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ASSESSMENT and QUALIFICATIONS ALLIANCE

## General Certificate of Secondary Education

# Mathematics 4301 and 4302

*Specifications A (Linear) and B (Modular)*

2008

# TEACHERS' GUIDE

Further copies of this specification booklet are available from:

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# Background Information

## 1 Introduction

Following a review of the National Curriculum requirements, and the establishment of the National Qualifications Framework, all the unitary awarding bodies revised their GCSE specifications for first examination in 2003. These specifications have been further revised for examination in 2008 to incorporate necessary changes in moving to two tiers of assessment. The new specifications will be used by schools and colleges for two-year courses starting in September 2006

### 1.1 Purpose

This Teachers' Guide has been provided to assist teachers in their preparation for the delivery of courses based on the revised AQA GCSE specifications 4301 and 4302. The guide should be read in conjunction with the specification document and the specimen assessment materials that accompany them. All documents are available in hard copy. The specifications and specimen assessment materials are also available on the AQA Website (www.aqa.org.uk).

### 1.2 Requirements at GCSE

#### Tiering

In GCSE Mathematics the scheme of assessment must include question papers targeted at two tiers of grades: A* to D (Higher), and C to G (Foundation).

Candidates should be entered at the tier appropriate to their attainment. In GCSE Mathematics (Modular) each candidate may enter for each individual module at a different tier of entry. However, the final range of grades available to a candidate is determined by the tier of entry for Module 5. Candidates who fail to achieve the mark for the lowest grade available at each tier of Module 5 will be recorded as U (unclassified).

#### Key Skills

All GCSE specifications must identify, as appropriate, opportunities for generating evidence on which candidates may be assessed in the 'main' Key Skills of *Communication, Application of Number* and *Information and Communication Technology* at the appropriate level(s). Also, where appropriate, they must identify opportunities for developing and generating evidence for addressing the 'wider' Key Skills of *Working with Others, Improving own Learning and Performance* and *Problem Solving*.

Opportunities for delivering Key Skills have been highlighted in the specifications.

## 1.3 Use of ICT

The subject content of all GCSEs must require candidates to make effective use of ICT and provide, where appropriate, assessment opportunities for ICT.

Within these specifications candidates will have opportunities to apply and develop their ICT capabilities through the practical use of ICT tools to support their learning. The level to which the use of ICT is developed will depend in part upon the teaching styles and methods used to deliver the subject content, the opportunities available at the centre and the abilities of the candidates.

However, ICT skills are not assessed by any component of these specifications.

Suitable opportunities may be found, within each section of the subject content, to use ICT skills to find and develop information and present findings in a variety of appropriate formats. Candidates should be provided with opportunities to support their work by:

- Using the Internet/CD-ROMS/databases/software packages to obtain, select and manipulate information: eg, using software packages to generate data for the AO1 task (further guidance is provided in Section 6.9); using the Internet and databases to collect and select data for the AO4 task (a list of websites is provided in Section 10.3);
- Using ICT to analyse and present data for the AO4 task (Section 6.9);
- Presenting results from investigations using ICT tools to amend and refine their work and enhance its quality and accuracy;
- Reviewing, modifying and evaluating their mark, reflecting critically on its quality as it progresses (eg, in writing up investigations for coursework tasks).

The National Curriculum for Mathematics identifies opportunities for the use of ICT as candidates learn Mathematics. The references for these opportunities are given below. Each statement is referenced to the corresponding statement in the Foundation or Higher Programme of Study in the National Curriculum. Thus, F2.1a refers to AO2, Foundation Programme of Study, statement 1a.

| **Specification Reference** | **ICT Opportunity** |
|---|---|
| F2.5f | Candidates could use a spreadsheet to construct formulae to model situations |
| F2.6d | Candidates could use a spreadsheet to calculate points and draw graphs to explore the effects of varying $m$ and $c$ in the graph of $y = mx + c$ |
| F4.5c | Candidates could use databases to present their findings |
| H2.6b – H2.6f | Candidates could generate functions from plots of data, (for example, from a science experiment) using simple curve fitting techniques on graphic calculators, or with graphics software |
| H2.6g | Candidates could use software to explore transformations of graphs |
| H3.3b – H3.3f | Candidates could use software to explore transformations and their effects on properties of shapes |
| H4.1c | Candidates could use databases or spreadsheets to present their findings and display their data |
| H4.5c | Candidates could use databases to present their findings |

## 1.4 Citizenship

Since 2002, students in England have been required to study Citizenship as a national curriculum subject. Each GCSE specification must signpost, where appropriate, opportunities for developing citizenship knowledge, skills and understanding

Coursework tasks, particularly those for AO4 *Handling Data*, promote the skills of enquiry and communication. They also encourage the skill of participation and responsible action in the educational establishment and/or communication.

| **Specification Reference** | **Aspect of Citizenship** |
|---|---|
| F3.1e; H3.1d; H3.1f; H3.1j; F4.1a; H4.1a; F4.1b; F4.1e; H4.1d; F4.1i; H4.1f; H4.1g; F4.5a; H4.5a; F4.5b; F4.5c; H4.5c; F4.5d; H4.5d; F4.5e; H4.5e; F4.5f; F4.5g; H4.5g; F4.5h; H4.5h; F4.5j | *Thinking skills,* through helping candidates to engage in social issues that require the use of reasoning, understanding and action through enquiry and evaluation |

| Specification Reference | Aspect of Citizenship |
|---|---|
| F2.2e; H2.2e; F2.3c; F2.3e; H.23e; F2.3m; H2.3j; F2.3n; H2.3j; H2.3k; F2.3q; H2.3p; F2.4a; F2.4c; F2.5f; H3.4a | *Financial capability,* through developing candidates' understanding of the nature and role of money |
| F2.2e; H2.2e; F2.3c; F2.3e; H2.3e; F2.3m; H2.3j; F2.3n; H2.3j; H2.3k | *Enterprise and entrepreneurial skills,* through developing candidates' understanding of the importance of these skills for a thriving economy and democracy |
| F3.4a; F3.4i | *Work related learning,* through helping candidates to appreciate the link between learning and work for a thriving economy and society |
| H3.3t; F2.6e | *Education for sustainable development,* through developing candidates' skills in, and commitment to, effective participation in the democratic and other decision-making processes that affect the quality, structure and health of environments and society and exploring values that determine people's actions within society, the economy and the environment. |

## 1.5 The Mathematics Criteria

- Internal assessment is compulsory.
- Internal assessment comprises two tasks:
  - (i) the AO4 task – a handling data task which counts as half of the AO4 weighting;
  - (ii) the AO1 task – an investigative task which assesses AO1 in the context of AO2 and/or AO3 and counts as half of the AO1 weighting.
- The other halves of the AO1 and AO4 weightings are assessed in the written papers.
- The overall subject content is the same as the 2007 specification. Changes are to accommodate only the different tiering arrangements.
- Some questions demanding the unprompted solution of multi-step problems are required.
- Grade descriptions have been modified slightly.

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# Specification at a Glance
## *Mathematics A*

- AQA offers two GCSE Mathematics specifications. Specification A is a traditional linear scheme; and is suitable for both pre-16 and post-16 candidates.
- There are two tiers of assessment, Foundation (C – G) and Higher (A* – D).

| GCSE Mathematics A (4301) | |
|---|---|
| **Paper 1** | |
| Written Paper (Non-Calculator) | 40% of total marks |
| Foundation Tier | 1 hour 30 minutes |
| Higher Tier | 2 hours |
| **Paper 2** | |
| Written Paper (Calculator) | 40% of total marks |
| Foundation Tier | 1 hour 30 minutes |
| Higher Tier | 2 hours |
| **Coursework** | |
| Both tiers | 20% of total marks |
| **Either**<br>Option T<br>Centre-set or AQA-set tasks<br>Centre-marked | **Or**<br>Option X<br>AQA-set tasks<br>AQA-marked |

| Foundation Tier |
|---|
| Option T 4301TF<br>Option X 4301XF |
| **Higher Tier** |
| Option T 4301TH<br>Option X 4301XH |

# Specification at a Glance
# *Mathematics B (Modular)*

AQA offers two GCSE Mathematics specifications. Specification B is modular and is suitable for both pre-16 and post-16 candidates.

- There are two tiers of assessment, Foundation (C – G) and Higher (A* – D).

| GCSE Mathematics B (4302) | |
|---|---|
| **Module 1** | |
| Written Paper | 11% of total marks |
| 2 x 25 minutes (Both tiers) | |
| Section A – Calculator<br>Section B – Non-calculator | |
| **Module 2** | |
| Coursework (AO4 task) | 10% of total marks |
| **Either**<br>Option T<br>Centre-set or AQA-set task<br>Centre-marked | **Or**<br>Option X<br>AQA-set task<br>AQA-marked |
| **Module 3** | |
| Written Paper | 19% of total marks |
| 2 x 40 minutes (Both tiers) | |
| Section A – Calculator<br>Section B – Non-calculator | |
| **Module 4** | |
| Coursework (AO1 task) | 10% of total marks |
| **Either**<br>Option T<br>Centre-set or AQA-set task<br>Centre-marked | **Or**<br>Option X<br>AQA-set task<br>AQA-marked |
| **Module 5** | |
| Written Paper | 50% of total marks |
| Paper 1 Non-Calculator | |
| Foundation Tier | 1 hour 15 minutes |
| Higher Tier | 1 hour 15 minutes |
| Paper 2 Calculator | |
| Foundation Tier | 1 hour 15 minutes |
| Higher Tier | 1 hour 15 minutes |

| Foundation Tier<br>Higher Tier |
|---|
| Modules 1, 3 and 5 are available in both tiers |

# Scheme of Assessment

# 3 Subject Content Guidance

## 3.1 National Criteria

Both specifications comply with the following:

The GCSE Subject Criteria for Mathematics;

The GCSE and GCE A/AS Code of Practice;

The GCSE Qualification Specific Criteria;

The Arrangements for the Statutory Regulation of External Qualifications in England, Wales and Northern Ireland: Common Criteria.

## 3.2 Marks per Grade

The Foundation Tier will have 50% of the assessment targeted at grades G and F and 50% targeted at grades E, D and C.

| 50% | | 50% | | |
|---|---|---|---|---|
| G | F | E | D | C |

The Higher Tier will have 50% of the assessment targeted at grades D and C and 50% targeted at grades B, A and A*.

| 50% | | 50% | | |
|---|---|---|---|---|
| D | C | B | A | A* |

### Units Mark

One question per tier will have the additional requirement that candidates 'State the units of your answer'.

This will be clearly written in the question.

Note that, due to the modular nature of Specification B, candidates may meet this requirement more than once. In Specification A it will occur once per pair of papers at each Tier.

Candidates will simply be required to state the units of the answer. A mark will be lost if they do not do this.

This mark is independent of the mathematics required to solve the question.

## 3.3 Accuracy

The accuracy to which candidates give answers can cause problems. In general there is no accuracy requirement unless stated. Answers given to three significant figures are always acceptable. The answers to Pythagoras and Trigonometry problems, in particular, could be given to three significant figures.

However, there are situations where accuracy is important.

Foundation Tier

Candidates will be required to round off a given value or a calculated answer to a given accuracy.

The required accuracies will be one, two or three decimal places or one significant figure.

Higher Tier

One question per tier will have the additional requirement that candidates 'Give your answer to a suitable/sensible/appropriate degree of accuracy'.

This will be clearly written in the question.

Note that due to the modular nature of Specification B candidates may meet this requirement more than once. In Specification A it will occur once per pair of papers at each tier.

This requirement assesses candidates' ability to give an answer to an accuracy that is sensible in the context of the problem. For example, questions about astronomical distances do not need answers given to several decimal places.

As a general guide, when this requirement is asked for, candidates should give answers to the same (or fewer) number of significant figures as the numbers used in the question.

## 3.4 Premature Rounding

Candidates often write intermediate values down, particularly in multi-step questions. These values, taken from a calculator display are sometimes rounded to two or three significant figures. Further calculation using these values (for example, using inverse sine with a two significant figure value) can lead to the final answers being outside the acceptable range of accuracy as determined by the examiner and the subsequent loss of a mark.

Candidates are advised to use the values as shown in the calculator display or to write down intermediate values to at least four significant figures. This will then ensure that the final answer will be within the acceptable range of accuracy.

## 3.5 Specification B – Division of content across modules

For the most part, division of content across the modules is clearly defined in the specification. In the Teachers' Guide, the module in which a particular specification reference is assessed is shown in the third column of the subject content guidance which begins on page 5. In a number of cases, more than one module is given alongside a particular reference. This is generally because a strict assignment of the reference to one module would lead to artificial and undesirable divisions in the subject, which we have sought to avoid. In the interests of clarity, the following notes attempt to give some detail on how and where these references will be assessed.

It should be noted that throughout the question papers, knowledge of the Key Stage 3 curriculum is assumed and, across the Higher tier, knowledge of the Foundation content is assumed.

References to *Using and Applying Number and Algebra* (F2.1 a-m and H2.1 a-m) may be assessed in both modules 3 and 5 as well as through the AO1 coursework, module 4.

References to *Using and Applying Shape, Space and Measures* may be assessed in module 5 as well as through the AO1 coursework, module 4.

References to *Using and Applying Handling Data* may be assessed in module 1 as well as through the AO4 coursework, module 2.

Several other *Number and Algebra* references appear in both modules 3 and 5. These are listed and clarified by tier below.

Foundation Tier

**F2.2a, H2.2a** This reference is about basic understanding of number, simple rounding and vocabulary such as factor, multiple and prime. This knowledge is expected across both modules. Questions which specifically ask for Least Common Multiple, Highest Common Factor or Prime Factor Decomposition will be found in module 3.

**F2.2b, H2.2b** Knowledge and use of squares and square roots and cubes and cube roots and index notation is expected in both modules 3 and 5.

**F2.2c** This reference is about basic understanding of equivalent fractions and is expected in both modules 3 and 5. Cancelling fractions in the context of probability is also expected in module 1. Questions assessing equivalent fractions will be tested in module 5,

Module 3 requires knowledge of equivalent fractions. Questions testing four rules of fractions will be assessed on module 3.

The ability to simplify fractions by cancelling is expected on all modules.

**F2.2d, H2.2d** This is a reference about basic decimal notation and conversion between fractions and decimals. This knowledge is required for both modules

Use of decimal notation will be tested on both modules at Foundation Tier but at Higher Tier will be assessed only on module 3.

**F2.3a, H2.3a, F2.3b, F2.3g, H2.3g, F2.3o** These are references about basic arithmetic. This knowledge is assumed for all modules and may be tested in both modules at Foundation tier

**F2.4a, H2.4a** This refers to using knowledge of number skills to solve problems which applies to all modules. Problems involving ratio and proportion, fractions and percentages are assessed in module 3. Problems involving basic "percentage of" problems could be assessed in either module. Problems involving measures, conversion of measures and compound measures may be asked in either module 3 or 5 as is presently the case.

Higher Tier

**H2.2a** This reference is about basic understanding of number, simple rounding and vocabulary such as factor, multiple and prime. This knowledge is expected across both modules. Questions which specifically ask for Least Common Multiple, Highest Common Factor or Prime Factor Decomposition will be found in module 3.

**H2.2c** This reference is about basic understanding of equivalent fractions and is expected in both modules 3 and 5. Cancelling fractions in the context of probability is also expected in module 1.

Questions assessing equivalent fractions will be tested in module 5,

Module 3 requires knowledge of equivalent fractions. Questions testing four rules of fractions will be assessed on module 3.

The ability to simplify fractions by cancelling is expected on all modules.

**H2.2d** This reference includes some basic knowledge of decimal notation which is assumed at this tier in all modules. Specific questions on recurring decimals and conversion to fractions will appear in module 3.

**H2.2e** This reference involves a basic understanding of percentage which is assumed at this level and may be required in any module. Specific Higher Tier percentage problems will appear in module 3.

**H2.3a** This reference covers a number of assumed basic arithmetic skills. Knowledge of index laws may be required in both modules. They will be numerical on module 3 but could be numerical or algebraic on module 5.

**H2.5b** The first part of this reference is about basic algebraic skills and understanding which is expected at this tier in all modules eg, knowledge of the commutative, distributive and associative laws. The second part of the reference from 'multiplying a single term over a bracket' onwards will specifically be assessed in module 5.

**H2.5c** This reference applies to both modules.

**H2.6e** This reference is about plotting graphs of quadratic functions and using them to solve equations. These skills may be assessed in both modules.

In the guidance tables which follow, shading has been used to highlight where specific parts of a reference apply to a particular module.

## 3.6 Subject Content Guidance

The following two sections give guidance for the interpretation of the subject content for the Foundation and Higher Tiers in Specifications A and B. This guidance is given in the form of notes on the National Curriculum statements.

The first column gives the reference to the appropriate statement in the Foundation or Higher Programme of Study in the National Curriculum. Thus, F2.1a refers to AO2, Foundation Programme of Study, statement 1a. Column two gives the corresponding statement of subject content, column three gives the Specification B module(s) in which the statement will be assessed and column four provides guidance in the form of notes on the statement, which are applicable to both specifications.

# Foundation Tier Guidance

## AO2 Number and algebra

### 1. Using and applying number and algebra

Pupils should be taught to:

| Foundation Tier | Notes |
|---|---|

**Problem solving**

| | | | |
|---|---|---|---|
| F2.1a | select and use suitable problem-solving strategies and efficient techniques to solve numerical and algebraic problems | B3, B4, B5 | Mini-investigations will not be set but candidates will be expected to make decisions and use the appropriate techniques to solve a problem.<br>Candidates should choose relevant information when some is redundant. |
| H2.1b | identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches | B3, B4, B5 | Candidates will be expected to give reasons for answers or to show working. If a question states, "You **must** show your working", marks will be lost if the instruction is ignored. |
| F2.1b H2.1c | break down a complex calculation into simpler steps before attempting to solve it | B3, B4, B5 | Multi-step problems will be set. At the lowest grades there will normally only be one interim step to be identified. At the higher grade problems may require more than one interim step. |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F2.1c | use algebra to formulate and solve a simple problem – identifying the variable, setting up an equation, solving the equation and interpreting the solution in the context of the problem | B4, B5 | Candidates should understand the role of a letter as an unknown, in setting up expressions and solving equations.<br>eg, The angles of a triangle are $2x$, $x + 30$ and $x + 70$. Find the value of $x$. (Diagram given).<br>eg, Jo is 3 years older than Sam. The sum of their ages is 15. By forming an equation find Jo's age. |
| F2.1d | make mental estimates of the answers to calculations; use checking procedures, including use of inverse operations; work to stated levels of accuracy | B3, B4 | Candidates should be able to round to one significant figure to make mental estimates of calculations. Candidates should be able to round answers to given accuracies of 1, 2 and 3 decimal places and 1 significant figure. |

**Communicating**

| | | | |
|---|---|---|---|
| F2.1e | interpret and discuss numerical and algebraic information presented in a variety of forms | B3, B4, B5 | Candidates will be expected to understand problems in context.<br>eg, Find the cost of 26 tins of polish costing 73p each. |
| F2.1f | use notation and symbols correctly and consistently within a given problem | B4, B5 | Correct use of numerical and algebraic notation is expected. For example $a + a = a2$ will not be accepted |
| F2.1g | use a range of strategies to create numerical, algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem | B3, B4, B5 | Candidates should be able to interpret for example, diagrams, information given in a real-life context, such as a 'Sale' poster and translate this into a mathematical problem. |
| F2.1h | present and interpret solutions in the context of the original problem | B3, B4, B5 | Candidates will be required to give sensible answers to questions.<br>eg, How many 4-seater taxis are needed to carry 14 passengers? |
| F2.1i | review and justify their choice of mathematical presentation | B3, B4, B5 | eg, Candidates should be able to explain patterns in words. |

| Foundation Tier | Notes |
|---|---|

**Reasoning**

| | | | |
|---|---|---|---|
| F2.1j | explore, identify, and use pattern and symmetry in algebraic contexts investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems | B4, B5 | For example, using simple codes that substitute numbers for letters. See further guidance on counter examples given in section on proof (Appendix C). |
| F2.1k | show step-by-step deduction in solving a problem | B3, B4, B5 | Candidates should always show their working, particularly in multi-step and using and applying questions, where marks are allocated for candidates showing a strategy to solve the problem. |
| F2.1l | understand the difference between a practical demonstration and a proof | B4, B5 | See section on proof (Appendix C). |
| F2.1m | recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem | B4, B5 | Candidates should assume that information given is exact unless the question states or implies otherwise. |

## 2. Numbers and the number system

Pupils should be taught to:

| Foundation Tier | Notes |
|---|---|

**Integers**

| | | | |
|---|---|---|---|
| F2.2a<br>H2.2a | use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use positive numbers and negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition | B3, B5 | Candidates could be asked to round a number to the nearest whole number, 10, 100 or 1000.<br>Abbreviations will not be used in examinations. The word 'least' will be used.<br>Candidates will be expected to identify eg, multiples, factors and prime number from lists.<br>There is no requirement that candidates use the prime factor decomposition method to solve HCF and LCM problems. Candidates can write out lists of multiples and factors to identify common factors and multiples. |

| Foundation Tier | Notes |
|---|---|

**Powers and roots**

| | | | |
|---|---|---|---|
| F2.2b<br>H2.2b | use the terms square, positive and negative square root, cube and cube root; use index notation for squares, cubes and powers of 10; use index laws for multiplication and division of integer powers; express standard index form both in conventional notation and on a calculator display | B3, B5 | Questions will not ask for 'positive square root' but will use the notation $\sqrt{25}$. When a square root is asked for, only the positive value will be required. However, candidates are expected to know that a square root can be negative.<br>If the solution to $x^2 = 25$ is required then both the negative and positive root are expected.<br>Powers of 10 will be used only up to $10^6$<br>Values of simple integer powers eg, $2^4$ will be tested.<br>The words cube, square and cube root may be used and should be understood.<br>Candidates may be asked to write $7^5 \text{ x } 7^2$ or $7^5 \div 7^2$ as a single power of 7.<br>Candidates are not expected to know the definition of standard form but should be aware that calculator displays can sometimes show values such as $1.7 \times 10^{-3}$ or $1.7^{-03}$ and know how to interpret these. |

**Fractions**

| | | | |
|---|---|---|---|
| F2.2c | understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator | B3, B5 | Candidates may be asked to give a fractional answer in its simplest form. When this requirement is not clearly stated candidates do not have to cancel fractional answers.<br>Candidates will require a basic knowledge of mixed numbers.<br>The term 'common denominator' will not be used. |

| Foundation Tier | Notes |
|---|---|

**Decimals**

| | | | |
|---|---|---|---|
| F2.2d<br>H2.2d | use decimal notation and recognise that each terminating decimal is a fraction recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals. | B3, B5 | eg, $0.137 = \frac{137}{1000}$<br>eg, $\frac{1}{7} = 0.142857142857\ldots$<br>Candidates should know that $0.\dot{3} = \frac{1}{3}$ and $0.\dot{6} = \frac{2}{3}$ and that other fractions give recurring decimals and know how to write<br>eg, $\frac{1}{6} = 0.16666\ldots.$ as $0.1\dot{6}$ |

**Percentages**

| | | | |
|---|---|---|---|
| F2.2e | understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of '; use percentage in real-life situations. | B3, B5 | eg, 10% means 10 parts per 100, and 15% of Y means $\frac{15}{100} \times Y$<br>$x$% of $y$ will be required.<br>VAT rates will be provided.<br>Percentage problems on non-calculator papers will involve percentages that can be worked out using multiples of 1% and 10%.<br>Some basic knowledge of percentages in every day life eg, commerce and business including rate of inflation, VAT, price index, interest rates and financial capability is required.<br>Note that problems involving compound interest will not be set at Foundation tier but simple interest on investments over 1 or more years could be assessed |

| Foundation Tier | | | Notes |
|---|---|---|---|

**Ratio**

| | | | |
|---|---|---|---|
| F2.2f | use ratio notation, including reduction to its simplest form and its various links to fraction notation | B3 | Candidates should be familiar with ratio notation (eg, 2:3) and should know how to reduce to simplest form. Questions asking for the ratio in the form 1:$n$ may be required.<br>Candidates should know that if, say, red balls and blue balls are in the ratio 3:4 then the fraction of red balls is $\frac{3}{7}$ |

## 3. Calculations

Pupils should be taught to:

| Foundation Tier | Notes |
|---|---|

**Number operations and the relationships between them**

| | | | |
|---|---|---|---|
| F2.3a<br>H2.3a | add, subtract, multiply and divide integers and then any number; multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers; use inverse operations | B3, B5 | The following should be known:<br>table facts up to $10 \times 10$;<br>squares up to $15 \times 15$;<br>non-calculator methods for adding and subtracting 3 digit numbers;<br>non-calculator methods for multiplying and dividing up to 3 digit numbers by up to 2 digit numbers.<br>Candidates should be able to interpret a remainder from a division problem.<br>Multiplication and division of integers and decimals by powers of 10 will be restricted to 10, 100 and 1000.<br>Questions may involve negative integers<br>eg, Find the reciprocal of $\frac{2}{3}$, 0.8<br>eg, Express 48 as the product of prime factors |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F2.3b | use brackets and the hierarchy of operations | B3, B5 | The BIDMAS or BODMAS convention should be known but the mnemonic need not be known. Candidate could be asked to insert brackets into a calculation to make it true. |
| F2.3c | calculate a given fraction of a given quantity expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal | B3 | For example, for scale drawings and construction of models, down payments, discounts.<br>eg, Work out $\frac{3}{8}$ of 56 (non-calculator question)<br>eg, Find $\frac{2}{7}$ of 5467 (calculator question)<br>Questions involving mixed numbers may be set.<br>eg, Work out $1\frac{2}{5} + \frac{3}{4}$<br>eg, Work out $3\frac{5}{6} - 2\frac{1}{2}$<br>eg, Find $\frac{3}{8}$ of 520. (non-calculator question)<br>eg, Find $\frac{2}{7}$ of 520 people. (calculator question with answer given to a sensible degree of accuracy) |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F2.3d<br>H2.3d | understand and use unit fractions as multiplicative inverses multiply and divide a fraction by an integer, by a unit fraction and by a general fraction | B3 | eg, thinking of multiplication by $\frac{1}{5}$ as division by 5.<br>eg, $4 \times \frac{7}{8}$, $\frac{6}{11} \div 3$<br>eg, $\frac{3}{4} \times \frac{8}{9}$, $\frac{4}{15} \div \frac{4}{15}$<br>Multiplication and division problems with mixed numbers will not be assessed. |
| F2.3e | convert simple fractions of a whole to percentages of the whole and vice versa then understand the multiplicative nature of percentages as operators | B3 | The use of multipliers is expected although other methods are acceptable.<br>eg, 32% of £80 = $0.32 \times 80$<br>For example, a 15% increase in value Y, is calculated as $1.15 \times Y$ |
| F2.3f | divide a quantity in a given ratio | B3 | eg, Divide £500 in ratio 3:2 |

| Foundation Tier | | | Notes |
|---|---|---|---|

**Mental methods**

| | | | |
|---|---|---|---|
| F2.3g<br>H2.3g | recall all positive integer complements to 100; recall all multiplication facts to $10 \times 10$, and use them to derive quickly the corresponding division facts; recall integer squares from $11 \times 11$ to $15 \times 15$ and the corresponding square roots, recall the cubes of 2, 3, 4, 5 and 10, and the fraction-to-decimal conversion of familiar simple fractions | B3, B5 | For example $37 + 63 = 100$<br><br>It will be acceptable to give the decimal equivalent of $\frac{1}{3}$ and $\frac{2}{3}$ as 0.33 and 0.66 or 0.67<br><br>For example, decimal and percentage equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{8}$ should be known.<br><br>Mental methods will not be tested in the written papers but a quick recall of basic number facts is expected. |
| F2.3h | round to the nearest integer and to one significant figure; estimate answers to problems involving decimals | B3 | Candidates should be able to round off a calculator display. Money answers should be written as £3.60 not £3.6<br><br>Candidates should be able to round to a given number of decimal places and also to the nearest 10, 100 or 1000.<br><br>When estimating with whole numbers or decimal numbers less than 1, the numbers should be rounded to 1s.f. before the estimation is done. |
| F2.3i | develop a range of strategies for mental calculation; add and subtract mentally numbers with up to one decimal place; multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments | B3 | Knowledge of the terms 'commutative', 'associative' and 'distributive' is not required and the term 'factorise' in the context of number need not be known.<br><br>eg, $\frac{2000}{0.4} = \frac{20000}{4} = 5000$ |

| Foundation Tier | Notes |
|---|---|

**Written methods**

| | | | |
|---|---|---|---|
| F2.3j | use standard column procedures for addition and subtraction of integers and decimals | B3 | Candidates may use any algorithm in non-calculator papers. |
| F2.3k | use standard column procedures for multiplication of integers and decimals, understanding where to position the decimal point by considering what happens if they multiply equivalent fractions; solve a problem involving division by a decimal (up to 2 d.p.) by transforming it to a problem involving division by an integer | B3 | Candidates may use any algorithm in non-calculator papers.<br>eg, $408 \div 0.17 = 40800 \div 17$<br>eg, $0.02 \times 0.3 = 0.006$, $0.4^2 = 0.16$ |
| F2.3l | use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer | B3 | If asked to calculate $\frac{4}{9} \times \frac{3}{8}$, candidates will not be penalised if they fail to cancel before carrying out the multiplication. They may not gain full marks, however, if they fail to cancel $\frac{12}{72}$ |
| F2.3m | solve simple percentage problems, including increase and decrease | B3 | For example, simple interest, VAT, annual rate of inflation, income tax, discounts.<br>Use of a multiplier is expected, although other methods will be accepted.<br>eg, Decrease 76 kg by 12% is calculated as $0.88 \times 76$ |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F2.3n | solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution | B3 | For example, given that $m$ identical items cost £$y$, then one item costs £$\frac{y}{m}$ and n items cost £ $(n \times \frac{y}{m})$, the number of items that can be bought for £$z$ is $z \times \frac{m}{y}$<br>eg, 8 pencils cost £2.40. How much do 11 cost? (non-calculator question)<br>eg, 8 pencils cost £2.56. How many can be bought for £4.80? (calculator question) |
| H2.3n | use $\pi$ in exact calculations, without a calculator | B3 | Candidates should be aware that $6\pi$ is an exact answer and $6 \times 3.14159\ldots$ is an estimate.<br>In the non-calculator papers candidates could be asked to give the area of a circle of radius 3cm in terms of $\pi$. |

**Calculator methods**

| | | | |
|---|---|---|---|
| F2.3o | use calculators effectively and efficiently; know how to enter complex calculations and use function keys for reciprocals, squares and powers | B3, B5 | eg, Work out $3.2 + 5.4^2$<br>eg, Work out $3.2 + \frac{1}{5.4}$ |
| F2.3p | enter a range of calculations, including those involving standard index form and measures | B3 | For example, time calculations in which fractions of an hour must be entered as fractions or as decimals.<br>Candidates will not be expected to calculate with Standard Form numbers or enter them into a calculator. |
| F2.3q<br>H2.3p | understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation | B3 | To avoid problems with premature rounding candidates should use the full calculator display. If they wish to write down an intermediate value during a calculation it should be to at least 4 significant figures. |

## 4. Solving numerical problems

Pupils should be taught to:

| Foundation Tier | | Notes | |
|---|---|---|---|
| F2.4a<br>H2.4a | draw on their knowledge of operations, inverse operations and the relationships between them, and of simple integer powers and their corresponding roots, and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, fractions, percentages and measures and conversion between measures, and compound measures defined within a particular situation | B3, B5 | The terms 'commutative', 'associative' and 'distributive' will not be used in the examination.<br>Knowledge of the term 'root' is required.<br>Knowledge of the term 'inverse operation' is required and candidates should know the inverse operations of the four rules: square, square root, cube and cube root.<br>Compound measures may be expressed in the form metres per second, $m/s$, $m\ s^{-1}$. Candidates would be expected to understand speed and know the relationship between speed, distance and time. Units may be any of those in common usage such as miles per hour or metres per second. The values used in the question will make the required unit clear.<br>Other compound measures that are non-standard would be defined in the question eg, population density is population/$km^2$.<br>Conversion between measures would involve knowledge of the connection between metric units. Conversions between imperial units will be given. |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F2.4b | select appropriate operations, methods and strategies to solve number problems, including trial and improvement where a more efficient method to find the solution is not obvious | B3 | Trial and improvement should not be used when a standard algorithm is expected. |
| F2.4c<br>H2.4b | estimate answers to problems; use a variety of checking procedures, including working the problem backwards, and considering whether a result is of the right order of magnitude | B3 | eg, The heights of 7 men are 150, 151, 148, 133, 138, 142, 140 cm. Give a reason why 143.143 cm is not an appropriate answer for their mean height.<br>For example if the answer to $4x = 2$ is calculated as 2, checking should show that $4 \times 2 = 8 \neq 2$. |
| F2.4d | give solutions in the context of the problem to an appropriate degree of accuracy, interpreting the solution shown on a calculator display, and recognising limitations on the accuracy of data and measurements | B3 | Candidates should be able to round to a given degree of accuracy. This could be 1 s.f., 1, 2 or 3 d.p, to the nearest penny, etc.<br>An understanding that rounded values may be inaccurate by up to half a unit is necessary. |

## 5. Equations, formulae and identities

Pupils should be taught to:

| Foundation Tier | Notes |
|---|---|

**Use of symbols**

| | | | |
|---|---|---|---|
| F2.5a<br>H2.5a | distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations defined quantities or variables in formulae, general, unspecified and independent numbers in identities, and in functions they define new expressions or quantities by referring to known quantities. | B5 | eg, knowing that $5x + 1 = 16$ is an equation.<br>eg, knowing that $x^2 + 1 = 82$ is an equation.<br>eg, knowing that $V = IR$ is a formula.<br>eg, knowing that $3x + 2x \equiv 5x$, for all values of $x$ is an identity.<br>eg, knowing that $(x + 1)^2 \equiv x^2 + 2x + 1$, for all values of $x$ is an identity.<br>Knowledge of the identity symbol $\equiv$ is required.<br>eg, knowing that $y = 2x$ is a function.<br>eg, knowing that $y = 7 – 2x$ is a function.<br>Candidates will be expected to know the standard convention such as<br>$2x$ for $2 \times x$ and $\frac{1}{2}x$ or $\frac{x}{2}$ Candidates who write $2 \times x$, $x \times 2$, $x \div 2$<br>or $x/2$ will not be penalised, but $x2$ will not be accepted for $2x$.<br>eg, Write an expression for the cost of $x$ sweets at 12p each. |

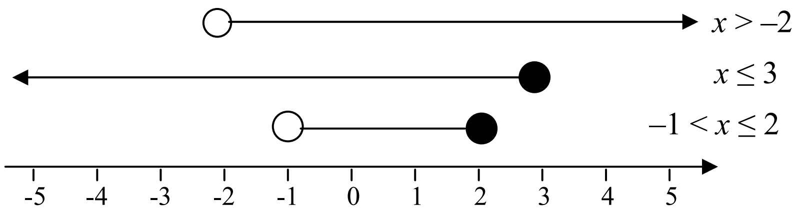
| Foundation Tier | | Notes | |
|---|---|---|---|
| F2.5b<br>H2.5b | understand that the transformation of algebraic expressions obeys and generalises the rules of generalised arithmetic, expand the product of two linear expressions; manipulate algebraic expressions by collecting like terms by multiplying a single term over a bracket, and by taking out common factors; distinguish in meaning between the words 'equation', 'formula', 'identity' and 'expression' | B5 | For example, $x + 5 - 2x - 1 = 4 - x$<br>Candidates will be expected to know the term 'factorise' in the context of an algebraic simplification.<br>eg, Factorise $2a + 4b$<br>eg, Simplify $2x + 2 - 2x + 5$<br>eg, Simplify $2(x - 2) + 5(2x - 3)$<br>eg, The expression $x^2 + 2x + 1$ has three terms. The equation $3x - 2 = 7$ can be solved, $A = lb$ is a formula. $3n + 2n \equiv 5n$ as it is true for all $n$. Knowledge of the identity symbol $\equiv$ is required.<br>eg, Expand and simplify $(x + 3)(x - 4)$<br>Candidates will be expected to know the meaning of 'solve' in relation to linear equations. (eg, Solve the equation $2x - 15 = 3$) |

**Index notation**

| | | | |
|---|---|---|---|
| F2.5c | use index notation for simple integer powers, and simple instances of index laws substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$ | B5 | eg, Evaluate $2^5$<br>Evaluate $2x^3$ when $x = -2$, $3x^3 + 4$ when $x = \frac{1}{2}$ or $-3$<br>For example, $x^3 \times x^2 = x^5$; $x^8 \div x^2 = x^6$ |

| Foundation Tier | Notes |
|---|---|

**Inequalities**

| | | | |
|---|---|---|---|
| F2.5d | solve simple linear inequalities in one variable, and represent the solution set on a number line | B5 | Candidates should know the difference between >, <, ≤ and ≥<br>For example Solve $2x + 3 < 7$<br>Candidates should know the convention of an open circle for a strict inequality and a closed circle for an included boundary.<br>eg,  |

**Equations**

| | | | |
|---|---|---|---|
| H2.5e | set up simple equations, solve simple equations by using inverse operations or by transforming both sides in the same way | B5 | For example, find the angle $a$ in a triangle with angles $a$, $a$ +10, $a$ + 20<br>For example, $5x = 7$; $11 - 4x = 2$; $3(2x+ 1) = 8$, $2(1 - x) = 6(2 + x)$,<br>$4x^2 = 49$, $3 = \frac{12}{x}$ |

| Foundation Tier | Notes |
|---|---|

**Linear Equations**

| | | | |
|---|---|---|---|
| F2.5e | solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution | B5 | eg, Solve $7 - 5x = 20$<br>eg, Solve $5x + 7 = 2$<br>eg, Solve $2(x - 4) = 7$<br>eg, Solve $6x + 7 = x + 13$<br>eg, Solve $5x + 17 = 3(x + 6)$<br>eg, Solve $\frac{15 - x}{4} = 2$ |

**Formulae**

| | | | |
|---|---|---|---|
| F2.5f | use formulae from mathematics and other subjects expressed initially in words and then using letters and symbols; substitute numbers into a formula; derive a formula and change its subject | B5 | eg, formula for area of a triangle, area of a parallelogram, area of a circle, wage earned = hours worked × hourly rate or wage earned = hours worked × hourly rate plus bonus, volume of a prism. Substitute numbers into a formula; derive a formula (eg, convert temperature between degrees Fahrenheit and degrees Celsius, find the perimeter of a rectangle given its area A and the length l of one side).<br>Formula to be rearranged will need at most two operations to rearrange. Formula where a power of the subject appears will not be required.<br>eg, Rearrange $x + y = 7$ to make $x$ the subject.<br>eg, Rearrange $C = 2\pi r$ to make $r$ the subject<br>eg, Rearrange $y = 2x + 3$ to make $x$ the subject. |

| Foundation Tier | Notes |
|---|---|

**Numerical Methods**

| | | | |
|---|---|---|---|
| H2.5m | use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them | B5 | For example, Solve $x^3 - x = 900$<br><br>Solve $\frac{1}{x} = x^2 - 5$<br><br>Answers will be expected to 1 d.p. Candidates will be expected to test the mid-value of the 1 d.p. interval to establish which 1 d.p. value is nearest to the solution. |

## 6. Sequences, functions and graphs

Pupils should be taught to:

| Foundation Tier | Notes |
|---|---|

**Sequences**

| | | | |
|---|---|---|---|
| F2.6a<br>H2.6a | generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); use linear expressions to describe the nth term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated | B5 | Candidates should be able to explain how a sequence continues. The nth terms of linear sequences will be required. Candidates will **not** be expected to find the nth term of a non-linear sequence.<br><br>However, candidates should be familiar with the idea of a non-linear sequence and the fact that the nth term can be generated by an expression of the form $\frac{1}{2}n(n + 1)$, for example.<br><br>They should also know that the $n$th term of the square number sequence is given by $n^2$.<br><br>Candidates should be able to generate simple sequences of eg, odd or even numbers, square integers and sequences derived from diagrams. |

| Foundation Tier | | Notes | |
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**Graphs of linear functions**

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|---|---|---|---|
| F2.6b | use the conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for $m$ and $c$) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which $y$ is given explicitly in terms of $x$ or implicitly. | B5 | eg, explicitly such as $y = 2x + 3$ or implicitly such as $x + y = 5$.<br>Partially completed tables of values may sometimes be given but candidates should be able to plot the graph of $y = 3x – 1$, say, with no further assistance. Knowledge that $m$ is the gradient and $c$ is the $y$-intercept will be expected.<br>Candidates will not be expected to find the equation of a given line. |
| F2.6c | construct linear functions from real-life problems and plot their corresponding graphs; discuss and interpret graphs modelling real situations; understand that the point of intersection of two different lines in the same two variables that simultaneously describe a real situation is the solution to the simultaneous equations represented by the lines; draw line of best fit through a set of linearly related points and find its equation | B5 | For example, currency conversion graphs, distance-time graphs, graphs describing trends, graphs of height or weight against age, graphs of quantities that vary against time, such as employment, graphs of costs of units of gas.<br>Candidates should be able to read from graphs. For example, find the cost of a bill for so many units of gas or find the number of units for a given cost. They should understand that the intercept in such a graph represents the fixed charge, for example.<br>Candidates will be expected to find the velocity for sections of a distance time graph and should understand that the steeper the line the greater the speed. |

**Interpret graphical information**

| | | | |
|---|---|---|---|
| F2.6e | interpret information presented in a range of linear and non-linear graphs | B5 | Candidates should be able to interpret graphs showing real-life situations such as the depth of water in containers as they are filled at a steady rate.<br>Candidates may be given non-linear graphs from real life situations to interpret eg, the height of a ball plotted against time. |

| Foundation Tier | | | Notes |
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**Gradients**

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| F2.6d | find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for $m$ and $c$); investigate the gradients of parallel lines | B5 | Candidates will not be expected to find the equation of a line when the graph is given but should know that lines of the form $y = 2x + c$, for example, are parallel and lines of the form $y = ax + 1$ all pass through the same point on the $y$-axis. |

**Quadratic functions**

| | | | |
|---|---|---|---|
| H2.6e | generate points and plot graphs of simple quadratic functions, then more general quadratic functions find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function | B5 | eg, $y = x^2$, $y = 3x^2 + 4$<br>eg, $y = x^2 - 2x + 1$<br>If candidates are required to draw a graph then a table may be given in which some $y$ values may have to be calculated.<br>Quadratic graphs are expected to be drawn as a curve.<br>Candidates will be expected to know that the roots of an equation $f(x) = 0$ can be found where the graph of the function intersects the $x$-axis and that the solution of $f(x) = a$ is found where $y = a$ intersects with $f(x)$. |

# AO3: Shape, space and measures

## 1. Using and applying shape, space and measures

Pupils should be taught to:

| Foundation Tier | Notes |
|---|---|

**Problem solving**

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|---|---|---|---|
| F3.1a<br>H3.1a | select problem-solving strategies and resources, including ICT tools, to use in geometrical work, and monitor their effectiveness; consider and explain the extent to which the selections they made were appropriate | B4, B5 | Mini-investigations will not be set but candidates will be expected to make decisions and use the appropriate techniques to solve a problem drawing on well known facts, such as the sum of angles in a triangle. |
| F3.1b | select and combine known facts and problem-solving strategies to solve complex problems | B4, B5 | Multi-step problems will be set.<br>eg, Find the base angle of an isosceles triangle (apex angle and diagram given). |
| F3.1c<br>H3.1c | identify what further information is needed to solve a geometrical problem; break complex problems down into a series of tasks; develop and follow alternative lines of enquiry | B4, B5 | Redundant information may sometimes be used, for example, the slant height of a parallelogram. Candidates should be able to identify which information given is needed to solve the given problem. |

| Foundation Tier | Notes |
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**Communicating**

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| F3.1d | interpret, discuss and synthesise geometrical information presented in a variety of forms | B4, B5 | Candidates will be expected to interpret information from diagrams such as angles, equal lines marked, parallel lines marked, |
| F3.1e<br>H3.1d | communicate mathematically with emphasis on a critical examination of the presentation and organisation of results, and on effective use of symbols and geometrical diagrams | B4, B5 | Candidates will be expected to use correct mathematical notation and produce a logical solution to a given problem. |
| F3.1f | use geometrical language appropriately | B4, B5 | Use of colloquial terminology may be penalised. For example, words such as 'flip' will not be accepted for reflection. |
| F3.1g | review and justify their choices of mathematics presentation | B4, B5 | Candidates should be able to choose an appropriate method when several methods are possible. |

**Reasoning**

| | | | |
|---|---|---|---|
| F3.1h | distinguish between practical demonstrations and proofs | B4, B5 | Candidates may be required to give specific examples or more general proofs |
| F3.1i | apply mathematical reasoning, explaining and justifying inferences and deductions | B4, B5 | See further guidance given in section on proof (Appendix C). |
| F3.1j | show step-by-step deduction in solving a geometrical problem | B4, B5 | Candidates should be able to explain reasons using words or diagrams |
| F3.1k | state constraints and give starting points when making deductions | B4, B5 | Candidates should realise when an answer is inappropriate. |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F3.1l | recognise the limitations of any assumptions that are made; understand the effects that varying the assumptions may have on the solution | B4, B5 | eg, Candidates should assume that lengths and angles given are exact unless the question states otherwise. |
| F3.1m | identify exceptional cases when solving geometrical problems | B4, B5 | |

## 2. Geometrical reasoning

Pupils should be taught to:

| Foundation Tier | | Notes | |
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**Angles**

| | | | |
|---|---|---|---|
| F3.2a | recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex | B5 | Candidates should be able to justify an answer with explanations such as 'angles on a straight line', etc.<br>The notations angle $ABC$, $\angle ABC$ or $A\hat{B}C$ will be used, although questions might sometimes refer to angle $B$ where there is no ambiguity. |
| F3.2b | distinguish between acute, obtuse, reflex and right angles; estimate the size of an angle in degrees | B5 | Candidates should know and understand the terms acute, obtuse, reflex and right angle. |

| Foundation Tier | Notes |
|---|---|

**Properties of triangles and other rectilinear shapes**

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|---|---|---|---|
| F3.2c<br>H3.2a | distinguish between lines and line segments; use parallel lines, alternate angles and corresponding angles; understand the consequent properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices | B5 | Explanations should use the correct terminology. Colloquial terms such as 'Z', 'F' angles are not acceptable as reasons.<br>Acceptable terms for the interior, co-interior or allied<br>See further guidance on proof (Appendix C).<br>Candidates should know that a straight line drawn between two points or vertices is a line segment. |
| F3.2d | use angle properties of equilateral, isosceles and right-angled triangles; understand congruence; explain why the angle sum of a quadrilateral is 360 degrees | B5 | Candidates should be able to recognise congruent shapes when rotated, reflected or in different orientations.<br>Angle sum of a quadrilateral is based on the angle sum of a triangle which can be taken as a fact. |
| F3.2e | use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle | B5 | Candidates will not be expected to deduce the formula for area of rectangles, parallelograms and triangles in examinations but will be expected to know them.<br>Questions involving compound shapes made up of rectangles and triangles will be assessed. |
| F3.2f<br>H3.2c | recall the essential properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties | B5 | Questions may be set that test candidates' knowledge of these properties. The properties of a kite should also be known.<br>The formula for the area of a trapezium is given on the formula sheet.<br>Candidates should know the side, angle and diagonal properties of quadrilaterals. |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F3.2g | calculate and use the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons; calculate and use the angles of regular polygons. | B5 | Candidates should learn or know how to work out the angle sum of polygons up to a hexagon. Questions involving the interior and exterior angles of regular polygons may be set.<br>Questions could include octagons and decagons |
| F3.2h | understand, recall and use Pythagoras' theorem | B5 | Questions may be set in context, for example, a ladder against a wall, but questions will always include a diagram of a right angled triangle with two sides marked and the third side to be found. Quoting the formula will not gain credit. It must be used with the appropriate numbers, eg, $x^2 = 7^2 + 5^2$, $x^2 = 12^2 - 9^2$ or $x^2 + 9^2 = 12^2$ |

**Properties of circles**

| | | | |
|---|---|---|---|
| F3.2i | recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that inscribed regular polygons can be constructed by equal division of a circle | B5 | Questions asking for the angle at the centre of a regular polygon may be set. |

**3-D shapes**

| | | | |
|---|---|---|---|
| F3.2j | explore the geometry of cuboids (including cubes), and shapes made from cuboids | B5 | Candidates should understand Planes of symmetry of a cuboid and be able to find the surface area of a cuboid.<br>Questions could include simple isometric drawing of cuboids (including cubes) and shapes made from cuboids |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F3.2k<br>H3.2i | use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms and cylinders | B5 | Isometric drawings should be understood. Dotted lines will indicate hidden edges.<br>Answers in terms of $\pi$ may be required. (non-calculator question)<br>Questions asking for the surface area of a cylinder will not be set but candidates should have an understanding of the net of a cylinder.<br>The volume of a prism is given on the formula sheet.<br>Formula for the volume of a cylinder is not given. Candidates should know this. |

## 3. Transformations and coordinates

Pupils should be taught to:

| Foundation Tier | | Notes | |
|---|---|---|---|
| **Specifying transformations** | | | |
| F3.3a<br>H3.3a | understand that rotations are specified by a centre and an (anticlockwise) angle; rotate a shape about the origin, or any other point; measure the angle of rotation using right angles, simple fractions of a turn or degrees; understand that reflections are specified by a mirror line, at first using a line parallel to an axis, then a mirror line such as $y = x$ or $y = -x$; understand that translations are specified by a distance and direction(or a vector), and enlargements by a centre and positive scale factor | B5 | The direction of rotation will always be given.<br>Column vector notation should be understood.<br>Lines of symmetry will be restricted to $x = a$, $y = a$, $y = x$ and $y = -x$<br>Scale factors for enlargements can be fractional. |

**Properties of transformations**

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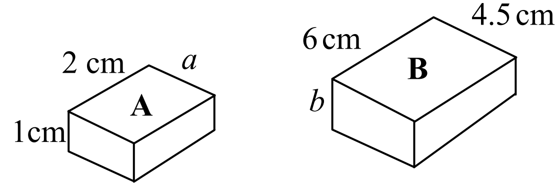
| F3.3b<br>H3.3b | recognise and visualise rotations, reflections and translations, including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations, recognising that these transformations preserve length and angle, so that any figure is congruent to its image under any of these transformations; distinguish properties that are preserved under particular transformations | B5 | When describing transformations, the minimum requirement is<br>Reflection described by a mirror line<br>Translations described by a vector or a clear description such as 3 squares to the right, 5 squares down.<br>Rotations described by centre, direction (unless a half turn) and an amount of turn (as a fraction of a whole or in degrees).<br>Candidates will always be asked to describe a **single** transformation but could be asked to do a combined transformation on a single shape.<br>Questions will be set on line symmetry and order of rotational symmetry, including tessellations. |
| F3.3c | recognise, visualise and construct enlargements of objects using positive scale factors greater than one, then positive scale factors less than one; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not | B5 | Scale factors for enlargements will be restricted to positive integers or simple unit fractions. Enlargements may be drawn on a grid, or on a coordinate grid, where the centre of enlargement will always be at a point of intersection of two grid lines.<br>Questions assessing the properties of similar shapes, including similar triangles, will not be set. |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F3.3d | recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments and apply this to triangles; understand the implications of enlargement for perimeter; use and interpret maps and scale drawings; understand the implications of enlargement for area and for volume; distinguish between formulae for perimeter, area and volume by considering dimensions; understand and use simple examples of the relationship between enlargement and areas and volumes of shapes and solids | B5 | Scales will be given as, for example, 1 cm represents 10 km, or 1:100.<br>Area and volume scale factors are not required. Questions involving the effect of enlargement on area and volume will involve a diagram.<br>eg, These boxes are similar.<br><br>(a) What is the length *a* ?<br>(b) What is the length *b* ?<br>(c) What is the ratio of the volume of box A to box B ?<br>Candidates should be able to distinguish between formula for length, area and volume. eg, Given three formulae, identify one as a length, one as an area and one as a volume. Formula used will be within the experience of candidates such as $2l + 2w$, $\pi r^2 h$ etc. |

| Foundation Tier | Notes |
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**Coordinates**

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| F3.3e | understand that one coordinate identifies a point on a number line, two coordinates identify a point in a plane and three coordinates identify a point in space, using the terms '1-D', '2-D' and '3-D'; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; find the coordinates of points identified by geometrical information; find the coordinates of the midpoint of the line segment *AB*, given points *A* and *B*, then calculate the length *AB*. | B5 | Questions asking for the mid-point will always be accompanied by a grid.<br>The formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ need not be known.<br>For example, find the coordinates of the fourth vertex of a parallelogram with vertices at (2, 1) (–7, 3) and (5, 6).<br>eg, identify the coordinates of the vertex of a cuboid on a 3-D grid |

**Vectors**

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| F3.3f | understand and use vector notation for translations | B5 | eg, Candidates could be asked to translate a shape by $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ |

## 4. Measures and construction

Pupils should be taught to:

| Foundation Tier | Notes |
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**Measures**

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|---|---|---|---|
| F3.4a | interpret scales on a range of measuring instruments, including those for time and mass; know that measurements using real numbers depend on the choice of unit; recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction; convert measurements from one unit to another; know rough metric equivalents of pounds, feet, miles, pints and gallons; make sensible estimates of a range of measures in everyday settings | B5 | Conversion between measures would involve knowledge of the connection between metric units. Conversions between imperial units will be given but the rough metric equivalents to common imperial measures should be known.<br>These will be restricted to 8 km $\approx$ 5 miles, 1 litre $\approx$ 1.75 pints, 1 kg $\approx$ 2.2 lbs, 1 gallon $\approx$ 4.5 litres, 1 foot $\approx$ 30 cm.<br>Other conversions will be given in the question<br>eg, Give the upper and lower limits of a length of 11 cm measured to the nearest centimetre.<br>Lower limit = 10.5 cm,<br>Upper limit = 11.5 cm.<br>The notation $10.5 \le \text{length} < 11.5$ should be understood. |
| F3.4b | understand angle measure using the associated language | B5 | For example, use bearings to specify direction.<br>Bearings will always be given as a 3-figure bearing. The eight points of the compass (N, NE, E, SE, S, SW, W, NW) and their equivalent bearings should be known |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F3.4c<br>H3.4a | understand and use compound measures, including speed and density | B5 | For example, what distance is covered travelling at 40mph for 3 hours.<br>Speed may be expressed in the form metres per second, (m/s). Candidates would be expected to understand these, and also units in common usage such as miles per hour (mph) or kilometres per hour (km/h).<br>Calculations involving distance or time will be restricted to $\frac{1}{4}$ hour, $\frac{1}{3}$ hour, $\frac{1}{2}$ hour , $\frac{2}{3}$ hour or a whole number of hours. |

**Construction**

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| F3.4d | measure and draw lines to the nearest millimetre, and angles to the nearest degree; draw triangles and other 2-D shapes using a ruler and protractor, given information about their side lengths and angles; understand, from their experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; construct cubes, regular tetrahedral, square-based pyramids and other 3-D shapes from given information | B5 | Knowledge of SSS, SAS, ASA and RHS terminology will not be required but candidates should be able to recognise when two triangles are congruent.<br>Candidates will be expected to draw a net of a 3-D shape and also to recognise a shape from a given net.<br>eg, Construct a triangle with side of 6 cm, 7 cm and 8 cm<br>When constructing triangles, compasses should be used to measure lengths rather than rulers. Construction arcs need to be seen for full marks to be awarded. |
| F3.4e | use straight edge and compasses to do standard constructions, including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle | B5 | Candidates will be expected to show clear evidence that a straight edge and compasses have been used to do constructions. |

| Foundation Tier | | | Notes |
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**Mensuration**

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| F3.4f | find areas of rectangles, recalling the formula, understanding the connection to counting squares and how it extends this approach; recall and use the formulae for the area of a parallelogram and a triangle; find the surface area of simple shapes using the area formulae for triangles and rectangles; calculate perimeters and areas of shapes made from triangles and rectangles | B5 | Questions on area and perimeter using compound shapes formed from two or more rectangles may be set.<br>Questions may include the perimeter of simple shapes<br>Questions may include areas of parallelograms and trapezia |
| F3.4g | find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids | B5 | The formula $V = lwh$ should be known,<br>The formula Volume of prism = cross-sectional area × length is given on the formula sheet. |
| F3.4h | find circumferences of circles and areas enclosed by circles, recalling relevant formulae | B5 | Circumference and area formula for a circle should be known.<br>Perimeters and areas of semi-circles or simple fractions of a circle, eg, quarter circles, could be assessed. |
| F3.4i | convert between area measures, including square centimetres and square metres, and volume measures, including cubic centimetres and cubic metres | B5 | eg, A rectangle has sides of 30 cm and 40 cm. Find the area. Give your answer in $m^2$. |

| Foundation Tier | | | Notes |
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**Loci**

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|---|---|---|---|
| F3.4j | find loci, both by reasoning and by using ICT to produce shapes and paths | B5 | Loci will be restricted to at most two constraints.<br>eg, Find the overlapping area of two transmitters, with ranges of 30km and 40km respectively.<br>Loci problems may be set in practical contexts such as finding the position of a radio transmitter. Constructions expected are the perpendicular bisector of two points, accurate construction of a circle and the angle bisector. Questions involving bearings may also be required. |

# AO4: Handling data

## 1. Using and applying handling data

Pupils should be taught to:

| Foundation Tier | Notes |
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**Problem solving**

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| F4.1a | carry out each of the four aspects of the handling data cycle to solve problems:<br>(i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed<br>(ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources<br>(iii) process and represent the data: turn the raw data into usable information that gives insight into the problem<br>(iv) interpret and discuss the data: answer the initial question by drawing conclusions from the data | B1, B2 | Teachers may wish to use this section of the subject content as a basis for AO4 coursework. |

| Foundation Tier | | Notes | |
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| F4.1b<br>H4.1b | identify what further information is needed to pursue a particular line of enquiry; select the problem-solving strategies to use in statistical work, and monitor their effectiveness (these strategies should address the scale and manageability of the tasks, and should consider whether the mathematics and approach used are delivering the most appropriate solutions) | B1, B2 | |
| F4.1c | select and organise the appropriate mathematics and resources to use for a task | B1, B2 | |
| F4.1d | review progress while working; check and evaluate solutions | B1, B2 | |

**Communicating**

| | | | |
|---|---|---|---|
| F4.1e | interpret, discuss and synthesise information presented in a variety of forms | B1, B2 | Candidates should be familiar with data presented in a variety of forms such as lists of raw data, lists of ordered data, tables, frequency tables, bar charts, pie charts and should be aware of the links between them. |
| F4.1f | communicate mathematically, including using ICT, making use of diagrams and related explanatory text | B1, B2 | Candidates should know and be able to draw a variety of statistical diagrams. |
| F4.1g | examine critically, and justify, their choices of mathematical presentation of problems involving data | B1, B2 | Candidates should be able to recognise when diagrams contain errors or omissions |

**Reasoning**

| Foundation Tier | | Notes | |
|---|---|---|---|
| F4.1h | apply mathematical reasoning, explaining and justifying inferences and deductions | B1, B2 | Candidates should be able to show methods clearly, explaining their answers. |
| H4.1e | identify exceptional or unexpected cases when solving statistical problems | B1, B2 | |
| F4.1i<br>H4.1f | explore connections in mathematics and look for relationships between variables when analysing data | B1, B2 | eg, Describing the relationship from a line of best fit |
| F4.1j | recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis | B1, B2 | Candidates should know and recognise when answers are inappropriate |

## 2. Specifying the problem and planning

Pupils should be taught to:

| Foundation Tier | | Notes | |
|---|---|---|---|
| F4.2a | see that random processes are unpredictable | B1 | Candidates should understand that the outcome of a random event cannot be predicted and that the probability of a fair coin landing on heads is 0.5 (or $\frac{1}{2}$), even if the previous six throws have given heads. Probability may be expressed as fractions, decimals or percentages. |
| F4.2b<br>H4.2b | identify key qquestions that can be addressed by statistical methods | B1 | Standard statistical terminology such as average, range, data, etc, should be understood. Candidates should know that questions involving comparison and analysis of data will need to be solved using statistical methods |
| F4.2c | discuss how data relate to a problem, identify possible sources of bias and plan to minimise it | B1 | Candidates may be asked to criticise survey questions or comment on the results of experimental data (relative frequency). |
| F4.2d | identify which primary data they need to collect and in what format, including grouped data, considering appropriate equal class intervals | B1 | Candidates should be familiar with terms such as raw data, ordered data, discrete data and continuous data. |
| F4.2e<br>H4.2e | design an experiment or survey; decide what primary and secondary data to use | B1 | For example, candidates may be asked to give an appropriate question for a survey or to criticise given questions<br>Candidates should understand the terms 'primary' and 'secondary' data. |

## 3. Collecting data

Pupils should be taught to:

| Foundation Tier | | Notes | |
|---|---|---|---|
| F4.3a | design and use data-collection sheets for grouped, discrete and continuous data; collect data using various methods, including observation, controlled experiment, data logging, questionnaires and surveys | B1 | Candidates may be asked to design a data collection sheet in the written examination but this specification reference is better assessed in AO4 coursework.<br>Candidates should know, eg, that data logging is when data is collected automatically by machine, eg, the numbers of cars in a car park at any time. |
| F4.3b | gather data from secondary sources, including printed tables and lists from ICT-based sources | B1 | Reading and analysing data from tables, charts and lists will be required. |
| F4.3c | design and use two-way tables for discrete and grouped data | B1 | eg, Design a two-way table to analyse the colour and make of vehicles. |

## 4. Processing and representing data

Pupils should be taught to:

| Foundation Tier | | Notes | |
|---|---|---|---|
| F4.4a | draw and produce, using paper and ICT, pie charts for categorical data, and diagrams for continuous data, including line graphs for time series, scatter graphs, frequency diagrams and stem-and-leaf diagrams | B1 | Includes knowledge and use of pictograms and bar-charts<br>Includes frequency polygons, histograms with equal class intervals, and frequency diagrams for grouped discrete data.<br>Pie charts should be labelled. If a frequency diagram is required, then it can be an equal interval histogram or a frequency polygon. If a stem and leaf diagram is given a key will be provided. If candidates are asked to draw a stem and leaf diagram they should give a key |
| F4.4b | calculate mean, range and median of small data sets with discrete then continuous data; identify the modal class for grouped data | B1 | Includes knowledge and use of the mode<br>Data in questions will always be given in tabular form with space provided for the addition of an extra column or columns for working. |
| F4.4c | understand and use the probability scale | B1 | Candidates should be familiar with the words impossible, unlikely, evens, likely, certain and their positions on the probability scale. Knowledge that the scale runs from 0 to 1 is expected. |
| F4.4d | understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency | B1 | Questions using coins and dice will be set. Questions using playing cards or involving gambling will not be set. Other situations such as taking counters from bags will also be used. Candidates will be required to know the meaning of 'at random' and this term will be used in questions.<br>Questions may include the addition of simple probabilities |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F4.4e | list all outcomes for single events, and for two successive events, in a systematic way | B1 | Candidates should be familiar with sample space diagrams. Lists or sample space diagrams may be given as answers. |
| F4.4f | identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1. | B1 | eg, The probability that a person is left-handed is 0.19. What is the probability that a person is not left-handed? |
| F4.4g | find the median for large data sets and calculate an estimate of the mean for large data sets with grouped data | B1 | Data in questions will always be given in tabular form with space provided for the addition of extra columns for working. |
| F4.4h | draw lines of best fit by eye, understanding what these represent | B1 | Lines of best fit need not go through the mean point but should pass as close to as many data points as possible. |
| H4.4j | use relevant statistical functions on a calculator or spreadsheet | B1 | Candidates may use statistical functions on their calculator.<br>Standard deviation is no longer tested in written examinations but the mean of a discrete or grouped frequency table is. Marks for this are usually awarded for the processes and the sole use of a calculator is not advised as all marks may be lost if the wrong values (for mid-points, for example) are used and there is no evidence of method. |

## 5. Interpreting and discussing results

Pupils should be taught to:

| Foundation Tier | | Notes | |
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| F4.5a | relate summarised data to the initial questions | B1 | This reference may be best assessed within AO4 coursework |
| F4.5b | interpret a wide range of graphs and diagrams and draw conclusions | B1 | Candidates should be familiar with pictograms, bar charts, bar line graphs, pie charts, stem and leaf diagrams, equal width histograms, frequency polygons, two-way tables, scatter graphs and line graphs. Candidates should know that lines joining points, such as on a graph plotting average temperature against month, have no meaning.<br>Candidates will be expected to interpret eg, the median and the range from stem and leaf diagrams. |
| F4.5c | look at data to find patterns and exceptions | B1 | For example, identifying a 'rogue' value on a scatter graph. |
| F4.5d | compare distributions and make inferences, using the shapes of distributions and measures of average and range | B1 | Questions comparing distributions will be restricted to comparisons of an average and range. |
| F4.5e | consider and check results and modify their approach if necessary | B1 | This reference may be best assessed within AO4 coursework |
| F4.5f<br>H4.5f | appreciate that correlation is a measure of the strength of the association between two variables; distinguish between positive, negative and zero correlation using lines of best fit; appreciate that zero correlation does not necessarily imply 'no relationship' but merely 'no linear relationship' | B1 | Candidates will not be expected to make statements about how reliable the correlation is but should be aware that some data can form a perfect linear relationship and that other data may not. They should also be aware that using the line of best fit to predict values beyond the plotted range may not be reliable. |
| F4.5g | use the vocabulary of probability to interpret results involving uncertainty and prediction | B1 | For example, 'there is some evidence from this sample that...' |

| Foundation Tier | | Notes | |
|---|---|---|---|
| F4.5h | compare experimental data and theoretical probabilities | B1 | Knowledge of the term 'relative frequency' is required. |
| F4.5i | understand that if they repeat an experiment, they may - and usually will - get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics | B1 | Candidates should know that, for example, throwing a dice will not always give a rectangular distribution but that the more trials are carried out then the better the reliability of the results. |
| F4.5j | discuss implications of findings in the context of the problem | B1 | Candidates may be asked to comment on the implications of their calculations. |
| F4.5k | interpret social statistics including index numbers; time series and survey data | B1 | For example, population growth<br>For example, the National Census<br>Candidates should understand the difference between a sample and a census.<br>eg, Candidates should know that if the index number for a base year is 100 and that eg, an index of 120 represents a 20% increase on the base year. |

# Higher Tier Guidance

## AO2 Number and algebra

### 1. Using and applying number and algebra

Pupils should be taught to:

| Higher Tier | Notes |
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**Problem solving**

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| H2.1a | select and use appropriate and efficient techniques and strategies to solve problems of increasing complexity, involving numerical and algebraic manipulation | B3, B4, B5 | Mini-investigations will not be set but candidates will be expected to make decisions and use the appropriate techniques to solve a problem logically.<br>Candidates should choose relevant information when some is redundant. |
| H2.1b | identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches | B3, B4, B5 | Candidates will be expected to give reasons for answers or show working. If a question states, "You **must** show your working", marks will be lost if the instruction is ignored. |
| H2.1c | break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods | B3, B4, B5 | Multi-step problems will be set. Credit can be gained in this type of question for showing a suitable strategy. |

| Higher Tier | | Notes | |
|---|---|---|---|
| H2.1d | make mental estimates of the answers to calculations; present answers to sensible levels of accuracy; understand how errors are compounded in certain calculations | B3, B4, B5 | Candidates should be able to round to make mental estimates of calculations.<br>In complex calculations, such as the solution of a trigonometric problem, candidates should not round off intermediate calculations as this could affect the accuracy of the final answer. Any intermediate values should be written to at least 4 significant figures.<br>Candidates should be able to round to any given accuracy.<br>As a general rule, when candidates are asked to give an answer to a suitable degree of accuracy solutions should be given to the same accuracy as the numbers used in the question. |

**Communicating**

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|---|---|---|---|
| H2.1e | discuss their work and explain their reasoning using an increasing range of mathematical language and notation | B3, B4, B5 | Candidates will be expected to use correct mathematical notation and derive a solution to a given problem logically |
| H2.1f | use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem | B3, B4, B5 | Candidates should be able to interpret for example, diagrams, information given in a real-life context, such as a 'Sale' poster and translate this into a mathematical problem. |
| H2.1g | present and interpret solutions in the context of the original problem | B3, B4, B5 | Candidates will be required to give sensible answers to questions.<br>eg, How many 4-seater taxis are needed to carry 14 passengers? |
| H2.1h | use notation and symbols correctly and consistently within a given problem | B4, B5 | eg, Candidates should be able to explain patterns in words.<br>Correct use of numerical and algebraic notation is expected.<br>eg, $a + a = a2$ will not be accepted. |
| H2.1i | examine critically, improve, then justify their choice of mathematical presentation, present a concise, reasoned argument | B3, B4, B5 | Questions may be set that can be made easier by standard mathematical techniques such as use of brackets, factorisation and cancelling. |

| Foundation Tier | Notes |
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**Reasoning**

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|---|---|---|---|
| H2.1j | explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems | B4, B5 | For example, using simple codes that substitute numbers for letters.<br>See further guidance given in section on proof (Appendix C)<br>Candidates may be required to find a counter-example to disprove a statement. |
| H2.1k | understand the difference between a practical demonstration and a proof | B3, B4, B5 | See further guidance given in section on proof (Appendix C) |
| H21.l | show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning | B3, B4 | See further guidance given in section on proof (Appendix C)<br>Candidates should always show working. |
| H2.1m | recognise the significance of stating constraints and assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem | B3, B4, B5 | Candidates should assume that information given is exact unless the question states or implies otherwise. |

## 2. Numbers and the number system

Pupils should be taught to:

| Higher Tier | Notes |
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**Integers**

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|---|---|---|---|
| H2.2a | use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition | B3, B5 | Abbreviations will not be used in examinations. The word 'least' will be used.<br>Candidates could be asked to round a number to any power of 10 up to $10^6$.<br>Candidates will be expected to identify eg, multiples, factors and prime numbers from lists.<br>Questions may be set that ask candidates to do prime factor decomposition and find HCF or LCMs. There is no obligation on candidates to use prime factor decomposition to find HCF or LCMs. The writing out of multiples or factors is an acceptable method and may be more efficient. |

| Higher Tier | Notes |
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**Powers and roots**

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| H2.2b | use the terms square, positive square root, negative square root, cube and cube root; use index notation and index laws for multiplication and division of integer powers; use standard index form, expressed in conventional notation and on a calculator display | B3 | Definition of standard index form is $a \times 10^n$ where $1 \leq a < 10$ and $n$ is an integer.<br>The term 'standard form' will be used in the examination.<br>Questions will not ask for positive square root but will use the notation $\sqrt{25}$. When a square root is asked for only the positive value will be required. Candidates who give the negative root or both answers will not be penalised, but if the value is to be used in further calculation they may lose marks by using the negative root.<br>If the solution to $x^2 = 25$ is required then both the negative and positive root are expected.<br>Powers of 10 up to $10^6$ should be understood.<br>Values of simple integer powers eg, $2^4$ will be tested.<br>The words cube, square and cube root may be used and should be understood.<br>Candidates should be aware that calculator displays can sometimes show values such as $1.7 \times 10^{-3}$ or $1.7^{-3}$ and know how to interpret these. |

**Fractions**

| | | | |
|---|---|---|---|
| H2.2c | understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator | B3, B5 | Candidates may be asked to give a fractional answer in its simplest form. When this requirement is not clearly stated candidates do not have to cancel fractional answers.<br>Candidates will require knowledge of mixed numbers.<br>The term 'common denominator' will not be used. |

| Higher Tier | Notes |
|---|---|

**Decimals**

| | | | |
|---|---|---|---|
| H2.2d | use decimal notation and recognise that each terminating decimal is a fraction; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals | B3, B5 | eg, $0.137 = \frac{137}{1000}$<br>eg, $\frac{1}{7} = 0.142857142857\ldots$<br>Candidates should know that $0.\dot{3} = \frac{1}{3}$ and $0.\dot{6} = \frac{2}{3}$ and that other fractions give recurring decimals and know how to write<br>eg, $\frac{1}{6} = 0.16666\ldots.$ as $0.1\dot{6}$<br>Candidates could be asked to write a recurring decimal as a rational number and should know a method for converting a recurring decimal to a rational number. |

| Higher Tier | Notes |
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**Percentages**

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|---|---|---|---|
| F2.2e<br>H2.2e | understand that 'percentage' means 'number of parts per 100' and use this to compare proportions; interpret percentage as the operator 'so many hundredths of'; use percentage in real-life situations | B3, B5 | For example, 10% means 10 parts per 100, and 15% of $Y$ means $\frac{15}{100} \times Y$<br>$x$% of $y$ will be required.<br>VAT rates will be provided.<br>Percentage problems on non calculator papers will involve percentages that can be worked out using multiples of 1% and 10%.<br>Some basic knowledge of percentages in every day life eg, commerce and business including rate of inflation, VAT, price index, interest rates and financial capability is required.<br>Note that problems involving simple interest for more than one year will not be set but that compound interest on investments up to 20 years could be assessed.<br>The use of a percentage multiplier is expected. |

**Ratio**

| | | | |
|---|---|---|---|
| H2.2f | use ratio notation, including reduction to its simplest form and its various links to fraction notation | B3 | For example, in maps and scale drawings, paper sizes and gears. Candidates will be expected to know that a line divided in the ratio 1:2 will be split into $\frac{1}{3}$ and $\frac{2}{3}$ of its length.<br>Candidates should know that if say red balls and blue balls are in the ratio 3:4 then the fraction of red balls is $\frac{3}{7}$<br>Candidates should be familiar with ratio notation, for example 2:3, and should know how to reduce to simplest form. Questions asking for the ratio in the form 1:$n$ may be required. |

## 3. Calculations

Pupils should be taught to:

| Higher Tier | Notes |
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**Number operations and the relationships between them**

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| H2.3a | multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer, fractional and negative powers; use inverse operations, understanding that the inverse operation of raising a positive number to power $n$ is raising the result of this operation to power $\frac{1}{n}$ | B3, B5 | eg, Find the reciprocal of $\frac{2}{3}$<br>eg, Find the exact value of $3^{-2}$ (non-calculator question)<br>eg, Find the value of $32^{\frac{2}{5}}$ (non-calculator question)<br>eg, Find the value of $64^{-\frac{2}{3}}$ (non-calculator question)<br>eg, Find the value of $9^0$ (non-calculator question)<br>eg, Solve $x^{\frac{1}{2}} = 9$ (non-calculator question)<br>eg, Find $x$ if $x^6 = 64$ (non-calculator question)<br>eg, Express 48 as the product of prime factors<br><br>The following should be known: table facts up to $10 \times 10$ squares up to $15 \times 15$<br><br>non-calculator methods for adding and subtracting 3 digit numbers; non-calculator methods for multiplying and dividing up to 3 digit numbers by up to 2 digit numbers.<br>Candidates should be able to interpret a remainder from a division problem. Multiplication and division of integers and decimals by powers of 10 will be restricted to 10, 100 and 1000. |

| Higher Tier | | Notes | |
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| H2.3b | use brackets and the hierarchy of operations | B3 | The BIDMAS or BODMAS convention should be known but this mnemonic need not be known. Candidate could be asked to insert brackets into a calculation to make it true. |
| H2.3c | calculate a given fraction of a given quantity expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal; distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals); convert a recurring decimal to a fraction | B3 | For example, for scale drawings and construction of models, down payments, discounts.<br>eg, Find $0.\dot{3}\dot{9}$ as a fraction. (non-calculator question)<br>eg, Find $0.\dot{4}3\dot{2}$ as a fraction. (calculator question)<br>eg, Find $\frac{3}{8}$ of 56. (non-calculator question)<br>eg, Find $\frac{2}{7}$ of 5467. (calculator question)<br>Questions involving mixed numbers may be set.<br>eg, Work out $1\frac{2}{5} + \frac{3}{4}$<br>eg, Work out $3\frac{5}{6} - 2\frac{1}{2}$<br>eg, Work out $\frac{5}{6} - \frac{1}{2}$<br>eg, Write $\frac{7}{20}$ as a decimal. |

| Higher Tier | | Notes | |
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| H2.3d | understand and use unit fractions as multiplicative inverses multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction | B3 | eg, thinking of multiplication by $\frac{1}{5}$ as division by 5.<br><br>eg, $4 \times \frac{7}{8}$, $\frac{6}{11} \div 3$<br><br>eg, $\frac{3}{4} \times \frac{8}{9}$, $\frac{4}{15} \div \frac{2}{5}$<br><br>Multiplication and division problems with mixed numbers may be assessed.<br><br>eg, Work out $1\frac{2}{5} \times 1\frac{3}{4}$<br><br>eg, Work out $3\frac{5}{6} \div 2\frac{1}{2}$ |

<table>
<tr><th colspan="2">Higher Tier</th><th colspan="2">Notes</th></tr>
<tr><td>H2.3e</td><td>convert simple fractions of a whole to percentages of the whole and vice versa then understand the multiplicative nature of percentages as operators calculate an original amount when given the transformed amount after a percentage change; reverse percentage problems</td><td>B3</td><td>For example, analysing diets, budgets or the costs of running, maintaining and owning a car.<br><br>Finding $x$% of $y$ will not be asked for, but an $x$% increase of $y$ may be asked for in the context of a real life problem, or as introduction to compound interest type problems.<br><br>The use of a percentage multiplier is recommended.<br><br>For example, a 15% increase in the value $Y$, is calculated as $1.15 \times Y$.<br><br>For example, a 15% increase in value $Y$, followed by a 15% decrease is calculated as $1.15 \times 0.85 \times Y$.<br><br>eg, A seal colony is decreasing at 12% per annum. If the original population is 2000, after how many years will the population have fallen to half its original number?<br><br>eg, If £550 is invested at 10% per annum compound interest, how much will there be after 2 years? (non-calculator question)<br><br>eg, After a 7% decrease the cost of a TV is £232.50. What was the original price? (calculator question)<br><br>For example, given that a meal in a restaurant costs £36 with VAT at 17.5%, its price before VAT is calculated as £$\frac{36}{1.175}$</td></tr>
<tr><td>H2.3f</td><td>divide a quantity in a given ratio</td><td>B3</td><td>eg, If a quantity of money is divided in the ratio 5:3 and the smaller share is £90, how much is the larger share? (non-calculator question)<br><br>eg, If a quantity of money is divided in the ratio 5:3 and the smaller share is £34.50, how much was the original quantity? (calculator question)</td></tr>
</table>

| Higher Tier | | Notes | |
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**Mental methods**

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| H2.3g | recall integer squares from $2 \times 2$ to $15 \times 15$ and the corresponding square roots, the cubes of 2, 3, 4, 5 and 10, the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers $n$ the corresponding rule for negative numbers $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number $n$ | B3 | Mental methods will not be tested in the written papers but a quick recall of basic number facts is expected.<br>For example, $10° = 1$, $9^{-1} = \frac{1}{9}$<br>For example, $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$<br>For example, $25^{\frac{1}{2}} = 5$ and $64^{\frac{1}{3}} = 4$<br>It will be acceptable to give the decimal equivalent of $\frac{1}{3}$ and $\frac{2}{3}$ as 0.33 and 0.66 or 0.67 |

| Higher Tier | | Notes | |
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| H2.3h | round to a given number of significant figures; develop a range of strategies for mental calculation; derive unknown facts from those they know; convert between ordinary and standard index form representations converting to standard index form to make sensible estimates for calculations involving multiplication and/or division | B3 | For example, $0.1234 = 1.234 \times 10^{-1}$<br>Candidates should be able to round to a given number of decimal places.<br>Candidates could be asked to give an answer to 1, 2 or 3 significant figures, but should be able to round to any given number of significant figures<br>Questions requiring the answer to be given to a sensible degree of accuracy will be set. In these cases candidates will have to make their own decisions. An answer given to no more accuracy than the least accurate data in the question is expected.<br>eg, Find 23% of 106 000. Give your answer to a sensible degree of accuracy.<br>An answer of 24 000 or 24 400 would be expected. 23% is only to 2 s.f. but this would not count as a rounded value.<br>What percentage is 230 of 1460? Give your answer to a sensible degree of accuracy.<br>An answer of 16% (2 s.f.) would be expected as 230 is given to 2 s.f. and would be regarded as a rounded value. |
| F2.3i | develop a range of strategies for mental calculation; add and subtract mentally numbers with up to one decimal place; multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments | B3 | Knowledge of the terms 'commutative', 'associative' and 'distributive' is not required and the term 'factorise' in the context of number need not be known |

| Higher Tier | Notes |
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**Written methods**

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| F2.3k | division by decimal (up to 2 d.p.) by division using an integer; understand where to position the decimal point by considering what happens if they multiply equivalent fractions, eg, given that…work out… | B3 | Candidates may use any algorithm in non-calculator papers.<br>For example $\frac{2000}{0.4} = \frac{20000}{4} = 5000$<br>eg, $408 \div 0.17 = 40800 \div 17$<br>eg, $0.02 \times 0.3 = 0.006$, $0.4^2 = 0.16$ |
| H2.3i | use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer | B3 | Candidates may use any algorithm in non-calculator papers.<br>If calculating $\frac{4}{9} \times \frac{3}{8}$, in say a probability question, candidates will not be penalised if they fail to cancel before carrying out the multiplication. They may not gain full marks, however, if they fail to cancel $\frac{12}{72}$ |
| H2.3j | solve percentage problems including percentage increase and decrease and reverse percentages | B3 | Candidates will be expected to be familiar with the terms 'simple interest', 'VAT' and 'annual rate of inflation' 'income tax' and 'discount' and to know what they mean.<br>In questions involving VAT, the amount of VAT will be given, as will the value of the annual rate of inflation.<br>eg, Mrs Smith has a salary of £30 000 at the start of 2004. This is increased at the end of the year in line with annual rate of inflation. In 2005 this was 3.2% and in 2006 it was 2.9%. What is Mrs Smith's salary at the end of 2000?<br>eg, A computer costs £1116.25 including 17.5% VAT. Schools do not pay VAT. How much would the computer cost a school? |

| Higher Tier | | Notes | |
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| F2.3n | solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution | B3 | For example, given that m identical items cost £y, then one item costs £$\frac{y}{m}$ and n items cost £(n × $\frac{y}{m}$), the number of items that can be bought for £z is z × $\frac{m}{y}$<br>eg, 8 pencils cost £2.40. How much do 11 cost? (non-calculator question)<br>eg, 8 pencils cost £2.56. How many can be bought for £4.80? (calculator question) |
| H2.3k | represent repeated proportional change using a multiplier raised to a power | B3 | Candidates will be expected to be familiar with the term 'compound interest' and know what it means.<br>eg, A seal colony is decreasing at 12% per annum. If the original population is 2000, after how many years will the population have fallen to half the original number? (calculator question: such problems are likely to appear on the calculator paper due to the repetitive nature of the calculations)<br>eg, If £550 is invested at 10% per annum compound interest, how much will there be after 2 years? (non-calculator question)<br>eg, The value of the investment after 3 years is $a$ × 550, what is the value of $a$? (non-calculator question) – an answer of $1.1^3$ or 1.331 would be accepted. |
| H2.3l | calculate an unknown quantity from quantities that vary in direct or inverse proportion | B3 | eg, Two men can mow a meadow in 2 hours. How long would they take to mow a meadow that is twice as big?<br>eg, Two men can mow a meadow in 2 hours. How long would three men, working at the same rate, take to mow the meadow? |

| Higher Tier | | Notes | |
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| H2.3m | calculate with standard index form | B3 | Questions may be set in context. For example, populations, astronomical data, size of atoms. Numbers used in non-calculator papers will be straightforward.<br>eg, $(4 \times 10^7) + (5 \times 10^6) = 45 \times 10^6 = 4.5 \times 10^7$ (non-calculator question)<br>eg, $2.4 \times 10^7 \times 5 \times 10^3 = 12 \times 10^{10} = 1.2 \times 10^{11}$ (non calculator question)<br>eg, $(2.5 \times 107) \div (5 \times 103) = 5 \times 103$ (non-calculator question) |

| Higher Tier | | Notes | |
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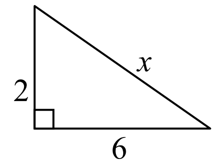
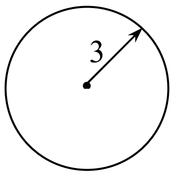
| H2.3n | use surds and π in exact calculations, without a calculator;<br><br>rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ | B3, B5 | Questions requiring answers in surds or in terms of π will appear in non-calculator papers.<br><br>eg, Find the exact value of *x*.<br><br>An answer of $\sqrt{40}$ would not need to be simplified unless this was asked for.<br><br>Find the area of the circle. Leave your answer in terms of π.<br><br>Candidates may be required to calculate a circumference or area in non-calculator papers but the value of π will be given (π = 3.14)<br><br>Candidates will need to understand the term 'rationalise the denominator'.<br>Denominators in such questions will always be in the form $\sqrt{a}$ . |

| Higher Tier | | | Notes |
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**Calculator methods**

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| H2.3o | use calculators effectively and efficiently, knowing how to enter complex calculations; use an extended range of function keys, including trigonometry and statistical functions relevant across this programme of study | B3 | Efficient use of a calculator will be expected. Candidates may be asked to calculate expressions such as $\frac{2.3+4.9}{5.2-1.7}$ using memory or brackets.<br>Candidates will not need to show intermediate working in trigonometry problems, for example.<br>eg, 52 × sin 37 can be calculated and the answer written down, (with appropriate rounding if required). |
| F2.3p<br>H2.3p | enter a range of calculations, including those involving measures<br>understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation | B3 | For example, time calculations in which fractions of an hour must be entered as fractions or as decimals.<br>Candidates should use the calculator display whenever possible to make further progress in a calculation but if intermediate values have to be written down they should write the value to at least 4 significant figures so that the final answer is within the acceptable range of accuracy. |
| H2.3q | use calculators, or written methods, to calculate the upper and lower bounds of calculations, particularly when working with measurements | B3 | Candidates need to realise that a number written as 9.7 correct to 1 decimal place can actually lie anywhere between $9.65 \le x < 9.75$.<br>eg, Calculate the upper bound of the area of a rectangle with dimensions 4.5 cm by 7.5 cm, both values given to 1 decimal place.<br>Candidates should know how to combine the upper and lower bounds when using the four rules to work out upper and lower bounds of calculations.<br>eg, A car travels 60 miles (to the nearest 10 miles) at an average speed of 55 mph (to 2 significant figures). What are the limits of the time of the journey? |

| Higher Tier | | Notes | |
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| H2.3r | use standard index form display and know how to enter numbers in standard index form | B3 | Use of the EXP or EE button on a calculator is expected in calculator papers. Candidates are expected to interpret the calculator display and to write their answers in correct standard index notation. |
| H2.3s | use calculators for reverse percentage calculations by doing an appropriate division | B3 | For example, when finding the original quantity after an increase of 8%, candidates are expected to divide by 108 and multiply by 100, or to divide by 1.08.<br>Reverse 'compound interest' problems will not be set. |
| H2.3t | use calculators to explore exponential growth and decay using a multiplier and the power key | B3 | For example, in science or geography.<br>eg, The population of a town is growing at 2% per annum. It is currently 3500.<br>Estimate the population after 20 years. |

## 4. Solving numerical problems

Pupils should be taught to:

| Higher Tier | | Notes | |
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| H2.4a | draw on their knowledge of operations and inverse operations (including powers and roots), and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, repeated proportional change, fractions, percentages and reverse percentages, inverse proportion, surds, measures and conversion between measures, and compound measures defined within a particular situation | B3 | The terms 'commutative', 'associative' and 'distributive' will not be used in the examination.<br><br>Knowledge of the term 'root' is required.<br><br>Knowledge of the term 'inverse operation' is required and candidates should know the inverse operations of the four rules: square, square root, cube and cube root.<br><br>Compound measures may be expressed in the form metres per second, $m/s$, $m\ s^{-1}$. Candidates would be expected to understand these and other standard compound measures, such as those for density. Units may be any of those in common usage or specifically mentioned in the specification such as speed (eg, miles per hour) or density (eg, grams/$\text{cm}^3$).<br><br>Other compound measures that are non-standard would be defined in the question eg, population density is population/$\text{km}^2$.<br><br>eg, Evaluate $(4+\sqrt{3})(4-\sqrt{3})$.<br><br>Conversion between measures would involve knowledge of the connection between metric units. Conversions between imperial units will be given. |
| H2.4b | check and estimate answers to problems; select and justify appropriate degrees of accuracy for answers to problems; recognise limitations on the accuracy of data and measurements | B3 | eg, The heights of 7 men are 150, 151, 148, 133, 138, 142, 140cm. Give a reason why 143.143 cm is not an appropriate answer for their mean height.<br><br>eg, If the radius of a circle is 5.2 cm to 2 s.f., what is the minimum value its area could be? |

## 5. Equations, formulae and identities

Pupils should be taught to:

| Higher Tier | Notes |
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**Use of symbols**

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| H2.5a | distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations defined quantities or variables in formulae general, unspecified and independent numbers in identities and in functions they define new expressions or quantities by referring to known quantities. | B3, B5 | For example, knowing that $x^2 + 1 = 82$ is an equation.<br>For example, knowing that $V = IR$ is a formula.<br>For example, knowing that $(x + 1)^2 \equiv x^2 + 2x + 1$, for all values of $x$ is an identity.<br>Knowledge of the identity symbol $\equiv$ is required.<br>For example, knowing that $y = 2 – 7x$; $f(x) = x^2$ are functions.<br>For example, understanding that $y = \frac{1}{x}$ is not defined for $x = 0$<br>Candidates will be expected to know the standard conventions such as $2x$ for $2 \times x$, $\frac{1}{2}x$ or $\frac{x}{2}$<br>Candidates who write $2 \times x$, $x \times 2$, $x \div 2$ will not be penalised, but $x2$ will not be accepted for $2x$.<br>Candidates will be expected to understand and use the function notation eg, $f(x) = 2x + 3$ |

| Higher Tier | | Notes | |
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| H2.5b | understand that the transformation of algebraic entities obeys and generalises the well-defined rules of generalised arithmetic; expand the product of two linear expressions; manipulate algebraic expressions by collecting like terms; multiplying a single term over a bracket, taking out common factors; factorising quadratic expressions including the difference of two squares and cancelling common factors in rational expressions. | B3, B5 | For example, $a(b + c) = ab + ac$<br>eg, Expand and simplify $(2x+3)(3x-4)$.<br>Candidates will be expected to know the meaning of 'simplify' eg, Simplify $3x-2+4(x+5)$ and factorise eg, Factorise $3x^2y-9y$.<br>Knowledge of the phrase 'difference of two squares' will not be expected but candidates may be asked to factorise expressions such as $x^2 - 9$ or $4x^2 - y$<br>Candidates will be expected to know the meaning of 'solve' in relation to linear and non-linear equations. (eg, Solve the equation $x^2+2x-15=0$)<br>Knowledge of the term 'factorisation' would not be required in the context of a numerical problem but candidates should know how to cancel fractions, for example, and would be expected to explain why $4x^2+8x-12=0$ can be simplified to $x^2+2x-3=0$ and understand why<br>$\frac{3x+12}{9x-15}$ can be simplified to $\frac{x+4}{3x-5}$<br>Candidates will be expected to know how to simplify.<br>eg, $\frac{1}{x} + \frac{3}{2-x}$<br>Candidates will be expected to know how to factorise<br>eg, $4x^2 + 6xy$ |

| Higher Tier | | Notes | |
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| H2.5c | know the meaning of and use the words 'equation', 'formula', 'identity' and 'expression' | B3, B5 | Candidates will be expected to know the terms 'equation', 'formula', 'identity' and 'expression'.<br>eg, The expression $x^2+2x+1$ has three terms.<br>The equation $x^2+2x+1=0$ can be solved.<br>$A=x^2+2x+1$ is a formula.<br>$(x+1)^2 \equiv x^2+2x+1$ is an identity that is true for all $x$ |

**Index notation**

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| H2.5d | use index notation for simple integer powers, and simple instances of index laws substitute positive and negative numbers into expressions such as $3x^2+4$ and $2x^3$ | B5 | Evaluate $2x^3$ when $x = -2$, $3x^3+4$ when $x = \frac{1}{2}$ or $-3$.<br>eg, Simplify $2x^{-2} \times 3x^4$<br>eg, Simplify $\frac{4ab^2 \times 3ab}{6ab}$<br>eg, Simplify $(2x^2)^3$<br>eg, Evaluate $2^5$<br>eg, Simplify $x^2 \times x^{-3}$, $x^6 \div x^{-2}$, $x^{-2} \div x^3$<br>eg, Simplify $2x^{-2} \times 3x^4$<br>eg, Simplify $\frac{4ab^2 \times 3ab}{6ab}$<br>eg, Simplify $(x^2)^3$ |

| Higher Tier | Notes |
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**Equations**

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| H2.5e | set up simple equations solve simple equations by using inverse operations or by transforming both sides in the same way | B5 | For example, find the angle $a$ in a triangle with angles $a$, $a + 10$, $a + 20$<br>For example, $5x = 7$, $11 - 4x = 2$, $3(2x + 1) = 8$, $2(1 - x) = 6(2 + x)$, $4x^2 = 49$, $3 = \frac{12}{x}$ |

**Linear Equations**

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| H2.5f | solve linear equations in one unknown, with integer or fractional coefficients, in which the unknown appears on either side or on both; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution | B5 | eg, Solve $2(x - 4) = 7$ (non-calculator or calculator question)<br>eg, $2x - 3 = \frac{x+2}{3}$<br>eg, Solve $5x + 17 = 3(x + 6)$ (non-calculator or calculator question)<br>eg, Solve $\frac{15-x}{4} = 2$ (non-calculator or calculator question)<br>eg, Solve $\frac{2x-3}{6} + \frac{x+2}{3} = \frac{2}{5}$ (non-calculator or calculator question)<br>eg, Solve $\frac{17-x}{4} = 2 - x$ |

| Higher Tier | Notes |
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**Formulae**

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| H2.5g | use formulae from mathematics and other subjects substitute numbers into a formula; change the subject of a formula including cases where the subject occurs twice, or where a power of the subject appears; generate a formula | B5 | For example, for the area of a triangle or a parallelogram, area enclosed by a circle, volume of a prism<br>Candidates will be expected to substitute into formulae and to understand the order of operations.<br>eg, Rearrange $x + y = 7$ to make $x$ the subject.<br>eg, Rearrange $c = 2\pi r$ to make $r$ the subject<br>eg, Rearrange $y = 2x + 3$ to make $x$ the subject.<br>eg, If $u = 2.5, a = -1.7$ *and* $t = 2$ find $s = ut + \frac{1}{2}at^2$<br>eg, Rearrange $2(x + y) = 5y - 3$ to make $x$ the subject.<br>eg, Rearrange $A = 2\pi rh + \pi rh$ to make $r$ the subject.<br>For example, find the perimeter of a rectangle given its area $A$ and the length $l$ of one side.<br>For example volume of a cone. |

| Higher Tier | Notes |
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**Direct and inverse proportion**

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| H2.5h | set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relate algebraic solutions to graphical representation of the equations | B3 | Questions will be restricted to the following proportionalities: <br> $y \propto x,\ y \propto x^2, y \propto x^3,\ y \propto \sqrt{x},\ y \propto \sqrt[3]{x}, y \propto \frac{1}{x},\ y \propto \frac{1}{x^2},$ <br> $y \propto \frac{1}{x^3},\ y \propto \frac{1}{\sqrt{x}},\ y \propto \frac{1}{\sqrt[3]{x}}$ <br> The expected approach would be to set up an equation using a constant of proportionality. Find this and then use the equation to find a value of $y$ given $x$, or $x$ given $y$. Other methods may be used and can be given full credit. <br> eg, The weight of a sphere is proportional to the cube of its radius. When $r = 5$cm, $W = 500$g. Find the weight of a sphere with $r = 10$cm. |

**Simultaneous linear equations**

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| H2.5i | Find the exact solutions of two simultaneous equations in two unknowns by eliminating a variable and interpret the equations as lines and their common solution as the point of intersection | B5 | Solution by substitution of one variable into the other equation will be accepted as a method and may be the better method if equations are given, for example, as $y = 2x + 3$, $3x + 4y = 1$. Solution by graphical method may sometimes be an acceptable alternative approach but if the question clearly states 'Use an algebraic method', then this is expected. |

| Higher Tier | | Notes | |
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**Inequalities**

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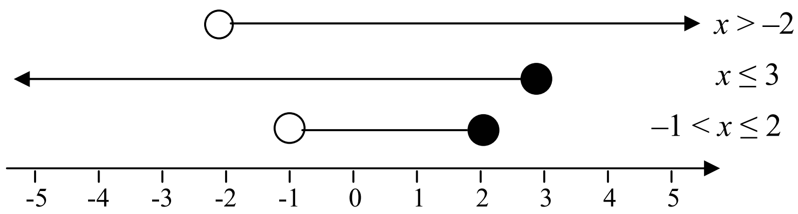
| H2.5j | solve linear inequalities in one variable, and represent the solution set on a number line; solve several linear inequalities in two variables and find the solution set | B5 | Candidates should know the difference between >, <, ≤ and ≥<br>For example Solve $2x + 3 < 7$.<br>Candidates should know the convention of an open circle for a strict inequality and a closed circle for an included boundary.<br>eg,  <br>Candidates should be familiar with the notation $-2 < x \leq 3$.<br>In 2 dimensions candidates should be able to plot and read inequalities such as $2x + 3y < 6$, $y \geq x + 2$, $y \leq 3$, $x > -4$ and find a set of inequalities that describe an enclosed region of the plane.<br>Boundary lines for strict inequalities should be dashed and included inequalities should be solid.<br>There is no convention for an overlapping region being shaded on the required area or shaded on the unwanted area. Candidates will be asked to mark the required region clearly with an R, for example. |

| Higher Tier | | | Notes |
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**Quadratic equations**

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| H2.5k | solve quadratic equations by factorisation, completing the square and using the quadratic formula | B5 | eg, Solve $x^2 + 5x + 6 = 0$<br>eg, Solve $2x^2 + 9x - 5 = 0$ (non-calculator question)<br>eg, Solve $x^2 - 2x - 1 = 0$ giving your answer to 2 d.p. (calculator question)<br>eg, Solve $x^2 - 2x - 1 = 0$ giving your answer in the form $a \pm \sqrt{b}$. (non-calculator question)<br>eg, Write $x^2 + 4x - 9$ in the form $(x + a)^2 - b$<br>Hence solve the equation $x^2 + 4x - 9 = 0$, giving answers to 2 d.p. (calculator question) / giving answers in the form $a \pm \sqrt{b}$ (non-calculator question)<br><br>Trial and improvement will not be accepted as a valid method.<br><br>Candidates need not know that $b^2 - 4ac$ is the discriminant but should be aware that some quadratic equations have no solution. |

| Higher Tier | Notes |
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**Simultaneous linear and quadratic equations**

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| H2.5l | solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other or where the second is of the form $x^2 + y^2 = r^2$ | B5 | The expected method for solving one linear and one non-linear equation will be to substitute a variable from the linear equation into the non-linear equation.<br>For example, solve the simultaneous equations $y = 11x - 2$ and $y = 5x^2$.<br>The non linear equation will be of the form $y = ax^2 + bx + c$, where $b$ and $c$ are integers (including zero), or $x^2 + y^2 = r^2$, where $r$ is not necessarily an integer. |

**Numerical Methods**

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|---|---|---|---|
| H2.5m | use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them | B5 | For example, Solve $x^3 - x = 900$, Solve $\frac{1}{x} = x^2$<br>Answers will be expected to 1 d.p. Candidates will be expected to test the mid-value of the 1 d.p. interval to establish which 1 d.p. value is nearest to the solution. |

## 6. Sequences, functions and graphs

Pupils should be taught to:

| Higher Tier | Notes |
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**Sequences**

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|---|---|---|---|
| H2.6a | generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the *n*th term of an arithmetic sequence, justifying its form by reference to the activity or context from which it was generated | B5 | Candidates should be able to explain how a sequence continues.<br>The nth terms of linear sequences will be required.<br>Candidates will **not** be expected to find the nth term of a non-linear sequence.<br>However, candidates should be familiar with the idea of a non-linear sequence and the fact that the nth term can be generated by an expression of the form $\frac{1}{2}n\,(n + 1)$, for example.<br>They should also know that the nth term of the square number sequence is given by $n^2$. |

**Graphs of linear functions**

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| H2.6b | use conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for $m$ and $c$) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which $y$ is given explicitly in terms of $x$ (for example, $y = 2x + 3$), or implicitly (for example, $x + y = 7$); no table or axes given | B5 | eg, explicitly such as $y = 2x + 3$ or implicitly such as $x + y = 5$<br>Partially completed tables of values may sometimes be given but candidates should be able to plot the graph of $y = 3x – 1$, say, with no further assistance. Knowledge that $m$ is the gradient and $c$ is the $y$-intercept will be expected.<br>Know that the graph of $2x + 3y = 1$ is a straight line graph. |

| Higher Tier | | Notes | |
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| H2.6c | find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for $m$ and $c$); understand that the form $y = mx + c$ represents a straight line and that $m$ is the gradient of the line and $c$ is the value of the $y$ – intercept; explore the gradients of parallel lines and lines perpendicular to each other | B5 | For example, know that the lines represented by the equations $y = -5x$ and $y = 3 - 5x$ are parallel, each having gradient $(-5)$ and that the line with equation $y = \frac{x}{5}$ is perpendicular to these lines and has gradient $\frac{1}{5}$<br>Find the equation of a straight line when the graph is given.<br>Knowledge of the condition $m_1 m_2 = -1$ will not be required but candidates will be expected to recognise that perpendicular lines have gradients that are negative reciprocals of each other.<br>Questions involving perpendicular lines could include a graph which may be used in the solution of the problem and candidates can use graph paper to assist in their solution.<br>eg, 'Find the equation of the line perpendicular to $y = 2x + 3$ passing through (0, 5)' will not include a graph.<br>eg, 'Find the equation of the line perpendicular to the mid-point of '$A(-1, 3)$ and $B(3, 1)$' will include a graph. |

**Interpret graphical information**

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|---|---|---|---|
| H2.6d | construct linear functions and plot the corresponding graphs arising from real-life problems discuss and interpret graphs modelling real situations | B5 | For example, distance - time graph for a particle moving with constant speed, the depth of water in a container as it empties.<br>For example, the velocity-time graph for a particle with constant acceleration<br>Candidates will be expected to find the velocity for sections of a distance–time graph, and should understand that the steeper the line the faster the speed. They should also understand the significance of a negative gradient. Candidates will not be expected to find the area under a curved graph nor to find the acceleration by measuring the gradient of a tangent. Units of acceleration are not required. |

**Quadratic functions**

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| H2.6e | generate points and plot graphs of simple quadratic functions then more general quadratic functions find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function; find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions | B3,B5 | For example, $y = x^2$, $y = 3x^2 + 4$<br>For example, $y = x^2 - 2x + 1$<br>If candidates are required to draw a graph then a table may be given in which some $y$ values may have to be calculated.<br>Quadratic graphs are expected to be drawn as a curve.<br>Candidates will be expected to know that the roots of an equation $f(x) = 0$ can be found where the graph of the function intersects the $x$-axis and that the solution of $f(x) = a$ is found where $y = a$ intersects with $f(x)$.<br>Candidates will be expected to know that the solution of an equation such as $x^2 + 3x - 2 = 0$ can be found from the intersection of two graphs such as $y = x^2 + 2x - 1$ and $y = 1 - x$. The non linear graph will always be quadratic. |

**Other functions**

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|---|---|---|---|
| H2.6f | plot graphs of simple cubic functions the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the exponential function $y = k^x$ for integer values of $x$ and simple positive values of $k$ the circular functions $y = \sin x$ and $y = \cos x$, using a spreadsheet or graph plotter as well as pencil and paper; recognise the characteristic shapes of all these functions | B5 | Candidates would be expected to recognise a sketch of the cubic, for example, $y = x^3$, and reciprocal graphs (including negative values of $x$). They would also be expected to sketch a graph of $y = \sin x$, and $y = \cos x$ between 0 and 360°, and know that the maximum and minimum values for sin and cos are 1 and −1. They would also be expected to know that the graphs of sin and cos are periodic.<br>If candidates are required to draw an exponential graph, for example, $y = 2x$, $y = (\frac{1}{2})^x$, then a table will be given in which some $y$ values may have to be calculated. Graphs are expected to be drawn as a curve. Joining points with straight lines will not get full credit. |

| Higher Tier | | | Notes |
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**Transformation of functions**

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| H2.6g | apply to the graph of $y = \mathrm{f}(x)$ the transformations $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(ax)$, $y = \mathrm{f}(x + a)$, $y = a\mathrm{f}(x)$ for linear, quadratic, sine and cosine functions $\mathrm{f}(x)$ | B5 | $\mathrm{f}(x)$ will be restricted to a simple quadratic, $y = ax^2 + bx + c$, where one of $b$ or $c$ will be zero, $y = \sin(x)$ or $y = \cos(x)$ |

**Loci**

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| H2.6h | construct the graphs of simple loci including the circle $x^2 + y^2 = r^2$ for a circle of radius $r$ centred at the origin of coordinates; find graphically the intersection points of a given straight line with this circle and know that this corresponds to solving the two simultaneous equations representing the line and the circle | B5 | Candidates will be expected to recognise that $x^2 + y^2 = a^2$ is a circle, centre origin and radius $a$.<br>A grid will be provided.<br>eg, (a) Draw the graph of the set of points which are equidistant from the $x$ and $y$-axes.<br>(b) Write down the equation of your graph.<br><br>eg, (a) The $y$ coordinate of point $P$ is twice its $x$ coordinate. Write down one possible pair of coordinates for point $P$.<br>(b) On the grid, draw the graph of the set of points $P$.<br>(c) Give the equation of the set of points $P$.<br>Candidates should know that a line that intersects the curve twice represents the solution of a quadratic with two roots and that when a line is a tangent to a curve it represents the solution of a quadratic with a repeated root. |

# AO3: Shape, space and measures

## 1. Using and applying shape, space and measures

Pupils should be taught to:

| Higher Tier | Notes |
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**Problem solving**

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| H3.1a | select the problem-solving strategies to use in geometrical work, and consider and explain the extent to which the selections they made were appropriate | B4, B5 | Mini-investigations will not be set but candidates will be expected to make decisions and use the appropriate techniques to solve a problem drawing on well known facts, such as the sum of angles in a triangle. |
| H3.1b | select and combine known facts and problem-solving strategies to solve more complex geometrical problems | B4, B5 | Multi-step problems will be set.<br>eg, In the triangle $ABC$, angle $A\hat{B}C$ is obtuse, angle $B\hat{A}C = 32^\circ$, $AC = 10$ cm. $BC = 6$ cm.<br>Calculate the area of triangle $ABC$ (diagram given) |
| H3.1c | develop and follow alternative lines of enquiry, justifying their decisions to follow or reject particular approaches | B4, B5 | Redundant information may sometimes be used, for example, the slant height of a parallelogram. Candidates should be able to identify which information given is needed to solve the given problem. |

**Communicating**

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| H3.1d | communicate mathematically, with emphasis on a critical examination of the presentation and organisation of results, and on effective use of symbols and geometrical diagrams | B4, B5 | Candidates will be expected to use correct mathematical notation and produce a logical solution to a given problem. |
| H3.1e | use precise formal language and exact methods for analysing geometrical configurations | B4, B5 | For example candidates may be asked to give reasons why angles have certain values in a problem using circle theorems. |
| F3.1g | review and justify their choices of mathematics presentation | B4, B5 | Candidates should be able to choose an appropriate method when several methods are possible |

**Reasoning**

| | | | |
|---|---|---|---|
| F3.1h | distinguish between practical demonstrations and proofs | B4, B5 | Candidates may be required to give specific examples or more general proofs. |
| H3.1f | apply mathematical reasoning, progressing from brief mathematical explanations towards full justifications in more complex contexts | B4, B5 | See further guidance given in section on proof (Appendix C) |
| H3.1g | explore connections in geometry; pose conditional constraints of the type 'If… then…'; and ask questions 'What if…?' or 'Why?' | B4, B5 | See further guidance given in section on proof (Appendix C) |
| H3.1h | show step-by-step deduction in solving a geometrical problem | B4, B5 | Candidates should be able to explain reasons using words or diagrams. |
| H3.1i | state constraints and give starting points when making deductions | B4, B5 | Candidates should realise when an answer is inappropriate. |
| H3.1j | understand the necessary and sufficient conditions under which generalisations, inferences and solutions to geometrical problems remain valid | B4, B5 | eg, Candidates should assume that lengths and angles given are exact unless the question states otherwise. |

## 2. Geometrical reasoning

Pupils should be taught to:

| Higher Tier | Notes |
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**Properties of triangles and other rectilinear shapes**

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| H3.2a | distinguish between lines and line segments; use parallel lines, alternate angles and corresponding angles; understand the consequent properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices | B5 | Explanations should use the correct terminology. Colloquial terms such as 'Z', 'F' angles are not acceptable as reasons.<br>Acceptable terms for the interior, co-interior or allied<br>See further guidance on proof (Appendix C). Candidates should know that a straight line drawn between two points or vertices is a line segment. |
| H3.2b | use angle properties of equilateral, isosceles and right-angled triangles; explain why the angle sum of a quadrilateral is 360 degrees | B5 | Angle sum of a quadrilateral is based on the angle sum of a triangle which can be taken as a fact. |
| F3.2e | use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle | B5 | Candidates will not be expected to deduce the formula for area of rectangles, parallelograms and triangles in examinations but will be expected to know them.<br>Questions involving compound shapes made up of rectangles and triangles will be assessed. |
| H3.2c | recall the definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties | B5 | Questions may be set that test candidates' knowledge of these properties. The properties of a kite should also be known.<br>The formula for the area of a trapezium is given on the formula sheet.<br>Candidates should know the side, angle and diagonal properties of quadrilaterals. |

| Higher Tier | Notes |
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**Properties of triangles and other rectilinear shapes**

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|---|---|---|---|
| H3.2d | calculate and use the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons; calculate and use the angles of regular polygons. | B5 | Candidates should learn or know how to work out the angle sum of polygons up to a hexagon. Questions involving the interior and exterior angles of regular polygons may be set.<br>Questions could include octagons and decagons. |
| H3.2e | understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and compass constructions | B5 | Candidates can justify congruence by a variety of methods but their justifications must be complete. The use of SSS notation etc, is not expected but will make the justification of congruence easier.<br>See further guidance on proof (Appendix C). |
| H3.2f | understand, recall and use Pythagoras' theorem in 2-D, then 3-D problems; investigate the geometry of cuboids including cubes, and shapes made from cuboids, including the use of Pythagoras' theorem to calculate lengths in three dimensions | B5 | In 2 dimensions questions may be set in context, for example, a ladder against a wall, but questions will always include a diagram of a right angled triangle with two sides marked and the third side to be found. Quoting the formula will not gain credit. It must be used with the appropriate numbers, eg, $x^2 = 7^2 + 5^2$, $x^2 = 12^2 - 9^2$ or $x^2 + 9^2 = 12^2$.<br>In three dimensions candidates should identify a right angled triangle that contains the required information and then use Pythagoras' theorem (or trigonometry) to solve the problem. The use of the rule $d = \sqrt{a^2 + b^2 + c^2}$ is not required as problems will always be solvable using a combination of triangles.<br>eg, Find the length of the diagonal $AB$ in the cuboid with dimensions 9 cm, 40 cm and 41 cm. (diagram given) |

| Higher Tier | | Notes | |
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| H3.2g | understand similarity of triangles and of other plane figures, and use this to make geometric inferences; understand, recall and use trigonometry relationships in right-angled triangles, and use these to solve problems, including those involving bearings, then use these relationships in 3-D contexts, including finding the angles between a line and a plane (but not the angle between two planes or between two skew lines); calculate the area of a triangle using $\frac{1}{2}ab\ \sin C$; draw, sketch and describe the graphs of trigonometric functions for angles of any size, including transformations involving scaling in either or both the $x$ and $y$ directions; use the sine and cosine rules to solve 2-D and 3-D problems | B5 | Candidates should know that there are many solutions to $\sin x = 0.5$, for example. Questions may require candidates to solve simple trigonometry functions, for example<br>$\cos x = -0.5$ The domain for $x$ will always be defined, for example $0^\circ \leq x \leq 360^\circ$.<br>Candidates should be aware of the ambiguous case for the sine rule.<br>Transformations of $\mathrm{f}(x) = \tan x$ will not be required.<br>In three dimensions candidates should identify a right angled triangle that contains the required information and then use trigonometry (or Pythagoras' theorem) to solve the problem. Although the sine and cosine rule can sometimes be used to solve 3-D problems they will always be solvable by a combination of right angled triangles. |

**Properties of circles**

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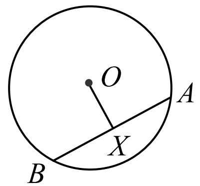
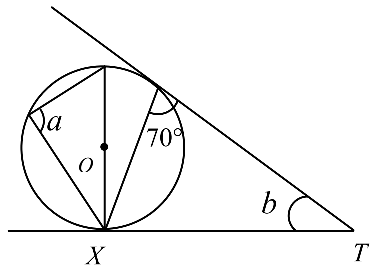
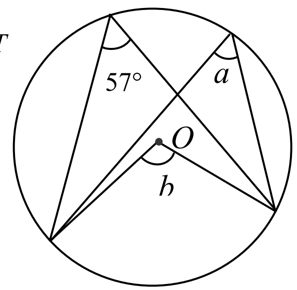
| H3.2h | recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that the tangent at any point on a circle is perpendicular to the radius at that point; understand and use the fact that tangents from an external point are equal in length; explain why the perpendicular from the centre to a chord bisects the chord; understand that inscribed regular polygons can be constructed by equal division of a circle; prove and use the facts that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, the angle subtended at the circumference by a semicircle is a right angle, that angles in the same segment are equal, and that opposite angles of a cyclic quadrilateral sum to 180 degrees; prove and use the alternate segment theorem | B5 | Questions asking for the angle at the centre of a regular polygon may be set.<br>See further guidance given in section on proof (Appendix C)<br>When asked to give reasons for angles any clear indication that the correct theorem is being referred to is acceptable. For example, angles on same chord, angle at centre twice angle at edge angle on diameter is $90^\circ$, opposite angle in cyclic quad. Alt seg theory. |

| Higher Tier | | Notes | |
|---|---|---|---|
| H3.2h | recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that the tangent at any point on a circle is perpendicular to the radius at that point; understand and use the fact that tangents from an external point are equal in length; explain why the perpendicular from the centre to a chord bisects the chord; understand that inscribed regular polygons can be constructed by equal division of a circle; prove and use the facts that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, the angle subtended at the circumference by a semicircle is a right angle, that angles in the same segment are equal, and that opposite angles of a cyclic quadrilateral sum to 180 degrees; prove and use the alternate segment theorem | B5 | Candidates will be expected to know the meaning of the terms 'sector' and 'segment'. Questions involving these terms will always be accompanied by a diagram where the appropriate region will be shown.<br>At the middle grade questions based on circle theorems will not involve complicated diagrams. At most, two straightforward applications of the circle theorems will be asked for in one diagram.<br>eg, $O$ is the centre of the circle<br>$O\hat{X}A = 90^\circ$<br>Explain why $OX$ bisects $AB$.<br><br><br>eg, $O$ is the centre of the circle. Find angles $a$ and $b$<br><br><br>eg, $O$ is the centre of the circle.<br><br><br>Find the sizes of angles $a$ and $b$.<br>At the higher grades questions may be set that involve complicated diagrams and require a proof. |

| Higher Tier | Notes |
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**3-D shapes**

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| H3.2i | use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms, pyramids, cylinders, cones and spheres; solve problems involving more complex shapes and solids, including segments of circles and frustums of cones | B5 | Formulae for the surface area and volume of a cylinder will not be given<br>Questions involving the use of given formula, such as the surface area of a sphere, may not be accompanied by a diagram. More complex problems involving, for example, a hemisphere on top of a cylinder will always be accompanied by a diagram.<br>Answers in terms of $\pi$ may be required. (non-calculator question) |

## 3. Transformations and coordinates

Pupils should be taught to:

| Higher Tier | Notes |
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**Specifying transformations**

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| H3.3a | understand that rotations are specified by a centre and an (anticlockwise) angle; use any point as the centre of rotation; measure the angle of rotation, using right angles, fractions of a turn or degrees; understand that reflections are specified by a (mirror) line; understand that translations are specified by giving a distance and direction (or a vector), and enlargements by a centre and a positive scale factor | B5 | The direction of rotation will always be given.<br>Column vector notation should be understood.<br>Lines of symmetry will be restricted to $x = a$, $y = a$, $y = x$ and $y = -x$.<br>Scale factors for enlargements can be fractional (and/or negative) |

| Higher Tier | | | Notes |
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**Properties of transformations**

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|---|---|---|---|
| H3.3b | recognise and visualise rotations, reflections and translations including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations; use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations; distinguish properties that are preserved under particular transformations | B5 | When describing transformations, the minimum requirement is<br>Reflection described by a mirror line<br>Translations described by a vector or a clear description such as 3 squares to the right, 5 squares down.<br>Rotations described by centre, direction (unless a half turn) and an amount of turn (as a fraction of a whole or in degrees).<br>Candidates will always be asked to describe a **single** transformation but could be asked to do a combined transformation on a single shape.<br>Candidates could be asked to describe a single transformation equivalent to combination of transformations.<br>Questions will be set on line symmetry and order of rotational symmetry, including tessellations. |
| H3.3c | recognise, visualise and construct enlargements of objects; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not, then use positive fractional and negative scale factors | B5 | In the context of enlargements only. Enlargements may be about any point on a coordinate grid or may involve the enlargement of a plane shape not drawn on a coordinate grid.<br>Candidates may be asked to find the scale factor and centre of enlargement. |

| Higher Tier | | Notes | |
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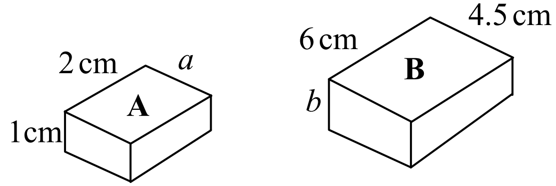
| H3.3d | recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments; understand the implications of enlargement for perimeter; use and interpret maps and scale drawings; understand the difference between formulae for perimeter, area and volume by considering dimensions; understand and use the effect of enlargement on areas and volumes of shapes and solids | B5 | Candidates will be expected to know if a formula is consistent, and to be able to explain why a formula represents, for example, volume. It will be sufficient to say that the formula is the product of three lengths or dimensions, for example $L^3$.<br><br>Candidates will be expected to know the connection between the linear, area and volume scale factors of similar shapes and solids. Questions may be asked that exploit the relationship between weight and volume, area and cost of paint etc.<br><br>Scales will be given as, for example, 1 cm represents 10 km, or 1:100.<br><br>eg, These boxes are similar.<br><br><br><br><br>What is the ratio of the volume of box A to box B ?<br><br>eg, What is the ratio of the surface area of two similar cones with base radii 3cm and 12cm respectively? |

| Higher Tier | | | Notes |
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**Coordinates**

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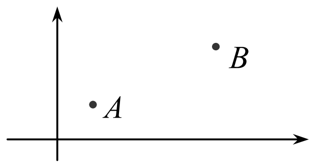
| H3.3e | understand that one coordinate identifies a point on a number line, that two coordinates identify a point in a plane and three coordinates identify a point in space, using the terms '1-D', '2-D' and '3-D'; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; (for example, identify the co-ordinates of a cuboid drawn on a 3-D grid) find the coordinates of points identified by geometrical information (for example, find the coordinates of the fourth vertex of a parallelogram with vertices at (2, 1) (–7, 3) and (5, 6)); find the coordinates of the midpoint of the line segment *AB*, given the points *A* and *B*, then calculate the length *AB* | B5 | The formulae $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, and $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$ need not be known.<br>Diagrams may be given.<br>eg, The diagram shows the position of $A(1, 1)$ and $B(7, 5)$<br><br>Calculate the length of the line segment *AB*;<br>(for example, find the coordinates of the fourth vertex of a parallelogram with vertices at (2, 1) (−7, 3) and (5, 6));<br>Questions using 3-D coordinates will be set<br>(for example, identify the coordinates of a cuboid on a 3-D grid)<br>eg, identify the coordinates of the vertex of a cuboid on a 3-D grid<br>eg, identify the coordinates of the mid-point of a line segment in 3-D |

| Higher Tier | | | Notes |
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**Vectors**

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| H3.3f | understand and use vector notation; calculate, and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector; calculate the resultant of two vectors; understand and use the commutative and associative properties of vector addition; solve simple geometrical problems in 2-D using vector methods | B5 | Column vectors may be used to describe translations.<br><br>Use of bold type and arrows such as $\mathbf{a} = \overrightarrow{OA}$ will be used to represent vectors in geometrical problems. |

## 4. Measures and construction

Pupils should be taught to:

| Higher Tier | | | Notes |
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**Measures**

| | | | |
|---|---|---|---|
| H3.4a | use angle measure know that measurements using real numbers depend on the choice of unit; recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction; convert measurements from one unit to another; understand and use compound measures, including speed and density | B5 | For example, use bearings to specify direction<br><br>Conversions between imperial units will be given but the rough metric equivalents to common imperial measures should be known. These will be restricted to 8 km ≈ 5 miles, 1 litre ≈ 1.75 pints,<br><br>1 kg ≈ 2.2 lbs, 1 gallon ≈ 4.5 litres, 1 foot ≈ 30 cm.<br><br>eg, Give the upper and lower limits of a length of 11 cm measured to the nearest centimetre.<br>Lower limit = 10.5 cm,<br>Upper limit = 11.5 cm.<br><br>The notation $10.5 \leq \text{length} < 11.5$ should be understood.<br><br>Units of speed will be given as miles per hour (mph) , kilometres per hour (km/h), or metres per second , m/s, m $s^{-1}$. Candidates who express speed in alternative units such as metres per minute will not be penalised providing the units are clearly stated.<br><br>Density will be given as gm/$cm^3$ or kg/$m^3$. Candidates who express density in alternative units such as grams per cubic metre will not be penalised providing the units are clearly stated. |

| Higher Tier | Notes |
|---|---|

**Construction**

| | | | |
|---|---|---|---|
| F3.4d<br>H3.4b | draw approximate constructions of triangles and other 2-D shapes, using a ruler and protractor, given information about side lengths and angles; understand, from their experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; construct specified cubes, regular tetrahedra, square-based pyramids and other 3-D shapes | B5 | Knowledge of SSS, SAS, ASA and RHS terminology will be required as candidates could be asked to explain why two triangles are congruent.<br>Candidates will be expected to draw a net of a 3-D shape and also to recognise a shape from a given net. |
| H3.4c | use straight edge and compasses to do standard constructions including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle | B5 | Candidates will be expected to show clear evidence that a straight edge and compasses only have been used to do constructions. |

**Mensuration**

| | | | |
|---|---|---|---|
| F3.4f<br>F3.4i<br>H3.4d | calculate perimeters and areas of shapes made from triangles and rectangles; find the surface area of simple shapes by using the formulae for the areas of triangles and rectangles; find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids; convert between area measures, including square centimetres and square metres, and volume measures, including cubic centimetres and cubic metres; find circumferences of circles and areas enclosed by circles, recalling relevant formulae; calculate the lengths of arcs and the areas of sectors of circles | B5 | Candidates will be expected to know the meaning of the terms 'sector', 'segment', 'major' and 'minor'. Questions involving the terms 'major' and 'minor' may be set but will always be accompanied by a diagram.<br><br>Questions on area and perimeter using compound shapes formed from two or more rectangles may be set.<br><br>eg, A rectangle has sides of 30 cm and 40 cm. Find the area. Give your answer in $m^2$.<br>Circumference and area formula for a circle should be known.<br>Perimeters and areas of semi-circles or simple fractions of a circle, eg, quarter circles, could be assessed. |

| Higher Tier | | | Notes |
|---|---|---|---|

**Loci**

| | | | |
|---|---|---|---|
| H3.4e | find loci, both by reasoning and by using ICT to produce shapes and paths | B5 | For example, a region bounded by a circle and an intersecting line<br>Loci problems may be set in practical contexts such as finding the position of a radio transmitter. Constructions expected are the perpendicular bisector of two points, accurate construction of a circle and the angle bisector. Questions involving bearings may also be required. |

# AO4: Handling data

## 1. Using and applying handling data

Pupils should be taught to:

| Higher Tier | Notes |
|---|---|

**Problem solving**

| | | | |
|---|---|---|---|
| H4.1a | carry out each of the four aspects of the handling data cycle to solve problems:<br>(i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed<br>(ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources<br>(iii) process and represent the data: turn the raw data into usable information that gives insight into the problem<br>(iv) interpret and discuss the data: answer the initial question by drawing conclusions from the data | B1, B2 | Teachers may wish to use this section of the subject content as a basis for AO4 coursework. |
| H4.1b | select the problem-solving strategies to use in statistical work, and monitor their effectiveness (these strategies should address the scale and manageability of the tasks, and should consider whether the mathematics and approach used are delivering the most appropriate solutions) | B1, B2 | |

| Higher Tier | | | Notes |
|---|---|---|---|

**Communicating**

| | | | |
|---|---|---|---|
| H4.1c | communicate mathematically, with emphasis on the use of an increasing range of diagrams and related explanatory text, on the selection of their mathematical presentation, explaining its purpose and approach, and on the use of symbols to convey statistical meaning | B1, B2 | Candidates should know and be able to draw and interpret a variety of statistical diagrams and use statistical notation accurately. |

**Reasoning**

| | | | |
|---|---|---|---|
| H4.1d | apply mathematical reasoning, explaining and justifying inferences and deductions, justifying arguments and solutions | B1, B2 | Candidates will be required to make comparisons between statistical data presented in a variety of formats, such as scatter graphs, stem and leaf diagrams, tables, cumulative frequency diagrams, box plots, histograms, etc. |
| H4.1e | identify exceptional or unexpected cases when solving statistical problems | B1, B2 | Candidates should be able to recognise 'rogue' data and the effect this may have on measures of location or dispersion. |
| H4.1f | explore connections in mathematics and look for relationships between variables when analysing data | B1, B2 | Candidates may be asked to describe connections between bivariate data.<br>eg, describing the relationship from a line of best fit.. |
| H4.1g | recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis | B1, B2 | Candidates should be able to recognise when diagrams contain errors or omissions.<br>Candidates should know and recognise when answers are inappropriate. |

## 2. Specifying the problem and planning

Pupils should be taught to:

| Higher Tier | | Notes | |
|---|---|---|---|
| H4.2a | see that random processes are unpredictable | B1 | Candidates should understand that the outcome of a random event cannot be predicted and that the probability of a fair coin landing on heads is 0.5, even if the previous six throws have given heads. Probabilities may be expressed as fractions, decimals or percentages. |
| H4.2b | identify key questions that can be addressed by statistical methods | B1 | Standard statistical terminology such as average, range, data, etc, should be understood. Candidates should know that questions involving comparison and analysis of data will need to be solved using statistical methods. |
| H4.2c | discuss how data relate to a problem, identify possible sources of bias and plan to minimise it | B1 | Candidates should understand the term 'bias'.<br>Candidates may be asked to criticise survey questions or comment on the results of experimental data (relative frequency). |
| H4.2d | identify which primary data they need to collect and in what format, including grouped data, considering appropriate equal class intervals; select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling | B1 | Candidates should understand the terms 'sampling', 'investigate a population', 'random' and 'stratified sampling'. |
| H4.2e | design an experiment or survey; decide what primary and secondary data to use | B1 | Candidates should understand the terms 'primary' and 'secondary' data.<br>For example, candidates may be asked to give an appropriate question for a survey or to criticise given questions. |

## 3. Collecting data

Pupils should be taught to:

| Higher Tier | | Notes | |
|---|---|---|---|
| H4.3a | collect data using various methods, including observation, controlled experiment, data logging, questionnaires and surveys | B1 | Candidates could be asked about sampling methods and ways of collecting data, although this reference may better be assessed in AO4 coursework.<br>Candidates should know, eg, that data logging is when data is collected automatically by machine, eg, the numbers of cars in a car park at any time. |
| H4.3b | gather data from secondary sources, including printed tables and lists from ICT-based sources | B1 | Candidates will be expected to know that as the number of trials increases then the relative frequency will approach the theoretical probability.<br>Reading and analysing data from tables charts and lists will be required. |
| H4.3c | design and use two-way tables for discrete and grouped data | B1 | eg, Design a two-way table to analyse the colour and make of vehicles. |
| H4.3d | deal with practical problems such as non-response or missing data | B1 | This reference may be better assessed in AO4 coursework. |

## 4. Processing and representing data

Pupils should be taught to:

| Higher Tier | | Notes | |
|---|---|---|---|
| H4.4a | draw and produce, using paper and ICT, pie charts for categorical data, and diagrams for continuous data, including line graphs (time series), scatter graphs, frequency diagrams, stem-and-leaf diagrams, cumulative frequency tables and diagrams, box plots and histograms for grouped continuous data | B1 | Includes frequency polygons, histograms with equal class intervals and frequency diagrams for grouped discrete data.<br>Questions involving back to back stem and leaf diagrams will not be set.<br>Cumulative frequency diagrams should be drawn so that the upper class boundary is plotted against the cumulative frequency. Data in tables will always be given in the form $0 < x \le 10$, $10 < x \le 20$, etc.<br>No keys showing area allocated to quantities will be provided on histograms. Candidates are expected to plot the frequency density (frequency ÷ class width) between the lower and upper class boundaries. They will have to choose their own scales for the side axis. Full credit will be given for alternative approaches providing the area of the bar is proportional to the frequency.<br>Candidates should know that the lower quartile is the value which cuts the total area in the ratio 1:3, the median is the value which cuts the total area in the ratio 1:1 and the upper quartile is the value that cuts the total area in the ratio 3:1. These values will always be integers and can be found by simple linear interpolation.<br>Pie charts should be labelled. If a frequency diagram is required then it can be an equal interval histogram or a frequency polygon. If a stem and leaf diagram is given a key will be provided. If candidates are asked to draw a stem and leaf diagram they should give a key.<br>Data in tables will always be given in the form $0 < x \le 10$, $10 < x \le 20$, etc. |

| Higher Tier | | Notes | |
|---|---|---|---|
| H4.4a | | B1 | Box plots will consist of five significant pieces of data: the least and greatest values (also known as the whiskers), and the box consisting of a rectangle going from the lower to upper quartile with the median marked as a divider across the box. Scales will always be given. Candidates will be expected to compare distributions and the relationship between a box plot and a cumulative frequency diagram should be known. |
| H4.4b | understand and use estimates or measures of probability from theoretical models, or from relative frequency | B1 | Candidates will be expected to know that as the number of trials increases then the relative frequency will approach the theoretical probability. |
| H4.4c | list all outcomes for single events, and for two successive events, in a systematic way | B1 | Candidates should be familiar with sample space diagrams. Lists or sample space diagrams may be given as answers. |
| H4.4d | identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1. | B1 | eg, The probability that a person is left-handed is 0.19. What is the probability that a person is not left-handed? |
| H4.4e | find the median, quartiles and interquartile range for large data sets and calculate the mean for large data sets with grouped data | B1 | Includes knowledge and the use of the mode<br><br>Candidates will be required to find the median and quartiles from a frequency table or list of discrete data. The use of the median being the $\frac{n+1}{2}$th value in the table will be expected.<br><br>Candidates will be required to find the median and quartiles from cumulative frequency graphs.<br>The median is found by reading off at $\frac{n}{2}$ and the quartiles at $\frac{n}{4}$ and $\frac{3n}{4}$.<br>Candidates will be expected to use the mid-point for estimating the mean of a grouped frequency table. |

| Higher Tier | | Notes | |
|---|---|---|---|
| H4.4f | calculate an appropriate moving average | B1 | These problems will be set in context and involve the trend as a quantity against time (eg, quarterly sales, values of shares, etc). Candidates should understand the concept of an $n$-point moving average (where $n$ is a integer). These may be accompanied by a graph.<br>The first point of the moving average graph is plotted at the middle of the data values. For example, the first point of a 3-point moving average is plotted against the second-time value.<br>Candidates will be expected to comment on and use the trends shown by the moving average, and use it to predict further values. |
| H4.4g | know when to add or multiply two probabilities: if $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$, whereas if $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$ | B1 | Candidates are required to understand the term 'mutually exclusive' but this will not be used in a written paper.<br>The terms 'independent' and 'dependent' may be used in a written papers but candidates will not be asked to show that two events are independent. |
| H4.4h | use tree diagrams to represent outcomes of compound events, recognising when events are independent | B1 | Simple cases of conditional probability may be asked. For example, when picking socks at random from a drawer containing red and black socks, the probability of getting a black sock alters as each sock is withdrawn. No more than three events will be used in the tree diagram when there are two outcomes to each event and no more than two events will be used when there are three or more outcomes to an event.<br>Candidates may use AND/OR to calculate probabilities. |
| H4.4i | draw lines of best fit by eye, understanding what these represent | B1 | Lines of best fit need not go through the mean point but should pass as close to as many data points as possible. |

| Higher Tier | | Notes | |
|---|---|---|---|
| H4.4j | use relevant statistical functions on a calculator or spreadsheet | B1 | Candidates may use statistical functions on their calculator.<br>Standard deviation is no longer tested in written examinations but the mean of a discrete or grouped frequency table is. Marks for this are usually awarded for the processes and the sole use of a calculator is not advised as all marks may be lost if the wrong values (for mid-points, for example) are used and there is no evidence of method. |

## 5. Interpreting and discussing results

Pupils should be taught to:

| Higher Tier | | Notes | |
|---|---|---|---|
| H4.5a | relate summarised data to the initial questions | B1 | |
| H4.5b | interpret a wide range of graphs and diagrams and draw conclusions; identify seasonality and trends in time series | B1 | This will be tested in context. For example, ice cream sales will show seasonal variation.<br>Candidates should be familiar with bar charts, bar line graphs, pie charts, stem and leaf diagrams, equal width histograms, frequency polygons, two way tables, scatter graphs and line graphs. Candidates should know that lines joining points, such as on a graph plotting average temperature against month, may have no meaning.<br>Candidates will be expected to interpret, eg, the median and the range from stem and leaf diagrams. |
| H4.5c | look at data to find patterns and exceptions | B1 | For example, identifying a 'rogue' value on a scatter graph. |

| Higher Tier | | Notes | |
|---|---|---|---|
| H4.5d | compare distributions and make inferences, using shapes of distributions and measures of average and spread, including median and quartiles; understand frequency density | B1 | Comparisons between two distributions should be based on measures of an average and spread.<br>Candidates should know when each of the three measures of location is valid and be able to justify their choice. They should understand that the interquartile range measures spread of the middle 50% of the data, is centred on the median, and that it eliminates extreme values from the measure of spread. |
| H4.5e | consider and check results, and modify their approach if necessary | B1 | This reference may be best assessed within AO4 coursework |
| H4.5f | appreciate that correlation is a measure of the strength of the association between two variables; distinguish between positive, negative and zero correlation using lines of best fit; appreciate that zero correlation does not necessarily imply 'no relationship' but merely 'no linear relationship' | B1 | Candidates will not be expected to make statements about how reliable a correlation is but should be aware that some data can form a perfect linear relationship and other data may not. They should also be aware that using the line of best fit to predict values beyond the plotted range may not be reliable. |
| H4.5g | use the vocabulary of probability to interpret results involving uncertainty and prediction | B1 | For example, 'there is some evidence from this sample that...' |
| H4.5h | compare experimental data and theoretical probabilities | B1 | Knowledge of the term 'relative frequency' is required. |
| H4.5i | understand that if they repeat an experiment, they may - and usually will - get different outcomes, and that increasing sample size generally leads to better estimates of probability and population parameters | B1 | Candidates should know that, for example, throwing a dice will not always give a rectangular distribution but that the more trials are carried out then the better the reliability of results. |

| Higher Tier | | Notes | |
|---|---|---|---|
| F4.5k | interpret social statistics including index numbers time series and survey data | B1 | For example, the General Index of Retail Prices<br>For example, population growth<br>For example, the National Census<br>Candidates should understand the difference between a sample and a census.<br>eg, candidates should know that if the index number for a base year is 100 and the, eg, an index of 120 represents a 20% increase on the base year. |

# 4 Coursework Issues

## 4.1 Introduction

This section provides guidance for teachers who are using AQA GCSE Mathematics Specification A and Specification B (Modular). The GCSE coursework requirements (amended in 2003) require all candidates to submit two tasks covering:

- AO1 - Using & applying coursework
- AO4 - Handling data coursework

Each task is worth 10% of the overall assessment

There are two alternative approaches to the assessment of the coursework modules:

- **Option T** centres may choose from a bank of coursework tasks provided by AQA or they set their own coursework tasks; centres then mark the coursework tasks with moderation of candidates' coursework by AQA;
- **Option X** centres choose from the bank of coursework tasks provided by AQA and candidates' coursework is marked by AQA.

Apart from the choice of coursework tasks and the method of assessment, the nature of the coursework is the same for Option T and Option X. The following details apply to both Option T and Option X. It is not necessary to use the same option for both tasks.

The details for the coursework are common to both GCSE Mathematics Specifications. (A list of the AQA tasks can be found on the AQA Website and in the specification).

As GCSE coursework in mathematics is to be withdrawn, the advice in this section will only apply until 2008. Once the revised GCSE requirements for 2009 are known, this guide will be updated.

## 4.2 Assessment Criteria

There are two different sets of Assessment Criteria for the Assessment of GCSE Mathematics coursework tasks. The Assessment Criteria for *Using and applying mathematics* are used for the AO1 task, and the Assessment Criteria for *Handling data* for the AO4 task.

### A01 Task

For the AO1 task candidates are expected to submit one task only. This investigational task must be set in the context of AO2 and/or AO3. The Assessment Criteria for the AO1 task are given in Section 5.3. The coursework task is expected to take approximately two weeks to complete, including lesson and homework time. The AO1 task will be marked out of a total of 24 marks.

AO4 Task

For the *Handling data* task candidates are expected to submit one task only. It will not be possible for the AO4 task to be used as the AO1 task. Tasks based on probability only, without data handling, are unlikely to score well on these criteria and should be avoided. Simulation activities are acceptable provided that they lead to statistical tasks rather than probability tasks. Candidates may choose to use statistical information from the Internet or other sources. The Assessment Criteria for the AO4 task are given in Section 6.5. The coursework task is expected to take approximately two weeks to complete, including lesson and homework time. The AO4 task will be marked out of a total of 24 marks.

## 4.3 Role of the Teacher

Whichever coursework option is used (T or X), the role of the teacher is the same. The objective is to provide opportunities for a candidate to demonstrate his/her skills and knowledge in AO1 and AO4, and to ensure that the work produced is representative of the highest level of which he/she is capable.

Coursework should be an integrated part of the teaching and learning process. There should be interaction between teacher and candidate, and the teacher should feel free to intervene as appropriate in the development of the task. Task-specific guidance to individual candidates must be recorded on the *Candidate Record Form*.

It is anticipated that a candidate will have had the opportunities to explore a variety of investigations and handling data tasks before embarking on the work to be submitted for assessment. The candidate should have studied the relevant areas of mathematics before attempting a task.

## 4.4 Presentation of Tasks

When undertaking a task it is important that a candidate plans the initial response, chooses appropriate techniques, reviews progress and develops the task further. Tasks should be presented in a way which allows this to happen, and consideration should be given to the following points.

- A class discussion or brainstorm is a valuable way to start a task; candidates' ideas can be shared and used as a basis to start an individual investigation.
- A candidate's work might include a plan which is altered as the task proceeds, this is entirely acceptable.
- Teachers should ensure that data is collected which is suitable for analysis by the techniques appropriate to the level of study.

- Any task sheet used should act as a stimulus for a candidate to make his/her own decisions. Do not use tasks which are simply a list of instructions because this would deny the candidate access to higher scores.

Details of the tasks will be published annually, in the spring term, on the AQA Website.

## 4.5 Monitoring Progress

A key aspect of the teacher's role is to make sure that a candidate makes sensible decisions in developing the coursework tasks and in presenting the work. It is expected that much of the work will be done in class, where monitoring is easier

The following points give guidance about appropriate teacher intervention.

- Individual candidates should be given the opportunity to explain a particular feature of their own work to the teacher. This can help the candidate to develop his/her own communication and reasoning skills. This can also help the teacher in establishing the candidate's individual ownership of the task.
- Teachers should encourage candidates to review and extend their own work by asking appropriate questions such as:

  Why did you choose that representation/calculation?

  What do you notice?

  Can you explain how you got that result?

  Have you tested/justified your results?

  Are your findings always true?

  Why did you choose that sample size/method?

  What have you done to avoid possible bias?

  What do you notice from your graph?

  How effective was your strategy?

  What do your findings tell you about the hypothesis?

See also Appendices D and E in this guide for *Student Support Sheets*.

- When specific mathematical/statistical techniques are introduced by candidates, the opportunity can be used to clarify the language and reinforce the standard notation and conventions.
- Candidates should be encouraged to avoid unproductive and repetitive work.

### 4.6 Closing the Task

Candidates should be given a final deadline for completion of a task. Some candidates will need support from the teacher to reach a final conclusion and there should be opportunities to get that before the final deadline is reached.

The progress of a task can be monitored for the whole class by giving interim deadlines; this might be particularly useful for the AO4 task where candidates need to know how long to spend on data collection.

Teachers should provide feedback during a task, and tackle any issues of major concern raised before the work is submitted by the candidate. Whichever assessment option is chosen (T or X), it is important that the candidates' work is annotated with teacher comments where key evidence needs to be clarified; this can include oral evidence.

The specification requires that teachers must record details of any additional assistance given to candidates which affects the mark in any strand on the *Candidate Record Form*. (See Specification A, Section 15 and Specification B, Sections 19 and 24).

# 5 Using and Applying Mathematics (AO1 task)

### 5.1 Using and Applying Mathematics (AO1 task)

Candidates will be assessed in terms of their attainment in each of the following three strands which correspond to the three areas of the Programme of Study for *Using and applying mathematics* at National Curriculum Key Stages 3 and 4.

| Strand | | Maximum mark |
|---|---|---|
| 1 | Making and monitoring decisions to solve problems | 8 |
| 2 | Communicating mathematically | 8 |
| 3 | Developing skills of mathematical reasoning | 8 |
| | Maximum total mark | 24 |

The score in each of the three strands should be that which reflects the best performance by the candidate in that strand. These marks should be totalled to give a mark out of 24.

The criteria are to be used as best fit indicative descriptions and the statements within them are not to be taken as hurdles. It is necessary, however, for the majority of the statement to be met for the mark to be awarded.

The mark descriptions within a strand are designed to be broadly hierarchical. This means that, in general, a description at a particular mark subsumes those at lower marks.

Therefore, the mark awarded may not be supported by direct evidence of achievement of lower marks in each strand. It is assumed that tasks which allow higher marks will involve a more sophisticated approach and/or treatment.

The AO1 coursework task must be set in the context of AO2 (*Number and algebra*) and/or AO3 (*Shape, space and measures*).

In these criteria, there is an intended approximate link between 7 marks and grade A, 5 marks and grade C and 3 marks and grade F.

## 5.2 Choice of AO1 Task

The AO1 task provides an opportunity to use and apply the content of mathematics from AO2 and/or AO3 in the National Curriculum. Illustrations of the application of AO1 Assessment Criteria in extracts of candidates' work are provided in Section 5.7. The following section gives some additional guidance for AO1 tasks.

The AQA-set tasks cover a range of levels so centres can choose tasks appropriate to the ability of their candidates. Details of the tasks will be published annually, in the spring term, on the AQA Website.

### Strand 1: Making and monitoring decisions to solve problems

This strand is about deciding what needs to be done, then doing it. The strand requires candidates to select an appropriate approach, obtain information and introduce their own questions which develop the task further. For the higher marks candidates need to analyse alternative mathematical approaches and apply, independently and extensively, a range of appropriate techniques.

### Strand 2: Communicating Mathematically

This strand is about communicating what is being done using words, tables, diagrams and symbols. Candidates should consider the appropriateness of their chosen presentation and amend this as necessary. For the higher marks candidates will need to use mathematical symbols accurately, concisely and efficiently in presenting a reasoned argument.

### Strand 3: Developing skills of mathematical reasoning

This strand is about testing, explaining and justifying what has been done and requires the candidate to search for patterns and provide generalisations. Generalisations should then be tested, justified and explained. For the higher marks candidates will need to provide a sophisticated and rigorous justification, argument or proof which demonstrates a mathematical insight into the problem.

## 5.3 AO1 Assessment Criteria

The Assessment Criteria for AO1 are presented on the following pages.

## (AO1 task) – Assessment criteria for Using and Applying Mathematics

| | Strand 1<br>Making and monitoring decisions to solve problems | Strand 2<br>Communicating mathematically | Strand 3<br>Developing skills of mathematical reasoning |
|---|---|---|---|
| 1 | Candidates try different approaches and find ways of overcoming difficulties that arise when they are solving problems. They are beginning to organise their work and check results. | Candidates discuss their mathematical work and are beginning to explain their thinking. They use and interpret mathematical symbols and diagrams. | Candidates show that they understand a general statement by finding particular examples that match it. |
| 2 | Candidates are developing their own strategies for solving problems and are using these strategies both in working within mathematics and in applying mathematics to practical contexts. | Candidates present information and results in a clear and organised way, explaining the reasons for their presentation. | Candidates search for a pattern by trying out ideas of their own. |
| 3 | In order to carry through tasks and solve mathematical problems, candidates identify and obtain necessary information; they check their results, considering whether these are sensible. | Candidates show understanding of situations by describing them mathematically using symbols, words and diagrams. | Candidates make general statements of their own, based on evidence they have produced, and give an explanation of their reasoning. |
| 4 | Candidates carry through substantial tasks and solve quite complex problems by breaking them down into smaller, more manageable tasks. | Candidates interpret, discuss and synthesise information presented in a variety of mathematical forms. Their writing explains and informs their use of diagrams. | Candidates are beginning to give a mathematical justification for their generalisations; they test them by checking particular cases. |
| 5 | Starting from problems or contexts that have been presented to them, candidates introduce questions of their own, which generate fuller solutions. | Candidates examine critically and justify their choice of mathematical presentation, considering alternative approaches and explaining improvements they have made. | Candidates justify their generalisations or solutions, showing some insight into the mathematical structure of the situation being investigated. They appreciate the difference between mathematical explanation and experimental evidence. |

| | **Strand 1**<br>**Making and monitoring decisions to solve problems** | **Strand 2**<br>**Communicating mathematically** | **Strand 3**<br>**Developing skills of mathematical reasoning** |
|---|---|---|---|
| 6 | Candidates develop and follow alternative approaches. They reflect on their own lines of enquiry when exploring mathematical tasks; in doing so they introduce and use a range of mathematical techniques. | Candidates convey mathematical meaning through consistent use of symbols. | Candidates examine generalisations or solutions reached in an activity, commenting constructively on the reasoning and logic employed, and make further progress in the activity as a result. |
| 7 | Candidates analyse alternative approaches to problems involving a number of features or variables. They give detailed reasons for following or rejecting particular lines of enquiry. | Candidates use mathematical language and symbols accurately in presenting a convincing reasoned argument. | Candidates' reports include mathematical justifications explaining their solutions to problems involving a number of features or variables. |
| 8 | Candidates consider and evaluate a number of approaches to a substantial task. They explore extensively a context or area of mathematics with which they are unfamiliar. They apply independently a range of appropriate mathematical techniques. | Candidates use mathematical language and symbols efficiently in presenting a concise reasoned argument. | Candidates provide a mathematically rigorous justification or proof of their solution to a complex problem, considering the conditions under which it remains valid. |

## 5.4 Further Exemplification of AO1 Assessment Criteria

The further exemplification of the AO1 assessment criteria is presented in the following pages. It consists of the following:

**Column 1** gives the strand and the mark under consideration so 1/5 refers to a mark of 5 under strand 1 (making and monitoring decisions).

This annotation should be used on candidates' work when indicating where a particular mark has been awarded.

**Column 2** reproduces the criteria statement provided by QCA to all awarding bodies.

**Column 3** gives the minimum requirements for the award of a mark. These are agreed across all awarding bodies in England, Wales and Northern Ireland.

**Column 4** gives teachers' notes provided by AQA following discussion between senior moderators and other awarding bodies. These notes are intended to give guidance to both teachers and moderators in making consistent assessment and in supporting the work of candidates.

| | Making and monitoring decisions to solve problems | Minimum Requirements | Teachers' Notes |
|---|---|---|---|
| 1/1 | Candidates try different approaches and find ways of overcoming difficulties that arise when they are solving problems. They are beginning to organise their work and check results. | The candidate can, with help, understand a simple task and produce some information or results. | One random example is found<br>An award of 1/1 generally implies an award of 2/1 and 3/1**,** as a minimum |
| 1/2 | Candidates are developing their own strategies for solving problems and are using these strategies both in working within mathematics and in applying mathematics to practical contexts. | The candidate interprets a simple task showing **some evidence** of their own planning and obtains a number of results, but no conclusion. | Three examples are found, although they may contain some errors.<br>An award of 1/2 generally implies an award of 2/2 and 3/2, as a minimum |
| 1/3 | In order to carry through tasks and solve mathematical problems, candidates identify and obtain necessary information; they check their results, considering whether these are sensible. | The candidate obtains what is required to solve a simple task, finding and checking necessary information. | Three correct results are seen.<br>An award of 1/3 implies an award of 3/2**,** as a minimum |
| 1/4 | Candidates carry through substantial tasks and solve quite complex problems by breaking them down into smaller, more manageable tasks. | The candidate carries through a substantial task without additional direction, by breaking it down into smaller more manageable sub-tasks at least one of which is solved. | The task, as presented, should be open-ended enabling the candidate to break it down and hence achieve 3/3. There must be at least 3 correct results seen **and a system.** |
| 1/5 | Starting from problems or contexts that have been presented to them, candidates introduce questions of their own, which generate fuller solutions. | The candidate independently extends the task by changing one feature in order to give a fuller solution. | The extension of the task must lead to one feature being altered that leads to a further solution. This must lead to a second award of 3/3. A feature is some aspect of the task such as a variable, constraint or condition. |
| 1/6 | Candidates develop and follow alternative approaches. They reflect on their own lines of enquiry when exploring mathematical tasks; in doing so they introduce and use a range of mathematical techniques. | The candidate reflects on their line of enquiry and uses an additional relevant technique to extend the task further. | Developing the task using an algebraic, trigonometric or graphical approach leading to 2/6 **may** imply 1/6. After an award of 1/5 drawing further generalisations together to give an overarching solution is worthy of 1/6. The work must be explained and convincing. Work must be at the appropriate level |
| 1/7 | Candidates analyse alternative approaches to problems involving a number of features or variables. They give detailed reasons for following or rejecting particular lines of enquiry. | The candidate works on complex task(s) involving at least 3 features and gives reasons for following or rejecting lines of enquiry. | Three features or variables must be in evidence and manipulated to reach a solution. The work as a whole must be at the appropriate level. Please note the link between 7 marks and grade A. |
| 1/8 | Candidates consider and evaluate a number of approaches to a substantial task. They explore extensively a context or area of mathematics with which they are unfamiliar. They apply independently a range of appropriate mathematical techniques. | The candidate applies, independently and extensively, appropriate mathematical techniques to solve a complex problem. | The work must be at the appropriate level. The candidate needs to show their understanding explicitly. |

In strand 1: a maximum of 3 marks is available for "simple" tasks; a maximum of 6 marks is available for "substantial" tasks; the full range of marks is available for "complex" tasks

**In these criteria there is an intended approximate link between 7 marks and grade A, 5 marks and grade C and 3 marks and grade F**

| | Communicating Mathematically | Minimum Requirements | Teachers' Notes |
|---|---|---|---|
| 2/1 | Candidates discuss their mathematical work and are beginning to explain their thinking. They use and interpret mathematical symbols and diagrams. | The candidate shows some evidence of their thinking. | This may be oral (supported by teacher annotation) or written, and could take the form of random calculations or drawings etc.<br>An award of 1/1 generally implies this mark, as a minimum |
| 2/2 | Candidates present information and results in a clear and organised way, explaining the reasons for their presentation. | The candidate presents some information or results in a clear or organised way. | This could include listing and/or diagrams.<br>An award of 1/2 generally implies this mark, as a minimum |
| 2/3 | Candidates show understanding of situations by describing them mathematically using symbols, words and diagrams. | The candidate shows some understanding of the task by describing a feature of the task mathematically by using words and symbols **or** words and diagrams **or** symbols and diagrams. | Words can be headings, statements or connectives. Symbols and diagrams could be shown by a list with 'lettered' headings.<br>2/3 can be awarded if a table is seen. |
| 2/4 | Candidates interpret, discuss and synthesise information presented in a variety of mathematical forms. Candidates` writing explains and informs their use of diagrams. | The candidate brings together more than one form of mathematical presentation with a **linking** commentary. | This is not a series of displays or diagrams with no purpose. The linking commentary must allow the reader to understand what the candidate has noticed e.g. from their table, identifying differences. |
| 2/5 | Candidates examine critically and justify their choice of mathematical presentation, considering alternative approaches and explaining improvements they have made. | The candidate gives some explanation for their choice of presentation. The presentation may be symbolic or diagrammatic. | The introduction of a formula does not justify 6 marks in this strand. However, substitution into their derived algebraic formula would support 2/5. One example would be sufficient |
| 2/6 | Candidates convey mathematical meaning through consistent use of symbols. | The candidate **conveys** mathematical meaning through the **sustained use of symbolism*** at the appropriate level. | At the appropriate level variables need to be defined and symbols must be correctly and consistently used in a number of cases. Manipulation of expressions would support 2/6 e.g. multiplication of brackets leading to a quadratic or higher order expression, quadratic factorisation, transposition of formulae and trigonometric symbolism. Correct, convincing manipulation with at least 3 correct examples |
| 2/7 | Candidates use mathematical language and symbols accurately in presenting a convincing reasoned argument. | The candidate presents a convincing reasoned argument through the use of mathematical language and symbolism, which is generally accurate. | There should be increased emphasis on accuracy. Incorrect algebra cannot lead to a convincing argument. |
| 2/8 | Candidates use mathematical language and symbols efficiently in presenting a concise reasoned argument. | The candidate produces an elegant argument. | A long-winded argument is not of an elegant nature. If the argument could be more succinct, a mark of 2/8 is not appropriate. |

To qualify for a mark of 4, 6, 8 on any strand the content of the task must meet, or go beyond, the relevant aspects of the grade descriptors for grades F, C and A respectively.

* Symbolism might include for example: 1 Algebra

2 Trigonometry

**In these criteria there is an intended approximate link between 7 marks and grade A, 5 marks and grade C and 3 marks and grade F**

| | Developing the skills of mathematical reasoning | Minimum Requirements | Teachers' Notes |
|---|---|---|---|
| 3/1 | Candidates show that they understand a general statement by finding particular examples that match it. | The candidate produces a simple example that shows an understanding of the task. | This mark is generally implied by an award of 1/1, as a minimum. |
| 3/2 | Candidates search for a pattern by trying out ideas of their own. | The candidate gathers sufficient data from which a simple observation may be made. | This mark is generally implied by an award of 1/2 as a minimum. |
| 3/3 | Candidates make general statements of their own, based on evidence they have produced, and give an explanation of their reasoning. | The candidate makes a general statement based on **their** results. | Their results need not be correct for the task but should be consistent with their data. A general statement may be in words and might be as simple as *'goes up in two's'* or *'all odd numbers'* If the next number is predicted, candidates must say why. |
| 3/4 | Candidates are beginning to give a mathematical justification for their generalisations; they test them by checking particular cases. | The candidate tests their generalisation by checking a further case. | An award of 3/4 cannot be given without 3/3. The test must be on their generalisation and involve new data. The candidate must comment on the outcome. |
| 3/5 | Candidates justify their generalisations or solutions, showing some insight into the mathematical structure of the situation being investigated. They appreciate the difference between mathematical explanation and experimental evidence. | The candidate produces a sensible argument stating **why** the results occur by relating these results to the mathematical situation e.g. physical, geometrical or graphical. | The candidate gives an explanation about why the results occur. This may be algebraic, graphical or diagrammatic. The use of difference tables or equivalent resulting in a quadratic expression, properly explained and fully correct is sufficient for 3/5. |
| 3/6 | Candidates examine generalisations or solutions reached in an activity, commenting constructively on the reasoning and logic employed, and make further progress in the activity as a result. | The candidate uses reasoning and logic to make further progress in the activity. | To consider the award of this mark the candidate should have extended the task.<br>Drawing further generalisations together to give an overarching solution, which is <u>fully explained</u>, is worthy of 3/6. |
| 3/7 | Candidates' reports include mathematical justifications, explaining their solutions to problems involving a number of features or variables. | The candidate gives a general result or conclusion with justification for parts of the overall solution, co-ordinating at least 3 features. | ***This mark cannot be awarded without the award or 7 or 8 marks in strand 1.*** Please note the link between 7 marks and grade A. |
| 3/8 | Candidates provide a mathematically rigorous justification or proof of their solution to a complex problem, considering the conditions under which it remains valid. | | The candidate must provide a rigorous proof with work at the appropriate level. The candidate needs to show their understanding explicitly **especially with regard to validity** |

**In these criteria there is an intended approximate link between 7 marks and grade A, 5 marks and grade C and 3 marks and grade F**

## 5.5 Further comments on AO1

The following additional comments from moderators' and examiners' reports might be useful to centres in preparing and assessing candidates for the using and applying mathematics coursework:

### Strand 1: Making and monitoring decisions to solve problems

An award of mark 5 can only be given where the task is independently extended beyond the original problem set.

An award of mark 6 is appropriate where a candidate 'pulls together' their various algebraic investigations at a level commensurate with grade B work.

The inclusion of an algebraic formula is, on its own, insufficient to suggest an award of mark 6.

An award of mark 7 can only be given where the candidate co-ordinates three features or variables at a level commensurate with grade A work.

An award of mark 8 is appropriate where a candidate explores a task **extensively** and **independently**… similar work is unlikely to be independent

### Strand 2: Communicating Mathematically

Candidates should not waste time drawing tables and/or graphs unless they are relevant, commented upon and interpreted.

All candidates should be encouraged to make use of algebra to provide a commentary for the work.

An award of mark 4 requires candidates to consider their representations (tables or graphs) and make some appropriate and correct comment.

An award of mark 5 can be given (as best fit) where candidates make use of algebra rather than simply making an algebraic statement.

Substitution into the candidate's own derived formula might be sufficient to suggest an award of mark 5.

An award of mark 6 can only be given where candidates show **sustained** evidence of correct and convincing algebraic manipulation, factorisation or transposition.

The use of algebra for proving and justifying must be accurate and convincing to confidently award marks of 6 and above.

Centres are advised to check the accuracy of algebraic manipulation and ensure that all working is clearly shown.

Pattern spotting to produce a solution is not sufficiently rigorous to award marks of 6 and above in this strand

Strand 3: Developing skills of mathematical reasoning

Where generalisations are written down it is important that they are adequately explained in the text to confirm the candidate's own understanding.

Testing should be undertaken on candidate's own generalisations and make use of new data with a comment to say whether the test has worked or not.

An award of mark 5 can only be given where candidates justify why a generalisation works… repeated numerical substitution is unlikely to provide such a justification.

An award of mark 7 under this strand can only be given where strand 1 has been awarded a mark of 7 or 8.

An award of mark 8 would usually require the candidate to give some consideration to the conditions under which their proof remains valid.

## 5.6 Extracts from AO1 tasks

The next section shows some examples of work to illustrate further each strand mark. There are three extracts to illustrate the marking of each strand. These have been word-processed for ease of reading.

For the purposes of brevity, the candidates' work on the following pages is presented as a series of snapshots to illustrate particular assessment points.

These examples are set in isolation to illustrate a point; they do not represent a complete piece of work. Such a small piece of work alone will not gain the mark shown as marks are awarded for the whole piece of work. The complete piece of work, in each instance, would provide a better flavour for the mark awarded, since both coherence and sustainability are likely to produce a fuller picture.

| Mark | Strand 1 | Strand 2 | Strand 3 |
|---|---|---|---|
| 3 | Extract A | Extract D | Extract G |
| 4 | Extract B | Extract E | Extract H |
| 5 | Extract C | Extract F | Extract I |

There are no extracts to illustrate marks of 6, 7 or 8 since these marks are awarded on the quality and coherence of reasoning and development shown **throughout** the task.

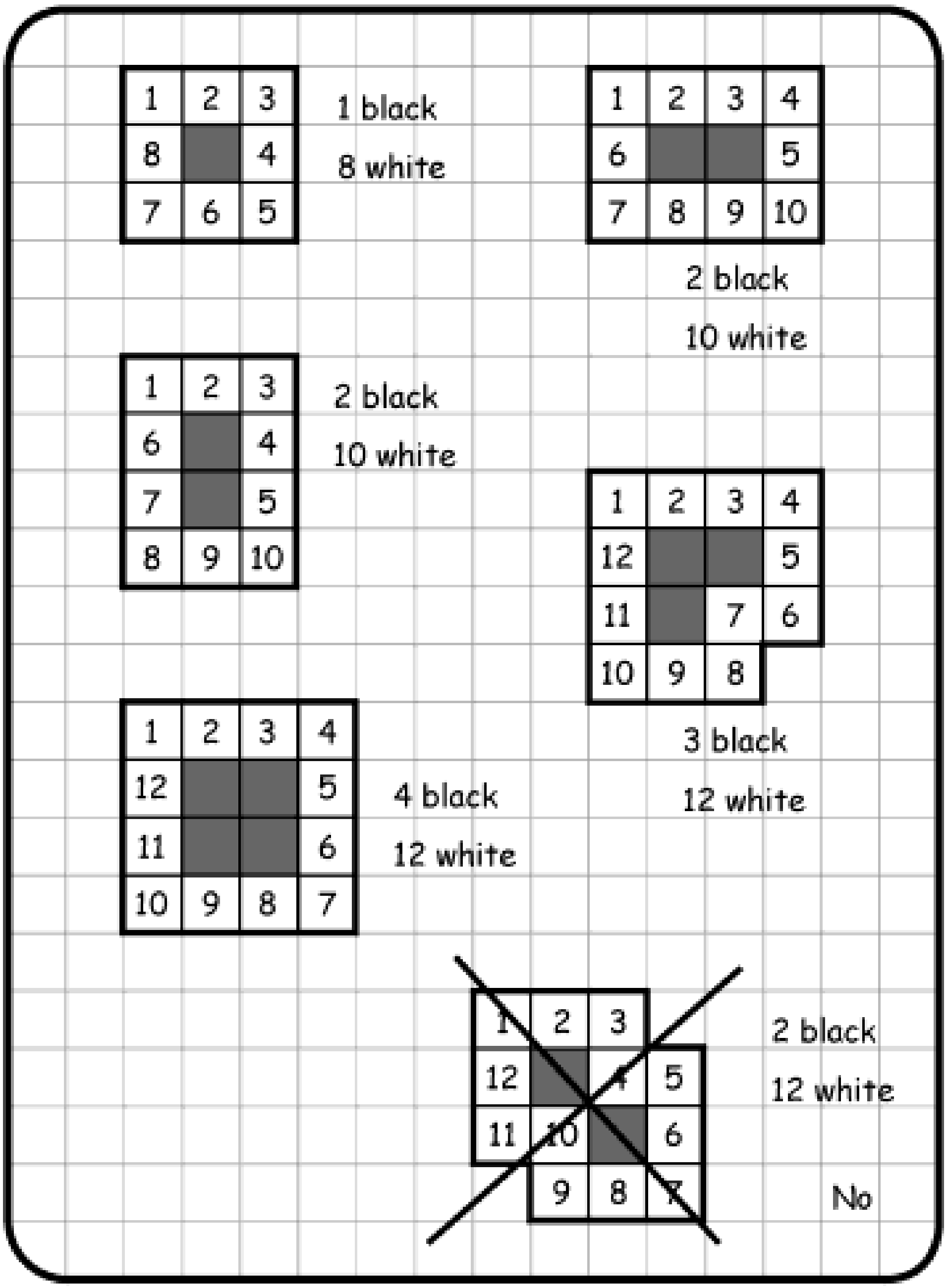
Extract A

Strand 1: Making and Monitoring decisions to solve problems

Context: Black tiles are surrounded by white tiles. Investigate.

For a mark of 3 you need to:

Identify and obtain the necessary information to solve the problem.




Notes

**Mark 3**

*In order to carry through tasks and solve mathematical problems, candidates identify and obtain necessary information; they check their results, considering whether these are sensible.*

**Assessment**

The candidate is exploring the relationship between the number of black tiles and the number of white tiles. They have obtained information for four different arrangements although there is no system to their work.

The fact that the candidate has identified and obtained this information is evidence of the award given.

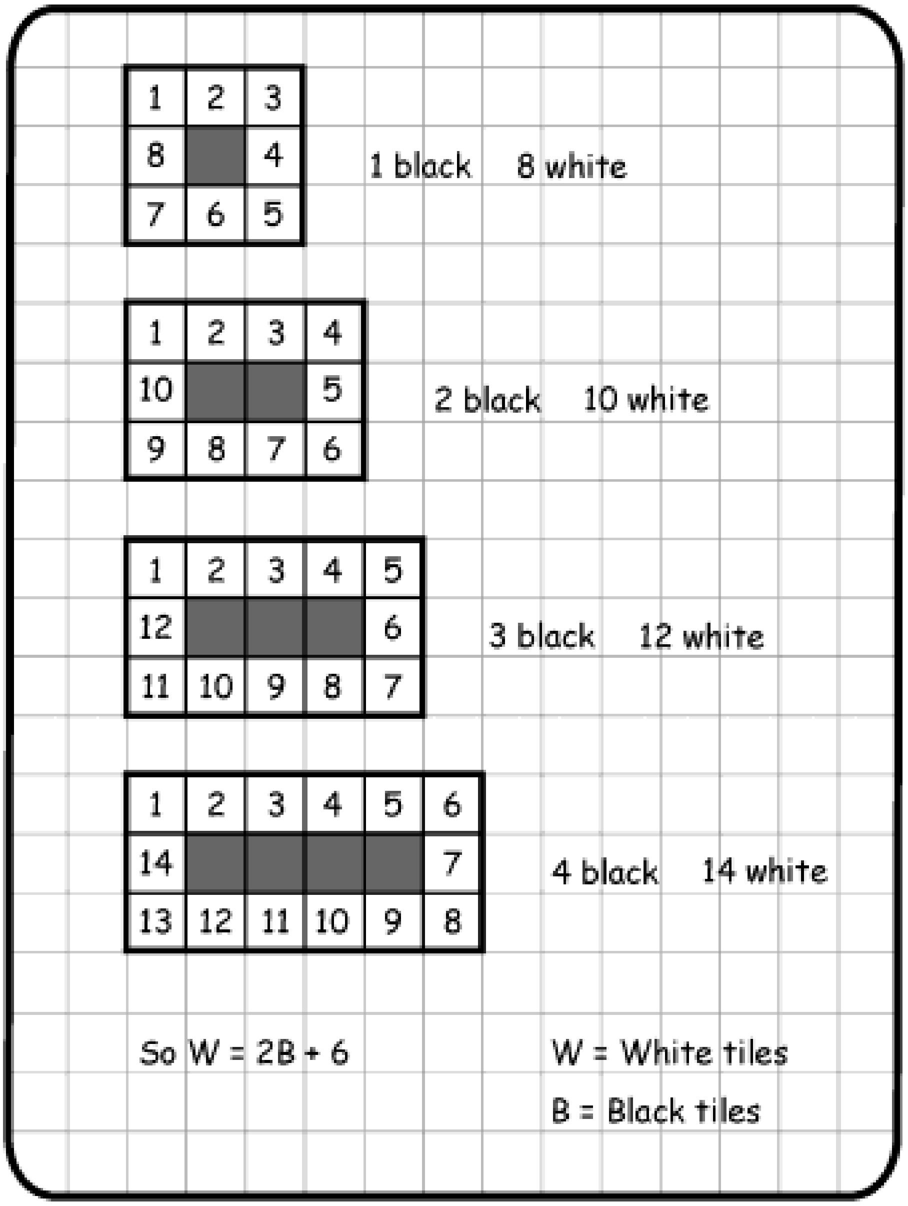
***THE CANDIDATE'S WORK IS AWARDED MARK 3 IN STRAND 1***

Extract B — Strand 1: Making and Monitoring decisions to solve problems

Context: Black tiles are surrounded by white tiles. Investigate.

For a mark of 4 you need to:

Break down the given task to solve it in a methodical fashion.




Notes

**Mark 4**

*Candidates carry through substantial tasks and solve quite complex problems by breaking them down into smaller, more manageable tasks.*

**Assessment**

The candidate is exploring the relationship between the number of black tiles and the number of white tiles. They have decided to concentrate on horizontal rows of black tiles producing results for 1, 2, 3 and 4 black tiles.

The candidate has broken down the task and their systematic approach to the work has allowed a solution to this smaller task. This is evidence of the award given.

***THE CANDIDATE'S WORK IS AWARDED MARK 4 IN STRAND 1***

## Extract C

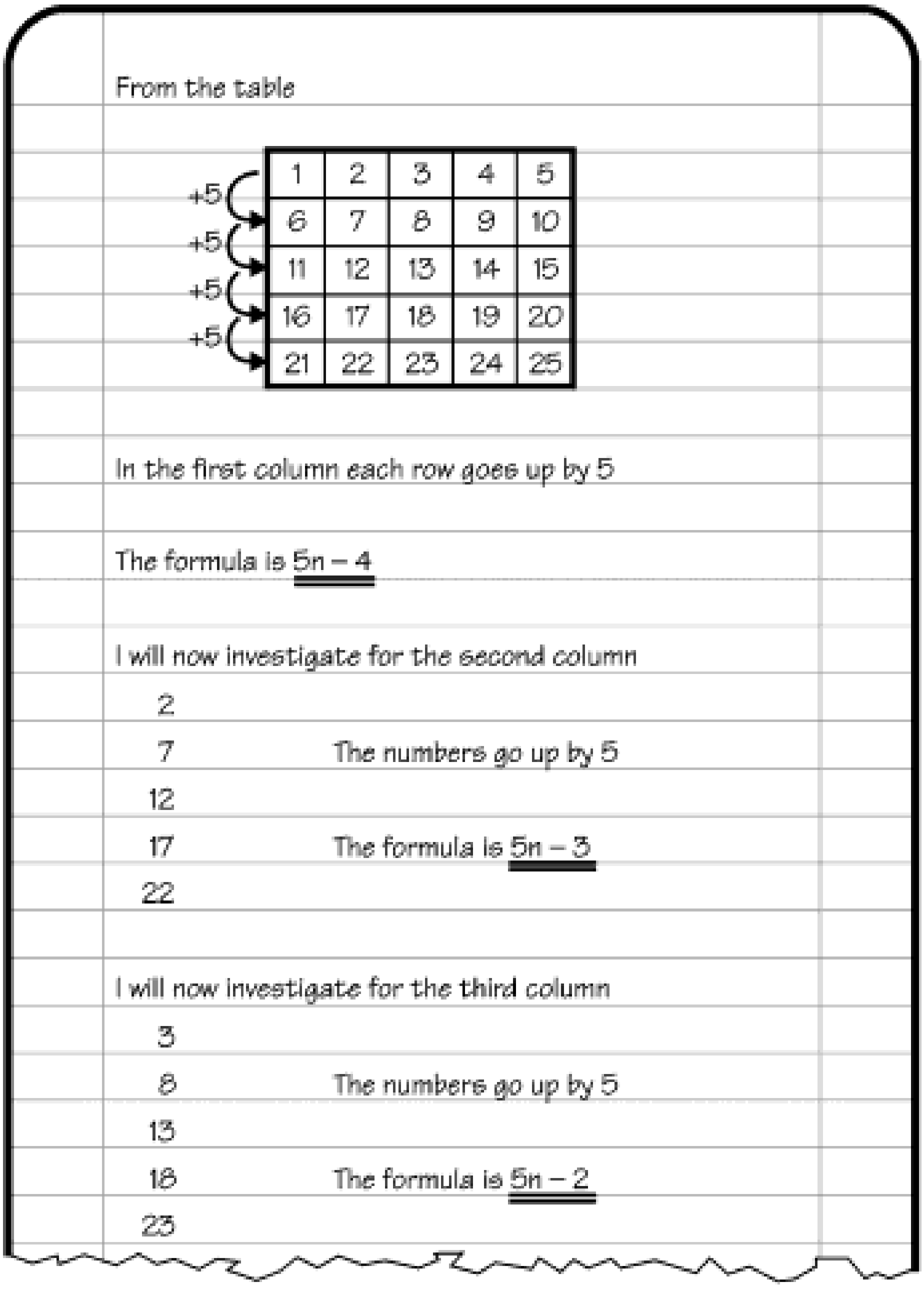
Strand 1: Making and Monitoring decisions to solve problems

Context: The numbers 1 to 25 have been put in a 5 × 5 grid, as illustrated.

Investigate different patterns in the grid.

For a mark of 5 you need to:

Introduce your own relevant questions to develop the task beyond that given.


From the table

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 |

+5 +5 +5 +5

In the first column each row goes up by 5

The formula is 5n – 4

I will now investigate for the second column

2
7 The numbers go up by 5
12
17 The formula is 5n – 3
22

I will now investigate for the third column

3
8 The numbers go up by 5
13
18 The formula is 5n – 2
23

### Notes

**Mark 5**

*Starting from problems or contexts that have been presented to them, candidates introduce questions of their own, which generate fuller solutions.*

**Assessment**

The candidate is exploring patterns in the 5 × 5 table and has come up with a formula for finding the numbers in the first column. The work is developed to produce similar formulae for the other columns but none of this 'develops' the task further, since it repeats the mathematics used in the first part.

The candidate subsequently attempts to pull the information together and identify the relationship between the number, the row number and the column number. The resulting formula provides a 'fuller solution' to the task (which was not suggested in the original question) and is evidence of a mark of 5 under this strand.

The development of the task in other ways (such as looking at diagonals or different sized squares) might also provide evidence for a mark of 5 being awarded if it provides a 'fuller solution'.

***THE CANDIDATE'S WORK IS AWARDED MARK 5 IN STRAND 1***

Extract C, cont...

Strand 1: Making and Monitoring decisions to solve problems

I will now investigate the relationship between the number, the row number and the column number.

| | |
|---|---|
| Column 1 | $5n - 4$ |
| Column 2 | $5n - 3$ |
| Column 3 | $5n - 2$ |
| Column 4 | $5n - 1$ |
| Column 5 | $5n$ |

Column x $\quad 5n - (5 - x)$

$5n - 5 + x$

$5n + x - 5$

$\uparrow$ row $\quad \uparrow$ column

The formula is $\underline{\underline{5n + x - 5}}$

eg row 3 column 2 $5 \times 3 + 2 - 5 = 12$ ✓

row 5 column 4 $5 \times 5 + 4 - 5 = 24$ ✓

Extract D

Strand 2: Communicating mathematically

Context: An open box is made using a net from a 10 cm × 10 cm square.

Investigate the volume of the box.

For a mark of 3 you need to:

Show your understanding by illustrating information using symbols, words and diagrams.

Putting all of this information in a table.

| Length of square (cm) | Volume of box ($cm^3$) |
|---|---|
| 0 | 0 |
| 1 | 64 |
| 2 | 72 |
| 3 | 48 |
| 4 | 16 |
| 5 | 0 |

Notes

**Mark 3**

*Candidates show understanding of situations by describing them mathematically using symbols, words and diagrams.*

**Assessment**

The candidate has collected together some results and illustrated these results in a table which is properly headed to show what the information represents. The table provides evidence for an award of mark 3 under this strand.

A diagram or a graph of the situation would also provide similar evidence.

***THE CANDIDATE'S WORK IS AWARDED MARK 3 IN STRAND 2***

Extract E Strand 2: Communicating mathematically

Context: Matchsticks are used to form triangles.

Investigate.

For a mark of 4 you need to:

Use *appropriate* forms of presentation with linking explanation and interpretation.

| No. of triangles T | No. of match sticks M |
|---|---|
| 3 | 7 |
| 4 | 9 |
| 5 | 11 |

+ 2

+ 2

From the table, I notice that as the number of triangles increases by 1 then the number of match sticks increase by 2.

Notes

**Mark 4**

*Candidates interpret, discuss and synthesise information presented in a variety of mathematical forms. Their writing explains and informs their use of diagrams.*

**Assessment**

The candidate has collected together some results and provided a table with some accompanying commentary. The commentary explains what the candidate has noticed in their table, and provides a useful stepping stone to the provision of a general solution and is evidence for a mark 4 under this strand.

Appropriate commentary accompanying a diagram or a graph would also provide similar evidence.

***THE CANDIDATE'S WORK IS AWARDED MARK 4 IN STRAND 2***

Extract F

Strand 2: Communicating mathematically

Context: An open box is made using a net from a 10 cm × 10 cm square.

Investigate the volume of the box.

For a mark of 5 you need to:

Consider and discuss alternative approaches to your presentation which enable you to make further progress with the task.

Putting all of this information in a table.

| Length of square (cm) | Volume of box ($cm^3$) | |
|---|---|---|
| 0 | 0 | |
| 1 | 64 | + 64 |
| 2 | 72 | + 8 |
| 3 | 48 | − 24 |
| 4 | 16 | − 32 |
| 5 | 0 | − 16 |

I can't see any pattern in the table except that the volumes are all in the 4 × table and the 8 × table.

I shall try graphing the information to see if this will help me to see a relationship between the length of the square and the volume of the cuboid.

Notes

**Mark 5**

*Candidates examine critically and justify their choice of mathematical presentation, considering alternative approaches and explaining improvements they have made.*

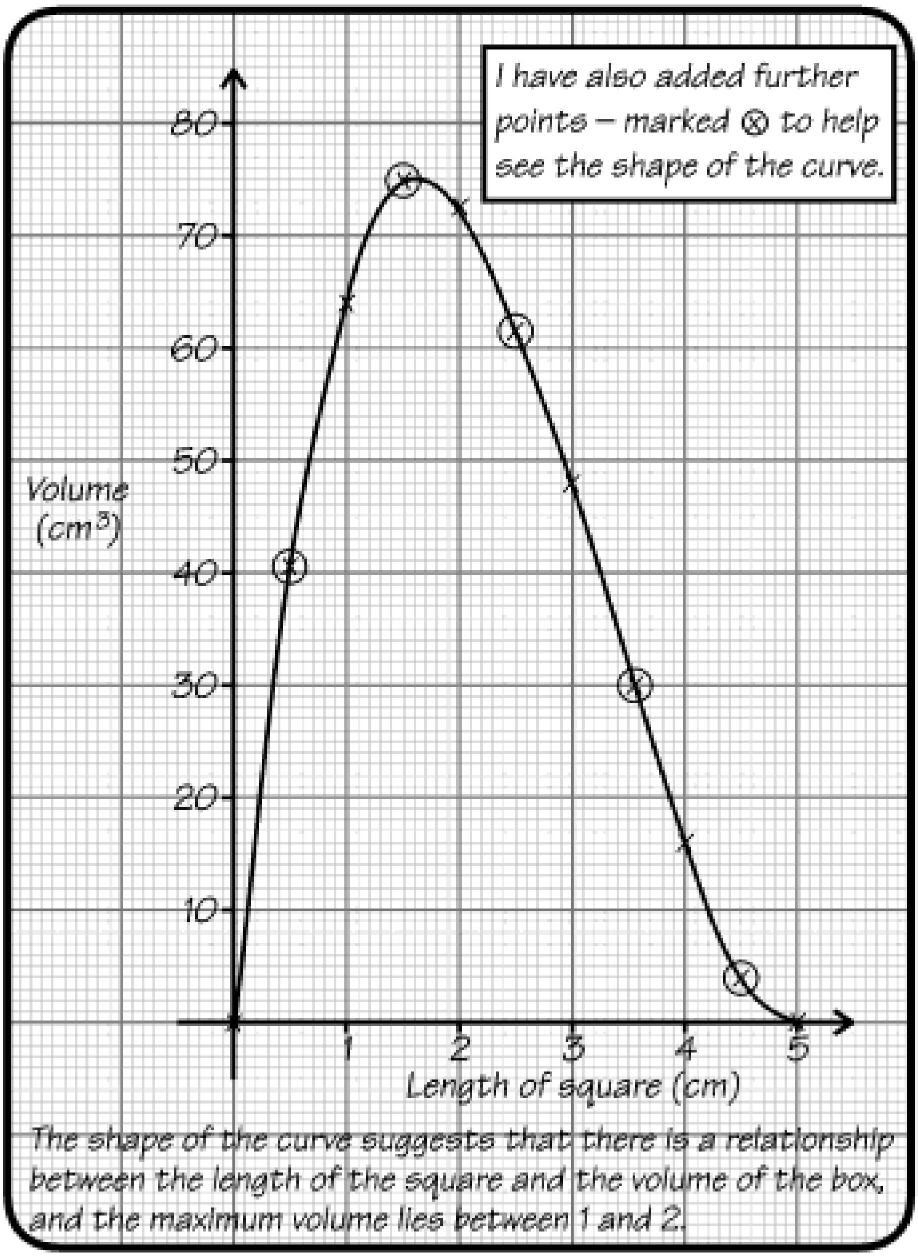
**Assessment**

The candidate has collected together some results and provided a table, although they are unable to come up with any relationships other than the fact that the volumes are multiples of 4 and 8.

The candidate decides to draw a graph to see if there is any relationship and even adds further points.

Extract F, cont…

Strand 2: Communicating mathematically




**Notes**

The provision of a graphical representation as an alternative approach to the problem to find a relationship, the addition of further points, and the recognition that the maximum volume lies between 1 and 2 are sufficient to provide evidence for an award of mark 5 here.

Alternatively, an award of mark 5 is appropriate where the candidate makes use of some (grade C) type algebra, for example when substituting to test a generalisation. In practice, an award of mark 5 is often given when higher (grade B) algebra is used but not sustained.

***THE CANDIDATE'S WORK IS AWARDED MARK 5 IN STRAND 2***

Extract G

Strand 3: Developing skills of mathematical reasoning

Context: Matchsticks are used to form triangles.

Investigate.

For a mark of 3 you need to:

Make a generalisation based on your own evidence.

| No. of triangles $T$ | No. of match sticks $M$ | |
|---|---|---|
| 3 | 7 | |
| 4 | 9 | + 2 |
| 5 | 11 | + 2 |

*From the table, I notice that as the number of triangles increases by 1 then the number of match sticks increase by 2.*

*So the number of match sticks is twice the number of triangles plus one.*

Notes

**Mark 3**

*Candidates make general statements of their own, based on evidence they have produced, and give an explanation of their reasoning.*

**Assessment**

The candidate has collected together some results and used the table to come up with a relationship linking the number of triangles and the number of matchsticks. The provision of the generalisation defined is evidence of an award of mark 3 under this strand. Provision of algebraic results is also evidence for an award of 3 marks.

***THE CANDIDATE'S WORK IS AWARDED MARK 3 IN STRAND 3***

Extract H

Strand 3: Developing skills of mathematical reasoning

Context: Black tiles are surrounded by white tiles.

Investigate.

For a mark of 4 you need to:

Confirm the generalisation by testing a further example.

Formula W = 2B + 6

W = white tiles

B = black tiles

I will test this for 5 black tiles

W = 2B + 6

W = 2 × 5 + 6

W = 16 - see diagram

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 16 | | | | | | 8 |
| 15 | 14 | 13 | 12 | 11 | 10 | 9 |

5 black tiles

16 white tiles

So my formula is correct $\boxed{W = 2B + 6}$

Notes

**Mark 4**

*Candidates are beginning to give a mathematical justification for their generalisations; they test them by checking particular cases.*

**Assessment**

The candidate has come up with a generalisation linking the number of white tiles and the number of black tiles which is subsequently tested on a **further** example. The test which confirms the generalisation provides evidence for the award.

***THE CANDIDATE'S WORK IS AWARDED MARK 4 IN STRAND 3***

Extract I

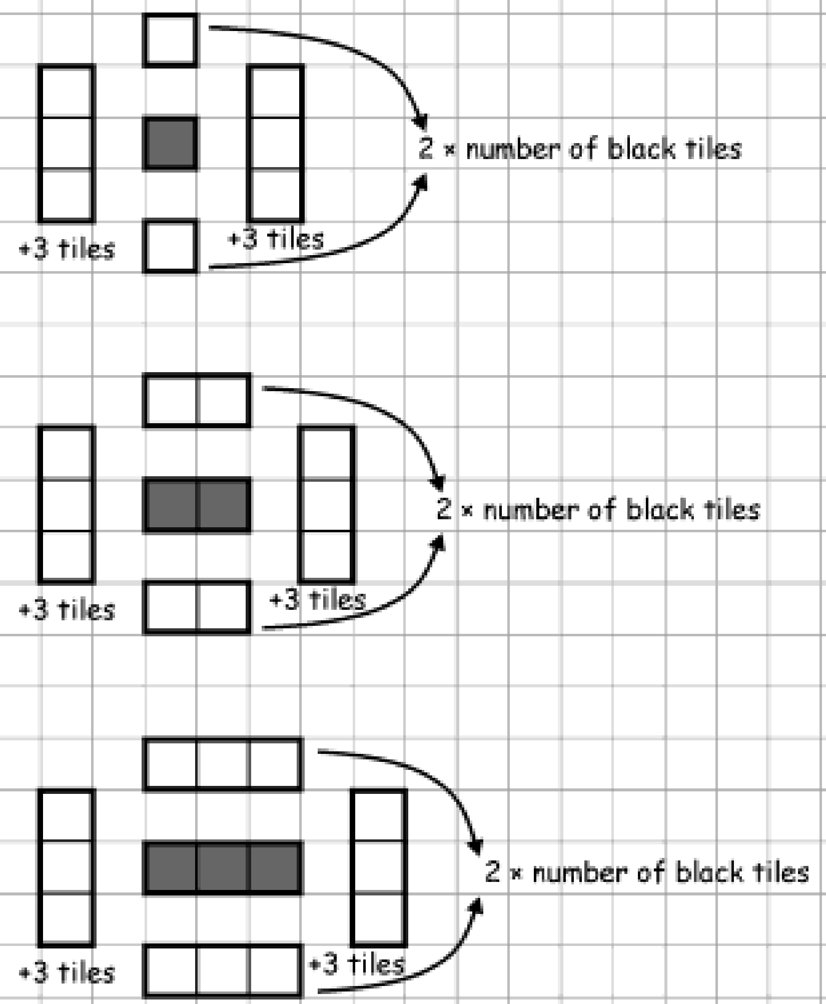
Strand 3: Developing skills of mathematical reasoning

Context: Black tiles are surrounded by white tiles.

Investigate.

For a mark of 5 you need to:

Provide a justification for the generalisation by showing mathematical insight into the problem.




As you can see, no matter how many black tiles I have, there will always be 2 × number of black tiles + 3 tiles + 3 tiles (ie 2 × number of black tiles + 6 tiles).

In algebra W = 2 × B + 6 where W = number of white tiles
or W = 2B + 6 B = number of black tiles

Notes

**Mark 5**

*Candidates justify their generalisations or solutions, showing some insight into the mathematical structure of the situation being investigated. They appreciate the difference between mathematical explanation and experimental evidence.*

**Assessment**

The candidate has provided a generalisation linking the number of white tiles and the number of black tiles which is subsequently tested and then asks themselves the question "Why does this work?"

The question is answered using an 'exploded diagram' which fully justifies the formula $W = 2B + 6$. The justification, which is more than just a test, provides evidence for the award.

***THE CANDIDATE'S WORK IS AWARDED MARK 5 IN STRAND 3***

5.7 Student Support Sheets (AO1)

Student Guidelines for AO1 tasks and a Writing Framework for AO1 tasks are provided in Appendix D. These *Student Support Sheets* may be photocopied and provided to candidates.

# 6 Handling Data (AO4 task)

## 6.1 Handing data (AO4 task)

Candidates will be assessed in terms of their attainment in each of the following three strands which correspond to the Programme of Study for *Handling data* at National Curriculum Key Stages 3 and 4.

| Strand | | Maximum Mark |
|---|---|---|
| 1 | Specify the problem and plan | 8 |
| 2 | Collect, process and represent data | 8 |
| 3 | Interpret and discuss results | 8 |
| | Maximum total mark | 24 |

The score in each of the three strands should be that which reflects the best performance by the candidate in that strand. These marks should be totalled to give a mark out of 24.

The criteria are to be used as best fit indicative descriptions and the statements within them are not to be taken as hurdles. This means candidates' work should be assessed in relation to the criteria taken as holistic descriptions of performance. The first consideration is: which of the descriptions in each strand best describes the work in a candidate's task. Once that is established, the final step is to decide between the lower and the higher tier mark available for that description; this decision may well involve looking again at the criteria above and below the selected best-fitting criterion. It is not appropriate to take each statement in each description and regard it as a separate assessment criterion. Nor is it necessary to consider whether the majority of the statements within a criterion have been met.

A mark of 0 should be awarded if a candidate's work fails to satisfy the requirements for 1 mark.

Descriptions for higher marks subsume those for lower marks.

Where there are references to 'meeting the level detailed in the handling data paragraph of the grade description for grade *X*', work which uses no technique beyond the specified grade is indicative of the lower of the two marks. Work using techniques beyond the specified grade is indicative of the higher of the two marks.

In these criteria, there is an intended approximate link between 7 marks and grade A, 5 marks and grade C and 3 marks and grade F.

## 6.2 Choice of AO4 tasks

The AO4 task provides an opportunity to use and apply the content of mathematics from Ma4 in the National Curriculum. Illustrations of the application of AO4 Assessment Criteria in extracts of candidates' work are provided in Section 5.6. The following section gives some additional guidance for AO4 tasks

**Strand 1: Specifying the problem and planning**

This strand is about choosing a problem, deciding what needs to be done and then doing it. The strand requires the candidate to provide clear aims, consider the collection of data, identify practical problems and explain how they might overcome them. For the higher marks, candidates need to decide upon a suitable sampling method, explain what steps were taken to avoid possible bias and provide a well structured report.

**Strand 2: Collecting, processing and representing the data**

This strand is about collecting data and using appropriate statistical techniques and calculations to process and represent the data. Diagrams should be appropriate and calculations appropriate and mostly correct. Use of a technique implies more than simply presenting a diagram or calculation. Candidates should draw conclusions from their representations which refer back to the original aims of the task. For the higher marks, candidates need to accurately use higher level statistical techniques/calculations from the Higher tier GCSE mathematics specification content.

**Strand 3: Interpreting and discussing the results**

This strand is about commenting, summarising and interpreting data. The discussion should link back to the original problem and provide an evaluation of the work as a whole. For the higher marks, candidates need to provide sophisticated and rigorous interpretations of their data and provide an analysis of how significant their findings are.

The AQA-set tasks cover a range of levels so centres can choose tasks appropriate to the ability of their candidates. Details of the tasks will be published annually, in the spring term, on the AQA Website.

To undertake a successful task the candidate needs to have an overall plan, which involves collecting, processing and representing data followed by interpreting and discussing results. The following points are useful to remember when setting handling data tasks.

- In order to provide candidates with as many statistical tools as possible it is advisable to cover the content of AO4 prior to undertaking the handling data task.
- It is important to remember that quantitative data offers more opportunities for demonstrating the use of statistical techniques than qualitative data.

## 6.3 Guidance on the stages involved in carrying out an AO4 task

A *Handling data* task can be divided into eight distinctive stages.

**Stage 1: Choosing a topic**

Experience shows that a candidate often performs better if he/she chooses a topic that interests him/her. However, the task must allow the use and interpretation of a range of statistical techniques. Topics which involve numerical values (quantitative data) are often more suitable for analysis; for example 'the relationship between heights and handspans'. On the other hand, investigating the 'colour of the eyes' (qualitative data) in a class may not produce a situation which is sufficiently mathematical.

**Stage 2: Forming hypotheses**

The aims of the task should be clearly stated and the plan should be developed in statistical terms; for example, 'Taller people have bigger hands' is a simple starting hypothesis but the plan would discuss what is meant by 'big' in this context and how data may be reliably collected. It may then discuss the most appropriate way to establish whether a positive correlation exists. The work could possibly be extended to look at specific age groups or gender. There may be several hypotheses for any task which need to be linked together to produce a substantial task worth marks of 5 or more in strand 1. A better approach, however, may be to start with a simple hypothesis and develop it, adding detail as the work goes on.

**Stage 3: Planning**

It is important that the plan meets the aims of the task. At this stage, a candidate may foresee some practical problems associated with some parts of the task, such as the amount of data or the sampling methods used to collect data.

**Stage 4: Identifying and collecting data**

It is important to identify the kind of data which is required for a reliable investigation in order to accept or reject hypotheses. It is often useful to do a trial run before a complete survey is undertaken. This will save time as a candidate may find that the data collected is incomplete or includes useless information. This will also identify how the strategy can be improved early on.

The Assessment Criteria for AO4 tasks put great emphasis on the collection of relevant and reliable data. In the case of using AQA-set tasks, the collection of data is still the candidate's responsibility and will constitute part of the assessment. For these reasons teachers are advised to discuss the importance of data collection with candidates. The following is intended to be typical of the kind of guidance which may be useful to a candidate.

- Choose an appropriate sample size. The bigger the sample size the more reliable the results; however, it also should be small enough to be manageable. A sample size of 30 is often reasonable for most tasks but should not be accepted as appropriate for all tasks without some consideration.

- Choose your method of collecting data. If you obtain your data from another source this is called 'secondary data'. If you collect the data through observation, questionnaires or experiments it is called 'primary data'. In both cases you need to choose an appropriate sample size for your task.
- Your method of sampling should avoid 'bias' so that the results will be representative of the whole population under investigation. Samples can be chosen by different methods such as, random sampling and stratified sampling.
- When writing a questionnaire, it is important to consider the following points:
  - questions should not be biased;
  - questions should give a clear choice and preferably, a limited number of responses;
  - questions should not upset or embarrass people;
  - questions should be relevant and appropriate for the hypotheses set;
  - a questionnaire should not take a long time to complete;
  - always do a trial run using a small sample and modify your questionnaire in the light of the responses.

Stage 5: Presenting data and calculation of statistical values

Appropriate and relevant methods must be chosen for representing the data. This also applies to calculations of statistical values such as mean, mode or median. Before using any of these techniques, a candidate should always consider "will this help me find out more information about my hypotheses?" If the answer is no, it may not need to be done.

Stage 6: Analysis of the results

This is a very important part of the task. The results and diagrams should be interpreted in relation to the original aims. The candidate should state clearly whether the hypothesis is accepted or rejected and give reasons for the decisions made.

Stage 7: Limitations

In the write up, the limitations encountered should be discussed and in some cases these may be addressed by further investigation. This may include:

- inappropriate sample size;
- an inappropriate method of sampling;
- biased data.

Stage 8: Conclusions

All the results should be drawn together and analysed in terms of the stated aims. If appropriate, limitations of the work should be discussed. The conclusion should be accompanied by an evaluation of how the work was undertaken. This evaluation might usefully focus on the success, or otherwise, of the planning, collecting, processing, representing, interpreting and concluding phases of the work.

| | | |
|---|---|---|
| 6.4 | Provision of databases by centres | It is acceptable for centres to provide suitable databases for candidates. Such databases can be found on the internet and, in particular, a number of these can be found at the *Census at School* website at www.censusatschool.ntu.ac.uk.<br><br>Where databases are used they must be large enough to ensure that many different questions can be addressed by the candidates. It is also useful if candidates are first invited to collect some data so they can see, more clearly, potential problems and comment on these in their work. |
| 6.5 | AO4 Assessment Criteria | The Assessment Criteria for AO4 are presented on the next pages. |

## (AO4 task) – Assessment criteria for Handling data

| | Strand 1<br>Specify the problem and plan | Strand 2<br>Collect, process and represent data | Strand 3<br>Interpret and discuss results |
|---|---|---|---|
| 1-2 | Candidates choose a simple well-defined problem. Their aims have some clarity. The appropriate data to collect are reasonably obvious. An overall plan is discernible and some attention is given to whether the plan will meet the aims. The structure of the report as a whole is loosely related to the aims. | Candidates collect data with limited relevance to the problem and plan. The data are collected or recorded with little thought given to processing. Candidates use calculations of the simplest kind. The results are frequently correct. Candidates present information and results in a clear and organised way. The data presentation is sometimes related to their overall plan. | Candidates comment on patterns in the data. They summarise the results they have obtained but make little attempt to relate the results to the initial problem. |
| 3-4 | Candidates choose a problem involving routine use of simple statistical techniques and set out reasonably clear aims. Consideration is given to the collection of data. Candidates describe an overall plan largely designed to meet the aims and structure the project report so that results relating to some of the aims are brought out. Where appropriate, they use a sample of adequate size. | Candidates collect data with some relevance to the problem and plan. The data are collected or recorded with some consideration given to efficient processing. Candidates use straightforward and largely relevant calculations involving techniques meeting the level detailed in the handling data paragraph of the grade description for grade F. The results are generally correct. Candidates show understanding of situations by describing them using statistical concepts, words and diagrams. They synthesise information presented in a variety of forms. Their writing explains and informs their use of diagrams, which are usually related to their overall plan. They present their diagrams correctly, with suitable scales and titles. | Candidates comment on patterns in the data and any exceptions. They summarise and give a reasonably correct interpretation of their graphs and calculations. They attempt to relate the summarised data to the initial problem, though some conclusions may be incorrect or irrelevant. They make some attempt to evaluate their strategy. |

| | Strand 1<br>Specify the problem and plan | Strand 2<br>Collect, process and represent data | Strand 3<br>Interpret and discuss results |
|---|---|---|---|
| 5-6 | Candidates consider a more complex problem. They choose appropriate data to collect and state their aims in statistical terms with the selection of an appropriate plan. Their plan is designed to meet the aims and is well described. Candidates consider the practical problems of carrying out the survey or experiment. Where appropriate, they give reasons for choosing a particular sampling method. The project report is well structured so that the project can be seen as a whole. | Candidates collect largely relevant and mainly reliable data. The data are collected in a form designed to ensure that they can be used. Candidates use a range of more demanding, largely relevant calculations that include techniques meeting the level detailed in the handling data paragraph of the grade description for grade C. The results are generally correct and no obviously relevant calculation is omitted. There is little redundancy in calculation or presentation. Candidates convey statistical meaning through precise and consistent use of statistical concepts that is sustained throughout the work. They use appropriate diagrams for representing data and give a reason for their choice of presentation, explaining features they have selected. | Candidates comment on patterns in the data and suggest reasons for exceptions. They summarise and correctly interpret their graphs and calculations, relate the summarised data to the initial problem and draw appropriate inferences. Candidates use summary statistics to make relevant comparisons and show an informal appreciation that results may not be statistically significant. Where relevant, they allow for the nature of the sampling method in making inferences about the population. They evaluate the effectiveness of the overall strategy and make a simple assessment of limitations. |
| 7-8 | Candidates work on a problem requiring creative thinking and careful specification. They state their aims clearly in statistical terms and select and develop an appropriate plan to meet these aims giving reasons for their choice. They foresee and plan for practical problems in carrying out the survey or experiment. Where appropriate, they consider the nature and size of sample to be used and take steps to avoid bias. Where appropriate, they use techniques such as control groups, or pre-tests of questionnaires or data sheets, and refine these to enhance the project. The project report is well structured and the conclusions are related to the initial aims. | Candidates collect reliable data relevant to the problem under consideration. They deal with practical problems such as non-response, missing data or ensuring secondary data are appropriate. Candidates use a range of relevant calculations that include techniques meeting the level detailed in the handling data paragraph of the grade description for grade A. These calculations are correct and no obviously relevant calculation is omitted. Numerical results are rounded appropriately. There is no redundancy in calculation or presentation. Candidates use language and statistical concepts effectively in presenting a convincing reasoned argument. They use an appropriate range of diagrams to summarise the data and show how variables are related. | Candidates comment on patterns and give plausible reasons for exceptions. They correctly summarise and interpret graphs and calculations. They make correct and detailed inferences from the data concerning the original problem using the vocabulary of probability. Candidates appreciate the significance of results they obtain. Where relevant, they allow for the nature and size of the sample and any possible bias in making inferences about the population. They evaluate the effectiveness of the overall strategy and recognise limitations of the work done, making suggestions for improvement. They comment constructively on the practical consequences of the work. |

## 6.6 Further Exemplification of AO4 Assessment Criteria

The further exemplification of the AO4 assessment criteria is presented in the following pages. It consists of the following:

**Column 1** gives the strand and the mark under consideration so 1/5 refers to a mark of 5 under strand 1 (specify and plan).

This annotation should be used on candidates' work when indicating where a particular mark has been awarded.

**Column 2** reproduces the criteria statement provided by QCA to all awarding bodies.

**Column 3** gives the minimum requirements for the award of a mark. These are agreed across all awarding bodies in England, Wales and Northern Ireland.

**Column 4** gives teachers' notes provided by AQA following discussion between senior moderators and other awarding bodies. These notes are intended to give guidance to both teachers and moderators in making consistent assessment and in supporting the work of candidates.

## SPECIFY and PLAN

Notes: 1. In these criteria there is an intended approximate link between 7 marks and grade A, 5 marks and grade C and 3 marks and grade F.
2. Candidates must provide evidence of their plan being implemented.
3. If secondary data is provided it must be in sufficient quantity to allow sampling to take place.

| | | Minimum requirements | Teachers' Notes |
|---|---|---|---|
| **1/1** | Candidates choose a simple well-defined problem. Their aims have some clarity. The appropriate data to collect are reasonably obvious. An overall plan is discernible and some attention is given to whether the plan will meet the aims. The structure of the report as a whole is loosely related to the aims. | • The candidate shows they understand a simple problem.<br>• There is an implicit plan. | May be shown by collecting or using some data. |
| **1/2** | | | |
| **1/3** | Candidates choose a problem involving routine use of simple statistical techniques and set out reasonably clear aims. Consideration is given to the collection of data. Candidates describe an overall plan largely designed to meet the aims and structure the project report so that results relating to some of the aims are brought out. Where appropriate, they use a sample of adequate size. | • Candidates set out reasonably clear aims (or the purpose).<br>• Their planning is largely designed to meet the aims/purpose.<br>• They use data appropriate to the problem. | These aims can occur at any point within the work.<br><br>Appropriate data would allow a valid inference to be drawn meeting the stated aims/purpose. |
| **1/4** | | | |
| **1/5** | Candidates consider a more complex problem. They choose appropriate data to collect and state their aims in statistical terms with the selection of an appropriate plan. Their plan is designed to meet the aims and is well-described. Candidates consider the practical problems of carrying out the survey or experiment. Where appropriate, they give reasons for choosing a particular sampling method. The project report is well structured so that the project can be seen as a whole. | • Candidates consider a **substantial** problem stating their initial aims clearly at the beginning of the report.<br>• Their plan is explicitly stated to meet those aims.<br>• They choose an appropriate sample. | ***Candidates react to problems with the data as they are encountered and act upon them.***<br><br>A 'more complex' problem is defined as substantial i.e. one in which comparisons are made relating a number of features. This can be achieved by the candidate developing their work in greater depth or **linking** across related areas.<br><br>Initial aims may be revised or reviewed as the work develops.<br><br>Explanation of the method of sampling and how each piece of individual data was chosen might suggest a mark of 6. |
| **1/6** | | | |
| **1/7** | Candidates work on a problem requiring creative thinking and careful specification. They state their aims clearly in statistical terms and select and develop an appropriate plan to meet these aims giving reasons for their choice. They foresee and plan for practical problems in carrying out the survey or experiment. Where appropriate, they consider the nature and size of sample to be used and take steps to avoid bias. Where appropriate, they use techniques such as control groups, or pre-tests or questionnaires or data sheets, and refine these to enhance the project. The project report is well structured and the conclusions are related to the initial aims. | • Candidates work on a **demanding** problem.<br>• They state their aims clearly in statistical terms and give valid reasons for their choice of planning.<br>• They explain and act upon limitations of their chosen sample (eg bias), where appropriate. | ***Candidates anticipate problems with the data and prepare for them***<br><br>A demanding problem is defined as one which requires careful specification, sophisticated thinking and efficient planning.<br><br>The appropriate use of and justification for the rejection of sampling methods; the use of pre-testing; the identification of bias and whether the sample does in fact represent the larger population are indicative of 1/7 or above<br><br>The best work also involves creative thinking and evidence of extensive independent thought. |
| **1/8** | | | |

## COLLECT, PROCESS and REPRESENT

**NB: Hand drawn graphs should be on graph paper**

Notes:
1. In these criteria there is an intended approximate link between 7 marks and grade A, 5 marks and grade C and 3 marks and grade F.
2. The mark awarded to a particular technique should reflect the quality of use and understanding as well as its position within the Level Indicators.
3. The inclusion of statistical techniques outside the National Curriculum does not necessarily justify the award of higher marks.
4. 'Diagrams' include tables, charts and graphs. At 5-6 marks the diagrams used should be appropriate. At 7-8 marks the range of diagrams should be appropriate to the problem chosen and the statistical strategy chosen.
5. 'Redundancy' implies unnecessary and/or inappropriate diagrams or calculations. This includes techniques that are not used for any conclusion.

| | | Minimum requirements | Level Indicators/ Teachers' Notes |
|---|---|---|---|
| **2/1** | Candidates collect data with limited relevance to the problem and plan. The data are collected or recorded with little thought given to processing. Candidates use calculations of the simplest kind. The results are frequently correct. Candidates present information and results in a clear and organised way. The data presentation is sometimes related to their overall plan. | • Candidates collect or use data and record it. | One diagram or calculation is sufficient<br><br>eg. extract and interpret information in simple tables and lists; construct bar charts & pictograms and interpret. |
| **2/2** | | | eg. collect discrete data and record using frequency tables; understand and use mode and range of data; group data in equal class intervals; represent collected data in frequency diagrams and interpret; construct and interpret simple line graphs. |
| **2/3** | Candidates collect data with some relevance to the problem and plan. The data are collected or recorded with some consideration given to efficient processing. Candidates use straightforward and largely relevant calculations involving techniques meeting the level detailed in the handling data paragraph of the grade description for grade F. The results are generally correct. Candidates show understanding of situations by describing them using statistical concepts, words and diagrams. They synthesise information presented in a variety of forms. Their writing explains and informs their use of diagrams, which are usually related to their overall plan. They present their diagrams correctly, with suitable scales and titles. | • Candidates collect or use data with some relevance to the problem.<br>• They utilise statistical techniques/diagrams (see note 1 above) to process and represent the data.<br>• Their results are generally correct. | Candidates use **a minimum of two techniques** and/or diagrams with one at this level.<br><br>eg. from discrete data compare simple distributions using range and one of mean, mode, median; interpret diagrams, including pie charts, and draw conclusions. |
| **2/4** | | | eg. from continuous data create appropriate equal class intervals toconstruct and interpret frequency tables; construct and interpret stem and leaf diagrams**;** construct pie charts; draw conclusions from scatter diagrams and have a basic understanding of correlation. |
| **2/5** | Candidates collect largely relevant and mainly reliable data. The data are collected in a form designed to ensure that they can be used. Candidates use a range of more demanding, largely relevant calculations that include techniques meeting the level detailed in the handling data paragraph of the grade description for grade C. The results are generally correct and no obviously relevant calculation is omitted. There is little redundancy in calculation or presentation. Candidates convey statistical meaning through precise and consistent use of statistical concepts that is sustained throughout the work. They use appropriate diagrams for representing data and give a reason for their choice of presentation, explaining features they have selected. | • Candidates collect/sample largely relevant data.<br>• They utilise appropriate calculations/techniques/ diagrams (see note 1 above) within the problem.<br>• Their results are generally correct. | Candidates use **a minimum of two appropriate techniques** and/or diagrams with one at this level. These should allow progressively more or different information to be obtained<br><br>eg. take account of variability or bias; determine modal class and estimate mean, median & range of grouped data, selecting and using the most appropriate statistic, to compare distributions and make inferences by using frequency polygons or drawing a line of best fit on a scatter diagram |
| **2/6** | | | eg. interpret and construct cumulative frequency diagrams and/or box plots; estimate median and interquartile range and use to compare distributions and make inferences. |
| **2/7** | Candidates collect reliable data relevant to the problem under consideration. They deal with practical problems such as non-response, missing data or ensuring secondary data are appropriate. Candidates use a range of relevant calculations that include techniques meeting the level detailed in the handling data paragraph of the grade description for grade A. These calculations are correct and no obviously relevant calculation is omitted. Numerical results are rounded appropriately. There is no redundancy in calculation or presentation. Candidates use language and statistical concepts effectively in presenting a convincing reasoned argument. They use an appropriate range of diagrams to summarise the data and show how variables are related. | • Candidates collect/sample largely relevant data.<br>• They utilise appropriate and necessary calculations/techniques/ diagrams (see note 1 above) consistently within the problem.<br>• Their results are correct. | |
| **2/8** | | | eg. Interpret, construct and compare histograms; understand how different methods of sampling and different sample sizes affect reliability of conclusions drawn; select and justify a sample and method to investigate a population. |

## INTERPRET and DISCUSS

**Notes:** **1. In these criteria there is an intended approximate link between 7 marks and grade A, 5 marks and grade C and 3 marks and grade F.**
**2. The number of marks awarded at this strand is unlikely to exceed the mark at Strand 1 by more than 1.**
**3. The use of ICT is to be encouraged to allow candidates more time to analyse and interpret the data. (There is no requirement for the diagrams to be drawn by hand).**

| | | **Minimum requirements** | **Teachers' Notes** |
|---|---|---|---|
| **3/1** | Candidates comment on patterns in the data. They summarise the results they have obtained but make little attempt to relate the results to the initial problem. | • Candidates comment on their data. | A simple observation made would merit 3/1<br>An observation such as mean = 4.5 or mode is 3 etc… would merit 3/2 |
| **3/2** | | | |
| **3/3** | Candidates comment on patterns in the data and any exceptions. They summarise and give a reasonably correct interpretation of their graphs and calculations. They attempt to relate the summarised data to the initial problem, though some conclusions may be incorrect or irrelevant. They make some attempt to evaluate their strategy. | • Candidates summarise some of their data.<br>• They make a statement based on their diagrams or calculations, which is relevant to the problem. | The observation is linked to the aims |
| **3/4** | | | |
| **3/5** | Candidates comment on patterns in the data and suggest reasons for exceptions. They summarise and correctly interpret their graphs and calculations, relate the summarised data to the initial problem and draw appropriate inferences. Candidates use summary statistics to make relevant comparisons and show an informal appreciation that results may not be statistically significant. Where relevant, they allow for the nature of the sampling method in making inferences about the population. They evaluate the effectiveness of the overall strategy and make a simple assessment of limitations. | • Candidates summarise **and** correctly interpret their diagrams or calculations.<br>• They relate these interpretations back to the original problem.<br>• They evaluate their strategy. | Some relevant comparisons are likely to be included.<br>**It is important that candidates appreciate the difference between observation and interpretation eg the difference between 'there is a positive correlation' and 'a positive correlation which shows that…'**<br>An evaluation of strategy should include reflective comments on the strengths and/or weaknesses of their methodology (in identifying, collecting and processing data). Candidates justify their choice and comment on the effectiveness of diagrams or calculations.<br>Evaluation is more than stating 'a bigger sample size' Evidence for this may be gathered throughout the work. |
| **3/6** | | | |
| **3/7** | Candidates comment on patterns and give plausible reasons for exceptions. They correctly summarise and interpret graphs and calculations. They make correct and detailed inferences from the data concerning the original problem using the vocabulary of probability. Candidates appreciate the significance of results they obtain. Where relevant, they allow for the nature and size of the sample and any possible bias in making inferences about the population. They evaluate the effectiveness of the overall strategy and recognise limitations of the work done, making suggestions for improvement. They comment constructively on the practical consequences of the work. | • Candidates summarise and correctly interpret their results.<br>• They show an appreciation of the significance of these results.<br>• They recognise possible limitations in their strategy and suggest improvements. | The use of the phrase 'using the vocabulary of probability' (with statistical relevance) is intended to recognise that the best work will give some indication how likely or unlikely the events inferred from the data are. |
| **3/8** | | | |

## 6.7 Extracts from AO4 tasks

The next section shows some examples of work to illustrate further each strand mark. There are three extracts to illustrate the marking of each strand. These have been word-processed for ease of reading.

For the purposes of brevity, the candidates' work on the following pages is presented as a series of snapshots to illustrate particular assessment points.

These examples are set in isolation to illustrate a point; they do not represent a complete piece of work. Such a small piece of work alone will not gain the mark shown as marks are awarded for the whole piece of work. The complete piece of work, in each instance, would provide a better flavour for the mark awarded, since both coherence and sustainability are likely to produce a fuller picture.

| Mark | Strand 1 | Strand 2 | Strand 3 |
|---|---|---|---|
| 3 | Extract A | Extract D | Extract G |
| 4 | Extract B | Extract E | Extract H |
| 5 | Extract C | Extract F | Extract I |

There are no extracts to illustrate marks of 6, 7 or 8 since these marks are awarded on the quality and coherence of reasoning and development shown **throughout** the task.

Extract A Strand 1: Specifying the problem and planning

Task: Investigate letters and words

Candidate's work

I think that vowels are the most commonly used letters in the alphabet.

I am going to choose an article from a national newspaper and count the number of times each letter occurs.

```
Climbing Welsh Hills is hard work.
Most of us prefer to do it by car,
or just go to the pub.  Legal
eagle Mike Wood loves the hills.
He spends all his spare time in
his boots.  From the sands of
Spain to the mud of Macedonia Mike
has climbed every mountain.
```

I am going to show my information in a tally chart

| Letter | Tally | Freq. |
|---|---|---|
| A | 卌 卌 \|\| | 12 |
| B | \|\|\|\| | 4 |
| C | \|\|\|\| | 4 |
| D | 卌 \|\|\| | 8 |
| E | | |

Notes

**Mark 3-4**

*Candidates choose a problem involving routine use of simple statistical techniques and set out reasonably clear aims. Consideration is given to the collection of data. Candidates describe an overall plan largely designed to meet the aims and structure the project report so that results relating to some of the aims are brought out. Where appropriate, they use a sample of adequate size*

**Assessment**

The task, as set, allows the candidate to choose their own problem and state their own hypothesis.

The candidate makes the decision to investigate the hypothesis that vowels are the most commonly used letters in the alphabet.

They choose an article and produce a tally chart for each of the letters of the alphabet.

The work sets out reasonably clear aims, uses simple statistical techniques, and due consideration is given to the collection of data.

However, the planning is fairly simplistic, representations are basic and there is no overall scheme of planning.

The sample size is adequate for the award given.

***THE CANDIDATE'S WORK IS AWARDED MARK 3 IN STRAND 1***

**ICT links**

The work might be developed to look at word counts which take advantage of 'letter count' software.

## Extract B

Strand 1: Specifying the problem and planning

Task: Investigate letters and words

Candidate's work

I have decided to investigate my idea that words in magazine articles are longer than words in newspaper articles.

I am going to choose similar articles from a glossy magazine and an article from a national newspaper so that these can be fairly compared.

I have chosen a sample size of 100 which will be big enough for my work but easy to manage. From my tally charts I will work out the mean, median, mode and range to compare data.

### Newspaper

Climbing Welsh Hills is hard work. Most of us prefer to do it by car, or just go to the pub. Legal Eagle Mike Wood loves the hills. He spends all his spare time in his boots. From the sands of Spain to the mud of Macedonia Mike has climbed every mountain.

~~I~~ asked how he keeps in trim. The office worker showed me his plan. He jogs, pumps iron, and swims. At weekends he's on the bike peddling 50 miles to his secret hide-out where he has found lots of new things to do with ~~a~~ compass

### Magazine

Clambering over the magnificent granite monuments of Snowdonia is perhaps a little too energetic for the aging bureaucrat. Local solicitor Mike Wood has a passion for this particular form of entertainment and spends every available moment scaling majestic mountains. From Sardinia to Scandinavia he has conquered local hills.

When I enquired about his general fitness Mr Wood showed me his exercise schedule. He engages in running weight training at the local gymnasium. At weekends he may frequently be seen cycling the thirty miles to a remote valley in the Peak District where he has developed an advanced navigation technique.

### Newspaper

| Length | Tally | Freq. |
|---|---|---|
| 1 | II | 2 |
| 2 | 卌 卌 卌 卌 卌 III | 28 |
| 3 | 卌 卌 卌 卌 II | 22 |
| 4 | 卌 卌 卌 II | 17 |
| 5 | 卌 卌 卌 I | 16 |

### Magazine

| Length | Tally | Freq. |
|---|---|---|
| 1 | IIII | 4 |
| 2 | 卌 卌 卌 I | 16 |
| 3 | 卌 卌 卌 I | 16 |
| 4 | 卌 卌 | 10 |
| 5 | 卌 III | 8 |

### Notes

**Mark 3-4**

*Candidates choose a problem involving routine use of simple statistical techniques and set out reasonably clear aims. Consideration is given to the collection of data. Candidates describe an overall plan largely designed to meet the aims and structure the project report so that results relating to some of the aims are brought out. Where appropriate, they use a sample of adequate size*

**Assessment**

The task, as set, allows the candidate to choose their own problem and state their own hypothesis.

The candidate makes the decision to investigate the hypothesis that words in magazine articles are longer than words in newspaper articles.

The work sets out clear aims, considers sampling and uses simple statistical techniques which allows comparisons to be made (fulfilling the grade F descriptor) and gives due consideration to the collection of data.

***THE CANDIDATE'S WORK IS AWARDED MARK 4 IN STRAND 1***

Extract C Strand 1: Specifying the problem and planning

Task: Investigate letters and words

Candidate's work

I have decided to investigate my idea that words in magazine articles are longer than words in newspaper articles.

I am going to choose similar articles from a glossy magazine and an article from a national newspaper so that these can be fairly compared.

I have chosen a sample size of 100 which will be big enough for my work but easy to manage. From my tally charts I will use systematic sampling to ensure that the words are randomly chosen from the whole article and I will roll a die to decide where to start

**The candidate pursues the task as illustrated in Extract C concluding that there is little difference between the length of the words in the magazine article and the newspaper article. The task is subsequently developed as follows:**

I noticed that there was little difference between the length of the words in the magazine article and the lengths of words in the newspaper article but I have now decided to look at a variety of different magazine articles and a variety of different newspaper articles……

**The candidate then considers different articles from the same magazine and the same newspaper or else offers a collection of different magazines and a collection of different newspapers from which to collect the data. The candidate subsequently pursues this new task to a conclusion**

Alternatively

I noticed that there was little difference between the length of the words in the magazine article and the lengths of words in the newspaper article but I have now decided to look at the length of sentences in the magazine article and the lengths of sentences in the newspaper article ……

**The candidate then considers the lengths of the sentences of the same articles or else different articles. The candidate subsequently pursues this new task to a conclusion**

Notes

**Mark 5-6**

*Candidates consider a more complex problem. They choose appropriate data to collect and state their aims in statistical terms with the selection of an appropriate plan. Their plan is designed to meet the aims and is well described. Candidates consider the practical problems of carrying out the survey or experiment. Where appropriate, they give reasons for choosing a particular sampling method. The project report is well structured so that the project can be seen as a whole.*

**Assessment**

The candidate makes the decision to develop the task beyond the original task by developing the work in greater depth (ie, considering a number of different articles) or linking the work to sentence lengths.

It is not sufficient to state how the work is developed but there must be evidence of the plan being implemented.

***THE CANDIDATE'S WORK IS AWARDED MARK 5 IN STRAND 1***

For an award of 6 marks, the candidate should provide a well structured report and consider the practical problems associated with the experiment including sampling methods and how each piece of individual data was chosen.

**Cross curricular links**

The task provides opportunities to develop cross curricular links with other languages in looking at foreign newspapers.

Extract D

Strand 2: Collecting, processing and representing the data

Task: Investigate estimation

Candidate's work

I collected data from 30 candidates on their estimates of the height (in metres) of a classroom wall:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.6 | 2.5 | 2.6 | 3.1 | 3.2 | 3.0 | 2.5 | 2.4 | 3.2 | 2.2 | 3.2 | 3.6 | 3.1 | 3.0 | 2.9 |

| 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.7 | 2.8 | 2.7 | 2.4 | 2.5 | 2.5 | 3.2 | 3.1 | 2.6 | 2.5 | 2.5 | 3.3 | 2.8 | 2.9 | 3.0 |

| Height (m) | Frequency |
|---|---|
| 2.2 | 1 |
| 2.3 | 0 |
| 2.4 | 2 |
| 2.5 | 6 |
| 2.6 | 3 |
| 2.7 | 2 |
| 2.8 | 2 |
| 2.9 | 2 |
| 3.0 | 3 |
| 3.1 | 3 |
| 3.2 | 4 |
| 3.3 | 1 |
| 3.4 | 0 |
| 3.5 | 0 |
| 3.6 | 1 |

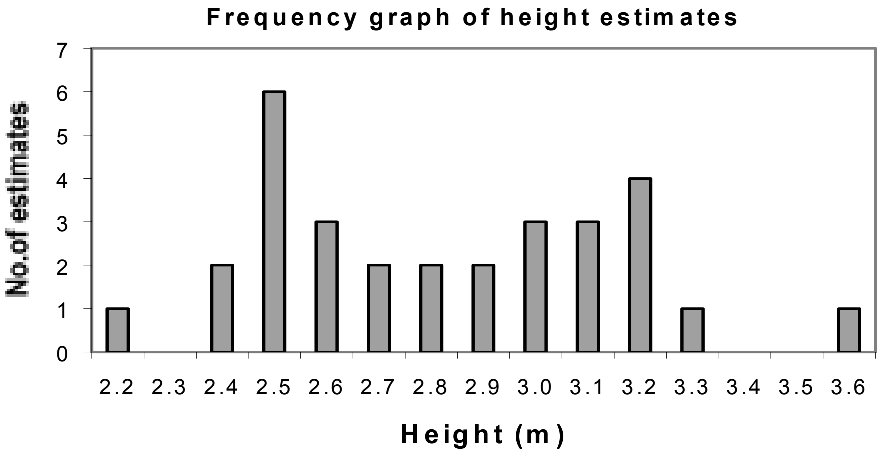
From my calculations:

Mean = $\dfrac{84.6}{30}$ = 2.82 m

Median = 2.8 m

Mode = 2.5 m

I can show this information as a graph:




Notes

**Mark 3-4**

*Candidates collect data with some relevance to the problem and plan. The data are collected or recorded with some consideration given to efficient processing. Candidates use straightforward and largely relevant calculations involving techniques of at least the level detailed in the handling data paragraph of the grade description for grade F. The results are generally correct. Candidates show understanding of situations by describing them using statistical concepts, words and diagrams. They synthesise information presented in a variety of forms. Their writing explains and informs their use of diagrams, which are usually related to their overall plan. They present their diagrams correctly, with suitable scales and titles.*

**Assessment**

The candidate interprets the task on investigating estimation as an opportunity to collect data on candidates' estimates of the height of a classroom wall.

The candidate gives some simple calculations of the mean, median and mode. The calculations are straightforward, 'largely relevant' and correct.

Together with the calculations, the provision of a graph with suitable scales and a title are sufficient for the award given.

***THE CANDIDATE'S WORK IS AWARDED MARK 3 IN STRAND 2***

**ICT links**

Computer-generated representations are also acceptable but candidates should remember to explain and analyse any such work.

Extract E

Strand 2: Collecting, processing and representing the data

Task: Investigate estimation

Candidate's work

I collected data from 40 children detailing their estimations of the length of a red line and the angle between two lines.

| Line (mm) | 60 | 55 | 59 | 61 | 40 | 45 | 65 | 59 | 47 | 60 | 55 | 50 | 40 | 59 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Angle (º) | 46 | 43 | 45 | 34 | 44 | 53 | 63 | 49 | 52 | 45 | 55 | 60 | 60 | 56 |

| Line (mm) | 50 | 52 | 46 | 50 | 50 | 42 | 60 | 44 | 49 | 52 | 50 | 50 | 70 | 45 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Angle (º) | 53 | 34 | 47 | 52 | 48 | 48 | 37 | 55 | 53 | 50 | 48 | 47 | 65 | 59 |

| Line (mm) | 41 | 45 | 48 | 54 | 48 | 50 | 61 | 45 | 30 | 40 | 47 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Angle (º) | 60 | 52 | 45 | 56 | 49 | 50 | 32 | 47 | 45 | 41 | 52 | 41 |

Mean:

Length of line = $\frac{2024}{40}$ = 50.6 mm

Size of angle $\frac{1971}{40}$ = 49.28 º

From my calculations:

| | Mean | Range |
|---|---|---|
| Length of line (mm) | 50.60 | 40 |
| Size of angle (º) | 49.28 | 33 |

From my findings, I can see that .........

Notes

**Mark 3-4**

*Candidates collect data with some relevance to the problem and plan. The data are collected or recorded with some consideration given to efficient processing. Candidates use straightforward and largely relevant calculations involving techniques of at least the level detailed in the handling data paragraph of the grade description for grade F. The results are generally correct. Candidates show understanding of situations by describing them using statistical concepts, words and diagrams. They synthesise information presented in a variety of forms. Their writing explains and informs their use of diagrams, which are usually related to their overall plan. They present their diagrams correctly, with suitable scales and titles.*

**Assessment**

The candidate interprets the task on investigating estimation as an opportunity to investigate the hypothesis that children's estimation of lengths is better than angles.

The candidate provides a more appropriate focus to the work and calculates the mean and range for the estimations on length and angle in order to compare and interpret the data.

The information is synthesised into the 2 × 2 table and the format allows a sensible comparison of the findings.

***THE CANDIDATE'S WORK IS AWARDED MARK 4 IN STRAND 2***

Extract F

Strand 2: Collecting, processing and representing the data

Task: Investigate estimation

Candidate's work

I collected data from 30 children and 30 adults recording their estimations of the length of a line and the timing of one minute.

Estimation of length of a line: Children

| Length (mm) | Mid Interval Value | Frequency Children | Cum. Freq. Children | M x $f_c$ |
|---|---|---|---|---|
| 40 - 45 | 42.5 | 4 | 4 | 170 |
| 45 - 50 | 47.5 | 8 | 12 | 380 |
| 50 - 55 | 52.5 | 9 | 21 | 472.5 |
| 55 - 60 | 57.5 | 4 | 25 | 230 |
| 60 - 65 | 62.5 | 3 | 28 | 187.5 |
| 65 - 70 | 67.5 | 1 | 29 | 67.5 |
| 70 - 75 | 72.5 | 1 | 30 | 72.5 |
| Totals | | | | 1580 |

Estimation of length of a line: Adults

| Length (mm) | Mid Interval Value | Frequency Adults | Cum. Freq. Adults | M x $f_a$ |
|---|---|---|---|---|
| 40 - 45 | 42.5 | 1 | 1 | 42.5 |
| 45 - 50 | 47.5 | 2 | 3 | 95 |
| 50 - 55 | 52.5 | 5 | 8 | 262.5 |
| 55 - 60 | 57.5 | 14 | 22 | 805 |
| 60 - 65 | 62.5 | 4 | 26 | 250 |
| 65 - 70 | 67.5 | 4 | 30 | 270 |
| 70 - 75 | 72.5 | 0 | 30 | 0 |
| Totals | | | | 1725 |

Children's mean = $\frac{1580}{30}$ = 52.67 mm

Adults' mean = $\frac{1725}{30}$ = 57.50 mm

Notes

**Mark 5-6**

*Candidates collect largely relevant and mainly reliable data. The data are collected in a form designed to ensure that they can be used. Candidates use a range of more demanding, largely relevant calculations that include techniques of at least the level detailed in the handling data paragraph of the grade description for grade C. The results are generally correct and no obviously relevant calculation is omitted. There is little redundancy in calculation or presentation. Candidates convey statistical meaning through precise and consistent use of statistical concepts that is sustained throughout the work. They use appropriate diagrams for representing data and give a reason for their choice of presentation, explaining features they have selected*

**Assessment**

The candidate interprets the task on investigating estimation as an opportunity to investigate the hypothesis that childrens' estimation of lengths is better than adults and adults' estimation of time is better than children.

The calculation of the mean of a group frequency distribution certainly supports this award. The results are relevant and correct.

Extract F , cont...

Estimation of the timing of one minute: Children

| Time (sec) | Mid Interval Value | Frequency Children | Cum. Freq. Children | $M \times f_c$ |
|---|---|---|---|---|
| 40 - 45 | 42.5 | 3 | 3 | 127.5 |
| 45 - 50 | 47.5 | 4 | 7 | 190 |
| 50 - 55 | 52.5 | 6 | 13 | 315 |
| 55 - 60 | 57.5 | 6 | 19 | 345 |
| 60 - 65 | 62.5 | 5 | 24 | 312.5 |
| 65 - 70 | 67.5 | 4 | 28 | 270 |
| 70 - 75 | 72.5 | 1 | 29 | 72.5 |
| 75 - 80 | 77.5 | 1 | 30 | 77.5 |
| Totals | | | | 1710 |

Estimation of the timing of one minute: Adults

| Time (sec) | Mid Interval Value | Frequency Adults | Cum. Freq. Adults | $M \times f_a$ |
|---|---|---|---|---|
| 40 - 45 | 42.5 | 1 | 1 | 42.5 |
| 45 - 50 | 47.5 | 1 | 2 | 47.5 |
| 50 - 55 | 52.5 | 3 | 5 | 157.5 |
| 55 - 60 | 57.5 | 8 | 13 | 460 |
| 60 - 65 | 62.5 | 7 | 20 | 437.5 |
| 65 - 70 | 67.5 | 5 | 25 | 337.5 |
| 70 - 75 | 72.5 | 4 | 29 | 290 |
| 75 - 80 | 77.5 | 1 | 30 | 77.5 |
| Totals | | | | 1850 |

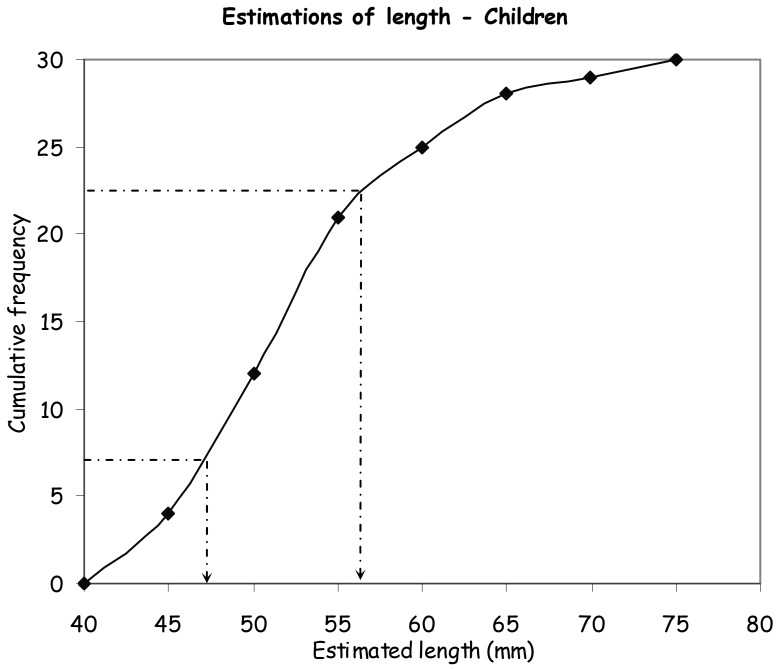
$$\text{Children's mean} = \frac{1710}{30} = 57.00\,\text{sec}$$

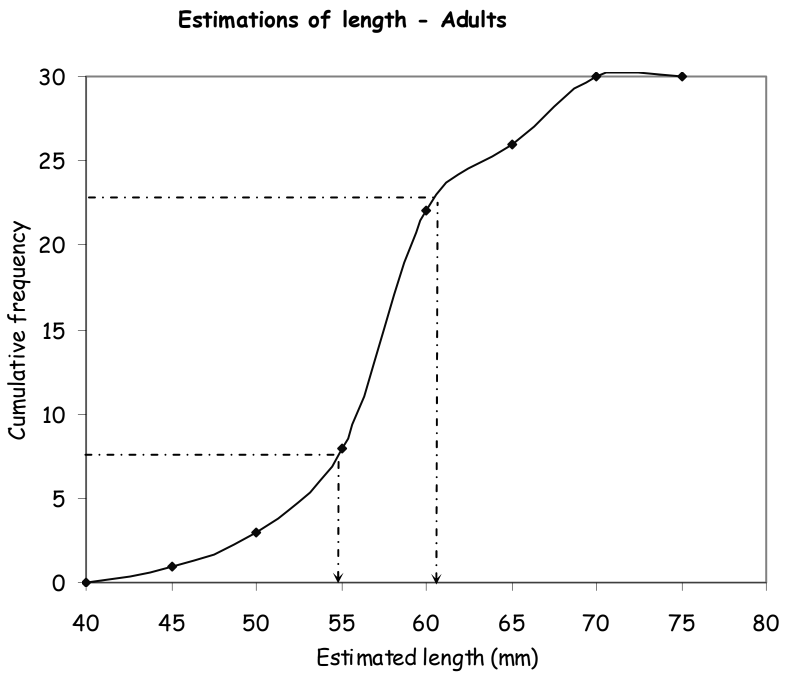
$$\text{Adults' mean} = \frac{1850}{30} = 61.67\,\text{sec}$$

Extract F , cont...

Candidate's work

I shall now show this information in a cumulative frequency graph to calculate the interquartile range which will help me to compare the information.




Notes

The provision of a cumulative frequency graph and the calculation of the interquartile range also allows this award. The results are relevant and correct.

Similarly, other calculations or techniques (e.g. box plots) would also provide evidence for this award.

***THE CANDIDATE'S WORK IS AWARDED MARK 5 IN STRAND 2***

For an award of 6 marks, the candidate needs to use a range of relevant techniques and justify their choice. It is essential for the higher marks that the candidate interprets (rather than describes) their findings.

If this is addressed later in the work, then a higher award might be given.

**Other investigations**

An alternative approach might be to look at the estimation skills of boys and girls.

Extract G Strand 3: Interpreting and discussing the results

Task: Investigate examination results

The candidate is provided with a class set of secondary data detailing KS3 levels in Mathematics, English and Science. There are 30 candidates in the class.

Candidate's work

| Candidate | Sex | Maths | English | Science |
|---|---|---|---|---|
| 4385 | M | 5 | 6 | 5 |
| 4386 | F | 4 | 4 | 4 |
| 4388 | F | 6 | 6 | 5 |
| … | … | … | … | … |
| … | … | … | … | … |
| 4435 | F | 3 | 3 | 3 |
| 4452 | M | 4 | 4 | 4 |
| 4465 | F | 4 | 5 | 5 |

I have calculated the mean, median and mode in each subject area:

Mean Maths $= \frac{118}{30} = 3.9$ Mean English $= \frac{133}{30} = 4.3$ Mean Science $= \frac{114}{30} = 3.8$

From my calculations:

| Mathematics | English | Science |
|---|---|---|
| Mean = 3.9 | Mean = 4.3 | Mean = 3.8 |
| Median = 4 | Median = 5 | Median = 4 |
| Mode = 4 | Mode = 5 | Mode = 4 |

From my findings, I can say that English candidates' results are higher than Mathematics and Science. Therefore, my hypothesis is proven because.....

Notes

**Mark 3-4**

*Candidates comment on patterns in the data and any exceptions. They summarise and give a reasonably correct interpretation of their graphs and calculations. They attempt to relate the summarised data to the initial problem, though some conclusions may be incorrect or irrelevant. They make some attempt to evaluate their strategy.*

**Assessment**

The candidate has decided to compare KS3 levels in Mathematics, English and Science by calculating averages.

The award is given for summarising the results with some valid comments which are correct for their calculations and linked back to their hypothesis.

***THE CANDIDATE'S WORK IS AWARDED MARK 3 IN STRAND 3***

NOTE:

It is acceptable for schools to provide a suitable database for candidates to use for this work. The database needs to be sufficiently large enough to ensure many different questions can be addressed by candidates.

Extract H

Strand 3: Interpreting and discussing the results

Task: Investigate examination results

The candidate is provided with a set of secondary data detailing KS3 levels in Mathematics, English and Science.

Candidate's work

I have calculated the mean and range of each subject area for males and females.

Mean (Females)

Mean Maths = $\frac{195}{50}$ = 3.9  Mean English = $\frac{225}{50}$ = 4.3  Mean Science = $\frac{190}{50}$ = 3.8

Range (females)

Mathematics = 4  English = 5  Science = 3

From my calculations

| | Males | | Females | |
|---|---|---|---|---|
| | Mean | Range | Mean | Range |
| Mathematics | 4.0 | 3 | 3.9 | 4 |
| English | 4.4 | 4 | 4.5 | 5 |
| Science | 3.8 | 3 | 3.8 | 3 |

From my findings, I can say that English results are higher than Mathematics and Science so pupils are doing better in English… perhaps because candidates use English in every lesson so it is easier. However, in terms of gender, boys do better than girls in mathematics. Therefore, my hypothesis is not proven because…

**Subsequent comment also considers the accuracy of the result in view of the sample size and the range.**

Notes

**Mark 3-4**

*Candidates comment on patterns in the data and any exceptions. They summarise and give a reasonably correct interpretation of their graphs and calculations. They attempt to relate the summarised data to the initial problem, though some conclusions may be incorrect or irrelevant. They make some attempt to evaluate their strategy.*

**Assessment**

The candidate has decided to compare KS3 levels in Mathematics, English and Science by calculating the mean and the range for males and females individually thus offering a development of the original task.

The work is linked back to the original hypothesis and subsequent comment on sample size or the range of the data shows some evaluation of their strategy.

The award is given for summarising the results with valid comments which are correct for their calculations.

***THE CANDIDATE'S WORK IS AWARDED MARK 4 IN STRAND 3***

Alternative lines of enquiry might include comparison with other classes/schools or comparing different ethnic groups.

Extract I

Strand 3: Interpreting and discussing the results

Task: Investigate examination results

The candidate is provided with data on candidates KS3 levels and their GCSE grades achieved two years later in Mathematics, English and Science.

Candidate's work

I am going to see if there is any relationship between candidates' KS3 levels and GCSE grades.

My hypothesis is that girls improve more than boys.

I calculated the mean and range of each subject area for males and females in KS3 and GCSE by converting the GCSE grades to points: (A* to 8, A to 7, etc).

From my calculations:

| | Male mean | Female mean |
|---|---|---|
| Mathematics KS3 | 2.6 | 3.0 |
| Mathematics GCSE | 4.10 | 4.30 |
| Difference | 1.5 | 1.3 |

In Mathematics the boys have improved more than the girls as the difference in the mean score is higher for the boys than the girls.

| | Male mean | Female mean |
|---|---|---|
| English KS3 | 2.9 | 3.5 |
| English GCSE | 4.36 | 5.16 |
| Difference | 1.46 | 1.96 |

In English the girls have improved more than the boys as the difference in the mean score is higher for the girls than the boys.

| | Male mean | Female mean |
|---|---|---|
| Science KS3 | 2.5 | 3.3 |
| Science GCSE | 4.17 | 4.77 |
| Difference | 1.67 | 1.47 |

In Science the boys have improved more than the girls as the difference in the mean score is higher for the boys than the girls.

Notes

**Mark 5-6**

*Candidates comment on patterns in the data and suggest reasons for exceptions. They summarise and correctly interpret their graphs and calculations, relate the summarised data to the initial problem and draw appropriate inferences. Candidates use summary statistics to make relevant comparisons and show an informal appreciation that results may not be statistically significant. Where relevant, they allow for the nature of the sampling method in making inferences about the population. They evaluate the effectiveness of the overall strategy and make a simple assessment of limitations.*

**Assessment**

The candidate decides to pursue the investigation by considering the relationship between KS3 levels and GCSE grades in terms of girls and boys. They comment on the findings in their data and, in their later work, suggesting reasons for exceptions, ie, inability to use all data and high number of absentees among girls at GCSE. They summarise and correctly interpret their calculations and draw appropriate inferences linked to their original hypothesis.

Candidates use summary statistics to make relevant comparisons and make a simple assessment of limitations.

***THE CANDIDATE'S WORK IS AWARDED MARK 5 IN STRAND 3***

An attempt to provide and link a number of these aspects and a fuller evaluation of the effectiveness of the strategy and the sampling method might suggest an award of mark 6 under this strand.

If this is addressed later in the work, then a higher award might be given.

Extract I, cont…

Candidate's work

<u>Overall conclusion</u>

My hypothesis is not true as girls improve more than boys in English, although boys improved more than girls in Mathematics and Science.

**The work also makes use of a variety of different representations and calculations and concludes with an evaluation of the candidate's strategy including the comments that:**

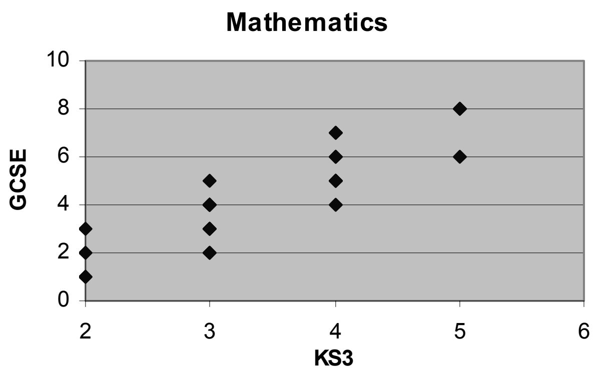
My conclusions are not so accurate because I was not able to use all of my data especially where candidates had missed their KS3 examination or their GCSE examination.
The GCSE data is not so accurate because a large number of girls were absent for the final examination and this may have had an effect on my results depending on the ability of the absent girls.

**Alternatively**

I am going to see if there is any relationship between candidates' KS3 levels and GCSE grades.

My hypothesis is that there is a relationship between candidates' KS3 levels and GCSE grades.

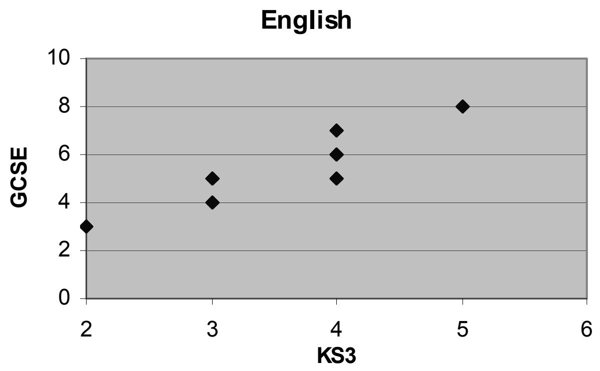
I will show the information on scatter graphs for each of the subjects.



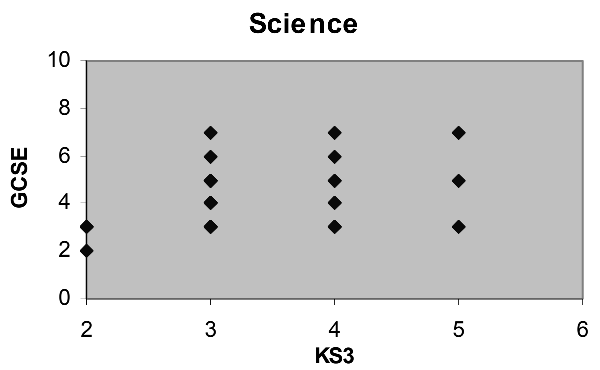

From my graph I notice that in Mathematics there is a weak correlation between KS3 and the GCSE results so that the better the performance in KS3 Mathematics then the better the performance in GCSE Mathematics.

Extract I, cont…

Candidate's work




From my graph, I notice that in English there is a strong correlation between the KS3 and the GCSE results so that the better the performance in KS3 English then the better the performance in GCSE English.




From my graph, I notice that in Science there is no correlation between the KS3 and the GCSE results so there would seem to be no link between performance in KS3 Science and GCSE Science.

<u>Overall conclusion</u>

My hypothesis is not true as there is no correlation between KS3 and GCSE results in Science although there is some correlation in Mathematics and English

The work also makes use of a variety of different representations and calculations and concludes with an evaluation of the candidates' strategy including the comments that:

My conclusions are not so accurate because I was not able to use all of my data especially where candidates had missed their KS3 examination or their GCSE examination.

My data does not take account of the spread of marks which would contribute to a particular level or grade so that a level 5 might be a weak 5 or a strong 5, etc.

## 6.8 Student Support Sheets (AO4)

Student Guidelines for AO4 Tasks and a Writing Framework for AO4 Tasks are provided in Appendix E. These *Student Support Sheets* may be photocopied and provided to candidates.

## 6.9 ICT in coursework

Some investigations can be carried out using the computer as a tool. Some programs in the SMILE series lead to *n*th term calculations and can be used to draw complex shapes which would be difficult by hand. An example of this would be 'Spirolaterals'. This investigation leads to a number of geometrical diagrams which have rotational symmetry and build up in a given sequence. In this case, the program will keep a table of results for different starting points, thus enabling the user to identify patterns more easily. However, it is important to remember that the computer is only being used as a tool. The first few drawings would normally be produced by hand before using the program to progress to more complex starting conditions which would be almost impossible to draw accurately by hand. Alternatively, the program '*Logo*' can be used to build up the shapes according to a rule, enabling the user to count the number of lines produced, etc.

A good graph drawing package such as '*Mouseplotter*' or '*Omni-Graph' could* also enable candidates to investigate features of different graphs and their related equations. Once again, the main advantage is in cutting down the time to draw the graphs.

The majority of the marks for mathematics coursework come from the method used to explain what is happening in the investigation. Therefore any write-up can be word-processed as long as all the relevant information is clearly obtained and the steps used clearly explained. The important thing is to use the ICT where appropriate and not simply for its own sake.

For the handling data investigation a spreadsheet (eg, *Excel*) can be a useful tool for storing data, doing statistical analysis and planning graphs. However, it is important to note that where formulae are used, they must be explained and shown in printouts. Using a spreadsheet can save a lot of time in drawing graphs – however, the candidate must be selective about the final printouts presented.

It is perfectly acceptable for candidates to download statistical data from the Internet for use in the AO4 task. A list of websites is provided in Section 10.3.

## 6.10 Cross-curricular coursework

A project carried out in another subject area may be used as evidence of mathematical attainment. For example, many Geography (also Biology, Psychology, Sociology) assignments are statistically based and might provide evidence for the AO4 task. It should be noted that work must be marked against the AO4 criteria.

## 6.11 GCSE Statistics

There is some overlap between the subject content of GCSE Mathematics Specifications A and B and AQA GCSE statistics.

It is permissible to submit the same coursework for the AO4 task and GCSE Statistics; however, both sets of assessment criteria must be applied to the coursework. In order that coursework is available for moderation for both specifications, it is necessary for the centre to make, for each candidate, a duplicate or photocopy. In this way the work will be available for submission to the moderator, should the same candidate's work be selected for moderation for both specifications. Candidates' work must be annotated to identify as precisely as possible, where in the work the relevant criteria have been satisfied. The appropriate completed *Candidate Record Form* must be attached to each piece of coursework.

Appendix F is a working document produced by senior moderators in Mathematics and Statistics and gives guidance on producing work which will ably meet both sets of criteria.

## 6.12 Support for coursework

Support for teachers in relation to the coursework unit takes several forms.

- **Annual meetings** will be held on a regional basis, usually in the autumn term, at which there will be discussion on the coursework requirements, examples of possible approaches and the application of the coursework criteria. Centres that are new to AQA or who have been adjusted will be automatically invited to these meetings.
- **Exemplar coursework** will be provided annually at centre standardising meetings to show not only the type of work that is expected, but also to illustrate the level of achievement needed to reach a particular grade. Copies can be obtained from the AQA GCSE Mathematics subject support office (mailto: mathematicsgcse@aqa.org.uk)
- **Coursework advisers** will be appointed by AQA to give centres advice on coursework. This can be sought by telephone or e mail, but the advice will be restricted to coursework issues.

# 7 Administration

## 7.1 Moderation procedures

Paragraph 5.20 of the *GCSE, GCE, VCE, GNVQ and AEA Code of Practice* states that "to ensure that standards are aligned within and across centres, the coursework marks submitted by each centre against the specified assessment criteria".

Moderation of work from a centre is carried out by an AQA moderator who will carry out a detailed scrutiny of all the work of a sample of candidates from the centre. On the basis of this inspection, the moderator will decide whether to:

- accept the centre's assessments;
- adjust the assessments to bring them into line with national standards;
- ask for a further sample;
- ask for the work of all candidates or request the centre to reassess or internally standardise their marks.

Normally a centre's judgement about the order of merit will be accepted. However, if major discrepancies are discovered, AQA reserves the right to alter the order of merit.

## 7.2 Annotation (Option T)

Paragraph 5.16 of the *GCSE, GCE, VCE, GNVQ and AEA Code of Practice* states that "The awarding body must require internal assessors to show clearly how credit has been assigned in relation to the criteria defined in the specification. The awarding body must provide guidance on how this is to be done".

This annotation will enable the moderator to see just where a teacher considers that a candidate has met the criteria in the specification.

Work could be annotated by one of the following methods:

- key pieces of evidence flagged throughout the work by annotation either in the margin or in the text;
- completion of the *Candidate Record Form* provided, using specific page references to indicate how the marks have been awarded.

Any information provided by the teacher/lecturer about how the task was undertaken or any comment to explain a candidate's thinking will be considered by the moderator in the assessment of the work.

## 7.3 Annotation (Option X)

Annotation is not required for coursework submitted under Option X but any information provided by the teacher/lecturer about how the task was undertaken or any comment to explain a candidate's thinking will be considered by the examiner in the assessment of the work.

# Course Organisation

## 8 Delivery of the Course Mathematics B *(Modular)*

### 8.1 Special Features

The modular course allows for external assessment of some areas of the Programme of Study early in the course. The areas of the Programme of Study are divided along Assessment Objective lines within the subject content of Module 1 and Module 3.

- GCSE Modular Mathematics can be taken over one year or two years.
- Specification B allows candidates to take modules early in the course on Handling Data (AO4) and the (mainly) number part of the Number and Algebra (AO2).
- There are common internal assessment tasks (coursework) with AQA GCSE Mathematics Specification A allowing for the interchange of tasks between the two specifications. Moderation of coursework is offered three times a year, in November, March and June. Centres may choose the most appropriate examination series to submit tasks for assessment. Centres may enter candidates for Module 2 and for Module 4 in different examination series. For example, centres may enter candidates for Module 2 in the summer of Year 10 and for Module 4 in the summer of Year 11. (Details of the structure and content of modules can be found in Specification B).
- Results are reported at the end of each module, enabling candidates to take greater responsibility for the planning and execution of their work.
- Modules 1 to 4 can be re-sat before final certification and the better result is taken.
- Candidates may enter each individual module at a different tier of entry allowing them to make the most of their strengths. (Please note: candidates may enter only for a single tier in each module, in a particular examination series.) The final range of grades available to candidates is determined by the tier of entry for Module 5. Module 5 is the certificating module and must be taken in the final examination series. The first examination series in which Module 5 is available is summer 2008. Again it should be noted that candidates entering Module 5 for this specification are prohibited from entering any other GCSE Mathematics specification that will be **certificated** in the same examination series.

- The modular nature of the specification is a good preparation for AS/A2 Mathematics or further study at GNVQ/AVCE level. The specification also offers links with Free Standing Maths Qualifications (FSMQs) and the Key Skill of Application of Number. There is an overlap between Module 1 of this specification and the GCSE Statistics specification.

The modular nature of the specification allows candidates who fail to obtain a GCSE grade C at KS4 to carry forward some of their module results into post-16 education.

## 8.2 Grading System

The qualification will be graded on an 8 point grade Scale A*, A, B, C, D, E, F, G. Candidates who fail to reach the minimum standard for grade G will be recorded as U (unclassified) and will not receive a qualification certificate.

The written paper modules are offered at two tiers of entry: Foundation tier and Higher tier. For candidates entered for the Foundation tier, grades C to G are available. For candidates entered for the Higher tier, grades A* to D are available. There is a safety net for candidates entered for the Higher tier, where an allowed grade E will be awarded where candidates just fail to achieve grade D. Candidates may enter for each individual module at a different tier of entry. However, the final range of grades available to a candidate is determined by the tier of entry of Module 5. Candidates who fail to achieve grade E on the Higher tier or grade G on the Foundation tier will be reported as U (unclassified).

(Written papers are the same for Option T and X).

## 8.3 Re-sits

Candidates can re-sit each of Modules 1 to 4 once before certification and the best mark is taken towards certification. For example, a candidate could enter Module 3 at Foundation Tier in the first examination series (eg, Year 10) and then enter at Higher Tier in the next examination series (eg, Year 11).

## 8.4 Availability of Assessment Units

Specification B is a modular assessment of GCSE Mathematics designed to be taken over a one year or two year course of study. To offer maximum flexibility to centres and to suit different teaching programmes, Modules 1 to 4 can be taken in any order and candidates can enter at different tiers for the different modules. Module 5 is the certificating module and must be taken in the final examination series. This is to meet the QCA requirement that at least 50% of the qualification is externally examined at the end of the course. The first examination series in which Module 5 and certification is available will be June 2008.

Examinations based on this specification will be available as follows.

| Series | Availability of Modules | | | | |
|---|---|---|---|---|---|
| | **Module 1** | **Module 2** | **Module 3** | **Module 4** | **Module 5** |
| **November 2006** | Both tiers | Both options | Both tiers | Both options | _ |
| **March 2007** | Both tiers | Both options | Both tiers | Both options | _ |
| **June 2007** | Both tiers | Both options | Both tiers | Both options | _ |
| **November 2007** | Both tiers | Both options | Both tiers | Both options | _ |
| **March 2008** | Both tiers | Both options | Both tiers | Both options | _ |
| ***June 2008** | Both tiers | Both options | Both tiers | Both options | Both tiers |
| ***November 2008** | Both tiers | Both options | Both tiers | Both options | Both tiers |

*Certification is available in this series.

## 8.6 Determination of the candidates' final grades

Uniform Mark Scale (expressed as mark ranges)

| Grade | Module 1 | Modules 2 & 4 | Module 3 | Module 5 |
|---|---|---|---|---|
| **A*** | 59 – 66 | 54 – 60 | 103 – 114 | 270 – 300 |
| **A** | 53 – 58 | 48 – 53 | 91 – 102 | 240 – 269 |
| **B** | 46 – 52 | 42 – 47 | 80 – 90 | 210 – 239 |
| **C** | 40 – 45 | 36 – 41 | 68 – 79 | 180 – 209 |
| **D** | 33 – 39 | 30 – 35 | 57 – 67 | 150 – 179 |
| **E** | 26 – 32 | 24 – 29 | 46 – 56 | 120 – 149 |
| **F** | 20 – 25 | 18 – 23 | 34 – 45 | 90 – 119 |
| **G** | 13 – 19 | 12 – 17 | 23 – 33 | 60 – 89 |
| **U** | 0 – 12 | 0 – 11 | 0 – 22 | 0 – 59 |

Candidates may enter Modules 1, 3 and 5 at different tiers of entry. However, it must be remembered that the final range of grades available to a candidate is determined by the tier of entry at Module 5.

# 9 Links with Other Qualifications

## 9.1 Introduction

The National Curriculum for Mathematics (1999) states that "Mathematics equips candidates with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem solving skills and the ability to think in abstract ways". Mathematical tools are used in all qualifications. Through Mathematics, candidates gain access to a language used in their other subjects.

Mathematical knowledge, understanding and skills are fundamental to the study for qualifications such as GCSE Chemistry, Physics, Science and Statistics. These qualifications may require a prior level of attainment in numeracy or specific background knowledge in Mathematics.

The National Curriculum for Mathematics identifies key links with English indicating connections between teaching requirements. It suggests that a requirement in Mathematics can build on the requirements in English in the same key stage.

## 9.2 Key Skills

Mathematics also clearly contributes to the development of the Key Skills. Examples are given below. (Details of the requirements are given in the Key Skills specifications.)

- *Communication*, through learning to express ideas and methods precisely, unambiguously and concisely.
- *Application of Number,* through using and applying the knowledge, skills and understanding of mathematics.
- *Information Technology*, through developing logical thinking; using graphics packages and spreadsheets to solve numerical, algebraic and graphical problems; using dynamic geometry packages to manipulate geometrical configurations; and using databases and spreadsheets to present and analyse data.
- *Working with Others*, through group activities and discussions of mathematical ideas.
- *Improving Own Learning and Performance*, through developing logical thinking, powers of concentration, analytical skills and reviewing approaches to solving problems.
- *Problem Solving*, through selecting and using methods and techniques, developing strategic thinking, and reflecting on whether the approach taken to a problem was appropriate.

(National Curriculum for Mathematics, 1999).

### 9.3 Key Skills Application of Number

The following qualifications in GCSE Mathematics provide exemption from the external test for *Application of Number*:

- GCSE A* to C examination performance provides exemption from the external test in Application of Number at Level 2.
- GCSE D to G examination performance provides exemption from the external test in Application of Number at Level 1.

# 10 Resources

### 10.1 Introduction

AQA has chosen Nelson Thornes to be our exclusive 'preferred partner' for the two tier GCSE Mathematics specifications.

*AQA Mathematics for GCSE* has been developed by Nelson Thornes in partnership with AQA to ensure resources are available which match perfectly the revised specifications.

#### Students' Books

Easy to handle Students' Books provide graded objectives and examination questions written by a GCSE examiner, allowing students to see exactly how to achieve higher grades.

**Learn Sections**

- contain concise examples with notes so students can focus on practice

**Apply Sections**

- provide a wealth of practice questions to help students apply newly-learnt mathematical concepts, with a variety of questions types.

**Explore Sections**

- provides investigative activities, suggesting ideal topics for coursework

**Access Section**

- provides plentiful questions increasing in order of difficulty to test students' understanding of the objectives of each chapter.

#### Homework Books

Provide one homework exercise for each topic and three mini coursework tasks.

| | |
|---|---|
| Teacher's Books | Designed to help all teachers deliver motivating and engaging lessons through a range of features.<br><br>Available for each Students' Book, these comprehensive Teacher's Books are designed to help all teachers deliver motivating engaging lessons through a range of features including:<br><br>• a planning timetable showing the specification reference and grade<br>• alternative methods of teaching topics covered in the Students Books, along with further examples and common errors to help ensure that students fully grasp new concepts<br>• answers for all questions in the Students' Books and Homework Books for easy reference |
| e-Mathematics | Available on CD ROM and online, e-Mathematics provides support and enrichment for your lessons.<br><br>A range of activities and electronic support to enhance lessons<br><br>• objectives for each chapter to introduce new topics<br>• starters and plenaries in three fun yet challenging formats<br>• support for selected *Learn* sections from the Students' Books through interactive activities, optional commentary and question banks<br>• worked examination questions helping to guide students trough questions step-by-step and show where marks are allocated<br>• two sample papers for each tier written by an examiner and including fully worked solutions and mark schemes. |
| Online Test and Assessment | A powerful online service to test, assess and monitor students with whole class or year group reporting.<br><br>• allows testing, assessment and monitoring of individuals and whole classes for assessment of learning<br>• online format means that tests can be completed by students anywhere – in the classroom, at home or in after-school clubs<br>• students can receive instant online feedback on tests so they can see how well they have performed<br>• automatically generated detailed graphical reports allow you to quickly and effectively analyse performance |

## 10.2 Contact details

**Nelson Thornes**

| | |
|---|---|
| Telephone | 01242 267272 |
| Fax | 01242 253695 |
| E mail | maths@nelsonthornes.com |
| Web site | www.nelsonthornes.com/aqamaths |

**AQA Mathematics Subject Support**

| | |
|---|---|
| Telephone | 0161 957 3852 |
| Fax | 0161 957 3873 |
| E mail | mathematicsgcse@aqa.org.uk |
| Web site | www.aqa.org.uk |

## 10.3 Websites

The following are offered, in good faith, as possible sources of data for coursework activities. They are in no particular order and some will be more suitable than others; teachers will need to judge their suitability.

| Organisation | Internet URL | Notes |
|---|---|---|
| **Statistics Sites** | | |
| Census at School | http://www.censusatschool.ntu.ac.uk | This site is an excellent place to find real data for use in conjunction with the data handling component in GCSE coursework. In particular, secondary data is available for the AQA set tasks 'Guestimate' and 'Where in the world' The site also includes a variety of other useful resources (including downloadable worksheets) and tips on how ICT can be used effectively to enhance learning and teaching resources for good practice in data handling. |
| Census Information | http://www.census.ac.uk | Everything you want to know about the UK Census is here (and a bit more besides). Lots of interesting information and plenty of data but you'll need to register to be able to use it…. Probably more effort than it is worth. |
| Centre for Innovation in Mathematics Teaching | http://www.geographyhigh.connectfree.co.uk | Geography High is an unusual concept…. A virtual reality school with only geography on the curriculum. The population room in the second year pupils' assembly hall is probably a good starting point for some interesting statistics. |
| National Statistics | http://www.statistics.gov.uk | National Statistics if the official UK statistics site with lots and lots of statistical information. You will need a couple of days just to navigate your way around the site but the 'themes' page is a useful starting pint for your journey. |

| Organisation | Internet URL | Notes |
|---|---|---|
| QCA National Qualifications (GCSE results) | http://www.qca.org.uk/rs/rer/bcse_results.asp | This QCA web site keeps statistical information on GCSE and short course examination results over the past few years. The data is provi9ded by the GCSE awarding bodies in England, Wales and Northern Ireland. |
| QCA National Qualifications (GCE results) | http://www.qca.org.uk/nq/subjects/a_level_results.asp | This QCA web site keeps statistical information on A level and AS level examination results over the past few years. The data is provided by the GCE awarding bodies and covers a variety of different subjects. |
| Tourism facts and figures | http://www.staruk.org.uk | An interesting site put together by various UK tourist boards with a whole host of interesting facts and figures…. Did you know that over 110 million day trips are made to Britain's coastline each year? |
| UK National Lottery Winning Numbers | http://lottery.merseyworld.com/Winning_index.html | It will not improve your chances on the lottery but it certainly will provide you with lots of statistical information about the National lottery winning numbers, jackpots and numbers of winners. |

**Other Websites**

| | | |
|---|---|---|
| ATM | http://www.atm.org.uk/ | The official website of Teachers of Mathematics. Includes details of ATM's publications, its activities, and its philosophy. It also provides information about the journals and about its membership. |
| BECTA | http://becta.org.uk | If you have never heard of BECTA then this site is worth a visit. BECTA is a Government agency for Information and Communication Technology and is responsible for developing the National Grid for learning NGfl. |

| Organisation | Internet URL | Notes |
|---|---|---|
| The Mathematical Association | http://www.m-a.org.uk | The official website of the Mathematical Association. Includes details of the MA's publications, its activities, and its organisation. It also provides information about the journals and about membership. |
| National Grid for Learning | http://www.ngfl.gov.uk/index.jsp/sectionId=1&categoryId=99 | The much acclaimed National Grid for Learning (NGfl) can be found here and includes lots of useful pages for schools, further education and higher education as well as the addresses of various community grids. You might even manage a visit to the Virtual Teacher Centre (VTC) with information on curriculum subjects, school management and professional development. |
| QCA | http://www.qca.org.uk/ages14-19/index.html | An absolute must if you want to keep up with everything new going on in education – the present emphasis is on the new curricula which you can download (if you have the time and patience). |
| The Standards Site | http://www.standards.dfee.gov.uk | The Standards Site providing on-line help for teachers in England and is dedicated to rising standards of achievement in schools (well that is what it says on the front page). |
| Teacher Training Agency | http://www.useyourheadteach.gov.uk | Provides lots of information ranging form how to become a teacher to research about teaching. Particularly useful for these thinking of entering or returning to teaching. |

# 11 Glossary of Terms

| | |
|---|---|
| Introduction | The following terms are commonly used by awarding bodies in specifications and associated documents. |
| Aims | The broad educational or vocational purposes of a qualification. |
| Assessment Objectives | The criteria used to evaluate candidates' attainments. |
| Coursework | Tasks set and undertaken during the course which are integral to the course of study. |
| Entry Codes | The codes to be used when entering candidates for each unit and each qualification. |
| External Assessment | A form of independent assessment in which an awarding body sets or defines assignments, tests or examinations, specifies the conditions under which they are to be taken (including details of supervision and duration), and assesses candidates' responses. |
| Internal Assessment | A form of assessment that does not meet the definition of external assessment for a general or vocational qualification. |
| Internal Standardisation | The requirement for centres to standardise assessment across different teachers and teaching groups to ensure that all candidates at each centre have been judged against the same standards. |
| Key Skills | Those generic skills that can enable people to perform well in education, training and life in general. They can help people to become members of a flexible workforce and equip them with the means to benefit from life-long learning. |
| Moderation | The process through which internal assessment is monitored by an awarding body to ensure that internal assessment is valid, reliable, fair and consistent with required standards. |

# Appendices

## A Changes in Tier Content

**Foundation topics**

These topics were formerly in the Intermediate and Higher tiers, but are now included in the Foundation tier.

| Ref | Topic | Example |
|---|---|---|
| **Number** | | |
| F2.1j | Understand counter-example | Zoe thinks that when you square a number it is always even.<br>Give an example to show that Zoe is wrong.<br>The phrase 'counter-example' will not be used at Foundation Tier. |
| H2.2a | Prime factors | Write 48 as the product of prime factors.<br>Give your answer in index form. |
| H2.2a | HCF and LCM | Find the HCF of 48 and 64 |
| H2.4b | Estimation | Estimate the answer to $\dfrac{710 \times 3.98}{0.19}$ |
| H2.3a | Reciprocals | Find the reciprocal of 0.5 |
| F2.3m | Percentage increase or decrease | The height of a plant increases from 50 cm to 64 cm<br>Work out the percentage increase. |
| F2.4d | Limits | A rope is 48 m long, to the nearest metre.<br>Write down the least value of the length.<br>Write down the greatest value of the length. |
| F2.3c | Subtraction and addition of mixed numbers | Work out $4\frac{1}{4} - 2\frac{2}{5}$ |
| H2.2b | Index laws for multiplication and division of powers | Write $5^3 \times 5^4$ as a single power of 5 |
| H2.2b | Negative square roots | $x^2 = 36$<br>Write down the two possible values of $x$ |
| H2.3f | Ratio | Share £420 in the ratio 3 : 4 |
| H2.3n | Use of π in exact calculations | A circle has radius 3 m<br>Work out the area of the circle.<br>Give your answer in terms of π |
| F2.3p<br>F2.3q | Enter standard form on a calculator.<br>Interpret calculator display. | This will not be assessed at Foundation tier. |

| Ref | Topic | Example |
|---|---|---|

**Algebra**

| Ref | Topic | Example |
|---|---|---|
| F2.5c | Simple instances of index laws | Simplify $x^3 \times x^2$<br>Simplify $x^8 \div x^4$ |
| H2.5b | Expanding and simplifying expressions | Expand and simplify $6(x + 2) - 4(x - 2)$ |
| H2.5b | Expanding brackets with powers | Expand $x(x^3 + 2x)$ |
| F2.5e | Solve linear equations that require prior simplification of brackets and have the variable on both sides | Solve $4(2x - 3) = 3x + 8$ |
| H2.6a | *n*th term of a linear sequence | Write down the *n*th term of 3, 7, 11, 15, 19,… |
| H2.5b | Form and solve a linear equation. | The angles of a triangle are $x + 30$, $2x + 10$ and $4x$<br>Find the value of $x$. |
| H2.5 m | Trial and Improvement | Use trial and improvement to solve $x^3 + 2x = 100$<br>Give your answer to one decimal place.<br>Starting value and / or table may be given. |
| H2.5b | Expand the product of two linear expressions | Expand and simplify $(x + 1)(x - 2)$ |
| F2.5f | Change subject of a formula | Make $x$ the subject of $y = 2x + 3$<br>Questions will require at most two operations to rearrange. |
| H2.6e | Generate and plot points of quadratic functions. | Complete the table and use it to plot a graph of $y = x^2 + 2x - 3$ |
| H2.6e | Find solutions of a quadratic equation from graph | Use the graph to solve $x^2 + 2x - 3 = 0$ |
| F2.6c | Draw line of best fit through set of linearly related points and find its equation | Finding the equation of a given or drawn line will not be assessed at Foundation Tier. |
| F2.5d | Solve simple inequalities | Solve $3x + 2 > 11$ |
| F2.5d | Inequalities on a number line | What inequality is shown?<br>Open circles will mean strict inequalities. Filled in circles will mean inclusive inequalities |
| F2.6c | Graphs from real-life situations | Drawing or interpreting a distance-time graph. |

| Ref | Topic | Example |
|---|---|---|
| **Shape, Space and Measure** | | |
| F3.4j | Loci | Show all points within 4 cm of point A.<br>At most two constraints. |
| F3.4e | Constructions<br>Perpendicular bisector<br>Angle bisector<br>Angle of 60°<br>Perpendicular to a line<br>Perpendicular from a line | Construct the perpendicular bisector of the points A and B. |
| F3.2g | Regular polygons | The exterior angle of a regular polygon is 30 °<br>How many sides does the polygon have? |
| F3.4h | Area and perimeter of semicircles | Find the perimeter of a semicircle with radius 4 cm. |
| F3.2h | Pythagoras | Find the missing length of a right-angled triangle. |
| H3.2i | Volume of a prism | Calculate the volume of a prism with a right-angled, triangular cross-section given.<br>First part of question finding area of triangle. |
| H3.2i | Volume of a cylinder | Find the volume of a cylinder with radius 5 cm and height 10 cm.<br>Surface area of cylinder will not be assessed at Foundation tier. |
| F3.3c | Enlargement with fractional scale factor | Enlarge a given triangle by a scale factor $\frac{1}{2}$ about the point (0, 1) |
| H3.4b | Vector notation for translation | Translate a shaded triangle by a given vector. |
| H3.4a | Compound measure, density | Find the density of a solid with a given volume and mass.<br>The word 'mass' will be used in density problems. |
| F3.3e | Mid-point of two given coordinates | Find the mid-point of A(–2, 6) and B(4, –4). |
| F3.4a | Limits of measurements to nearest unit. | The mass of a suitcase is 22 kg to the nearest kilogram.<br>What is the least that it could weigh? |
| F3.3d | Distinguish between formula for length area and volume | Which of these formulae are lengths?<br>Formulae will be ones that are within Foundation candidates' experience such as $2l + 2w$ or $2\pi r$. |
| F3.3d | Understand effect of enlargement on area and volume | Rectangle *ABCD* is 2 cm by 3 cm. Rectangle *PQRS* is twice the size of rectangle *ABCD*.<br>How many times greater is the area of rectangle *PQRS* than the area of rectangle *ABCD*? |

**Handling Data**

| | | |
|---|---|---|
| F4.5f | Describe correlation | Use a given scatter diagram to describe the relationship between two variables. |
| F4.5h | Relative frequency | Calculate the relative frequency from a table of frequencies. |
| F4.5i | More trials leads to better estimates | Which pupil's results are more reliable?<br>Explain your answer. |
| F4.4b | Mean of a grouped table | Calculate an estimate of the mean.<br>Grouped frequency table given. |
| F4.4a | Frequency diagrams for grouped data | Draw a frequency diagram for the data.<br>Grouped frequency table given. Diagram can be frequency polygon or histogram with equal intervals. |

## Higher topics

These topics were formerly in the Intermediate and Foundation tiers, but are now included in the Higher tier. The majority of topics listed here are 'subsumed' and will be used when doing more challenging problems which have always featured on the Higher tier. Other topics are discrete and may need to be carefully revised by Higher tier candidates.

| Ref | Topic | Example |
|---|---|---|

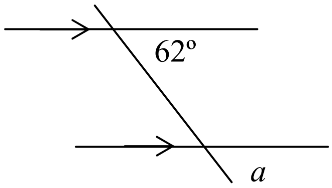
**Number**

| Ref | Topic | Example |
|---|---|---|
| H2.3h | Estimation | Estimate the value of $\frac{48\times 307}{57}$<br>Estimate the answer to $5.3 \times 19.8$ |
| H2.3o | Use calculators effectively and efficiently | Work out $\frac{4}{0.6^2}$<br>Work out $\frac{3.1\times 4.7}{8.3-5.6}$<br>Give your answers to one decimal place. |
| H2.3h | Linked calculations | You are given that $53 \times 821 = 43513$<br>Write down the value of $\frac{43513}{530}$ |
| H2.3j | New value after a percentage increase or decrease | The price of a washer was £450<br>What is the new price after a 4% reduction? |
| H2.3e | Fractions to percentages<br>($x$ as a fraction of $y \Rightarrow x$ as a percentage of $y$) | Write 90 out of 300 as a percentage. |
| H2.3c | Dividing fractions by fractions and whole numbers.<br>Multiplying fractions by whole numbers. | Work out $\frac{2}{3} \div \frac{1}{4}$<br>Work out $\frac{2}{3} \times 5$<br>Work out $\frac{2}{3} \div 4$ |
| H2.3c | Adding and subtracting fractions | Work out $\frac{2}{3} + \frac{4}{5}$<br>Work out $\frac{2}{3} - \frac{4}{5}$ |
| H2.3c | Distinguish between terminating and recurring decimals by performing short division | What is $\frac{7}{11}$ as a recurring decimal? |
| H2.3f | Simple ratio | Share £150 in the ratio 4 : 1 |

| Ref | Topic | Example |
|---|---|---|
| F2.3k | Division by decimal (up to two decimal places) by converting to division by integer. | Work out $31.2 \div 0.26$ |
| H2.3b | Brackets and hierarchy of operations | Put brackets in this calculation to make it correct<br>$2 + 3^2 \times 6 - 4 = 62$ |

**Algebra**

| | | |
|---|---|---|
| H2.5a | Algebraic expressions | John is $x$ years old.<br>Pam is 3 years younger than John.<br>Write an expression for Pam's age in terms of $x$. |
| H2.5d | Substitute positive and negative numbers into expressions | You are given that $a = -3$, $b = -4$.<br>Evaluate $a^2 - 2b$ |
| H2.5f | Solve linear equations where the variable appears on both sides of the equation | Solve $5x - 3 = 2x + 9$ |
| H2.5f | Solve linear equations that require prior simplification of brackets | Solve $4(2x - 3) = 28$ |
| H2.5f | Solve linear equations in one unknown with fractional coefficients | $\frac{x}{4} = 15$ |
| H2.5b | Factorisation | Factorise $x^2 + 5x$<br>Factorise $10a + 6$ |
| H2.5b | Expand and simplify | Multiply out $6(x - 7)$ |
| H2.6b | Plot linear graphs | Draw the graph of<br>$y = 3x - 1$ for $-3 \le x \le 3$<br>(no table given) |
| H2.6e | Generate and plot points of simple quadratic functions | Complete the table of $y = x^2 - 6$ for $-3 < x < 3$ and draw the graph |

| Ref | Topic | Example |
|---|---|---|
| **Shape, Space and Measure** | | |
| H3.2a | Angles in parallel lines and a transversal | What is the value of angle a. Give a reason for your answer?<br><br> |
| H2.3c | Definitions of quadrilaterals | Name a quadrilateral with:<br>2 pairs of equal sides<br>2 lines of symmetry<br>Rotational symmetry order 2 |
| H3.2i | Plan and elevation | Given an isometric drawing, draw the plan view and front elevation on a square grid. |
| H3.3a | Transformations | Describe a **single** transformation that takes shape A to shape B.<br>Draw a given **single** transformation.<br>Simple reflection, rotation, translation, enlargement with an integer scale factor. |
| H3.3d | Planes of symmetry | How many planes of symmetry does this solid have?<br>Sketch of cuboid with a square cross-section on one end. |
| H3.4a | Scale drawings | On the map, 1 cm represents 50 km.<br>Find the actual distance from A to B. |
| H3.4b | Speed, density | John travels 82.5 miles in 1 hour and 15 minutes.<br>What is his average speed? |
| H3.2g | Construct triangles | Construct the following triangle accurately.<br>Sketch of triangle with sides 8 cm, 6 cm and included angle of 75 °. 8 cm base line drawn. |
| H3.4d | Bearings | Measure the bearing of A from B. |
| H3.4d | Areas of compound shapes | Calculate the area of an L-shape. |
| H3.4d | Area of a trapezium | Calculate the area of a trapezium with dimensions of parallel sides and distance between them given. |
| H3.4d | Circumference and area of circles | Find the circumference and area of a circle with diameter 7 cm. |
| H3.4d | Convert between area and volume measures | Calculate the area of a rectangle of length 20 cm and width 30 cm.<br>Give your answer in square metres. |

| Ref | Topic | Example |
|---|---|---|

**Handling Data**

| Ref | Topic | Example |
|---|---|---|
| H4.3b | Design and use two way tables | The table shows the number of left-handed and right-handed boys and girls in year group.<br>A pupil is picked at random.<br>What is the probability that they are left-handed? |
| H4.4a | Stem and leaf diagrams | Calculate the mean of the data in the stem and leaf diagram. |
| H4.3a | Questionnaires | The following question is from a questionnaire on healthy eating.<br>Give two reasons why it is not a good question. |
| H4.4c | List outcomes of combined events | A dice and a coin are thrown together.<br>One outcome could be head and 6.<br>List all the possible outcomes. |
| H4.4d | Mutually exclusive outcomes. | Find the missing probability from a table of values. |
| F4.4b | Mean of a discrete frequency table | The table shows the number of children per family.<br>Calculate the mean. |
| H4.4b | Expectation | How many 4's would you expect in 100 throws?<br>Biased or fair four-sided spinner. |

B

# Student Support Sheet on Question Papers

## Question Paper Terminology

Listed below are examples of instructions given in questions and an amplification of their meaning.  The list is as comprehensive as possible but does not claim to be exhaustive.

| What we Say... | **What it means....** |
|---|---|
| Estimate | Round the numbers to 1 s.f. and use these to obtain an answer.<br>Find the mean of a grouped frequency table.<br>Average speed. |
| Explain | Use words to explain an  answer. |
| You **must** show your working | You will be penalised if you do not show your working. |
| Simplify | Collect terms together. |
| Simplify fully | Collect terms together and factorise the answer. |
| Show that | Use words, numbers or algebra to show an answer. |
| Prove | A rigid algebraic or geometric proof is required. |
| Work out | Normally means a calculation is involved but it may be possible to do it mentally. |
| Calculate | Will need a calculation that requires a calculator or a formal (such as column) method. |
| Measure | Use a ruler or protractor to measure a length or an angle. |
| Hence | Use the previous answer to proceed. |
| Hence, or otherwise | Use of the previous answer is expected but another method will be accepted. |
| Describe fully in transformations | Reflection – define the mirror line.<br>Translations – state vector.<br>Rotations – state centre, angle and direction.<br>Enlargement – state scale, factor and centre. |
| Factorise | Take out the common factor or factorise into two brackets if a quadratic. |

| | |
|---|---|
| Factorise fully | Usually means that there is more than one common factor, ie, indicates that there are at least two stages in the factorisation. |
| Use the graph | Do not calculate, read from the graph. Always worth putting lines on the graph to show where the answer came from. |
| Give an exact value | Give answer as a square root or surd form (non-calculator paper) |
| Give your answer in terms of $\pi$/in surd form | Give answer in terms of $\pi$ /in surd form (non-calculator paper) |
| Give answer to a sensible degree of accuracy | Normally no more accurate than values in the question. If question has values to 2 s.f. then give answer(s) to 2 s.f. or 1 s.f.<br>Trigonometrical answers accepted to 3 s.f. |
| Give answer to (2 d.p.) | Give answers to required accuracy. You will lose marks if you do not. |
| Not drawn accurately | Next to a diagram to discourage measuring of lengths or angles. |
| Drawn to scale | Next to diagram to encourage measuring by candidates. |
| Do an accurate drawing | Use compasses to draw lengths, protractors to measure angles (and a sharp pencil) |
| Use a ruler and compasses | A ruler may be needed to measure but more often than not we mean, use a straight edge and compasses. Used in constructions. |
| Use an algebraic method | Do not use trial and improvement. Working will be expected. |
| Do not use trial and improvement | An algebraic method is expected. Any sign of trial and improvement will be penalised. |
| Expand | Multiply out using distributive law. |
| Multiply out | Multiply out using distributive law. |
| Expand and simplify | Multiply out using distributive law and then collect terms. |
| Multiply out and simplify | Multiply out using distributive law and then collect terms. |
| Give a counter-example | Give a numerical or geometrical example that disproves a statement. |
| Solve | Find the values(s) of ($x$) that makes the equation true. |
| Make ($x$) the subject | Rearrange a formula. |
| Express, in terms of | Use given information to write an expression using only the letter(s) given. |
| Write down | Working out is not needed to get an answer. |

C

# Proof

Section 1

Within the written papers, the rigour and quality of proof required will depend on the grade at which it is being examined. The differing demands are reflected in the Assessment Criteria for *Using and Applying Mathematics* (AO1 task) Strand 3.

The table below matches the Assessment Criteria with the levels of proof as defined by Sue Waring in *Can You Prove It?* (Reproduced by permission of the Mathematical Association, 2000).

| Level | Level of Proof defined by Sue Waring | Assessment Criteria for UAM (AO1 task) Strand 3 | Mark |
|---|---|---|---|
| 0 | Pupils are ignorant of proof. | No evidence. | 0 |
| 1 | Pupils are aware of the notion of proof but consider that checking a few special cases is sufficient. | Candidates are beginning to give a mathematical justification for their generalisations; they test them by checking particular cases. | 4 |
| 2 | Pupils are aware that checking a few special cases is not a proof but are satisfied that either checking for more varied examples, or using a generic example, is enough for a proof. | Candidates justify their generalisations, showing some insight into the structure of the situation being investigated. They appreciate the difference between mathematical explanation and experimental evidence. | 5 |
| 3 | Pupils are aware of the need for a generalised proof and, although unable to construct a valid proof unaided, are likely to be able to understand the creation of a proof at an appropriate level of difficulty. | Candidates examine generalisations commenting constructively on the reasoning and logic employed. | 6 |
| 4 | Pupils are aware of the need for, and can understand the creation of, generalised proofs and are also able to construct such proofs in a limited number of familiar contexts. | Candidates' reports include mathematical justifications explaining their solutions to problems involving a number of features or variables. | 7 |
| 5 | Pupils are aware of the need for a generalised proof, can understand the creation of some formal proofs, and are able to construct proofs in a variety of contexts, including some unfamiliar. | Candidates provide a mathematically rigorous justification or proof of their solutions to a complex problem, considering the conditions under which it remains valid. | 8 |

As a UAM Strand 3 mark of 4 relates broadly to a Grade D, candidates working at Grade E or below would not be expected to have much understanding of mathematical proof.

An A* candidate would be expected to understand the need for a generalised proof as exemplified by Sue Waring's level 5 and the UAM Strand 3 mark 8.

Sections 1 and 2 of AO3 (shape, space and measures) give candidates the opportunity to appreciate geometrical proof. Candidates **will not** be required to reproduce the proofs listed in the specification (for example, H3.3h which says 'Prove and use…' the circle theorems). However, candidates will be expected to use these results in constructing a proof they have not met before. With their classes, teachers may find it helpful to use the versions of the proofs given in Section 2.

Foundation Tier

Candidates will need to know that 'a result' can be disproved by a single counter example and that even if a result works for many numerical examples it may not be valid for all values.

Example (at grade D)

Freda thinks that when you square a number you always get an even number answer. Give an example to show that Freda is wrong.

Answer

$3 \times 3 = 9$ and 9 is an odd number

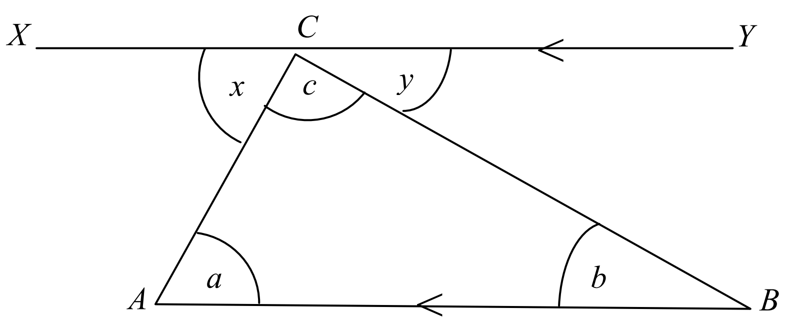
Example (at grade C)

The diagram shows a triangle $ABC$

$XYC$ is a straight line parallel to $AB$

Prove that the angles on a straight line add up to 180°

Explain your working




Answer

$a = x$ (alternate angle)

$b = y$ (alternate angle)

$a + b + c = 180°$ (Angles in triangle add up to 180°)

So $x + b + y = 180°$

Angles on a straight line add up to 180°

**Higher Tier**

At the lower grades candidates will not be expected to give rigorous proof and will be asked to 'Show that' a result is valid. This can be done by written explanation or annotation on diagrams for example.

Appropriate structure will be provided in the question so that candidates can explain a generalised result.

**Example (at grade C)**

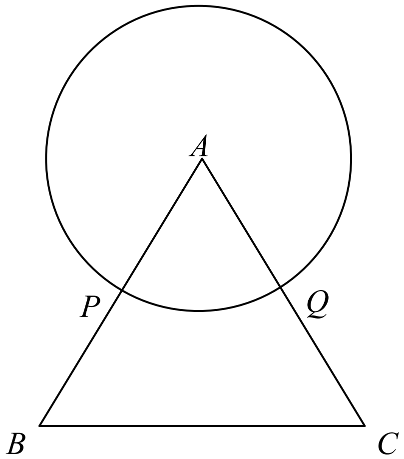
In the diagram, $A$ is the centre of the circle

$ABC$ is an isosceles triangle in which $AB = AC$

$AB$ cuts the circle at $P$ and $AC$ cuts the circle at $Q$

(a) Explain why $AP = AQ$

(b) Show that (or explain why $PB = QC$




**Answer**

A candidate may write:

(a) $AP$ and $AQ$ are radii

(b) $AB = AC$ So $AB - AP = AC - AQ$ (or in words) $PB = QC$

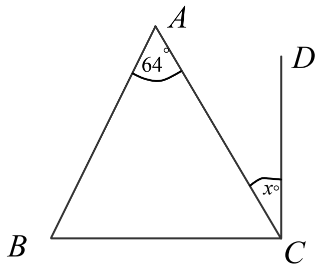
**Example (at grade B)**

(a) In Fig. 1, $AB = AC$ and angle $B\hat{A}C = 64°$

$DC$ is perpendicular to $BC$ and $A\hat{C}D = x°$

Calculate the value of angle $x$

Fig. 1 (not drawn accurately)



Example (at grade A)

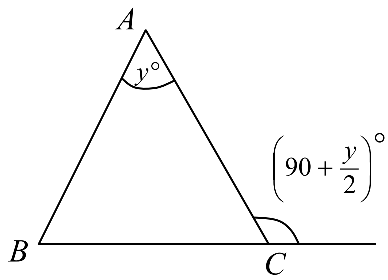
(b) In Fig. 2, $ABC$ is an isosceles triangle in which

$AB = AC$ and $B\hat{A}C = y\,°$

Using the result in part (a), or otherwise, show that the exterior angle of the triangle at

$C$ is $\left(90 \quad + \dfrac{y}{2}\right)^{\circ}$

Fig. 2 (not drawn accurately)




Answer

Candidates will probably mark base angles with letters.

Base angles of an isosceles triangle are equal

$x + x + y = 180°$ (angle sum of a triangle)

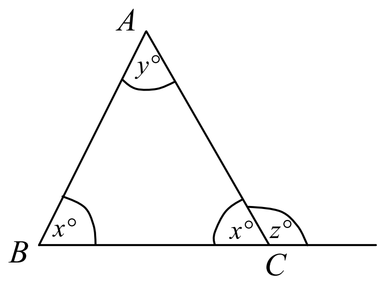
$2x = 180° - y$

$x = \left(90 - \dfrac{y}{2}\right)^{\circ}$

$x + z = 180$ (on a straight line)

$x + z = 180° \left(90 - \dfrac{y}{2}\right)^{\circ}$

$= \left(90 + \dfrac{y}{2}\right)^{\circ}$

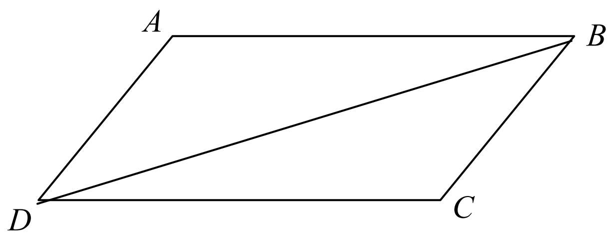


Example (at grade A*)

In questions designed to assess the highest grades, candidates would be expected to construct a proof which they may not have encountered before.

*ABCD* is a parallelogram. The line *BD* is drawn.

Prove that triangles *ABD* and *BCD* are congruent.




Answer

Candidates have a choice of proofs. The following are all valid starting points:

Identify two pairs of equal angles – stating reasons (two pairs of equal alternate angles, or one pair plus opposite angles of a parallelogram)

Identify the common side

Give the reason for congruence (AAS)

*AB* = *DC* (opposite sides of a parallelogram)

*AD* = *BC* (opposite sides of a parallelogram)

*DB* is a common side

Triangles *ABD* and *BCD* are congruent because of

Angle *ADB* = Angle *DBC* (alternate angles)

Example (at grade A*)

You are given that $(n + 3)^2 - (n + 1)^2 = 4(n + 2)$

Prove this result is true.

Answer

$$\begin{aligned}(n + 3)^2 - (n + 1)^2 &= n^2 + 6n + 9 - (n^2 + 2n + 1)\\ &= n^2 + 6n + 9 - n^2 - 2n - 1)\\ &= 4n + 8\\ &= 4(n + 2)\end{aligned}$$

Note that it is important that the algebra is exact, so candidates will need to show that they have used the minus sign in front of the bracket.

For example $(n + 3)^2 - n + 1)^2 = n^2 + 6n + 9 - (n^2 + 2n + 1)$

$= 4n + 8$ would lose a mark.

Also it is important that the final factorisation is shown.
An algebraic proof should start with the expression on the left hand side and end up with the expression on the right hand side.

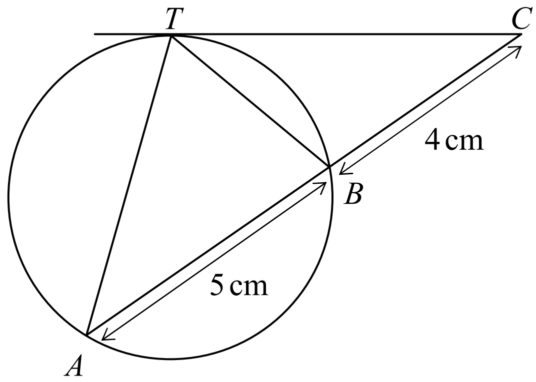
Example (at grade A*)

*CT* is a tangent to the circle at *T*.

$AB = 5$ cm and $BC = 4$ cm.

(a) Prove that triangles *BTC* and *TAC* are similar.

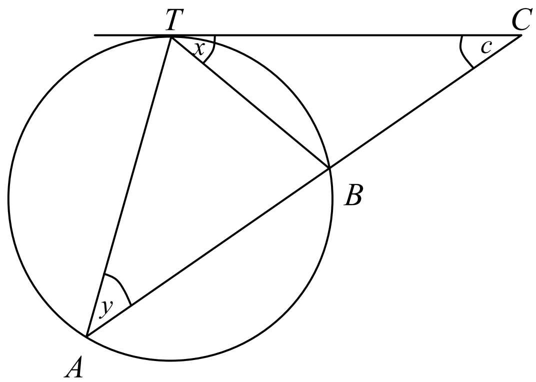
(b) Hence find the length of *CT*.




Answer

A candidate will probably mark, say, angles *x*, *y* and *c*.

In triangles *BTC* and *TAC*




$x = y$ (angle between tangent and chord equals angle in alternate segment)

Angle *c* is common to both triangles.

Therefore the triangles are equiangular and so are similar

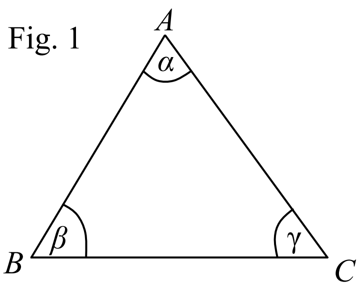
## Section 2: Exemplar Proofs

Teachers should refer to the specifications. At Foundation tier, for example, candidates are required to 'understand a proof that the angle sum of a triangle is 180°' and to 'understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices'. At the Higher tier, candidates are required to 'prove and use… (circle theorems)'. In any formal proof there needs to be an understanding of the assumptions. Versions of the proofs of the theorems in the specification have been reproduced here for the benefit of teachers. **Candidates will not be required to reproduce these proofs in examinations**. Teachers should note that other versions of the proofs are acceptable.

### Proof that the angle sum of a triangle is 180°

Take a triangle $ABC$ with angles $\alpha$, $\beta$ and $\gamma$ (Fig. 1)

Fig. 1



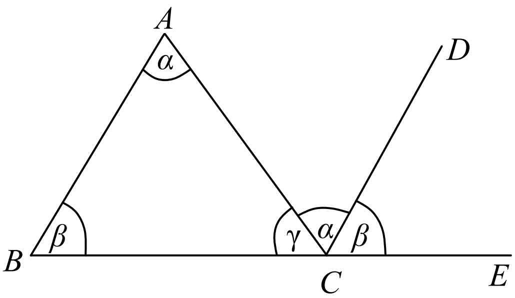

Draw a line $CD$ parallel to side $AB$ and extend $BC$ to $E$ (Fig. 2)

$A\hat{C}D = B\hat{A}C = \alpha$ (alternate angles)

$D\hat{C}E = A\hat{B}C = \beta$ (corresponding angles)

$BCE$ is a straight line so $\alpha + \beta + \gamma = 180°$

Fig. 2




### Proof that the exterior angle of a triangle is equal to the sum of the two opposite interior angles

In the diagram, angle $A = \alpha$, angle $B = \beta$ and angle $C = \gamma$.

$\alpha + \beta + \gamma = 180°$ (angle sum of a triangle)

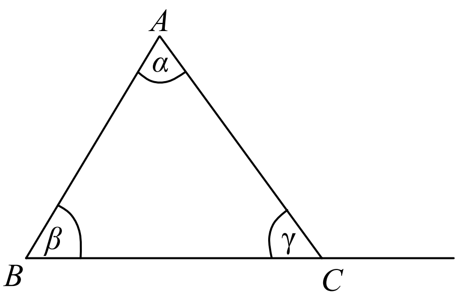
$\gamma = 180° - \alpha - \beta$

Exterior angle at $C = 180° - \gamma = 180° \; (180° - \alpha - \beta)$

$= 180° - 180° + \alpha + \beta$

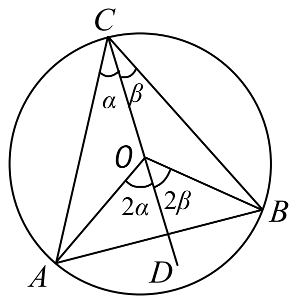
$= \alpha + \beta$

= sum of two opposite interior angles



Proof that the angle subtended by a chord at the centre of a circle is twice the angle subtended at the circumference in the same segment

In the diagram, $AB$ is a chord of circle centre $O$ and $C$ is a point on the circumference.




Draw a line from $C$ through $O$ to $D$

Let $A\hat{C}O = \alpha$; let $B\hat{C}O = \beta$

$C\hat{A}O = \alpha$ (triangle $OAC$ is isosceles)

Therefore $A\hat{O}D = 2\alpha$ (exterior angle equal to sum of opposite interior angles)

Similarly $D\hat{O}B = 2\beta$

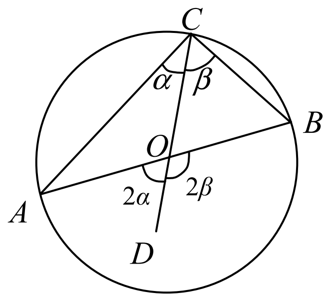
Hence, $A\hat{O}B = 2\alpha + 2\beta$

$= 2(\alpha + \beta)$

$= 2 \times A\hat{C}B$

Proof that the angle subtended by a diameter at the circumference is 90°

In the diagram, $AB$ is a diameter of circle centre O and $C$ is a point on the circumference.




Draw a line from $C$ through $O$ to $D$

Let $A\hat{C}O = \alpha$; let $B\hat{C}O = \beta$

$C\hat{A}O = \alpha$ (triangle $OAC$ is isosceles)

Therefore $A\hat{O}D = 2\alpha$ (exterior angle equal to sum of opposite interior angles)

Similarly $D\hat{O}B = 2\beta$

Hence, $A\hat{O}B = 2\alpha + 2\beta = 2(\alpha + \beta) = 180°$

Therefore, $\alpha + \beta = 90°$

So the angle at the circumference is $90°$

Proof that the opposite angles in a cyclic quadrilateral add to 180°

In the diagram, $ABCD$ is a quadrilateral drawn inside a circle centre $O$

Draw the radii $OA$ and $OC$

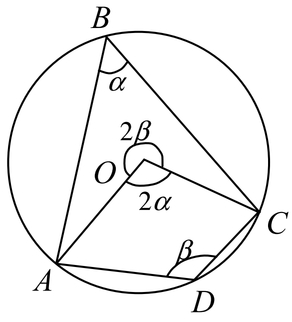
Let $A\hat{B}C = \alpha$; let $A\hat{D}C = \beta$

$A\hat{O}C$ (obtuse) $= 2\alpha$ (angle at centre is twice the angle at the circumference)

Similarly $A\hat{O}C$ (reflex) $= 2\beta$

Hence, $360° = 2\alpha + 2\beta = 2(\alpha + \beta)$

Therefore, $\alpha + \beta = 180°$




Proof that angles subtended by a chord at the circumference in the same segment are equal

In the diagram, $AB$ is a chord of the circle centre $O$, and $C$ is a point on the circumference

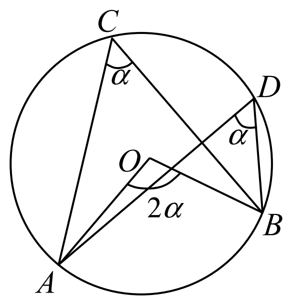
Let $A\hat{C}B = \alpha$

$A\hat{O}B = 2\alpha$ (angle at centre twice angle at circumference)

Let $D$ be another point on circumference

$A\hat{D}B = \alpha$ (angle at centre twice angle at circumference)

Hence, $A\hat{C}B = A\hat{D}B = \alpha$




Proof of the alternate segment theorem

In the diagram, $DB$ is a tangent at $B$ to the circle centre $O$

$A$ and $C$ are points on the circumference

Draw the radii $OB$ and $OC$

Let $O\hat{B}C = \beta$

$B\hat{O}C = 180 - 2\beta$ (triangle $OBC$ is isosceles)

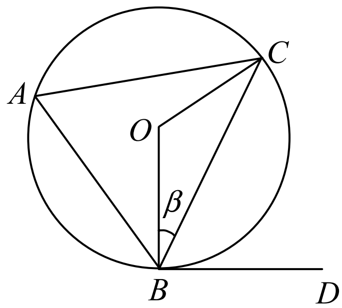
$$B\hat{A}C = \frac{180 - 2\beta}{2} = 90° - \beta$$

(angle at the centre is twice the angle at the circumference)

But $C\hat{B}D = 90° - O\hat{B}C = 90° - \beta$

(angle between tangent and radius is 90°)

Hence, $B\hat{A}C = C\hat{B}D$



D

# Student Support Sheet
# AO1 Coursework

Candidates guidelines for AO1 tasks

- Keep things as simple as possible, especially at the start of the task. Try out your ideas in order to spot potential difficulties or areas for further investigation.
- Adopt a systematic approach to your work.
- Collect any results that your work has produced and put them in a table.
- Present your data to provide the most information (ie, redefining your tables, drawing, diagrams, graphs, etc). Give reasons for your choice of presentation and comment on the data. Make suggestions on how you could improve the presentation and then carry out the required improvements.
- Analyse your results and state any observations, rules or patterns that you see. Try out your general rule on some new data.
- Give reasons to justify your rules, for example, by referring back to diagrams or graphs. Don't make statements or draw conclusions that you have not checked out, or that are not supported by your results.
- Don't be afraid to write up ideas that did not work. Try to modify these ideas in order to make further progress.
- Try to develop the task by adding something different to the original problem. Consider how the task could be changed in order to extend your investigation.
- Change one variable/feature at a time, exploring this thoroughly before moving on to another change.
- Write up your work in stages rather than leaving it all to the end.

| | |
|---|---|
| **Writing Framework for AO1 tasks** | If you can answer the following questions, then you should be able to write up your report. |
| Planning your task | What is your task about?<br>What do you have to do?<br>Have you got any questions of your own which you want to add to the original tasks?<br>What will you do first?<br>What information or data will you have to get in order to complete the task?<br>How much information do you think you will need to collect?<br>Where will you get the information from?<br>How will you know when you have got enough information?<br>How accurate does your data have to be?<br>How will you check that your data is accurate?<br>What will you do with the data in order to investigate your original task? |
| Working through the task | What sort of calculations will you need to do to get your solution?<br>How will you record your results?<br>Are there any special mathematical techniques you will need to use?<br>Can you give reasons for your choice of calculations/techniques?<br>Are you going to change anything to make the task different?<br>Is there more than one way of tackling this task?<br>Have you thought of trying a different method?<br>What will help you to decide the best way of completing this task?<br>What methods will you use to present your results?<br>Why will you use particular methods of presentation?<br>What advantages have these methods of presentation? |
| Results and conclusions | Have you written down all your results?<br>Have you checked your results?<br>Have you noticed anything about your results?<br>Have you discovered any rules which always seem to work for your results?<br>How will you check that your results are always true?<br>Is there any more work you could do to develop the task further?<br>Will you need to collect any more information?<br>Have you given reasons for all your answers/results? |

E

# Student Support Sheet
# AO4 Coursework

Candidates guidelines for AO4 tasks

- State a clear hypothesis: for example 'The lengths of words in a broadsheet newspaper are longer than the lengths of words in a tabloid newspaper'.
- Decide on the data needed, specify a suitable source for collecting data and methods used (eg, questionnaire).
- Carry out a trial run on a small sample. Try out your ideas in order to spot potential difficulties or areas for further investigation.
- Carry out improvements needed. Explain what you have done and why.
- Decide on an appropriate sample size. Give reasons for your choice.
- Decide on an appropriate sample method. Give reasons for your choice.
- Present your data to provide the most information (ie, consider tables, diagrams, graphs, etc). Comment and interpret the presentation of data with reference to your hypothesis.
- Make suggestions on how to improve the presentation of data and carry out the required improvements.
- Choose and calculate the best representations of your data (eg, the median). Give reason for your choice, for example by referring back to diagrams or graphs. Comment and interpret your results with reference to your hypothesis.
- Don't be afraid to write up ideas that did not work. Try to modify these ideas to find another idea or method. You may need to collect further data to enable you to move the task on.
- Don't make statements or draw conclusions that you have not checked out, or that are not supported by your results.
- Summarise your results in a report by referring to your tables/graphs and calculations in the light of your hypothesis.

| | |
|---|---|
| **Writing Framework for AO4 tasks** | If you can answer the following questions, then you should be able to write up your report. |
| Planning your task | What is your task about?<br>What is your hypothesis?<br>Have you got any hypotheses of your own which you want to add to the original ones?<br>What will you do first?<br>Will you need to design a questionnaire?<br>What information or data will you have to get in order to complete the task?<br>How much information do you think you will need to collect?<br>Where will you get the information from?<br>How will you know when you have got enough data?<br>How accurate does your data have to be?<br>How will you check that your data is accurate?<br>What will you do with the data in order to investigate the task? |
| Working through the task | What sort of calculations will you need to do?<br>How will you record your results?<br>Are there any special statistical techniques you will need to use?<br>Can you give reasons for your choice of techniques/calculations?<br>Are you going to change anything to make the task different?<br>Is there more than one way of investigating this task?<br>Have you thought of trying a different method?<br>What will help you to decide the best way of completing this task?<br>What methods will you use to present your results?<br>Why will you use particular methods of presentation?<br>What advantages have these methods of presentation? |
| Results and conclusions | Have you written down all your results?<br>Have you checked your results?<br>Have you noticed anything about your results?<br>Have you managed to draw any conclusions from your results?<br>Do your results have any limitations?<br>Is there any more work you could do to develop the task further?<br>Will you need to collect any more information?<br>Have you given reasons for all your answers/results? |

F

# Submitting a Common Data Handling and Statistics Task

Introduction

This is a working document that is an initial attempt to give advice on both the *Handling Data* and GCSE Statistics coursework such that one common piece of work will address both sets of criteria. When attempting to produce such a single piece of work, it is not necessary to cover all of the statements below, however they are there to act as a guide and, hopefully, a useful template for all candidates. The notes below should be read in conjunction with the coursework criteria for each subject and AQA's elaboration and further exemplification documents

The example given is only one of numerous starting points for this type of work.

AO4 Strand 1

**Statistics Strands 1 and 2**

- Must take a sample – giving reasons for method used.
- Why do it in the first place? eg, reasons for hypothesis.
- Which sampling method and **why** you are rejecting the others (eg, random, stratified, systematic).
- Sample size – giving reasons and justification.
- If a pre-test is suitable – give reasons and justifications and how/why you may change your future methods.
- May well compare primary data sample (eg, school) against database already known, if appropriate.
- MUST be able to see population lists with sample on it and, importantly, how you arrived at the sample (reasons for starting point if using systematic sampling).
- Aims and development can occur throughout and would actively encourage a running commentary.
- Consideration of results and further development eg, moving the task forward may and probably should occur as the task evolves

AO4 Strand 2

**Statistics Strands 3 and 4**

- Throughout this strand(s) satisfying the Statistics criteria would in most if not all instances, satisfy the equivalent Mathematics criteria.
- Justify all methods of recording, illustrating and calculating to fully interrogate the data (running commentary).
- Use, explain/interpret what you have produced in context.
- MUST comment on the data.
- Need to show a range of appropriate methods that show different aspects of the data (eg, range could be averages/mean, mode, median ⇒ Cumulative frequency/Box Plot ⇒ Normal curve/Spearman's. At lower levels: Bar charts ⇒ averages ⇒ scatter could get 3 to 5 in strand 3 Statistics).
- Range of appropriate presentation **and** calculations is essential.
- It needs to be appropriate eg, if scatter diagram has some correlation then it would be appropriate to move to Spearman's Rank. Find equation of line of best fit and use it – even moving to product moment.
- It is possible to jump to the most effective method saying why you reject the other possible appropriate methods (it would be unlikely to be seen except at the highest levels).
- For higher marks an increased emphasis on accuracy of both calculations and graphical representation is required (the use of graph paper is expected).

AO4 Strand 3

**Statistics Strand 5**

- Comment upon/interpret the data throughout.
- Must summarise and link to hypothesis.
- Interpret and not just make observations of results.
- For higher marks (5+ with statistics) must correctly interpret all measures used relating back to hypothesis (should have developed into more than one hypothesis/greater depth).
- **Must** evaluate strategy.
- For higher awards must have advanced detailed valid appreciation of the statistical significance of the results.
- Recognise the limitations.
- Full consideration of the effectiveness of the study.

Example

I want to see if temperature affects life expectancy.

My hypothesis is:

"The higher the average temperature, the lower the life expectancy"

Plan

- Database approximately 300 countries.
- Whole list printed and seen.
- Size of sample eg, 30 = 10% valid.
- Method of sample – up to you as long as reasons given eg, range of temperatures, may order them and systematically sample, or group and stratify.
- **Could** pre-test to see if database is close to world average
- What techniques are you going to use and why?

Gain results showing a range of techniques for both Presentation and Calculations

eg, Averages (mean, mode, median), simple histogram ⇒ Cumulative frequency, box plots ⇒ scatter diagrams, line of best fit, Spearman's ⇒ Standard Deviation

MUST HAVE INTERPRETATION

Produce histogram therefore increase sample size with reasons.

Give reasons why following one method and rejecting others.

May, having done this, develop the strategy further – reasons why eg, life expectancy versus G.D.P therefore task becomes substantial ⇒ demanding, candidate's own development.

Comment on results throughout linked to hypothesis and as a result develop problem further.

Link all findings together.

Comment on the validity of findings, their limitations and how you could improve them.

Evaluate the whole strategy; evaluation can include justifying their choice and effectiveness of their diagrams and calculations.