}$	B3	eg, Find the reciprocal of $\frac{2}{3}$ eg, Find the exact value of 3^{-2} (non-calculator question) eg, Find the value of $32^{\frac{2}{5}}$ (non-calculator question) eg, Find the value of $64^{-\frac{2}{3}}$ (non-calculator question) eg, Find the value of 9^0 (non-calculator question) eg, Solve $x^{\frac{1}{2}} = 9$ (non-calculator question) eg, Find x if $x^6 = 64$ (non-calculator question) eg, Express 48 as the product of prime factors The following should be known: table facts up to 10×10 squares up to 15×15 non-calculator methods for adding and subtracting 3 digit numbers; non-calculator methods for multiplying and dividing up to 3 digit numbers by up to 2 digit numbers. Candidates should be able to interpret a remainder from a division problem. Multiplication and division of integers and decimals by powers of 10 will be restricted to 10, 100 and 1000.

Higher Tier		Notes	
H2.3b	use brackets and the hierarchy of operations	B3	The BIDMAS or BODMAS convention should be known but this mnemonic need not be known. Candidate could be asked to insert brackets into a calculation to make it true.
H2.3c	calculate a given fraction of a given quantity expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal; distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals); convert a recurring decimal to a fraction	B3	For example, for scale drawings and construction of models, down payments, discounts. eg, Find $0.\dot{3}\dot{9}$ as a fraction. (non-calculator question) eg, Find $0.\dot{4}3\dot{2}$ as a fraction. (calculator question) eg, Find $\frac{3}{8}$ of 56. (non-calculator question) eg, Find $\frac{2}{7}$ of 5467. (calculator question) Questions involving mixed numbers may be set. eg, Work out $1\frac{2}{5} + \frac{3}{4}$ eg, Work out $3\frac{5}{6} - 2\frac{1}{2}$ eg, Work out $\frac{5}{6} - \frac{1}{2}$ eg, Write $\frac{7}{20}$ as a decimal.

Higher Tier		Notes	
H2.3d	understand and use unit fractions as multiplicative inverses multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction	B3	eg, thinking of multiplication by $\frac{1}{5}$ as division by 5. eg, $4 \times \frac{7}{8}$, $\frac{6}{11} \div 3$ eg, $\frac{3}{4} \times \frac{8}{9}$, $\frac{4}{15} \div \frac{2}{5}$ Multiplication and division problems with mixed numbers may be assessed. eg, Work out $1\frac{2}{5} \times 1\frac{3}{4}$ eg, Work out $3\frac{5}{6} \div 2\frac{1}{2}$

Higher Tier		Notes	
H2.3e	convert simple fractions of a whole to percentages of the whole and vice versa then understand the multiplicative nature of percentages as operators calculate an original amount when given the transformed amount after a percentage change; reverse percentage problems	B3	For example, analysing diets, budgets or the costs of running, maintaining and owning a car. Finding x% of y will not be asked for, but an x% increase of y may be asked for in the context of a real life problem, or as introduction to compound interest type problems. The use of a percentage multiplier is recommended. For example, a 15% increase in the value Y, is calculated as $1.15 \times Y$. For example, a 15% increase in value Y, followed by a 15% decrease is calculated as $1.15 \times 0.85 \times Y$. eg, A seal colony is decreasing at 12% per annum. If the original population is 2000, after how many years will the population have fallen to half its original number? eg, If £550 is invested at 10% per annum compound interest, how much will there be after 2 years? (non-calculator question) eg, After a 7% decrease the cost of a TV is £232.50. What was the original price? (calculator question) For example, given that a meal in a restaurant costs £36 with VAT at 17.5%, its price before VAT is calculated as £$\frac{36}{1.175}$
H2.3f	divide a quantity in a given ratio	B3	eg, If a quantity of money is divided in the ratio 5:3 and the smallest share is £90, how much is the largest share? (non-calculator question) eg, If a quantity of money is divided in the ratio 5:3 and the smallest share is £34.50, how much was the original quantity? (calculator question)

Higher Tier	Notes

Mental methods

H2.3g	recall integer squares from 2×2 to 15×15 and the corresponding square roots, the cubes of 2, 3, 4, 5 and 10, the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers n the corresponding rule for negative numbers $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n	B3	Mental methods will not be tested in the written papers but a quick recall of basic number facts is expected. For example, $10° = 1$, $9^{-1} = \frac{1}{9}$ For example, $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ For example, $25^{\frac{1}{2}} = 5$ and $64^{\frac{1}{3}} = 4$ It will be acceptable to give the decimal equivalent of $\frac{1}{3}$ and $\frac{2}{3}$ as 0.33 and 0.66 or 0.67.

Higher Tier		Notes	
H2.3h	round to a given number of significant figures; develop a range of strategies for mental calculation; derive unknown facts from those they know; convert between ordinary and standard index form representations converting to standard index form to make sensible estimates for calculations involving multiplication and/or division	B3	For example, $0.1234 = 1.234 \times 10^{-1}$ Candidates should be able to round to a given number of decimal places. Candidates could be asked to give an answer to 1, 2 or 3 significant figures, but should be able to round to any given number of significant figures. Questions requiring the answer to be given to a sensible degree of accuracy will be set. In these cases candidates will have to make their own decisions. An answer given to no more accuracy than the least accurate data in the question is expected. eg, Find 23% of 106 000. Give your answer to a sensible degree of accuracy. An answer of 24 000 or 24 400 would be expected. 23% is only to 2 s.f. but this would not count as a rounded value. What percentage is 230 of 1460? Give your answer to a sensible degree of accuracy. An answer of 16% (2s.f.) would be expected as 230 is given to 2 s.f. and would be regarded as a rounded value.
F2.3i	develop a range of strategies for mental calculation; add and subtract mentally numbers with up to one decimal place; multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments	B3	Knowledge of the terms 'commutative', 'associative' and 'distributive' is not required and the term 'factorise' in the context of number need not be known.

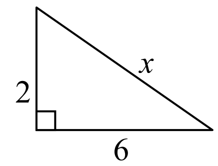
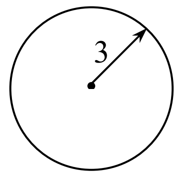
Higher Tier			Notes

Written methods

F2.3k	division by decimal (up to 2 d.p.) by division using an integer; understand where to position the decimal point by considering what happens if they multiply equivalent fractions, eg, given that…work out…	B3	Candidates may use any algorithm in non-calculator papers. For example $\frac{2000}{0.4} = \frac{20000}{4} = 5000$ eg, $408 \div 0.17 = 40800 \div 17$ eg, $0.02 \times 0.3 = 0.006$, $0.4^2 = 0.16$
H2.3i	use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer	B3	Candidates may use any algorithm in non-calculator papers. If calculating $\frac{4}{9} \times \frac{3}{8}$, in say a probability question, candidates will not be penalised if they fail to cancel before carrying out the multiplication. They may not gain full marks, however, if they fail to cancel $\frac{12}{72}$
H2.3j	solve percentage problems including percentage increase and decrease and reverse percentages	B3	Candidates will be expected to be familiar with the terms 'simple interest', 'VAT' and 'annual rate of inflation' 'income tax' and 'discount' and to know what they mean. In questions involving VAT, the amount of VAT will be given, as will the value of the annual rate of inflation. eg, Mrs Smith has a salary of £30 000 at the start of 2004. This is increased at the end of the year in line with annual rate of inflation. In 2005 this was 3.2% and in 2006 it was 2.9%. What is Mrs Smith's salary at the end of 2000? eg, A computer costs £1116.25 including 17.5% VAT. Schools do not pay VAT. How much would the computer cost a school?

Higher Tier		Notes	
F2.3n	solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution	B3	For example, given that m identical items cost £y, then one item costs £$\frac{y}{m}$ and n items cost £(n × $\frac{y}{m}$), the number of items that can be bought for £z is z × $\frac{m}{y}$ eg, 8 pencils cost £2.40. How much do 11 cost? (non-calculator question) eg, 8 pencils cost £2.56. How many can be bought for £4.80? (calculator question)
H2.3k	represent repeated proportional change using a multiplier raised to a power	B3	Candidates will be expected to be familiar with the term 'compound interest' and know what it means. eg, A seal colony is decreasing at 12% per annum. If the original population is 2000, after how many years will the population have fallen to half the original number? (calculator question: such problems are likely to appear on the calculator paper due to the repetitive nature of the calculations) eg, If £550 is invested at 10% per annum compound interest, how much will there be after 2 years? (non-calculator question) eg, The value of the investment after 3 years is a × 550, what is the value of a? (non-calculator question) – an answer of 1.1^3 or 1.331 would be accepted.
H2.3l	calculate an unknown quantity from quantities that vary in direct or inverse proportion	B3	eg, Two men can mow a meadow in 2 hours. How long would they take to mow a meadow that is twice as big? eg, Two men can mow a meadow in 2 hours. How long would three men, working at the same rate, take to mow the meadow?

Higher Tier		Notes	
H2.3m	calculate with standard index form	B3	Questions may be set in context. For example, populations, astronomical data, size of atoms. Numbers used in non-calculator papers will be straightforward. eg, $(4 \times 10^7) + (5 \times 10^6) = 45 \times 10^6 = 4.5 \times 10$ (non-calculator question) eg, $2.4 \times 10^7 \times 5 \times 10^3 = 12 \times 10^{10} = 1.2 \times 10^{11}$ (non calculator question) eg, $(2.5 \times 107) \div (5 \times 103) = 5 \times 103$ (non-calculator question)

Higher Tier		Notes	
H2.3n	use surds and π in exact calculations, without a calculator; rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	B3	Questions requiring answers in surds or in terms of π will appear in non-calculator papers. eg, Find the exact value of x.  An answer of $\sqrt{40}$ would not need to be simplified unless this was asked for. Find the area of the circle. Leave your answer in terms of π.  Candidates may be required to calculate a circumference or area in non-calculator papers but the value of π will be given $\pi = 3.14$) Candidates will need to understand the term 'rationalise the denominator'. Denominators in such questions will always be in the form $\sqrt{a}$.

	Higher Tier		Notes

Calculator methods

H2.3o	use calculators effectively and efficiently, knowing how to enter complex calculations; use an extended range of function keys, including trigonometry and statistical functions relevant across this programme of study	B3	Efficient use of a calculator will be expected. Candidates may be asked to calculate expressions such as $\frac{2.3+4.9}{5.2-1.7}$ using memory or brackets. Candidates will not need to show intermediate working in trigonometry problems, for example. eg, $52 \times \sin 37$ can be calculated and the answer written down, (with appropriate rounding if required).
F2.3p H2.3p	enter a range of calculations, including those involving measures understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation	B3	For example, time calculations in which fractions of an hour must be entered as fractions or as decimals. Candidates should use the calculator display whenever possible to make further progress in a calculation but if intermediate values have to be written down they should write the value to at least 4 significant figures so that the final answer is within the acceptable range of accuracy.
H2.3q	use calculators, or written methods, to calculate the upper and lower bounds of calculations, particularly when working with measurements	B3	Candidates need to realise that a number written as 9.7 correct to 1 decimal place can actually lie anywhere between $9.65 \le x < 9.75$. eg, Calculate the upper bound of the area of a rectangle with dimensions 4.5 cm by 7.5 cm, both values given to 1 decimal place. Candidates should know how to combine the upper and lower bounds when using the four rules to work out upper and lower bounds of calculations. eg, A car travels 60 miles (to the nearest 10 miles) at an average speed of 55 mph (to 2 significant figures). What are the limits of the time of the journey?

Higher Tier		Notes	
H2.3r	use standard index form display and know how to enter numbers in standard index form	B3	Use of the EXP or EE button on a calculator is expected in calculator papers. Candidates are expected to interpret the calculator display and to write their answers in correct standard index notation.
H2.3s	use calculators for reverse percentage calculations by doing an appropriate division	B3	For example, when finding the original quantity after an increase of 8%, candidates are expected to divide by 108 and multiply by 100, or to divide by 1.08. Reverse 'compound interest' problems will not be set.
H2.3t	use calculators to explore exponential growth and decay using a multiplier and the power key	B3	For example, in science or geography. eg, The population of a town is growing at 2% per annum. It is currently 3500. Estimate the population after 20 years.

4. Solving numerical problems

Pupils should be taught to:

Higher Tier		Notes	
H2.4a	draw on their knowledge of operations and inverse operations (including powers and roots), and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, repeated proportional change, fractions, percentages and reverse percentages, inverse proportion, surds, measures and conversion between measures, and compound measures defined within a particular situation	B3	The terms 'commutative', 'associative' and 'distributive' will not be used in the examination. Knowledge of the term 'root' is required. Knowledge of the term 'inverse operation' is required and candidates should know the inverse operations of the four rules: square, square root, cube and cube root. Compound measures may be expressed in the form metres per second, m/s, $m\ s^{-1}$. Candidates would be expected to understand these and other standard compound measures, such as those for density. Units may be any of those in common usage or specifically mentioned in the specification such as speed (eg, miles per hour) or density (eg, grams/cm^3). Other compound measures that are non-standard would be defined in the question eg, population density is population/km^2. eg, Evaluate $(4+\sqrt{3})(4-\sqrt{3})$. Conversion between measures would involve knowledge of the connection between metric units. Conversions between imperial units will be given.
H2.4b	check and estimate answers to problems; select and justify appropriate degrees of accuracy for answers to problems; recognise limitations on the accuracy of data and measurements	B3	eg, The heights of 7 men are 150, 151, 148, 133, 138, 142, 140cm. Give a reason why 143.143 cm is not an appropriate answer for their mean height. eg, If the radius of a circle is 5.2 cm to 2 s.f., what is the minimum value its area could be?

5. Equations, formulae and identities

Pupils should be taught to:

Higher Tier	Notes

Use of symbols

H2.5a	distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations defined quantities or variables in formulae general, unspecified and independent numbers in identities and in functions they define new expressions or quantities by referring to known quantities.	B3, B5	For example, knowing that $x^2 + 1 = 82$ is an equation. For example, knowing that $V = IR$ is a formula. For example, knowing that $(x + 1)^2 \equiv x^2 + 2x + 1$, for all values of x is an identity. Knowledge of the identity symbol $\equiv$ is required. For example, knowing that $y = 2 - 7x$; $f(x) = x^2$ are functions. For example, understanding that $y = \frac{1}{x}$ is not defined for $x = 0$ Candidates will be expected to know the standard conventions such as $2x$ for $2 \times x$, $\frac{1}{2}x$ or $\frac{x}{2}$ Candidates who write $2 \times x$, $x \times 2$, $x \div 2$ will not be penalised, but $x2$ will not be accepted for $2x$. Candidates will be expected to understand and use the function notation eg, $f(x) = 2x + 3$

Higher Tier		Notes	
H2.5b	understand that the transformation of algebraic entities obeys and generalises the well-defined rules of generalised arithmetic expand the product of two linear expressions manipulate algebraic expressions by collecting like terms, multiplying a single term over a bracket, taking out common factors; factorising quadratic expressions including the difference of two squares and cancelling common factors in rational expressions.	B3, B5	For example, $a(b + c) = ab + ac$ eg, Expand and simplify $(2x+3)(3x-4)$. Candidates will be expected to know the meaning of 'simplify' eg, Simplify $3x-2+4(x+5)$ and factorise eg, Factorise $3x^2y-9y$. Knowledge of the phrase 'difference of two squares' will not be expected but candidates may be asked to factorise expressions such as $x^2 - 9$ or $4x^2 - y$ Candidates will be expected to know the meaning of 'solve' in relation to linear and non-linear equations. (eg, Solve the equation $x^2+2x-15=0$) Knowledge of the term 'factorisation' would not be required in the context of a numerical problem but candidates should know how to cancel fractions, for example, and would be expected to explain why $4x^2+8x-12=0$ can be simplified to $x^2+2x-3=0$ and understand why $\frac{3x+12}{9x-15}$ can be simplified to $\frac{x+4}{3x-5}$ Candidates will be expected to know how to simplify. eg, $\frac{1}{x} + \frac{3}{2-x}$ Candidates will be expected to know how to factorise eg, $4x^2 + 6xy$

Higher Tier		Notes	
H2.5c	know the meaning of and use the words 'equation', 'formula', 'identity' and 'expression'	B3, B5	Candidates will be expected to know the terms 'equation', 'formula', 'identity' and 'expression'. eg, The expression x^2+2x+1 has three terms. The equation $x^2+2x+1=0$ can be solved. $A=x^2+2x+1$ is a formula. $(x+1)^2 \equiv x^2+2x+1$ is an identity that is true for all x

Index notation

H2.5d	use index notation for simple integer powers, and simple instances of index laws substitute positive and negative numbers into expressions such as $3x^2+4$ and $2x^3$	B5	Evaluate $2x^3$ when $x = -2$, $3x^3 + 4$ when $x = \frac{1}{2}$ or –3. eg, Simplify $2x^{-2} \times 3x^4$ eg, Simplify $\frac{4ab^2 \times 3ab}{6ab}$ eg, Simplify $(2x^2)^3$ eg, Evaluate 2^5 eg, Simplify $x^2 \times x^{-3}$, $x^6 \div x^{-2}$, $x^{-2} \div x^3$ eg, Simplify $2x^{-2} \times 3x^4$ eg, Simplify $\frac{4ab^2 \times 3ab}{6ab}$ eg, Simplify $(x^2)^3$

Higher Tier		Notes	

Equations

H2.5e	set up simple equations solve simple equations by using inverse operations or by transforming both sides in the same way	B5	For example, find the angle a in a triangle with angles a, $a + 10$, $a + 20$ For example, $5x = 7$, $11 - 4x = 2$, $3(2x + 1) = 8$, $2(1 - x) = 6(2 + x)$, $4x^2 = 49$, $3 = \frac{12}{x}$

Linear Equations

H2.5f	solve linear equations in one unknown, with integer or fractional coefficients, in which the unknown appears on either side or on both; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution	B5	eg, Solve $2(x - 4) = 7$ (non-calculator or calculator question) eg, $2x - 3 = \frac{x+2}{3}$ eg, Solve $5x + 17 = 3(x + 6)$ (non-calculator or calculator question) eg, Solve $\frac{15-x}{4} = 2$ (non-calculator or calculator question) eg, Solve $\frac{2x-3}{6} + \frac{x+2}{3} = \frac{2}{5}$ (non-calculator or calculator question) eg, Solve $\frac{17-x}{4} = 2 - x$

Higher Tier			Notes

Formulae

H2.5g	use formulae from mathematics and other subjects substitute numbers into a formula; change the subject of a formula including cases where the subject occurs twice, or where a power of the subject appears; generate a formula	B5	For example, for the area of a triangle or a parallelogram, area enclosed by a circle, volume of a prism. Candidates will be expected to substitute into formulae and to understand the order of operations. eg, Rearrange $x + y = 7$ to make x the subject. eg, Rearrange $c = 2\pi r$ to make r the subject eg, Rearrange $y = 2x + 3$ to make x the subject. eg, If $u = 2.5, a = -1.7$ *and* $t = 2$ find $s = ut + \frac{1}{2}at^2$ eg, Rearrange $2(x + y) = 5y - 3$ to make x the subject. eg, Rearrange $A = 2\pi rh + \pi rh$ to make r the subject. For example, find the perimeter of a rectangle given its area A and the length l of one side. For example volume of a cone.

	Higher Tier		Notes

Direct and inverse proportion

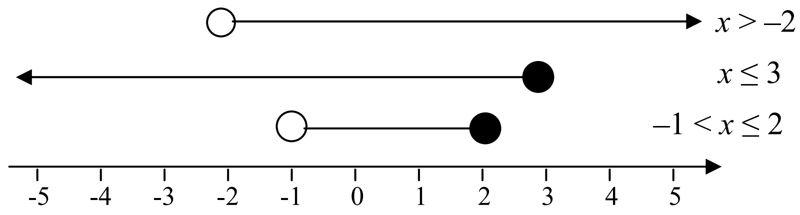
H2.5h	set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relate algebraic solutions to graphical representation of the equations	B3	Questions will be restricted to the following proportionalities: $y \propto x,\ y \propto x^2, y \propto x^3,\ y \propto \sqrt{x},\ y \propto \sqrt[3]{x}, y \propto \frac{1}{x},\ y \propto \frac{1}{x^2},$ $y \propto \frac{1}{x^3},\ y \propto \frac{1}{\sqrt{x}},\ y \propto \frac{1}{\sqrt[3]{x}}$ The expected approach would be to set up an equation using a constant of proportionality. Find this and then use the equation to find a value of y given x, or x given y. Other methods may be used and can be given full credit. eg, The weight of a sphere is proportional to the cube of its radius. When $r = 5$cm, $W = 500$g. Find the weight of a sphere with $r = 10$cm.

Simultaneous linear equations

H2.5i	Find the exact solutions of two simultaneous equations in two unknowns by eliminating a variable and interpret the equations as lines and their common solution as the point of intersection	B5	Solution by substitution of one variable into the other equation will be accepted as a method and may be the better method if equations are given, for example, as $y = 2x + 3$, $3x + 4y = 1$. Solution by graphical method may sometimes be an acceptable alternative approach but if the question clearly states 'Use an algebraic method', then this is expected.

	Higher Tier		Notes

Inequalities

H2.5j	solve linear inequalities in one variable, and represent the solution set on a number line; solve several linear inequalities in two variables and find the solution set	B5	Candidates should know the difference between >, <, ≤ and ≥ For example Solve $2x + 3 < 7$ Candidates should know the convention of an open circle for a strict inequality and a closed circle for an included boundary. eg,  Candidates should be familiar with the notation $-2 < x \leq 3$ In 2 dimensions candidates should be able to plot and read inequalities such as $2x + 3y < 6$, $y \geq x + 2$, $y \leq 3$, $x > -4$ and find a set of inequalities that describe an enclosed region of the plane. Boundary lines for strict inequalities should be dashed and included inequalities should be solid. There is no convention for an overlapping region being shaded on the required area or shaded on the unwanted area. Candidates will be asked to mark the required region clearly with an R, for example.

Higher Tier			Notes

Quadratic equations

H2.5k	solve quadratic equations by factorisation, completing the square and using the quadratic formula	B5	eg, Solve $x^2 + 5x + 6 = 0$ eg, Solve $2x^2 + 9x - 5 = 0$ (non-calculator question) eg, Solve $x^2 - 2x - 1 = 0$ giving your answer to 2 d.p. (calculator question) eg, Solve $x^2 - 2x - 1 = 0$ giving your answer in the form $a \pm \sqrt{b}$. (non-calculator question) eg, Write $x^2 + 4x - 9$ in the form $(x + a)^2 - b$ Hence solve the equation $x^2 + 4x - 9 = 0$, giving answers to 2 d.p. (calculator question) / giving answers in the form $a \pm \sqrt{b}$ (non-calculator question) Trial and improvement will not be accepted as a valid method. Candidates need not know that $b^2 - 4ac$ is the discriminant but should be aware that some quadratic equations have no solution.

	Higher Tier		Notes
Simultaneous linear and quadratic equations			
H2.5l	solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other or where the second is of the form $x^2 + y^2 = r^2$	B5	The expected method for solving one linear and one non-linear equation will be to substitute a variable from the linear equation into the non-linear equation. For example, solve the simultaneous equations $y = 11x - 2$ and $y = 5x^2$ The non linear equation will be of the form $y = ax^2 + bx + c$, where b and c are integers (including zero), or $x^2 + y^2 = r^2$, where r is not necessarily an integer.
Numerical Methods			
H2.5m	use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them	B5	For example, Solve $x^3 - x = 900$, Solve $\frac{1}{x} = x^2$ Answers will be expected to 1 d.p. Candidates will be expected to test the mid-value of the 1 d.p. interval to establish which 1 d.p. value is nearest to the solution.

6. Sequences, functions and graphs

Pupils should be taught to:

Higher Tier			Notes
Sequences			
H2.6a	generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the *n*th term of an arithmetic sequence, justifying its form by reference to the activity or context from which it was generated	B5	Candidates should be able to explain how a sequence continues. The nth terms of linear sequences will be required. Candidates will **not** be expected to find the nth term of a non-linear sequence. However, candidates should be familiar with the idea of a non-linear sequence and the fact that the nth term can be generated by an expression of the form $\frac{1}{2}n(n+1)$, for example. They should also know that the nth term of the square number sequence is given by n^2.
Graphs of linear functions			
H2.6b	use conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for *m* and *c*) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which *y* is given explicitly in terms of *x* (for example, $y = 2x + 3$), or implicitly (for example, $x + y = 7$); no table or axes given	B5	eg, explicitly such as $y = 2x + 3$ or implicitly such as $x + y = 5$ Partially completed tables of values may sometimes be given but candidates should be able to plot the graph of $y = 3x – 1$, say, with no further assistance. Knowledge that *m* is the gradient and *c* is the *y*-intercept will be expected. Know that the graph of $2x + 3y = 1$ is a straight line graph.

Higher Tier		Notes	
H2.6c	find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for m and c); understand that the form $y = mx + c$ represents a straight line and that m is the gradient of the line and c is the value of the y – intercept; explore the gradients of parallel lines and lines perpendicular to each other	B5	For example, know that the lines represented by the equations $y = -5x$ and $y = 3 - 5x$ are parallel, each having gradient (–5) and that the line with equation $y = \frac{x}{5}$ is perpendicular to these lines and has gradient $\frac{1}{5}$ Find the equation of a straight line when the graph is given. Knowledge of the condition $m_1 m_2 = -1$ will not be required but candidates will be expected to recognise that perpendicular lines have gradients that are negative reciprocals of each other. Questions involving perpendicular lines could include a graph which may be used in the solution of the problem and candidates can use graph paper to assist in their solution. eg, 'Find the equation of the line perpendicular to $y = 2x + 3$ passing through (0, 5)' will not include a graph. eg, 'Find the equation of the line perpendicular to the mid-point of '$A(-1, 3)$ and $B(3, 1)$' will include a graph.

Interpret graphical information

H2.6d	construct linear functions and plot the corresponding graphs arising from real-life problems discuss and interpret graphs modelling real situations	B5	For example, distance - time graph for a particle moving with constant speed, the depth of water in a container as it empties. For example, the velocity-time graph for a particle with constant acceleration Candidates will be expected to find the velocity for sections of a distance–time graph, and should understand that the steeper the line the faster the speed. They should also understand the significance of a negative gradient. Candidates will not be expected to find the area under a curved graph nor to find the acceleration by measuring the gradient of a tangent. Units of acceleration are not required.

Higher Tier			Notes

Quadratic functions

H2.6e	generate points and plot graphs of simple quadratic functions then more general quadratic functions find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function; find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions	B3, B5	For example, $y = x^2$, $y = 3x^2 + 4$ For example, $y = x^2 - 2x + 1$ If candidates are required to draw a graph then a table may be given in which some y values may have to be calculated. Quadratic graphs are expected to be drawn as a curve. Candidates will be expected to know that the roots of an equation $f(x) = 0$ can be found where the graph of the function intersects the x-axis and that the solution of $f(x) = a$ is found where $y = a$ intersects with $f(x)$. Candidates will be expected to know that the solution of an equation such as $x^2 + 3x - 2 = 0$ can be found from the intersection of two graphs such as $y = x^2 + 2x - 1$ and $y = 1 - x$. The non linear graph will always be quadratic.

Other functions

H2.6f	plot graphs of simple cubic functions the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the exponential function $y = k^x$ for integer values of x and simple positive values of k the circular functions $y = \sin x$ and $y = \cos x$, using a spreadsheet or graph plotter as well as pencil and paper; recognise the characteristic shapes of all these functions	B5	Candidates would be expected to recognise a sketch of the cubic, for example, $y = x^3$, and reciprocal graphs (including negative values of x). They would also be expected to sketch a graph of $y = \sin x$, and $y = \cos x$ between 0 and 360°, and know that the maximum and minimum values for sin and cos are 1 and –1. They would also be expected to know that the graphs of sin and cos are periodic. If candidates are required to draw an exponential graph, for example, $y = 2x$, $y = (\frac{1}{2})^x$, then a table will be given in which some y values may have to be calculated. Graphs are expected to be drawn as a curve. Joining points with straight lines will not get full credit.

	Higher Tier		Notes

Transformation of functions

H2.6g	apply to the graph of $y = \mathrm{f}(x)$ the transformations $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(ax)$, $y = \mathrm{f}(x + a)$, $y = a\mathrm{f}(x)$ for linear, quadratic, sine and cosine functions $\mathrm{f}(x)$	B5	$\mathrm{f}(x)$ will be restricted to a simple quadratic, $y = ax^2 + bx + c$, where one of b or c will be zero, $y = \sin(x)$ or $y = \cos(x)$

Loci

H2.6h	construct the graphs of simple loci including the circle $x^2 + y^2 = r^2$ for a circle of radius r centred at the origin of coordinates; find graphically the intersection points of a given straight line with this circle and know that this corresponds to solving the two simultaneous equations representing the line and the circle	B5	Candidates will be expected to recognise that $x^2 + y^2 = a^2$ is a circle, centre origin and radius a. A grid will be provided. eg, (a) Draw the graph of the set of points which are equidistant from the x and y-axes. (b) Write down the equation of your graph. eg, (a) The y coordinate of point P is twice its x coordinate. Write down one possible pair of coordinates for point P. (b) On the grid, draw the graph of the set of points P. (c) Give the equation of the set of points P. Candidates should know that a line that intersects the curve twice represents the solution of a quadratic with two roots and that when a line is a tangent to a curve it represents the solution of a quadratic with a repeated root.

AO3: Shape, space and measures

1. Using and applying shape, space and measures

Pupils should be taught to:

Higher Tier			Notes

Problem solving

H3.1a	select the problem-solving strategies to use in geometrical work, and consider and explain the extent to which the selections they made were appropriate	B5	Mini-investigations will not be set but candidates will be expected to make decisions and use the appropriate techniques to solve a problem drawing on well known facts, such as the sum of angles in a triangle.
H3.1b	select and combine known facts and problem-solving strategies to solve more complex geometrical problems	B5	Multi-step problems will be set. eg, In the triangle ABC, angle $A\hat{B}C$ is obtuse, angle $B\hat{A}C = 32^\circ$, $AC = 10$ cm. $BC = 6$ cm. Calculate the area of triangle ABC (diagram given)
H3.1c	develop and follow alternative lines of enquiry, justifying their decisions to follow or reject particular approaches	B5	Redundant information may sometimes be used, for example, the slant height of a parallelogram. Candidates should be able to identify which information given is needed to solve the given problem.

Higher Tier			Notes
Communicating			
H3.1d	communicate mathematically, with emphasis on a critical examination of the presentation and organisation of results, and on effective use of symbols and geometrical diagrams	B5	Candidates will be expected to use correct mathematical notation and produce a logical solution to a given problem.
H3.1e	use precise formal language and exact methods for analysing geometrical configurations	B5	For example candidates may be asked to give reasons why angles have certain values in a problem using circle theorems.
F3.1g	review and justify their choices of mathematics presentation	B5	Candidates should be able to choose an appropriate method when several methods are possible
Reasoning			
F3.1h	distinguish between practical demonstrations and proofs	B5	Candidates may be required to give specific examples or more general proofs.
H3.1f	apply mathematical reasoning, progressing from brief mathematical explanations towards full justifications in more complex contexts	B5	See further guidance given in section on proof (Appendix C)
H3.1g	explore connections in geometry; pose conditional constraints of the type 'If… then…'; and ask questions 'What if…?' or 'Why?'	B5	See further guidance given in section on proof (Appendix C),
H3.1h	show step-by-step deduction in solving a geometrical problem	B5	Candidates should be able to explain reasons using words or diagrams.
H3.1i	state constraints and give starting points when making deductions	B5	Candidates should realise when an answer is inappropriate.
H3.1j	understand the necessary and sufficient conditions under which generalisations, inferences and solutions to geometrical problems remain valid	B5	eg, Candidates should assume that lengths and angles given are exact unless the question states otherwise.

2. Geometrical reasoning

Pupils should be taught to:

Higher Tier			Notes
Properties of triangles and other rectilinear shapes			
H3.2a	distinguish between lines and line segments; use parallel lines, alternate angles and corresponding angles; understand the consequent properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices	B5	Explanations should use the correct terminology. Colloquial terms such as 'Z', 'F' angles are not acceptable as reasons. Acceptable terms for the interior, co-interior or allied See further guidance on proof (Appendix C). Candidates should know that a straight line drawn between two points or vertices is a line segment.
H3.2b	use angle properties of equilateral, isosceles and right-angled triangles; explain why the angle sum of a quadrilateral is 360 degrees	B5	Angle sum of a quadrilateral is based on the angle sum of a triangle which can be taken as a fact.
F3.2e	use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle	B5	Candidates will not be expected to deduce the formula for area of rectangles, parallelograms and triangles in examinations but will be expected to know them. Questions involving compound shapes made up of rectangles and triangles will be assessed.
H3.2c	recall the definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties	B5	Questions may be set that test candidates' knowledge of these properties. The properties of a kite should also be known. The formula for the area of a trapezium is given on the formula sheet. Candidates should know the side, angle and diagonal properties of quadrilaterals.

	Higher Tier		Notes

Properties of triangles and other rectilinear shapes

H3.2d	calculate and use the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons; calculate and use the angles of regular polygons.	B5	Candidates should learn or know how to work out the angle sum of polygons up to a hexagon. Questions involving the interior and exterior angles of regular polygons may be set. Questions could include octagons and decagons.
H3.2e	understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and compass constructions	B5	Candidates can justify congruence by a variety of methods but their justifications must be complete. The use of SSS notation etc, is not expected but will make the justification of congruence easier. See further guidance on proof (Appendix C).
H3.2f	understand, recall and use Pythagoras' theorem in 2-D, then 3-D problems; investigate the geometry of cuboids including cubes, and shapes made from cuboids, including the use of Pythagoras' theorem to calculate lengths in three dimensions	B5	In 2 dimensions questions may be set in context, for example, a ladder against a wall, but questions will always include a diagram of a right angled triangle with two sides marked and the third side to be found. Quoting the formula will not gain credit. It must be used with the appropriate numbers, eg, $x^2 = 7^2 + 5^2$, $x^2 = 12^2 - 9^2$ or $x^2 + 9^2 = 12^2$. In three dimensions candidates should identify a right angled triangle that contains the required information and then use Pythagoras' theorem (or trigonometry) to solve the problem. The use of the rule $d = \sqrt{a^2 + b^2 + c^2}$ is not required as problems will always be solvable using a combination of triangles. eg, Find the length of the diagonal AB in the cuboid with dimensions 9 cm, 40 cm and 41 cm. (diagram given)

Higher Tier		Notes	
H3.2g	understand similarity of triangles and of other plane figures, and use this to make geometric inferences; understand, recall and use trigonometry relationships in right-angled triangles, and use these to solve problems, including those involving bearings, then use these relationships in 3-D contexts, including finding the angles between a line and a plane (but not the angle between two planes or between two skew lines); calculate the area of a triangle using $\frac{1}{2}ab\ \sin C$; draw, sketch and describe the graphs of trigonometric functions for angles of any size, including transformations involving scaling in either or both the x and y directions; use the sine and cosine rules to solve 2-D and 3-D problems	B5	Candidates should know that there are many solutions to $\sin x = 0.5$, for example. Questions may require candidates to solve simple trigonometry functions, for example $\cos x = -0.5$ The domain for x will always be defined, for example $0^\circ \leq x \leq 360^\circ$. Candidates should be aware of the ambiguous case for the sine rule. Transformations of $f(x) = \tan x$ will not be required. In three dimensions candidates should identify a right angled triangle that contains the required information and then use trigonometry (or Pythagoras' theorem) to solve the problem. Although the sine and cosine rule can sometimes be used to solve 3-D problems they will always be solvable by a combination of right angled triangles.

Properties of circles

H3.2h	recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that the tangent at any point on a circle is perpendicular to the radius at that point; understand and use the fact that tangents from an external point are equal in length; explain why the perpendicular from the centre to a chord bisects the chord; understand that inscribed regular polygons can be constructed by equal division of a circle; prove and use the facts that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, the angle subtended at the circumference by a semicircle is a right angle, that angles in the same segment are equal, and that opposite angles of a cyclic quadrilateral sum to 180 degrees; prove and use the alternate segment theorem	B5	Questions asking for the angle at the centre of a regular polygon may be set. See further guidance given in section on proof (Appendix C) When asked to give reasons for angles any clear indication that the correct theorem is being referred to is acceptable. For example, angles on same chord, angle at centre twice angle at edge angle on diameter is 90°, opposite angle in cyclic quad. Alt seg theory.

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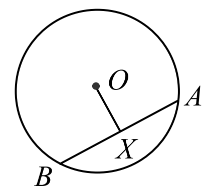
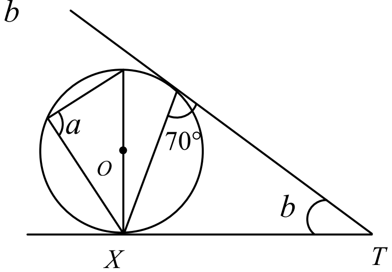
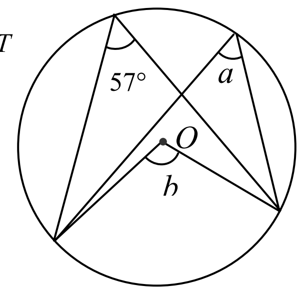
<tr><th colspan="2">Higher Tier</th><th colspan="2">Notes</th></tr>
<tr><td>H3.2h</td><td></td><td>B5</td><td>
Candidates will be expected to know the meaning of the terms 'sector' and 'segment'. Questions involving these terms will always be accompanied by a diagram where the appropriate region will be shown.
At the middle grade questions based on circle theorems will not involve complicated diagrams. At most, two straightforward applications of the circle theorems will be asked for in one diagram.
eg, O is the centre of the circle
$O\hat{X}A = 90^\circ$
Explain why OX bisects AB.


eg, O is the centre of the circle. Find angles a and b


eg, O is the centre of the circle.


Find the sizes of angles a and b.
At the higher grades questions may be set that involve complicated diagrams and require a proof.
</td></tr>
</table>

Higher Tier			Notes

3-D shapes

H3.2i	use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms, pyramids, cylinders, cones and spheres; solve problems involving more complex shapes and solids, including segments of circles and frustums of cones	B5	Formulae for the surface area and volume of a cylinder will not be given Questions involving the use of given formula, such as the surface area of a sphere, may not be accompanied by a diagram. More complex problems involving, for example, a hemisphere on top of a cylinder will always be accompanied by a diagram. Answers in terms of π may be required. (non-calculator question)

3. Transformations and coordinates

Pupils should be taught to:

Higher Tier			Notes

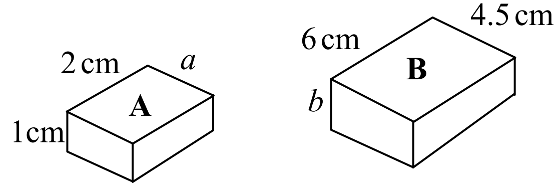
Specifying transformations

H3.3a	understand that rotations are specified by a centre and an (anticlockwise) angle; use any point as the centre of rotation; measure the angle of rotation, using right angles, fractions of a turn or degrees; understand that reflections are specified by a (mirror) line; understand that translations are specified by giving a distance and direction (or a vector), and enlargements by a centre and a positive scale factor	B5	The direction of rotation will always be given. Column vector notation should be understood. Lines of symmetry will be restricted to $x = a$, $y = a$, $y = x$ and $y = -x$. Scale factors for enlargements can be fractional (and/or negative)

Higher Tier			Notes

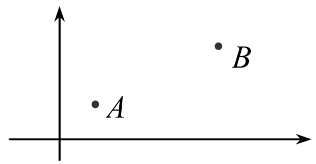
Properties of transformations

H3.3b	recognise and visualise rotations, reflections and translations including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations; use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations; distinguish properties that are preserved under particular transformations	B5	When describing transformations, the minimum requirement is Reflection described by a mirror line Translations described by a vector or a clear description such as 3 squares to the right, 5 squares down. Rotations described by centre, direction (unless a half turn) and an amount of turn (as a fraction of a whole or in degrees). Candidates will always be asked to describe a **single** transformation but could be asked to do a combined transformation on a single shape. Candidates could be asked to describe a single transformation equivalent to combination of transformations. Questions will be set on line symmetry and order of rotational symmetry, including tessellations.
H3.3c	recognise, visualise and construct enlargements of objects; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not, then use positive fractional and negative scale factors	B5	In the context of enlargements only. Enlargements may be about any point on a coordinate grid or may involve the enlargement of a plane shape not drawn on a coordinate grid. Candidates may be asked to find the scale factor and centre of enlargement.

Higher Tier		Notes	
H3.3d	recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments; understand the implications of enlargement for perimeter; use and interpret maps and scale drawings; understand the difference between formulae for perimeter, area and volume by considering dimensions; understand and use the effect of enlargement on areas and volumes of shapes and solids	B5	Candidates will be expected to know if a formula is consistent, and to be able to explain why a formula represents, for example, volume. It will be sufficient to say that the formula is the product of three lengths or dimensions, for example L^3. Candidates will be expected to know the connection between the linear, area and volume scale factors of similar shapes and solids. Questions may be asked that exploit the relationship between weight and volume, area and cost of paint etc. Scales will be given as, for example, 1 cm represents 10 km, or 1:100. eg, These boxes are similar.  What is the ratio of the volume of box A to box B ? eg, What is the ratio of the surface area of two similar cones with base radii 3cm and 12cm respectively?

Higher Tier			Notes

Coordinates

H3.3e	understand that one coordinate identifies a point on a number line, that two coordinates identify a point in a plane and three coordinates identify a point in space, using the terms '1-D', '2-D' and '3-D'; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; (for example, identify the co-ordinates of a cuboid drawn on a 3-D grid) find the coordinates of points identified by geometrical information (for example, find the coordinates of the fourth vertex of a parallelogram with vertices at (2, 1) (–7, 3) and (5, 6)); find the coordinates of the midpoint of the line segment *AB*, given the points *A* and *B*, then calculate the length *AB*	B5	The formulae $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, and $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ need not be known. Diagrams may be given. eg, The diagram shows the position of *A*(1, 1) and *B*(7, 5)  Calculate the length of the line segment *AB*; (for example, find the coordinates of the fourth vertex of a parallelogram with vertices at (2, 1) (−7, 3) and (5, 6)); Questions using 3-D coordinates will be set (for example, identify the coordinates of a cuboid on a 3-D grid) eg, identify the coordinates of the vertex of a cuboid on a 3-D grid eg, identify the coordinates of the mid-point of a line segment in 3-D

Higher Tier	Notes

Vectors

H3.3f	understand and use vector notation; calculate, and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector; calculate the resultant of two vectors; understand and use the commutative and associative properties of vector addition; solve simple geometrical problems in 2-D using vector methods	B5	Column vectors may be used to describe translations. Use of bold type and arrows such as $\mathbf{a} = \overrightarrow{OA}$ will be used to represent vectors in geometrical problems.

4. Measures and construction

Pupils should be taught to:

Higher Tier			Notes

Measures

H3.4a	use angle measure know that measurements using real numbers depend on the choice of unit; recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction; convert measurements from one unit to another; understand and use compound measures, including speed and density	B5	For example, use bearings to specify direction. Conversions between imperial units will be given but the rough metric equivalents to common imperial measures should be known. These will be restricted to 8 km $\approx$ 5 miles, 1 litre $\approx$ 1.75 pints, 1 kg $\approx$ 2.2 lbs, 1 gallon $\approx$ 4.5 litres, 1 foot $\approx$ 30 cm. eg, Give the upper and lower limits of a length of 11 cm measured to the nearest centimetre. Lower limit = 10.5 cm, Upper limit = 11.5 cm. The notation $10.5 \leq \text{length} < 11.5$ should be understood. Units of speed will be given as miles per hour (mph) , kilometres per hour (km/h), or metres per second , m/s, m s^{-1}. Candidates who express speed in alternative units such as metres per minute will not be penalised providing the units are clearly stated. Density will be given as gm/cm^3 or kg/m^3. Candidates who express density in alternative units such as grams per cubic metre will not be penalised providing the units are clearly stated.

	Higher Tier		Notes

Construction

F3.4d H3.4b	draw approximate constructions of triangles and other 2-D shapes, using a ruler and protractor, given information about side lengths and angles; understand, from their experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; construct specified cubes, regular tetrahedra, square-based pyramids and other 3-D shapes	B5	Knowledge of SSS, SAS, ASA and RHS terminology will be required as candidates could be asked to explain why two triangles are congruent. Candidates will be expected to draw a net of a 3-D shape and also to recognise a shape from a given net.
H3.4c	use straight edge and compasses to do standard constructions including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle	B5	Candidates will be expected to show clear evidence that a straight edge and compasses only have been used to do constructions.

Mensuration

F3.4f F3.4i H3.4d	calculate perimeters and areas of shapes made from triangles and rectangles; find the surface area of simple shapes by using the formulae for the areas of triangles and rectangles; find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids; convert between area measures, including square centimetres and square metres, and volume measures, including cubic centimetres and cubic metres; find circumferences of circles and areas enclosed by circles, recalling relevant formulae; calculate the lengths of arcs and the areas of sectors of circles	B5	Candidates will be expected to know the meaning of the terms 'sector', 'segment', 'major' and 'minor'. Questions involving the terms 'major' and 'minor' may be set but will always be accompanied by a diagram. Questions on area and perimeter using compound shapes formed from two or more rectangles may be set. eg, A rectangle has sides of 30 cm and 40 cm. Find the area. Give your answer in m^2. Circumference and area formula for a circle should be known. Perimeters and areas of semi-circles or simple fractions of a circle, eg, quarter circles, could be assessed.

Higher Tier			Notes

Loci

H3.4e	find loci, both by reasoning and by using ICT to produce shapes and paths	B5	For example, a region bounded by a circle and an intersecting line Loci problems may be set in practical contexts such as finding the position of a radio transmitter. Constructions expected are the perpendicular bisector of two points, accurate construction of a circle and the angle bisector. Questions involving bearings may also be required.

AO4: Handling data

1. Using and applying handling data

Pupils should be taught to:

Higher Tier			Notes

Problem solving

H4.1a	carry out each of the four aspects of the handling data cycle to solve problems: (i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed (ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources (iii) process and represent the data: turn the raw data into usable information that gives insight into the problem (iv) interpret and discuss the data: answer the initial question by drawing conclusions from the data	B1	
H4.1b	select the problem-solving strategies to use in statistical work, and monitor their effectiveness (these strategies should address the scale and manageability of the tasks, and should consider whether the mathematics and approach used are delivering the most appropriate solutions)	B1	

Higher Tier			Notes

Communicating

H4.1c	communicate mathematically, with emphasis on the use of an increasing range of diagrams and related explanatory text, on the selection of their mathematical presentation, explaining its purpose and approach, and on the use of symbols to convey statistical meaning	B1	Candidates should know and be able to draw and interpret a variety of statistical diagrams and use statistical notation accurately.

Reasoning

H4.1d	apply mathematical reasoning, explaining and justifying inferences and deductions, justifying arguments and solutions	B1	Candidates will be required to make comparisons between statistical data presented in a variety of formats, such as scatter graphs, stem-and-leaf diagrams, tables, cumulative frequency diagrams, box plots, histograms, etc.
H4.1e	identify exceptional or unexpected cases when solving statistical problems	B1	Candidates should be able to recognise 'rogue' data and the effect this may have on measures of location or dispersion.
H4.1f	explore connections in mathematics and look for relationships between variables when analysing data	B1	Candidates may be asked to describe connections between bivariate data. eg, describing the relationship from a line of best fit..
H4.1g	recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis	B1	Candidates should be able to recognise when diagrams contain errors or omissions. Candidates should know and recognise when answers are inappropriate.

2. Specifying the problem and planning

Pupils should be taught to:

Higher Tier		Notes	
H4.2a	see that random processes are unpredictable	B1	Candidates should understand that the outcome of a random event cannot be predicted and that the probability of a fair coin landing on heads is 0.5, even if the previous six throws have given heads. Probabilities may be expressed as fractions, decimals or percentages.
H4.2b	identify key questions that can be addressed by statistical methods	B1	Standard statistical terminology such as average, range, data, etc, should be understood. Candidates should know that questions involving comparison and analysis of data will need to be solved using statistical methods.
H4.2c	discuss how data relate to a problem, identify possible sources of bias and plan to minimise it	B1	Candidates should understand the term 'bias'. Candidates may be asked to criticise survey questions or comment on the results of experimental data (relative frequency).
H4.2d	identify which primary data they need to collect and in what format, including grouped data, considering appropriate equal class intervals; select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling	B1	Candidates should understand the terms 'sampling', 'investigate a population', 'random' and 'stratified sampling'.
H4.2e	design an experiment or survey; decide what primary and secondary data to use	B1	Candidates should understand the terms 'primary' and 'secondary' data. For example, candidates may be asked to give an appropriate question for a survey or to criticise given questions.

3. Collecting data

Pupils should be taught to:

Higher Tier		Notes	
H4.3a	collect data using various methods, including observation, controlled experiment, data logging, questionnaires and surveys	B1	Candidates could be asked about sampling methods and ways of collecting data. Candidates should know, eg, that data logging is when data is collected automatically by machine, eg, the numbers of cars in a car park at any time.
H4.3b	gather data from secondary sources, including printed tables and lists from ICT-based sources	B1	Candidates will be expected to know that as the number of trials increases then the relative frequency will approach the theoretical probability. Reading and analysing data from tables charts and lists will be required.
H4.3c	design and use two-way tables for discrete and grouped data	B1	eg, Design a two-way table to analyse the colour and make of vehicles.
H4.3d	deal with practical problems such as non-response or missing data	B1	

4. Processing and representing data

Pupils should be taught to:

Higher Tier		Notes	
H4.4a	draw and produce, using paper and ICT, pie charts for categorical data, and diagrams for continuous data, including line graphs (time series), scatter graphs, frequency diagrams, stem-and-leaf diagrams, cumulative frequency tables and diagrams, box plots and histograms for grouped continuous data	B1	Includes frequency polygons, histograms with equal class intervals and frequency diagrams for grouped discrete data. Questions involving back to back stem-and-leaf diagrams will not be set. Cumulative frequency diagrams should be drawn so that the upper class boundary is plotted against the cumulative frequency. Data in tables will always be given in the form $0 < x \leq 10$, $10 < x \leq 20$, etc. No keys showing area allocated to quantities will be provided on histograms. Candidates are expected to plot the frequency density (frequency ÷ class width) between the lower and upper class boundaries. They will have to choose their own scales for the side axis. Full credit will be given for alternative approaches providing the area of the bar is proportional to the frequency. Candidates should know that the lower quartile is the value which cuts the total area in the ratio 1:3, the median is the value which cuts the total area in the ratio 1:1 and the upper quartile is the value that cuts the total area in the ratio 3:1. These values will always be integers and can be found by simple linear interpolation. Pie charts should be labelled. If a frequency diagram is required then it can be an equal interval histogram or a frequency polygon. If a stem-and-leaf diagram is given a key will be provided. If candidates are asked to draw a stem-and-leaf diagram they should give a key. Data in tables will always be given in the form $0 < x \leq 10$, $10 < x \leq 20$, etc.

Higher Tier		Notes	
H4.4a		B1	Box plots will consist of five significant pieces of data: the least and greatest values (also known as the whiskers), and the box consisting of a rectangle going from the lower to upper quartile with the median marked as a divider across the box. Scales will always be given. Candidates will be expected to compare distributions and the relationship between a box plot and a cumulative frequency diagram should be known.
H4.4b	understand and use estimates or measures of probability from theoretical models, or from relative frequency	B1	Candidates will be expected to know that as the number of trials increases then the relative frequency will approach the theoretical probability.
H4.4c	list all outcomes for single events, and for two successive events, in a systematic way	B1	Candidates should be familiar with sample space diagrams. Lists or sample space diagrams may be given as answers.
H4.4d	identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1.	B1	eg, The probability that a person is left-handed is 0.19. What is the probability that a person is not left-handed?
H4.4e	find the median, quartiles and interquartile range for large data sets and calculate the mean for large data sets with grouped data	B1	Includes knowledge and the use of the mode Candidates will be required to find the median and quartiles from a frequency table or list of discrete data. The use of the median being the $\frac{n+1}{2}$ th value in the table will be expected. Candidates will be required to find the median and quartiles from cumulative frequency graphs. The median is found by reading off at $\frac{n}{2}$ and the quartiles at $\frac{n}{4}$ and $\frac{3n}{4}$. Candidates will be expected to use the midpoint for estimating the mean of a grouped frequency table.

Higher Tier		Notes	
H4.4f	calculate an appropriate moving average	B1	These problems will be set in context and involve the trend as a quantity against time (eg, quarterly sales, values of shares, etc). Candidates should understand the concept of an n-point moving average (where n is a integer). These may be accompanied by a graph. The first point of the moving average graph is plotted at the middle of the data values. For example, the first point of a 3-point moving average is plotted against the second-time value. Candidates will be expected to comment on and use the trends shown by the moving average, and use it to predict further values.
H4.4g	know when to add or multiply two probabilities: if A and B are mutually exclusive, then the probability of A or B occurring is $P(A) + P(B)$, whereas if A and B are independent events, the probability of A and B occurring is $P(A) \times P(B)$	B1	Candidates are required to understand the term 'mutually exclusive' but this will not be used in a written paper. The terms 'independent' and 'dependent' may be used in a written papers but candidates will not be asked to show that two events are independent.
H4.4h	use tree diagrams to represent outcomes of compound events, recognising when events are independent	B1	Simple cases of conditional probability may be asked. For example, when picking socks at random from a drawer containing red and black socks, the probability of getting a black sock alters as each sock is withdrawn. No more than three events will be used in the tree diagram when there are two outcomes to each event and no more than two events will be used when there are three or more outcomes to an event. Candidates may use AND/OR to calculate probabilities.
H4.4i	draw lines of best fit by eye, understanding what these represent	B1	Lines of best fit need not go through the mean point but should pass as close to as many data points as possible.

Higher Tier		Notes	
H4.4j	use relevant statistical functions on a calculator or spreadsheet	B1	Candidates may use statistical functions on their calculator. Standard deviation is no longer tested in written examinations but the mean of a discrete or grouped frequency table is. Marks for this are usually awarded for the processes and the sole use of a calculator is not advised as all marks may be lost if the wrong values (for midpoints, for example) are used and there is no evidence of method.

5. Interpreting and discussing results

Pupils should be taught to:

Higher Tier		Notes	
H4.5a	relate summarised data to the initial questions	B1	
H4.5b	interpret a wide range of graphs and diagrams and draw conclusions; identify seasonality and trends in time series	B1	This will be tested in context. For example, ice cream sales will show seasonal variation. Candidates should be familiar with bar charts, bar line graphs, pie charts, stem-and-leaf diagrams, equal width histograms, frequency polygons, two way tables, scatter graphs and line graphs. Candidates should know that lines joining points, such as on a graph plotting average temperature against month, may have no meaning. Candidates will be expected to interpret, eg, the median and the range from stem-and-leaf diagrams.
H4.5c	look at data to find patterns and exceptions	B1	For example, identifying a 'rogue' value on a scatter graph.

Higher Tier		Notes	
H4.5d	compare distributions and make inferences, using shapes of distributions and measures of average and spread, including median and quartiles; understand frequency density	B1	Comparisons between two distributions should be based on measures of an average and spread. Candidates should know when each of the three measures of location is valid and be able to justify their choice. They should understand that the interquartile range measures spread of the middle 50% of the data, is centred on the median, and that it eliminates extreme values from the measure of spread.
H4.5e	consider and check results, and modify their approach if necessary	B1	
H4.5f	appreciate that correlation is a measure of the strength of the association between two variables; distinguish between positive, negative and zero correlation using lines of best fit; appreciate that zero correlation does not necessarily imply 'no relationship' but merely 'no linear relationship'	B1	Candidates will not be expected to make statements about how reliable a correlation is but should be aware that some data can form a perfect linear relationship and other data may not. They should also be aware that using the line of best fit to predict values beyond the plotted range may not be reliable.
H4.5g	use the vocabulary of probability to interpret results involving uncertainty and prediction	B1	For example, 'there is some evidence from this sample that...'
H4.5h	compare experimental data and theoretical probabilities	B1	Knowledge of the term 'relative frequency' is required.
H4.5i	understand that if they repeat an experiment, they may - and usually will - get different outcomes, and that increasing sample size generally leads to better estimates of probability and population parameters	B1	Candidates should know that, for example, throwing a dice will not always give a rectangular distribution but that the more trials are carried out then the better the reliability of results.

Higher Tier		Notes	
F4.5k	interpret social statistics including index numbers time series and survey data	B1	For example, the General Index of Retail Prices. For example, population growth. For example, the National Census. Candidates should understand the difference between a sample and a census. eg, candidates should know that if the index number for a base year is 100 and the, eg, an index of 120 represents a 20% increase on the base year.

Course Organisation

4 Delivery of the Course Mathematics B *(Modular)*

4.1 Special Features

The modular course allows for external assessment of some areas of the Programme of Study early in the course. The areas of the Programme of Study are divided along Assessment Objective lines within the subject content of Module 1 and Module 3.

- GCSE Modular Mathematics can be taken over 1 year or 2 years.
- Specification B allows candidates to take modules early in the course on Handling Data (Module 1) and the (mainly) number part of the Number and Algebra (Module 3).
- Results are reported at the end of each module, enabling candidates to take greater responsibility for the planning and execution of their work.
- Modules 1 and 3 can be re-sat before final certification.
- Candidates may enter each individual module at a different tier of entry allowing them to make the most of their strengths. (Please note: candidates may enter only for a single tier in each module, in a particular examination series.) The final range of grades available to candidates is determined by the tier of entry for Module 5. Module 5 is the certificating module and must be taken in the final examination series. The first examination series in which Module 5 is available is summer 2009. Again it should be noted that candidates entering Module 5 for this specification are prohibited from entering any other GCSE Mathematics specification that will be **certificated** in the same examination series.
- The modular nature of the specification is a good preparation for AS/A-level Mathematics or further study at GNVQ/AVCE level. The specification also offers links with Free Standing Maths Units (FSMUs) and the Key Skill of Application of Number. There is an overlap between Module 1 of this specification and GCSE Statistics.

The modular nature of the specification allows candidates who fail to obtain a GCSE grade C at KS4 to carry forward some of their module results into post-16 education.

4.2 Grading System

The qualification will be graded on an 8 point grade Scale A*, A, B, C, D, E, F, G. Candidates who fail to reach the minimum standard for grade G will be recorded as U (unclassified) and will not receive a qualification certificate.

The modules are offered at two tiers of entry: Foundation tier and Higher tier. For candidates entered for the Foundation tier, grades C-G are available. For candidates entered for the Higher tier, grades A*-D are available. There is a safety net for candidates entered for the Higher tier, where an allowed grade E will be awarded where candidates just fail to achieve grade D. Candidates may enter for each individual module at a different tier of entry. However, the final range of grades available to a candidate is determined by the tier of entry of Module 5. Candidates who fail to achieve grade E on the Higher tier or grade G on the Foundation tier will be reported as U (unclassified).

4.3 Re-sits

Candidates can re-sit each tier of Modules 1 and 3 once before certification and the better of the two most recent marks are taken towards certification.

4.4 Availability of Assessment Units

Specification B is a modular assessment of GCSE Mathematics designed to be taken over a one- or two-year course of study. To offer maximum flexibility to centres and to suit different teaching programmes, Modules 1 and 3 can be taken in either order and candidates can enter at different tiers for the different modules. Module 5 is the certificating module and must be taken in the final examination series. This is to meet the QCA requirement that at least 50% of the qualification is externally examined at the end of the course. The first examination series in which Module 5 and certification is available will be June 2009.

Examinations based on this specification will be available as follows.

Series	Availability of Modules		
	Module 1	**Module 3**	**Module 5**
November 2007	Both tiers	Both tiers	_
March 2008	Both tiers	Both tiers	_
June 2008	Both tiers	Both tiers	_
November 2008	Both tiers	Both tiers	_
March 2009	Both tiers	Both tiers	_
***June 2009**	Both tiers	Both tiers	Both tiers
***November 2009**	Both tiers	Both tiers	Both tiers
March 2010	Both tiers	Both tiers	_

*Certification is available in this series.

4.5 Examples for routes through Modules

By studying the section on 'Availability of Modules' it is easy to see the flexibility available to candidates and centres. Specification B allows part of the GCSE assessment to be completed in Year 10 and part during Year 11 with re-sits built into the framework so that below-par results in Modules 1 and 3 can be discounted. A clear indication of grade and achievement can give incentive and motivation to otherwise underachieving candidates and can calm some examination nerves if some results have been banked. The modular specification is ideal for candidates entering at both tiers.

4.6 Determination of the candidates' final grades

Grade	Module 1	Module 3	Module 5
A*	97–108	146–162	297–330
A	86–96	130–145	264–296
B	76–85	113–129	231–263
C	65–75	97–112	198–230
D	54–64	81–96	165–197
E	43–53	65–80	132–164
F	32–42	49–64	99–131
G	22–31	32–48	66–98
U	0–21	0–31	0–65

5 Links with Other Qualifications

5.1 Introduction

The National Curriculum for Mathematics (1999) states that "Mathematics equips candidates with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem solving skills and the ability to think in abstract ways". Mathematical tools are used in all qualifications. Through Mathematics, candidates gain access to a language used in their other subjects.

Mathematical knowledge, understanding and skills are fundamental to the study for qualifications such as GCSE Chemistry, Physics, Science and Statistics. These qualifications may require a prior level of attainment in numeracy or specific background knowledge in Mathematics.

The National Curriculum for Mathematics identifies key links with English indicating connections between teaching requirements. It suggests that a requirement in Mathematics can build on the requirements in English in the same key stage.

5.2 Key Skills

Mathematics also clearly contributes to the development of the Key Skills. Examples are given below. (Details of the requirements are given in the Key Skills specifications.)

- *Communication*, through learning to express ideas and methods precisely, unambiguously and concisely.
- *Application of Number,* through using and applying the knowledge, skills and understanding of mathematics.
- *Information Technology*, through developing logical thinking; using graphics packages and spreadsheets to solve numerical, algebraic and graphical problems; using dynamic geometry packages to manipulate geometrical configurations; and using databases and spreadsheets to present and analyse data.
- *Working with Others*, through group activities and discussions of mathematical ideas.
- *Improving Own Learning and Performance*, through developing logical thinking, powers of concentration, analytical skills and reviewing approaches to solving problems.
- *Problem Solving*, through selecting and using methods and techniques, developing strategic thinking, and reflecting on whether the approach taken to a problem was appropriate.

(National Curriculum for Mathematics, 1999).

5.3 Key Skills Application of Number

Information about the generation of evidence for all the Key Skills, including Application of Number is provided in Section 8.

The following qualifications in GCSE Mathematics provide exemption from the external test for Application of Number:

- GCSE A* to C examination performance provides exemption from the external test in application of Number at Level 2.
- GCSE D to G examination performance provides exemption from the external test in application of Number at Level 1.

6 Resources

6.1 Introduction

AQA has chosen Nelson Thornes to be our exclusive 'preferred partner' for the two tier GCSE Mathematics specifications.

AQA Mathematics for GCSE has been developed by Nelson Thornes in partnership with AQA to ensure you have resources which match perfectly the revised specifications.

Students' Books

Easy to handle Students' Books provide graded objectives and examination questions written by a GCSE examiner, allowing students to see exactly how to achieve higher grades.

Learn Sections

- contain concise examples with notes so students can focus on practice

Apply Sections

- provide a wealth of practice questions to help students apply newly-learnt mathematical concepts, with a variety of questions types.

Explore Sections

- provide investigative activities.

Access Section

- provides plentiful questions increasing in order of difficulty to test students' understanding of the objectives of each chapter.

Homework Books	Provide one homework exercise for each topic.
Teacher's Books	Designed to help all teachers deliver motivating and engaging lessons through a range of features. Available for each Students' Book, these comprehensive Teacher's Books are designed to help all teachers deliver motivating engaging lessons through a range of features including: • a planning timetable showing the specification reference and grade • alternative methods of teaching topics covered in the Students Books, along with further examples and common errors to help ensure that students fully grasp new concepts • answers for all questions in the Students' Books and Homework Books for easy reference
e-Mathematics	Available on CD ROM and online, e-Mathematics provides support and enrichment for your lessons. A range of activities and electronic support to enhance lessons • objectives for each chapter to introduce new topics • starters and plenaries in three fun yet challenging formats • support for selected *Learn* sections from the Students' Books through interactive activities, optional commentary and question banks • worked examination questions helping to guide students trough questions step-by-step and show where marks are allocated • two sample papers for each tier written by an examiner and including fully worked solutions and mark schemes.
Online Test and Assessment	A powerful online service to test, assess and monitor students with whole class or year group reporting. • allows testing, assessment and monitoring of individuals and whole classes for assessment of learning • online format means that tests can be completed by students anywhere – in the classroom, at home or in after-school clubs • students can receive instant online feedback on tests so they can see how well they have performed • automatically generated detailed graphical reports allow you to quickly and effectively analyse performance

6.2 Contact details

Nelson Thornes

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E mail	maths@nelsonthornes.com
Web site	www.nelsonthornes.com/aqamaths

AQA Mathematics Subject Support

Telephone	0161 957 3852
Fax	0161 957 3873
E mail	mathematicsgcse@aqa.org.uk
Web site	www.aqa.org.uk

6.3 Websites

The following are offered, in good faith, as possible sources of data. They are in no particular order and some will be more suitable than others; teachers will need to judge their suitability.

Organisation	Internet URL	Notes
Statistics Sites		
Census at School	http://www.censusatschool.ntu.ac.uk	This site is an excellent place to find real data. The site also includes a variety of other useful resources (including downloadable worksheets) and tips on how ICT can be used effectively to enhance learning and teaching resources for good practice in data handling.
Census Information	http://www.census.ac.uk	Everything you want to know about the UK Census is here (and a bit more besides). Lots of interesting information and plenty of data but you'll need to register to be able to use it…. Probably more effort than it is worth.
Centre for Innovation in Mathematics Teaching	http://www.geographyhigh.connectfree.co.uk	Geography High is an unusual concept…. A virtual reality school with only geography on the curriculum. The population room in the second year pupils' assembly hall is probably a good starting point for some interesting statistics.
National Statistics	http://www.statistics.gov.uk	National Statistics if the official UK statistics site with lots and lots of statistical information. You will need a couple of days just to navigate your way around the site but the 'themes' page is a useful starting pint for your journey.
QCA National Qualifications (GCSE results)	http://www.qca.org.uk/rs/rer/bcse_results.asp	This QCA web site keeps statistical information on GCSE and short course examination results over the past few years. The data is provi9ded by the GCSE awarding bodies in England, Wales and Northern Ireland.

Organisation	Internet URL	Notes
QCA National Qualifications (GCE results)	**http://www.qca.org.uk/nq/subjects/a_level_results.asp**	This QCA web site keeps statistical information on A level and AS level examination results over the past few years. The data is provided by the GCE awarding bodies and covers a variety of different subjects.
Tourism facts and figures	**http://www.staruk.org.uk**	An interesting site put together by various UK tourist boards with a whole host of interesting facts and figures…. Did you know that over 110 million day trips are made to Britain's coastline each year?
UK National Lottery Winning Numbers	**http://lottery.merseyworld.com/Winning_index.html**	It will not improve your chances on the lottery but it certainly will provide you with lots of statistical information about the National lottery winning numbers, jackpots and numbers of winners.

Other Websites

ATM	http://www.atm.org.uk/	The official website of Teachers of Mathematics. Includes details of ATM's publications, its activities, and its philosophy. It also provides information about the journals and about its membership.
BECTA	http://becta.org.uk	If you have never heard of BECTA then this site is worth a visit. BECTA is a Government agency for Information and Communication Technology and is responsible for developing the National Grid for learning NGfl.
The Mathematical Association	http://www.m-a.org.uk	The official website of the Mathematical Association. Includes details of the MA's publications, its activities, and its organisation. It also provides information about the journals and about membership.

Organisation	Internet URL	Notes
National Grid for Learning	http://www.ngfl.gov.uk/index.jsp?sectionId=1&categoryId=99	The much acclaimed National Grid for Learning (NGfl) can be found here and includes lots of useful pages for schools, further education and higher education as well as the addresses of various community grids. You might even manage a visit to the Virtual Teacher Centre (VTC) with information on curriculum subjects, school management and professional development.
QCA	http://www.qca.org.uk/ages14-19/index.html	An absolute must if you want to keep up with everything new going on in education – the present emphasis is on the new curricula which you can download (if you have the time and patience).
The Standards Site	http://www.standards.dfee.gov.uk	The Standards Site providing on-line help for teachers in England and is dedicated to rising standards of achievement in schools (well that is what it says on the front page).
Teacher Training Agency	http://www.useyourheadteach.gov.uk	Provides lots of information ranging form how to become a teacher to research about teaching. Particularly useful for these thinking of entering or returning to teaching.

7 Glossary of Terms

Aims	The broad educational or vocational purposes of a qualification.
Assessment Objectives	The criteria used to evaluate candidates' attainments.
Entry Codes	The codes to be used when entering candidates for each unit and each qualification.
External Assessment	A form of independent assessment in which the awarding body sets or defines assignments, texts or examinations, specifies the conditions under which they are to be taken (including details of supervision and duration), and assesses candidates' responses.
Key Skills	Those generic skills that can enable people to perform well in education, training and life in general. They can help people to become members of a flexible workforce and equip them with the means to benefit from life-long learning.

8 Key Skills

8.1 What are Key Skills?

Key Skills are those generic skills that can enable people to perform well in education, training and life in general. They can help people to become members of a flexible workforce and equip them with the means to benefit from life-long learning.

8.2 How are Key Skills classified?

They are classified under the following titles:

The 'main' Key Skills

Communication
Application of Number
Information Technology (IT)

The 'wider' Key Skills

Working with Others
Improving Own Learning and Performance
Problem Solving

8.3 At what level are they available?

Each of the Key Skills is available at four levels (1-4) of increasing demand. Each level requires candidates to use the Key Skills in a discrete fashion, although it is possible to successfully demonstrate more than one Key Skill in a suitably co-ordinated and wide-ranging (multi-task) exercise.

Level 5 Key Skills are also available. At this level all six Key Skills are demonstrated through demanding exercises of a complex nature.

8.4 What do the units look like?

The units for the Key Skills of Communication, Application of Number, Information Technology, Working with Others, Improving Own Learning and Performance, and Problem Solving comprise three parts:

- Part A: What you need to know
- Part B: What you must do
- Part C: Guidance

Part A of a unit tells candidates what they need to learn and practise to feel confident about applying the Key Skills in their studies, work or other aspects of their life.

Part B of a unit describes the skills candidates must show. All of the candidate's work for this section will be assessed. Candidates must have evidence that they can do all the tasks listed in the bullet points.

Part C of a unit contains examples of the sort of evidence that candidates could produce to prove that they have the skills required.

Copies of the above mentioned units can be obtained from the QCA web site: www.qca.org.uk

8.5 What qualifications are available in Key Skills?

Candidates who complete the assessment requirements (internal and external) for a Key Skills unit at level 1, 2, 3 or 4 will be awarded a unit certificate.

Candidates who achieve unit certification in each of the three Key Skills of Communication, Application of Number and IT will be awarded the Key Skills Qualification. This qualification will be profiled to reflect the level achieved in each of the three Key Skills. Consequently candidates do not have to achieve the units at the same level in all three Key Skills.

The three 'wider' Key Skills of Working with Others, Improving Own Learning and Performance and Problem Solving do not, at present, form part of the Key Skills Qualification. Currently, it is proposed that they be individually certificated.

8.6 What must candidates do to achieve these qualifications?

The 'main' Key Skills

The scheme of assessment for each of the 'main' Key Skills units at each level is made up of two components. The internally-assessed portfolio of evidence and the externally-assessed test/task. The internally-assessed evidence will be externally moderated. External assessment will consist of tests, tasks or assignments developed and assessed by AQA. Candidates must pass both assessment components at the same level to be successful in a Key Skill unit at that level. Candidates may enter for the separate components on different assessment occasions.

The 'wider' Key Skills

To achieve unit accreditation, candidates must be successful in compiling a portfolio of work demonstrating evidence of achievement of Part B of the unit (i.e. 'What you must do'). Discussions are still taking place about whether external assessment requirements should be made in the 'wider' Key Skills.

8.7 How can these Mathematics specifications help candidates to gain Key Skills?

Candidates following a course of study based on AQA GCSE 4306 and 4307 can be offered opportunities to develop and generate evidence of achievement in the Key Skills of Application of Number, Communication and Information Technology and the 'wider' key skills of Improving Own Learning and Performance, Working with Others and Problem Solving. The work produced can form part, or even all, of the portfolio for each Key Skill. The level to which each Key Skill can be developed and demonstrated will depend upon the opportunities made available in the course at each centre, together with the ability of the candidate. Centres may choose to deliver Key Skills totally separately from the candidates' main programs of study.

Alternatively, they may wish to integrate Key Skills into all main programmes of study for all candidates, or a mixture of the two. Whatever delivery style a centre adopts it is hoped that candidates will appreciate for themselves the applicability of Key Skills to much of their subject specific work, and the transferability of the skills from one context to another.

Detailed signposting opportunities for the delivery and generation of evidence of achievement are given in Section 8.12 of this Teachers' Guide. They are provided to assist teachers in considering how Key Skills could best fit into their teaching strategy. In each case, unless otherwise stated, the signposting will be for Key Skills at Levels 1 and 2. As explained above, this does not mean that work focused at other levels, or candidates' responses demonstrating achievement at other levels, cannot be set or rewarded. Likewise, teachers may choose not to make use of each and every opportunity identified.

8.8	How do I know if the work produced meets the Key Skills requirements?	Teachers who have not been involved in Key Skills will be concerned that the work produced by their candidates is of the right standard for success. It is hoped that each centre will have one or more Key Skills Co-ordinators who will ensure internal standardisation of assessment of Key Skills across the centre. AQA intends to hold Annual Portfolio Standardising Meetings for Key Skills co-ordinators or other centre representatives in the Autumn term.
8.9	Key Skills exemptions	The regulatory authorities (QCA, ACCAC and CCEA) have been asked to ensure that candidates who have taken, or who are taking, English, Gaeilge, Welsh, Mathematics and ICT GCE, GCSE and GNVQ qualifications should not be involved in unnecessary double assessment when seeking their Key Skills Qualification.
8.10	Exemptions for Communication	**Qualifications in GCSE English or English Literature, Gaeilge and Welsh which provide exemption from the Key Skill of Communication** • GCSE A* to C examination performance provides exemption from the external test in this Key Skill at Level 2. • GCSE D to G examination performance provides exemption from the external test in this Key Skill at Level 1.
8.11	Exemptions for Application of Number	**Qualifications in GCSE Mathematics† which provide exemption from the Key Skill of Application of Number** • GCSE A* to C examination performance provides exemption from the external test in this Key Skill at Level 2. • GCSE D to G examination performance provides exemption from the external test in this Key Skill at Level 1.

8.12 Signposting of Key Skills in Mathematics

Application of Number Level 1

The work produced by candidates can be used as portfolio evidence for the Key Skill of Application of Number. Schemes of work can provide candidates with opportunities to learn and provide evidence of how to:

- interpret information;
- carry out calculations;
- interpret results and present findings.

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
as part of a task: analysing the task; identifying sources of information needed for the problem and the types of calculations chosen to get results.	**N1.1**	Interpret information from **two** different sources. At least **one** source must include a table, chart, graph or diagram	• obtain information you need to meet the purpose of your task; • identify suitable calculations to get the results you need. Examples of evidence: • a description of the smaller tasks and their purpose within the framework of the task/problem (F2.1b; F4.1a) • copies of source material (such as a table, chart diagram or line graph – these should be straightforward) (coursework tasks; F4.1e; F4.1f) • records of the information obtained and the types of calculations identified in order to get the results. Candidates must obtain the information themselves and they must choose the methods (F3.4a).

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
as part of a class work task: carrying out calculations accurately, showing methods, checking methods and results.	**N1.2**	Use your information to carry out calculations to do with: a amounts or sizes b scales or proportion c handling statistics	• carry out calculations to the levels of accuracy you have been given; • check your results make sense. Examples of evidence: • a record of the calculations for: • amounts and sizes (F2.1d; F2.2e) • scales and proportion (F3.4f; F3.4g; F2.2f; F2.3f) • handling data (F4.4b, mean and range of up to 10 items) • a record of how the calculations were checked (coursework tasks; F2.1d; F2.4c; F4.1d).

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
carrying out a class work task: interpreting solutions or drawing conclusions; presenting conclusions. Use should be made of at least one chart and one diagram.	**N1.3**	Interpret the results of your calculations and present your findings – in two different ways using charts or diagrams..	• Choose suitable ways to present your findings: • present your findings clearly; • describe how the results of your calculations meet the purpose of your task. Examples of evidence: • a description of the findings and how the results of the calculations meet the purpose of the tasks (F4.1a) • at least one chart and one diagram presenting the findings (F4.4a). A candidate who is carrying out a task/or problem which uses subject contact from the Foundation Tier should usually meet the requirements for Level 1, Application of Number.

Application of Number Level 2

The work produced by candidates can be used as portfolio evidence for the Key Skill of Application of Number. Schemes of work can provide candidates with opportunities to learn and provide evidence of how to:

- interpret information;
- carry out calculations;
- interpret results and present findings.

Candidates are required to carry out at least one substantial activity that includes straightforward tasks for N2.1, N2.2 and N2.3.

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
as part of a task: describing the task; choosing how to obtain the information needed; obtaining the relevant information; analysing data.	**N2.1**	Interpret information from a suitable source.	• choose how to obtain the information needed to meet the purpose of your activity; • obtain the relevant information; • select appropriate methods to get the results you need. Examples of evidence: • a description of the **substantial** activity (task/problem)that included straightforward tasks (H4.1a) • copies of source material (for example from written and graphical material, first-hand by measuring or observing) (H4.1c; H4.3a) • records of the information obtained and the methods selected for getting the results needed (F4.1a; H4.1a; H4.1b; F4.1c; F4.1d).

When Candidates are:	They will have the opportunity to produce the following Key Skill Evidence		
as part of a class work task: carrying out calculations accurately; showing methods; checking methods and results and indicating how this was done.	**N2.2**	Use your information to carry out calculations to do with: a amounts or sizes b scales or proportion c handling statistics d using formulae.	• carry out calculations, clearly showing your methods and levels of accuracy; • check your methods to identify and correct any errors, and make sure your results make sense. Examples of evidence: • a record of the calculations for: a amounts and sizes (H2.1d; H2.1c; H2.3e) b scales and proportion (F3.4a; H3.4d; H2.2f; H2.3f) c handling data (H4.5c [minimum 20 items]; F4.5d) d using formulae (H4.5a; F2.5f) • a record of how the calculations were checked (H2.1d; H2.4b; F4.1d).

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
carrying out a class work task: describing findings and methods; explaining how the results relate to the task. Use should be made of at least one graph, one chart and one diagram.	**N2.3**	Interpret the results of your calculations and present your findings.	• select effective ways to present your findings; • present your findings clearly and describe your methods; • explain how the results of your calculations meet the purpose. Examples of evidence: • a description of the findings and methods used (H4.1a) • notes on how the results from the calculations met the purpose of the activity (H4.1a). A candidate who is carrying out a task or problem which uses subject content from Higher Tier, Grade C or above, should usually meet the requirements for Level 2, Application of Number. The grade description for Grade C, given in Appendix A of the specification, gives a general indication of the standard of achievement to be shown by a candidate.

Communication Level 1

The work produced by candidates can be used as portfolio evidence for the Key Skill of Communication. Schemes of work can provide candidates with opportunities and provide evidence of how to:

- contribute to discussions;
- read and obtain information;
- write different types of documents.

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
as part of a one-to-one and group discussion: analysing the task; identifying sources needed for the problem; analysing data; drawing conclusions.	**C1.1**	Take part in either a **one to one** discussion or **group** discussion.	• provide information that is relevant to the subject and purpose of the discussion; • speak clearly in a way that suits the situation; • listen and respond appropriately to what others say. Examples of evidence: • record from an assessor who observed each discussion and noted how the requirements were met • an audio/video tape of the discussions also can be used.

When Candidates are:	They will have the opportunity to produce the following Key Skill Evidence		
as part of carrying out a task: analysing the task given; researching, selecting and using data appropriately.	**C1.2**	Read and obtain information from at least **one** document	• read relevant material; • identify accurately the main points and ideas in material; • use the information to suit your purpose. Examples of evidence: • a record of what was read and why, including a note or copy of the image (4F1a; 4F1c; 4F1e) • notes, highlighted text or answers to questions about the material read (F2.1e; F3.1d; F4.1a; F4.1b).

Communication Level 2

The work produced by candidates can be used as portfolio evidence for the Key Skill of Communication. Schemes of work can provide candidates with opportunities and provide evidence of how to:

- contribute to discussions;
- give a talk;
- read and obtain information;
- write different types of documents.

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
as part of classroom discussion: analysing the problem; identifying sources needed for the problem; analysing data; drawing conclusions	**C2.1a**	Take part in a group discussion.	• make clear and relevant contributions in a way that suits your purpose • listen and respond appropriately to what others say; • help to move the discussion forward. Examples of evidence: • a record from an assessor who observed the discussion and noted how the requirements were met; an audio/video tape of the discussions also can be used. There are many opportunities for candidates to take part in a discussion in classroom situations, for example, when studying (H2.1j; H3.1a; H3.1b; H3.1c; H4.1a).

When Candidates are:	They will have the opportunity to produce the following Key Skill Evidence		
as part of a classroom activity: giving a short talk on a solution of a mathematical problem, using images such as diagrams or models.	**C2.1b**	Give a talk of at least four minutes.	• speak clearly in a way that suits your subject, purpose and situation; • keep to the subject and structure your talk to help listeners follow what you are saying; • use an image to clearly illustrate your main points. Examples of evidence: • a record from an assessor who observed the task, or an audio/video tape of the talk • notes from preparing and giving the talk. The notes should be structured to help the listeners follow a line of thought • a copy of the image used which helped others understand the main points of the talk. There are many opportunities for candidates to make a short talk to an audience, for example, when studying (H2.1c; H2.1e; H2.1j; H2.1m; H3.1b; H3.1d; H4.1a).

Information and Communication Technology
Level 1

The work produced by candidates can be used as portfolio evidence for the Key Skill of Information Technology. Schemes of work can provide candidates with opportunities and provide evidence of how to:

- find and develop information;
- present information.

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
as part of a task: finding different types of information; using graphics packages and spreadsheets to help in interpreting the problem; using the INTERNET to access information which can be used to develop the task or to provide secondary data.	**ICT.1**	Find and select relevant information.	• find and select relevant information; • enter and bring in information, using formats that help development; • explore and develop information to meet your purpose. Examples of evidence: • printouts and copies of the different types of information selected for use (F3.1a; F4.1a; F4.1c; F4.1e; F4.1f) • a record from an assessor who observed the use of IT when exploring and developing information or working drafts with notes on how the requirements of the unit were met (F3.1a; F3.1b; F4.1a; F4.1c; F4.1e; F4.1f).

When Candidates are:	They will have the opportunity to produce the following Key Skill Evidence		
presenting findings from the investigation of a task; could be presenting to a group using an overhead projector, computer etc or, could be for presentation to a wider audience.	**ICT.2**	Enter and develop information to suit the task.	• use appropriate layouts for presenting information in a consistent way; • develop the presentation so it is accurate, clear and meets your purpose; • save information so it can be found easily. Examples of evidence: • working drafts showing how the presentation was developed or records from an assessor who saw the screen displays • printouts of a static or dynamic screen display of the final work including examples of text, images and numbers • records of how the information was saved. • The evidence for this requirement can be gathered from the short talk (C2.1b). There are also opportunities when studying (F3.1a; F3.1a; F4.1c; F4.1e; F4.1f).

Information and Communication Technology
Level 2

The work produced by candidates can be used as portfolio evidence for the Key Skill of Information Technology. Schemes of work can provide candidates with opportunities and provide evidence of how to:

- searching for and selecting information;
- developing information;
- presenting information.

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
identify sources of information; use search engines on the Internet to identify and select information which can be used to develop the task; use the Internet to provide secondary data.	**ICT.2.1**	Search for and select information to meet your needs. Use different information sources for each task and multiple search criteria in at least one case.	• identify the information you need and suitable sources; • carry out effective searches; • select information that is relevant to your purpose. Examples of evidence: • printouts of the relevant information with notes of sources and how searches were made or a record from the assessor who observed the use of IT when searching for information (H3.1a; H3.1b; H4.1a; H4.1c).

When Candidates are:	**They will have the opportunity to produce the following Key Skill Evidence**		
as part of a classroom task; creating tables and graphs to show results for an investigation; exploring the effects of changing information in a spreadsheet model to make and test predictions; linking and organising information to derive new information.	**ICT.2.2**	Enter and develop the information to suit the task and derive new information.	• enter and bring together information using formats that help development; • explore information as needed for your purpose; • develop information and derive new information as appropriate. Examples of evidence: • printouts or a record from an assessor who observed the use of IT with notes to show the explanation and development of information and the derivation of new information (H3.1a; H3.1c; H3.1d; H4.1a; H4.1b; H4.1c).

When Candidates are:	They will have the opportunity to produce the following Key Skill Evidence		
as part of a classroom task: presenting findings from an investigation; presenting to a group using an overhead projector including text, images and tables; combining the information in a suitable way using the method suitable for the purpose.	**ICT2.3**	Present combined information such as text with image, text with number, image with number.	• select and use appropriate layouts for presenting combined information in a consistent way; • develop the presentation to suit your purpose and the types of information; • ensure your work is accurate, clear and saved appropriately. Examples of evidence: • working drafts, or a record from an assessor who observed screen displays, with notes to show how content and presentation was developed • printouts, or prints or static or dynamic screen displays, of the final work, including examples of text, images and numbers • records of how the information was saved. • The evidence for this requirement can be gathered from the short talk (C2.1b). There are also opportunities when studying (H3.1a; H4.1a; H4.1c).

Appendices

A Changes in Tier Content

Grade C topics These topics are new to the Foundation tier.

Ref	Topic	Example
Number		
F2.1j	Understand counter-example	Zoe thinks that all when you square a number it is always even. Give an example to show that Zoe is wrong. The phrase 'counter-example' will not be used in Foundation Tier.
H2.2a	Prime factors	Write 48 as the product of prime factors. Give answer in index form.
H2.2a	HCF and LCM	Find the HCF of 48 and 64
H2.4b	Estimation	Estimate the answer to $710 \times \frac{3.98}{0.19}$
H2.3a	Reciprocals	Find the reciprocal of 0.5
F2.3m	Percentage increase or decrease	% increase from 50 cm to 64 cm
F2.4d	Limits	Least and greatest value of 48 m rounded to nearest metre
F2.3c	Subtraction and addition of mixed numbers	Work out $4\frac{1}{4} - 2\frac{2}{5}$
H2.2b	Index laws for multiplication and division of powers	Write $5^3 \times 5^4$ as a single power of 5
H2.2b	Negative square roots	$x^2 = 36$ Give possible values of x
H2.3f	Ratio	Share £420 in ratio 3:4
H2.3n	Use of π in exact calculations	Circle radius 3. What is area? Give answer in terms of π.
F2.3p F2.3q	Enter standard form on a calculator. Interpret calculator display	Will not be assessed at Foundation tier.

Ref	Topic	Example
Algebra		
F2.5c	Simple instances of index laws	Simplify $x^3 \times x^2$, $x^8 \div x^4$
H2.5b	Expanding and simplifying expressions	Expand and simplify $6(x + 2) - 4(x - 2)$
H2.5b	Expanding brackets with powers	Expand $x(x^3 + 2x)$
F2.5e	Solve linear equations that require prior simplification of brackets and have the variable on both sides	Solve $4(2x - 3) = 3x + 8$
H2.6a	*n*th term of a linear sequence	Write down the *n*th term of 3, 7, 11, 15, 19, ….
H2.5b	Form and solve a linear equation	Given the angles of a triangle are $x + 30$, $2x + 10$ and $4x$, find x
H2.5m	Trial and Improvement	Solve $x^3 + 2x = 100$ using trial and improvement Give answer to 1dp. Starting value and/or table may be given
H2.5b	Expand the product of two linear expressions	Expand and simplify $(x + 1)(x - 2)$
F2.5f	Change subject of a formula	Make x subject of $y = 2x + 3$ At most two operations to rearrange
H2.6e	Generate and plot points of quadratic functions	Complete table and use it to plot graph of $y = x^2 + 2x - 3$
H2.6e	Find solutions of a quadratic equation from graph	Use the graph to solve $x^2 + 2x - 3 = 0$
F2.6c	Draw line of best fit through set of linearly related points and find its equation	Finding the equation of a given or drawn line will not be assessed at Foundation Tier
F2.6e	Gradients of lines	Here are the equations of 6 lines. Which two are parallel?
F2.5d	Solve simple inequalities	Solve $3x + 2 > 11$

Ref	Topic	Example
F2.5d	Inequalities on a number line	What inequality is shown? Open circles will mean strict inequalities. Filled in circles will mean inclusive inequalities
F2.6c	Graphs from real-life situations	Drawing or interpreting a distance-time graph.

Shape, Space and Measure

F3.4j	Loci	Show all points within 4 cm of point A. At most two constraints.
F3.4e	Constructions Perpendicular bisector Angle bisector Angle of 60° Perpendicular to line Perpendicular from line	Construct the perpendicular bisector of the points A and B.
F3.2g	Regular polygons	Given exterior angle of 30°. How many sides has the polygon?
F3.4h	Area and perimeter of semicircles	Find the perimeter of a semicircle with radius 4 cm.
F3.2h	Pythagoras	Find the length x. Right angled triangle given with two sides given, Third side marked x.
H3.2i	Volume of a prism	Calculate the volume of the prism. Prism with right angled triangle cross section given. First part of question finding area of triangle.
H3.2i	Volume cylinder	Find the volume of a cylinder with radius 5 cm and height 10 cm. Surface area of cylinder will not be assessed at Foundation Tier.
F3.3c	Enlargement with fractional scale factor	Enlarge the given triangle by a scale factor $\frac{1}{2}$ about (0, 1)
H3.4b	Vector notation for translation	Translate the shaded triangle by given vector.

Ref	Topic	Example
H3.4a	Compound measure, density	Find the density of a solid with given volume and mass. NB word mass will be used in density problems.
F3.3e	Mid point of two given coordinates	Find mid-point of A(−2, 6) and B(4, −4)
F3.4a	Limits of measurements to nearest unit	Limits of mass of 32 kg to nearest kilogram.
F3.3d	Distinguish between formula for length area and volume	Which of these formulae are lengths? Formulae will be ones that are within Foundation experience such as $2l + 2w$.
F3.3d	Understand effect of enlargement on area and volume	Rectangle ABCD is 2 cm by 3 cm. Rectangle PQRS is twice the size of ABCD. How many times greater is the area of PQRS than the area of ABCD? Diagrams given.

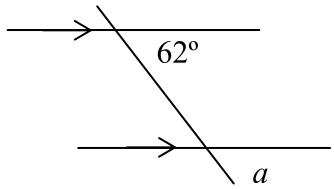
Handling Data

F4.5f	Describe correlation	Describe the relationship between two variables. Scatter diagram given.
F4.5h	Relative frequency	Calculate relative frequency from a table.
F4.5i	More trials leads to better estimates	Which pupils' results are more reliable? Explain your answer
F4.4b	Mean of a grouped table	Calculate an estimate of the mean of… Grouped table given.
F4.4a	Frequency diagrams for grouped data	Draw a frequency diagram for the data Grouped table given. Diagram can be frequency polygon or histogram with equal intervals.

Grade D topics — The majority of topics are 'subsumed' and used when doing problems at a higher grade.

Ref	Topic	Example
Number		
H2.3h	Estimation.	Estimate the value of $48 \times \frac{307}{57}$ Estimate the answer to 5.3×19.8
H2.3o	Use calculators effectively and efficiently	Work out $\frac{3.1 \times 4.7}{(8.3 - 5.6)}$ Give your answer to one decimal place
H2.3h	Linked calculations	Given that $53 \times 821 = 43513$, write down the value of $\frac{43513}{530}$
H2.3j	New value after a percentage increase or decrease	Price of a washer was £450. What is new price after a 4% reduction?
H2.3e	Fractions to percentages (x as a fraction of $y \Rightarrow x$ as a percentage of y)	90 out of 300 as a percentage.
H2.3c	Dividing fractions by fractions and whole numbers Multiplying fractions by whole numbers	$\frac{2}{3} \div \frac{1}{4}$ $\frac{2}{3} \times 5$ $\frac{2}{3} \div 4$
H2.3c	Adding and subtracting fractions	$\frac{2}{3} + \frac{4}{5}$, $\frac{2}{3} - \frac{1}{5}$
H2.3c	Distinguish between terminating and recurring decimals by performing short division	What is $\frac{7}{11}$ as a recurring decimal.
H2.3f	Simple ratio	Share £150 in ratio 4:1
F2.3k	Division by decimal (up to 2dp) by converting to division by integer	$31.2 \div 0.26$
H2.3b	Brackets and hierarchy of operations	Put brackets in this calculation to make it true $2 + 3^2 \times 6 - 4 = 50$

Ref	Topic	Example
Algebra		
H2.5a	Algebraic expressions	John is x years old. Pam is 3 years younger than John. Write an expression for Pam's age in terms of x.
H2.5d	Substitute positive and negative numbers into expressions	$a = -3$, $b = -4$. Evaluate $a^2 - 2b$
H2.5f	Solve linear equations where the variable appears on both sides of the equation	Solve $5x - 3 = 2x + 9$
H2.5f	Solve linear equations that require prior simplification of brackets	Solve $4(2x - 3) = 28$
H2.5f	Solve linear equations in one unknown with fractional coefficients	$\frac{x}{4} = 15$
H2.5b	Factorisation	Factorise $x^2 + 5x$ Factorise $10a + 6$
H2.5b	Expand and simplify	Multiply out $6(x - 7)$
H2.6b	Plot linear graphs	Draw the graph of $y = 3x - 1$ for $-3 \leq x \leq 3$ (no table given)
H2.6e	Generate and plot points of simple quadratic functions	Complete the table of $y = x^2 - 6$ for $-3 < x < 3$ and draw the graph

Ref	Topic	Example
Shape, Space and Measure		
H3.2a	Angles in parallel lines and a transversal	What is the value of angle a. Give a reason for your answer? 
H2.3c	Definitions of quadrilaterals	Name a quadrilateral with 2 pairs of equal sides 2 lines of symmetry Rotational symmetry order 2
H3.2i	Plan and elevation	Given isometric drawing, draw plan and front elevation on square grid.
H3.3a	Transformations	Describe the single transformation that takes shape A to shape B (simple reflection, rotation, translation, enlargement with whole number scale factor).
H3.3d	Scale drawings	Given that the map is to a scale of 1cm represents 50km, find the actual distance of A to B.
H3.4a	Speed, density	John travels 82.5 miles in 1 hour and 15 minutes. What is his average speed?
H3.4b	Construct triangles	Construct the following triangle accurately (sketch of triangle with sides 8 cm, 6 cm and included angle of 75°, 8 cm base line drawn)
H3.2g	Bearings	Measure the bearing of A from B (diagram to scale given)
H3.4d	Areas of compound shapes	Area of 'L' shape with some dimensions given.
H3.4d	Area of a trapezium	Calculate the area of trapezium with dimensions of parallel sides and distance between them given.
H3.4d	Circumference and area of circles	Find the circumference and area of a circle with diameter 7 cm.

Ref	Topic	Example
H3.4d	Covert between area and volume measures	Calculate the area of this rectangle. Give your answer in square metres. Picture of rectangle with sides of 20 cm and 30 cm given.

Handling Data

H4.3b	Design and use two way tables	This table shows the number of left handed and right handed boys and girls in year group. If a pupil is picked at random what is the probability they are left handed?
H4.4a	Stem-and-leaf diagrams	Calculate the mean of the data in the stem-and-leaf diagram
H4.3a	Questionnaires	The following question is from a questionnaire on 'Healthy eating'. Give two reasons why it is not a good question.
H4.4c	List outcomes of combined events	A dice and a coin are thrown together. One outcome could be Head and 6. List all the possible outcomes
H4.4d	Mutually exclusive outcomes	Find missing probability from a table of values.
F4.4b	Mean of a discrete frequency table	The table shows the number of children per family. Calculate the mean.
H4.4b	Expectation	How many 4s would you expect in 100 throws? (biased or fair 4 sided spinner).

B

Student Support Sheet

Question Paper Terminology

Listed below are examples of instructions given in questions and an amplification of their meaning. The list is as comprehensive as possible but does not claim to be exhaustive.

What we Say…	**What it means….**
Estimate	Round the numbers to 1 s.f. and use these to obtain an answer. Find the mean of a grouped frequency table. Average speed.
Explain	Use words to explain an answer.
You **must** show your working	You will be penalised if you do not show your working.
Simplify	Collect terms together.
Simplify fully	Collect terms together and factorise the answer.
Show that	Use words, numbers or algebra to show an answer.
Prove	A rigid algebraic or geometric proof is required.
Work out	Normally means a calculation is involved but it may be possible to do it mentally.
Calculate	Will need a calculation that requires a calculator or a formal (such as column) method.
Measure	Use a ruler or protractor to measure a length or an angle.
Hence	Use the previous answer to proceed.
Hence, or otherwise	Use of the previous answer is expected but another method will be accepted.
Describe fully in transformations	Reflection – define the mirror line. Translations – state vector. Rotations – state centre, angle and direction. Enlargement – state scale, factor and centre.
Factorise	Take out the common factor or factorise into two brackets if a quadratic.
Factorise fully	Usually means that there is more than one common factor, ie, indicates that there are at least two stages in the factorisation.

Use the graph	Do not calculate, read from the graph. Always worth putting lines on the graph to show where the answer came from.
Give an exact value	Give answer as a square root or surd form (non-calculator paper)
Give your answer in terms of π/in surd form	Give answer in terms of π /in surd form (non-calculator paper)
Give answer to a sensible degree of accuracy	Normally no more accurate than values in the question. If question has values to 2 s.f. then give answer(s) to 2 s.f. or 1 s.f. Trigonometrical answers accepted to 3 s.f.
Give answer to (2 d.p.)	Give answers to required accuracy. You will lose marks if you do not.
Not drawn accurately	Next to a diagram to discourage measuring of lengths or angles.
Not to scale	Next to diagram to discourage measuring of lengths.
Do an accurate drawing	Use compasses to draw lengths, protractors to measure angles (and a sharp pencil)
Use a ruler and compass	A ruler may be needed to measure but more often than not we mean, use a straight edge and compasses. Used in constructions.
Use an algebraic method	Do not use trail and improvement. Working will be expected.
Do not use trial and improvement	An algebraic method is expected. Any sign of trail and improvement will be penalised.
Expand	Multiply out using distributive law.
Multiply out	Multiply out using distributive law.
Expand and simplify	Multiply out using distributive law and then collect terms.
Multiply out and simplify	Multiply out using distributive law and then collect terms.
Give a counter-example	Give a numerical or geometrical example that disproves a statement.
Solve	Find the values(s) of (x) that makes the equation true.
Make (x) the subject	Rearrange a formula.
Express, in terms of	Use given information to write an expression using only the letter(s) given.
Write down	Working out is not needed to get an answer.

C

Proof

Section 1

Within the written papers, the rigour and quality of proof required will depend on the grade at which it is being examined. Questions on proof test reasoning, the third strand of using and applying mathematics, in number and algebra, shape, space and measures and handling data, of the specifications.

Candidates need to know the difference between verification and proof. They should be able to produce a deductive, step-by-step argument and also use a counter-example to show a statement is incorrect. In geometry, candidates should be able to construct a proof, giving reasons for statements in a step-by-step manner. However, they will not be expected to reproduce proofs of standard theorems, for example, those given in section 2.

Candidates will need to know that 'a result' can be disproved by a single counter example and that even if a result works for many numerical examples it may not be valid for all values.

Example (at grade E)

Freda thinks that when you square a number you always get an even number answer.

Give an example to show that Freda is wrong.

Answer

$3 \times 3 =$ and 9 is an odd number

Example (at grade E)

Tom is using multiples of threes and fours.

He notices $28 = 27 + 1$ and $16 = 15 + 1$

Tom says that multiples of four are always one more than a multiple of three.

Show that Tom is wrong.

Answer

$24 = 23 + 1$ and 23 is not a multiple of 3

Example (at grade D)

Show that the sum of **two** consecutive whole numbers is always odd.

Answer

Numbers are either odd + even = odd

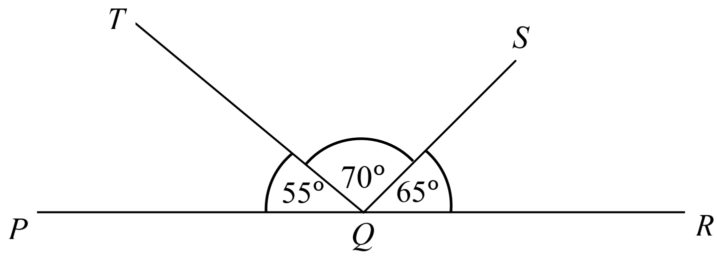
or even + odd = odd

[an algebraic method is fine but would not be expected at Foundation tier]

Let the numbers be n and $n + 1$. $n + n + 1 = 2n + 1$, which is odd.

Example (at grade D)

In the diagram, $\angle PQT = 55^\circ$, $\angle TQS = 70^\circ$ and $\angle SQR = 65^\circ$



Josh says that *PQR* is a straight line.

Is Josh correct?

You **must** explain your answer.

Answer

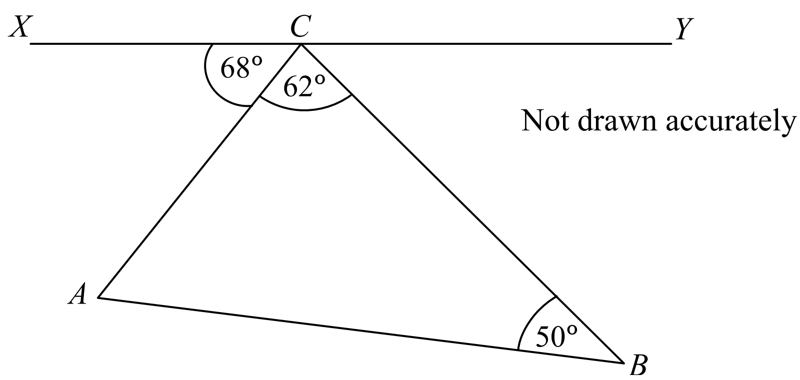
55 + 70 + 65 = 190

No, not straight. Angles should make 180°

Example (at grade C)

The diagram shows a triangle ABC and a straight line XCY.

$\angle XCA = 68^\circ$, $\angle ACB = 62^\circ$ and $\angle ABC = 50^\circ$



Prove that the line *XCY* is parallel to the line *AB*.

Answer

EITHER

$\angle BCY = 180^o - 68^o - 62^o$ (angles on a straight line = 180^o)

$= 50^o$

Since $\angle BCY = \angle ABC$ (= 50^o)

XCY is parallel to AB (converse of alternate angles equal)

[Although the word converse is desirable for the last reason it would not be expected.]

OR

$\angle CAB = 180^o - 62^o - 50^o$ (angle sum of a triangle = 180^o)

$= 68^o$

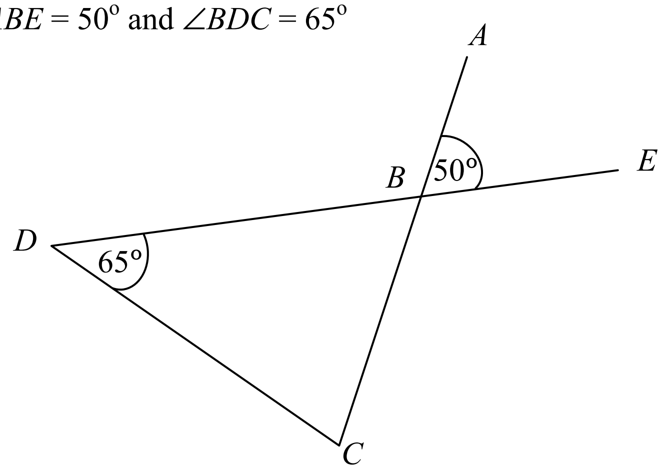
Since $\angle CAB = \angle XCA$ (= 68^o)

XCY is parallel to AB (alternate angles equal)

Example (at grade C)

In the diagram, ABC and EBD are straight lines.

$\angle ABE = 50^o$ and $\angle BDC = 65^o$



Show that triangle BCD is isosceles.

Give reasons with your working.

Answer

$\angle DBC = 50^o$ (vertically opposite angles equal)

$\angle BCD = 180^o - 50^o - 65^o$ (angle sum of a triangle = 180^o)

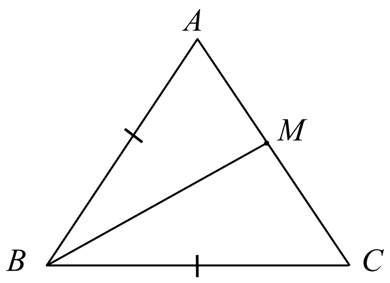
$= 65^o$

Triangle has two equal angles (of 65^o) and so is isosceles.

Example (at grade A)

ABC is an isosceles triangle with $AB = CB$.

M is the midpoint of AC.



Prove that triangles ABM and CBM are congruent.

Answer

$AB = BC$ (given, isosceles triangle)

$AM = MC$ (M is the midpoint of AC)

BM is common to both triangles

$\therefore \Delta ABM \equiv \Delta CBM$ [SSS]

[Other congruence results such as AAS, RHS, SAS should not be used as they use properties of isosceles triangles which are proved by congruency!]

Example (at grade A*)

(a) If n is a positive integer, explain why $2n + 1$ must be an odd number.

(b) Prove that the square of any odd number is always 1 more than a multiple of 8.

Answer

(a) 2 × number is even. Even + 1 is odd.

(b) $(2n + 1)^2 = (2n + 1)(2n + 1)$

$= 4n^2 + 2n + 2n + 1$

$= 4n^2 + 4n + 1$

$= 4n(n + 1) + 1$

n and $n + 1$ are consecutive numbers. One must be odd and one even.

$\therefore n(n + 1)$ is a multiple of 2

$\therefore 4n(n + 1)$ is a multiple of 8

$\therefore 4n(n + 1) + 1$ is '1 more than a multiple of 8'

Solutions to 'Proof' questions are never 'set in stone'. Alternatives to those given in the mark scheme that demonstrate the ability to think logically and explain clearly are always welcomed.

Here are two more solutions to part (b) of the above problem, the second of which not only has the necessary attributes but is also elegant in its simplicity.

Alternative 1, part (b)

If n is a positive integer then $4n$ must be even and $4n + 1$ must be odd.

$$(4n + 1)^2 = 16n^2 + 8n + 1$$

Subtracting 1 from the square of the odd number gives

$$(4n + 1)^2 - 1 = 16n^2 + 8n + 1 - 1$$
$$= 16n^2 + 8n$$
$$= 8n(2n + 1)\text{… which is clearly divisible by 8}$$

Note 1: The last step (factorising) is not essential.

2: Starting with $4n + 3$ and arriving at $16n^2 + 24n + 8$ is equally valid.

3: Starting with $4n + p$ (where p is any odd number) will lead to a correct conclusion.

Alternative 2, part (b)

Let n be an **odd** number.

Then n^2 is odd (odd × odd = odd).

$\therefore n^2 - 1$ is even

$n^2 - 1 = (n + 1)(n - 1)$

$(n - 1)$ and $(n + 1)$ must be **consecutive even** numbers

One of them will be a multiple of 2 and the other will be a multiple of 4

Their product must therefore be a multiple of 8

Note: The algebra skill needed is grade B but the thinking skill is A*.

Note: This was an actual response to this 'Proof' question when it was set in June 2004!

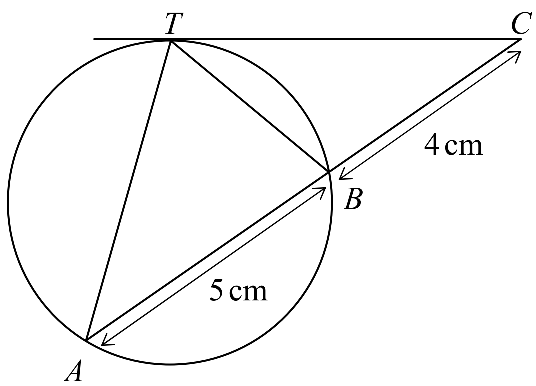
Example (at grade A*)

CT is a tangent to the circle at T.

$AB = 5$ cm and $BC = 4$ cm.

(a) Prove that triangles BTC and TAC are similar.

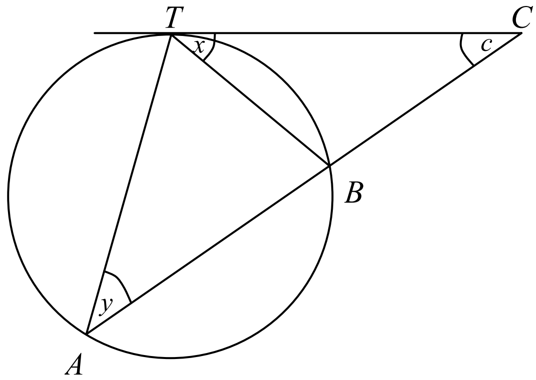
(b) Hence find the length of CT.



Answer

A candidate will probably mark, say, angles x, y and c.

In triangles BTC and TAC



$x = y$ (angle between tangent and chord equals angle in alternate segment)

Angle c is common to both triangles.

$\angle TBC = \angle ATC$ 3rd angles equal since sum of a triangle = 180°

Triangles $\dfrac{BTC}{TAC}$ are similar (equiangular)

Since triangles are similar,

$$\frac{BC}{TC} = \frac{TC}{AC}$$

$$BC \times AC = TC^2$$

$$TC^2 = 4 \times 9$$

$$= 36$$

$$TC = 6$$

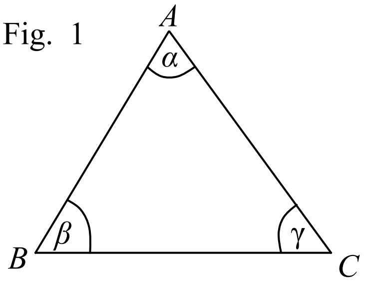
Section II: Exemplar Proofs

Teachers should refer to the specifications. At Foundation Tier, for example, candidates are required to 'understand a proof that the angle sum of a triangle is 180°' and to 'understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices'. At the Higher Tier, candidates are required to 'prove and use… (circle theorems)'. In any formal proof there needs to be an understanding of the assumptions. Versions of the proofs of the theorems in the specification have been reproduced here for the benefit of teachers. **Candidates will not be required to reproduce these proofs in examinations**. Teachers should note that other versions of the proofs are acceptable.

Proof that the angle sum of a triangle is 180°

Take a triangle *ABC* with angles α, β and γ (Fig. 1)

Fig. 1



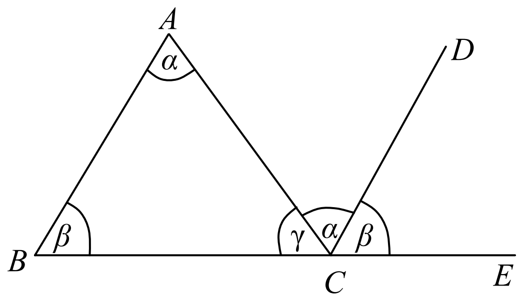
Draw a line *CD* parallel to side *AB* and extend *BC* to *E* (Fig. 2)

$A\hat{C}D = B\hat{A}C = \alpha$ (alternate angles)

$D\hat{C}E = A\hat{B}C = \beta$ (corresponding angles)

BCE is a straight line so $\alpha + \beta + \gamma = 180°$

Fig. 2



Proof that the exterior angle of a triangle is equal to the sum of the two opposite interior angles

In the diagram, angle $A = \alpha$, angle $B = \beta$ and angle $C = \gamma$.

$\alpha + \beta + \gamma = 180$ (angle sum of a triangle)

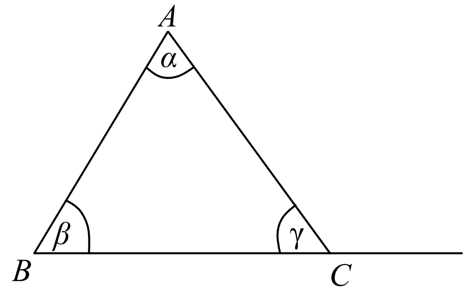
$\gamma = 180° - \alpha - \beta$

Exterior angle at $C = 180° - \gamma = 180° - (180° - \alpha - \beta)$

$= 180° - 180° + \alpha + \beta$

$= \alpha + \beta$

= sum of two opposite interior angles



Proof that the angle subtended by a chord at the centre of a circle is twice the angle subtended at the circumference in the same segment

In the diagram, AB is a chord of circle centre O and C is a point on the circumference.

Draw a line from C through O to D

Let $A\hat{C}O = \alpha$; let $B\hat{C}O = \beta$

$C\hat{A}O = \alpha$ (triangle OAC is isosceles)

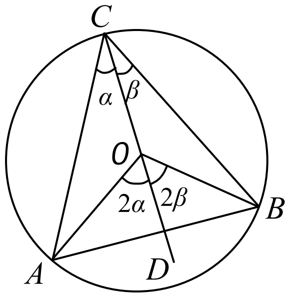
Therefore $A\hat{O}D = 2\alpha$ (exterior angle equal to sum of opposite interior angles)

Similarly $D\hat{O}B = 2\beta$

Hence, $A\hat{O}B = 2\alpha + 2\beta$

$= 2(\alpha + \beta)$

$= 2 \times A\hat{C}B$



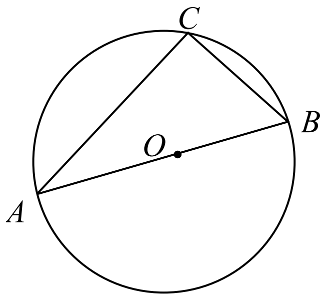
Proof that the angle in a semi-circle is a right angle

In the diagram, AB is a diameter of circle, centre O and C is a point on the circumference.

$\angle AOB = 2 \times \angle BCD$ (angle at centre = 2 × angle at circumference)

But $\angle AOB = 180°$ (AOB is a diameter which is a straight line)

$\therefore \angle ACB = 90°$



Proof that the opposite angles in a cyclic quadrilateral add to 180°

In the diagram, $ABCD$ is a quadrilateral drawn inside a circle centre O

Draw the radii OA and OC

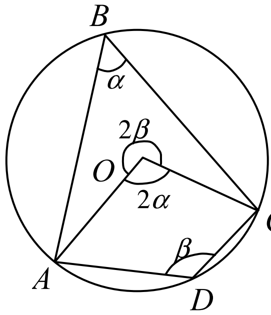
Let $A\hat{B}C = \alpha$; let $A\hat{D}C = \beta$

$A\hat{O}C$ (obtuse) $= 2\alpha$ (angle at centre is twice the angle at the circumference)

Similarly $A\hat{O}C$ (reflex) $= 2\beta$

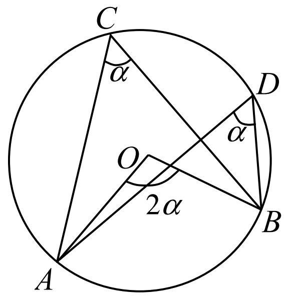
Hence, $360° = 2\alpha + 2\beta = 2(\alpha + \beta)$

Therefore, $\alpha + \beta = 180°$



Proof that angles subtended by a chord at the circumference in the same segment are equal

In the diagram, AB is a chord of the circle centre O, and C is a point on the circumference



Let $A\hat{C}B = \alpha$

$A\hat{O}B = 2\alpha$ (angle at centre twice angle at circumference)

Let D be another point on circumference

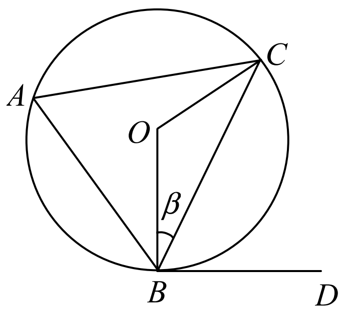
$A\hat{D}B = \alpha$ (angle at centre twice angle at circumference)

Hence, $A\hat{C}B = A\hat{D}B = \alpha$

Proof of the alternate segment theorem

In the diagram, DB is a tangent at B to the circle centre O

A and C are points on the circumference



Draw the radii OB and OC

Let $O\hat{B}C = \beta$

$B\hat{O}C = 180 - 2\beta$ (triangle OBC is isosceles)

$$B\hat{A}C = \frac{180 - 2\beta}{2} = 90° - \beta$$

(angle at the centre is twice the angle at the circumference)

But $C\hat{B}D = 90° - O\hat{B}C = 90° - \beta$

(angle between tangent and radius is 90°)

Hence, $B\hat{A}C = C\hat{B}D$