Teacher Support

GCSE Mathematics (4360)

Unit 2 – Number and Algebra (43602F and 43602H)

Feedback materials on the March 2011 examination

Summer 2011

Version 1.1

Contents

Question 4

4 Anil, Ben, Chloe, Dave and Emma play a game.
Here is some information about the number of points they score.

Anil scores 20 points.
Ben scores 12 more points than Anil.
Ben scores twice as many points as Chloe.
Dave scores 11 fewer points than Ben.

The total number of points scored is 100.

How many points did Emma score? *(3 marks)*

Mark scheme:

4	(B =) 32 (C =) their 32 ÷ 2 or 16 seen (D =) their 32 – 11 or 21 seen	B1 M1 M1	
	(E =) 11	A1 ft	100 – (20 + their B + their C + their D) ft dependent on both M's

This question was targeted at grade G and assessed AO3

The mean mark was 2.63, with over 45% of candidates scoring all 4 marks.

There was some misreading of the question and some errors in arithmetic, but the question proved to be accessible to all candidates.

Candidate A

How many points does Emma score?

20 + 12 = 32 bens

32 ÷ 2 = 16 chloe

32 − 11 = 21 Daves

20 Anil

$$\begin{array}{r} 32 \\ 16 \\ 21 \\ + \ 20 \\ \hline 89 \end{array}$$

Answer Anna has 11 points *(4 marks)*

This candidate gave a perfect answer, apart from writing Anna instead of Emma!

Mark awarded = 4

Candidate B

Anil scores 20 points.
Ben scores 12 more points than Anil.
Ben scores twice as many points as Chloe.
Dave scores 11 fewer points than Ben.

The total number of points scored is 100.

How many points does Emma score?

Ben 32
chloe 16
+ dave 21

69

$$\begin{array}{r} 100 \\ - \ 69 \\ \hline 31 \end{array}$$

Answer 31 *(4 marks)*

The only error this candidate makes is to omit Anil's 20 points when adding up the scores before doing the final subtraction.

Mark awarded = 3

Candidate C

How many points does Emma score?

Gail 20 18

Ben 32 21

Chloe 8 20

Dave 21 32

81

Answer 19 *(4 marks)*

This candidate scores marks for getting the points totals for Ben and Dave correct, but there is no working to show where Chloe's 8 points came from.

Although the arithmetic is correct using the candidates' values, the follow through accuracy mark cannot be awarded because only one of the method marks has been earned.

Mark awarded = 2

Candidate D

How many points does Emma score?

$20 + 12 = 32$

$\frac{1}{2}$ of $12 = 6$

$32 + 6 = 38 + 1 = 39$

Answer 61 *(4 marks)*

Ben = 32 points, scores 1 mark, but the candidate does not use the score of 32 to calculate the rest of the points totals.

12 is used instead of 32 to obtain totals of 6 and 1 respectively for Chloe and Dave.

To earn the two method marks their value for Ben must be used.

Mark awarded = 1

Question 5

5 There are 12 cans in one pack of cola.
One pack costs £6.50

Josh buys 10 packs.
He sells 90 of the cans for 80p each.
He sells the rest at half price.

How much profit does he make? *(5 marks)*

Mark scheme:

5	10 × 6.5 or (£)65 or 6500(p)	M1	
	90 × 80 or 7200(p) or (£)72	M1	
	(120 – 90) × 40 or 1200(p) or (£)12	M1	
	their 72 + their 12 – their 65	M1	SP (full) + SP (half) – CP
	19	A1	

This question was targeted at grade F and assessed AO2. It was also a functional question.

The mean mark was 2.57 and the question proved to be a good discriminator resulting in a fairly even distribution of marks.

There were some errors in arithmetic and some instances of candidates not reading the question carefully enough, but almost all were able to make an attempt.

Candidate E

How much profit does he make?

10 × £6.50 = £65

~~90 × 0.8~~ = 90 × 80 = 7200 ÷ 100 = £72

90 × 40 = 3600 ÷ 100 = £32

£72

£104

£104 − £65 = £39

Answer £ 39 *(5 marks)*

The first two steps are correct but then the candidate does not subtract 90 from 120 to find the number of cans that remain to be sold, instead multiplying 90 by 40.

There is a slip in the arithmetic as 3600p becomes £32, but thereafter the candidate's method is correct.

Mark awarded = 3

Candidate F

How much profit does he make?

6.50 × 10 = £65

120

90

30

3

90 × 80

= £72.00

£65.00

30 × 40

= £12.00

= 84

60

24

Answer £ 24 *(5 marks)*

With the exception of the very last step, all the correct calculations can be seen.

The candidate uses £60 instead of the £65 calculated earlier when working out the final profit. This can be treated as a mis-copy of their own values, rather than an error in their method.

Mark awarded = 4

Candidate G

How much profit does he make?

10 packs - £65.00 | 13.00
90 cans - £72.00 | 13.00
12 x 10 = 120 - 90 = 30 cans left | 13.00
30 cans for 40p = £12.00 | 13.00
12.00 72.00 | 13.00
84.00 | 12.00 | 65.00
-65.00 | 84.00
19.00

Answer £ 19.00 *(5 marks)*

All the correct calculations can be seen and the candidate scores full marks.

Mark awarded = 5

Candidate H

How much profit does he make?

60

6.50 x 10 = 75

12 x 10 = 120   120 - 90 = 30

90 x 80 = 72

30 x 40 = 1200   72 + 12 = 84

84 - 75 = 9

Answer £ 9 *(5 marks)*

There is an arithmetic error in the first step, although the method mark is earned for 6.50 × 10.

All the other calculations are correct, but the inevitable consequence of the earlier slip is an answer of £9 rather than £19.

Mark awarded = 4

Question 10

10b n represents an even number.

Explain why $(n + 1)(n - 1)$ is always odd. *(2 marks)*

Mark scheme:

10b	Recognises both brackets are odd	M1	oe for $n^2 - 1$, even × even = even
	odd × odd = odd	A1	oe for $n^2 - 1$, even – 1 = odd

This question was targeted at grade D and assessed AO2.

The mean mark was only 0.22, with only just over 20% of candidates scoring anything at all.

There were many instances of substituting numbers, and although some credit was given if this was accompanied by a partial explanation, it is not the recommended method to use for questions requiring explanation. An answer using the words odd and even is expected.

Candidate I

> n represents an even number.
>
> Explain why $(n + 1)(n - 1)$ is always odd.
>
> because n+1 = odd n-1 = odd and a odd × odd = odd.
>
> *(2 marks)*

This was one of the few fully correct answers.

Mark awarded = 2

Candidate J

> n represents an even number. answ
>
> Explain why $(n + 1)(n - 1)$ is always odd. If we substitute n for 4 which is an even number
>
> substitute 4
>
> 4+1 4-1
>
> 5 × 3 = 15 = odd number
>
> 4+1 = 5 × 4-1 = 3
>
> 5× 3 = 15 which is an odd number
>
> as an odd × odd = odd. *(2 marks)*

Although the candidate substitutes 4 into the expression, the candidate also explains that the result of the substitution (5 × 3 = 15) is such that it fits in with ‘odd × odd = odd’.

This was considered worthy of 1 mark, as a special case.

Mark awarded = 1

Candidate K

n represents an even number.

Explain why $(n + 1)(n - 1)$ is always odd.

an even + 1 is always an odd

and an even −1 is always an odd

(2 marks)

The candidate recognised that both brackets must be odd, but fails to go any further.

Mark awarded = 1

Candidate L

n represents an even number.

Explain why $(n + 1)(n - 1)$ is always odd.

because if you + or − 1 from a

even number a odd number is always

one above and under a even

number.

(2 marks)

Perhaps not phrased very explicitly, but nevertheless, the candidate does realise that both brackets will be odd.

Mark awarded = 1

Question 11

11b n is a positive whole number.
$6n - 1$ is **not** a prime number.

Work out a possible value for n. *(2 marks)*

Mark scheme:

11b	At least two correct substitutions evaluated correctly if answer not given	M1	5, 11, 17, 23, 29, 35, ...
	(n =) 6	A1	or other correct values eg 11 or 13 or 16 or 20

This question was targeted at grade E and assessed AO2.

The mean mark was 0.44. Substituting values for n was all that was required to score at least 1 mark.

Candidate M

Work out a possible value for *n*.

6 × 1 − 1 = 5
6 × 2 − 1 = 11
6 × 3 − 1 = 17
6 × 4 − 1 = 23
6 × 5 − 1 = 29
6 × 6 − 1 = 35

Answer n = 6 *(2 marks)*

A methodical and correct solution.

Mark awarded = 2

Candidate N

Work out a possible value for *n*.

6 × 3 = 18 − 1 = 17 ✗
6 × 4 = 24 − 1 = 23 ✗
6 × 5 = 35 − 1 = 34 ✓

Answer n = 5 *(2 marks)*

Two substitutions were evaluated correctly, but then a mistake working out 6 × 5 led to a wrong conclusion.

Mark awarded = 1

Candidate O

Work out a possible value for n.

$6 \times 7 = 36 - 1 = 35$

$5 \times 7 = 35$

$1 \times 35 = 35$

Answer ~~38~~ 7 (2 marks)

Only one complete substitution can be seen which is also incorrect.

Another example of poor knowledge of multiplication tables.

Mark awarded = 0

Candidate P

Work out a possible value for n.

$6 \times 3 - 1 = 17$ X $6 \times 11 - 1 = 65$ ✓

$6 \times 2 - 1 = 11$ X

$6 \times 5 - 1 = 29$ X

$6 \times 4 - 1 = 23$ X

Answer $6 \times 11 - 1 = 65$ (2 marks)

Trying 11 was rather a jump from trying 3, 2, 5 and 4, but it was effective.

Mark awarded = 2

Question 12

12 Here are three number cards.

A B C

There is a whole number on the back of each card.

The number on card A is three times the number on card B.
The number on card C is twice the number on card A.
The sum of the numbers on all three cards is 120.

Work out the number on each card. *(3 marks)*

Mark scheme:

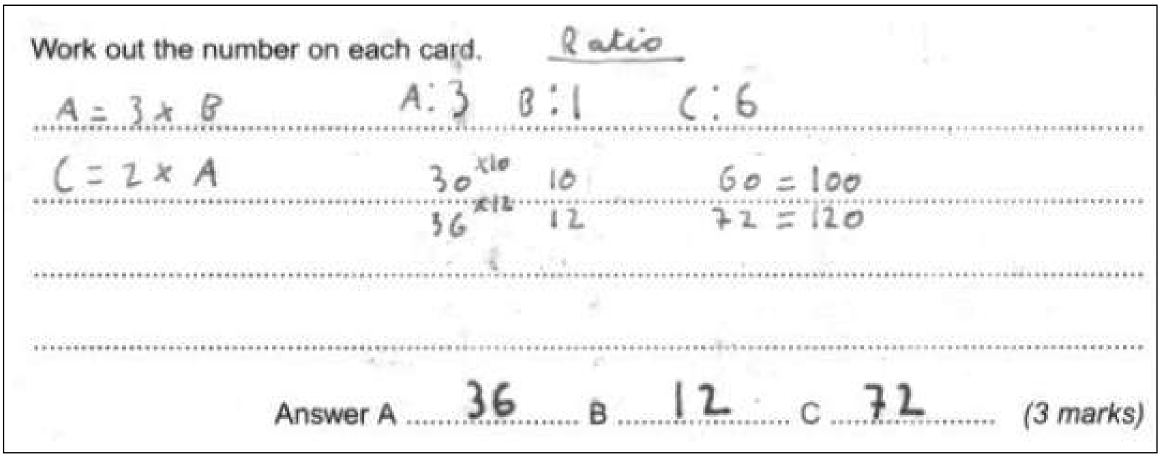
12	A = 36 B = 12 C = 72	B3	B2 for 2 conditions met eg A = 45 B = 15 C = 90 B1 for 1 condition met eg A = 30 B = 40 C = 50 SC2 for correct numbers in wrong order

This question was targeted at grade E and assessed AO3.

The mean mark was 1.35 and the question proved to be a good discriminator with an almost equal distribution of marks between 3 and 0.

Although a ratio method can be used, most candidates opted for trial and improvement, which, for a question like this, is appropriate and effective.

Candidate Q


Work out the number on each card. Ratio

A = 3 × B   A : 3   B : 1   C : 6

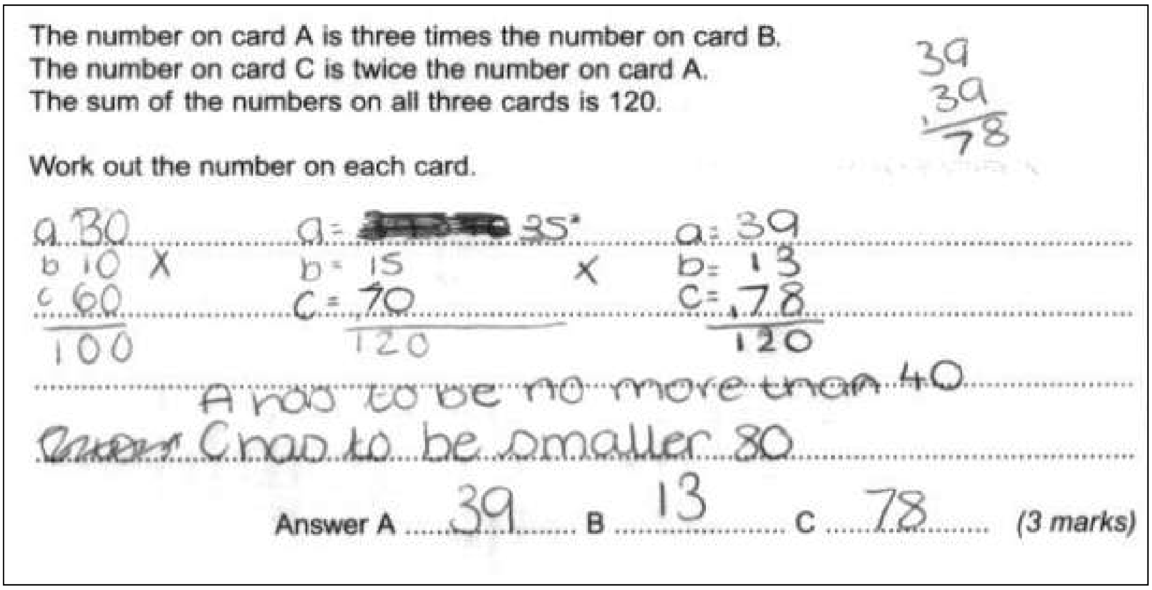
C = 2 × A $30^{\times 10}$ 10   60 = 100
$36^{\times 12}$ 12   72 = 120

Answer A 36 B 12 C 72 *(3 marks)*

This was a rare example of a candidate spotting the correct ratio of 3 : 1 : 6 and applying the correct technique to effect a solution to the problem. A neat and efficient method.

Mark awarded = 3

Candidate R


The number on card A is three times the number on card B.
The number on card C is twice the number on card A.
The sum of the numbers on all three cards is 120.

Work out the number on each card.

39
39
78

a 30   a = 35   a = 39
b 10 X   b = 15 X   b = 13
c 60   C = 70   C = 78
100   120   120

A has to be no more than 40
C has to be smaller 80

Answer A 39 B 13 C 78 *(3 marks)*

After two trials had been successfully rejected, the candidate tried 13 for B and correctly works out 39 and 78.

Errors in the arithmetic gave 39 + 13 + 78 = 130 not 120

Two conditions out of three have been met.

Mark awarded = 2

Candidate S

Work out the number on each card.

$\frac{1}{3} \div$ 120 =

135

6) 126

45

75

Answer A ~~[illegible]~~ 15 B 75 C ~~[illegible]~~ 30 *(3 marks)*

These values add up to 120 and the number on card C is twice the number on card A.

Two conditions satisfied out of three.

Mark awarded = 2

Candidate T

A	B	C
×3		×2

There is a whole number on the back of each card.

The number on card A is three times the number on card B.
The number on card C is twice the number on card A.
The sum of the numbers on all three cards is 120.

Work out the number on each card.

B	A	C
10	36	90 ×
20	60	120 ✓

Answer A 60 B 20 C 120 *(3 marks)*

This candidate satisfies two of the conditions, but fails to consider that the total of all three numbers needs to be 120.

Mark awarded = 2

Question 4

4* Bella wants to buy 12 tins of baked beans for a barbeque.
Two supermarkets have these special offers.

PriceSave	**CostCut**
Baked beans Normal price 50p **Special offer** 30% off all tins	**Baked beans** Normal price 48p **Special offer** Pay for 3 tins, get 1 free

Which is cheaper?
You **must** show your working. *(5 marks)*

Mark scheme:

4	$50(p) - \frac{30 \times 50(p)}{100}$ or $\frac{70 \times 50(p)}{100}$	M1	oe
	35(p) or (£)(0).35 420(p) or (£)4.2(0) 140(p) or (£)1.4(0)	A1	
	3 × 48(p) or 9 × 48(p) or 3 × 48(p)	M1	
	36(p) or (£)(0).36 432(p) or (£)4.32 144(p) or (£)1.44	A1	Note: for both A marks to be awarded they must be buying the same number of tins
	Correct conclusion from their working with all calculations shown	Q1	Strand (iii) Must have both Ms awarded and be comparing like with like

This was a common question with Q14 on the Foundation paper.

It was targeted at grade D and assessed AO2 and strand (iii) quality of written communication (QWC). It was also a functional question.

The mean mark at Foundation was 1.85, with over 52% of candidates scoring at least 2 marks.

The mean mark at Higher was 3.81, with over 68% of candidates scoring 4 or 5 marks.

The quality of candidates' written presentation was variable, sometimes making it hard for examiners to decide whether 1 tin, 4 tin or 12 tin strategy had been used. The question was generally well received with many very good answers.

Candidate A

Which is cheaper?
You **must** show your working.

Price Save	Cost Cut
1 tin = 50p	1 tin = 48p
12 tin = £6.00	1 tin, 1 tin, 1 tin, free tin
£6 10% = 60p	1 tin, 1 tin, 1 tin, free tin
30% = £1.80	1 tin, 1 tin, 1 tin, free tin
£6 − £1.80 = £4.20	50p × 9 = £4.50 − 9 × 2p
	= £4.22

Answer Price save (5 marks)

This is an almost perfect solution. The method for calculating the cost of 12 tins is clear and the only error is in the final subtraction where £4.50 – 9 × 2p should be £4.32 not £4.22

The candidate loses 1 accuracy mark. The Q mark is awarded because it is the correct conclusion from the candidates working, given that the method marks have been earned; the definition of a strand (iii) QWC mark.

Mark awarded = 4

__Candidate B__

Bella wants to buy 12 tins of baked beans for a barbeque.
Two supermarkets have these special offers.

PriceSave	**CostCut**
Baked beans	**Baked beans**
Normal price 50 p	Normal price 48 p
Special offer	**Special offer**
30% off all tins	Pay for 3 tins, get 1 free

Which is cheaper?
You **must** show your working.

PriceSave	costcut
50p − 30 % = 35p	48p × 3 = 146p + free tin
30% = 15p	48p × 4 = 146
50p − 15p = 35p	146 ÷ 4 = 36.5
1 tin = 35p	1 tin = 36.5p

Pricesave is cheaper by 1.5p

Answer pricesave (5 marks)

The candidate chooses to the 1 tin strategy. The discount at PriceSave is correctly calculated, giving the cost of 1 tin as 35p.

Although the method is correct, an error in arithmetic (48 × 3 = 146) means that the cost of 1 tin at CostCut is calculated as 36.5p instead of the correct answer of 36p.

This did not affect the candidate’s conclusion, so the Q mark was awarded.

Mark awarded = 4

Candidate C

Which is cheaper?
You **must** show your working.

30% of 50p = 15p
10% = 5p
15 x 12 = 1.80

48p x 3 = 1.44 (1 free)
1.44 x 3 = 4.32

Answer PriceSave is cheaper *(5 marks)*

Having correctly calculated 30% of 50p, this candidate did not subtract the 15p from 50p and used 15p as the price of 1 tin of beans at PriceSave. This was an example of incomplete method; they must use 70% of the normal price of a tin to score any marks for this part of the calculation, regardless of whether they work with 1 tin, 4 tins or 12 tins.

The answer of £4.32 at CostCut is correct.

The Q mark cannot be awarded because only 1 of the 2 method marks has been awarded.

Mark awarded = 2

Candidate D

*4 Bella wants to buy 12 tins of baked beans for a barbeque.
Two supermarkets have these special offers.

PriceSave	**CostCut**
Baked beans	**Baked beans**
Normal price 50 p	Normal price 48 p
Special offer	**Special offer**
30% off all tins	Pay for 3 tins, get 1 free

Which is cheaper?
You **must** show your working.

3
6 → 4 free
9
12

35
× 12
170
350
420
30%

Price Save → 50p ÷ 10 = 5p = 10% 5p × 3 = 15p off
50 − 15 = 35p per tin 35 × 12 = £4.20

get 4 free → 48p × 8 = £ 3.84

The Cost cut is cheaper by 36p

3
6 > 12 tins
9 get 4 free
12

Answer Cost cut by 36p (5 marks)

48
× 8
3.84

4.20
− 3.84
0.36

This candidate calculates the cost of 12 tins at PriceSave correctly.

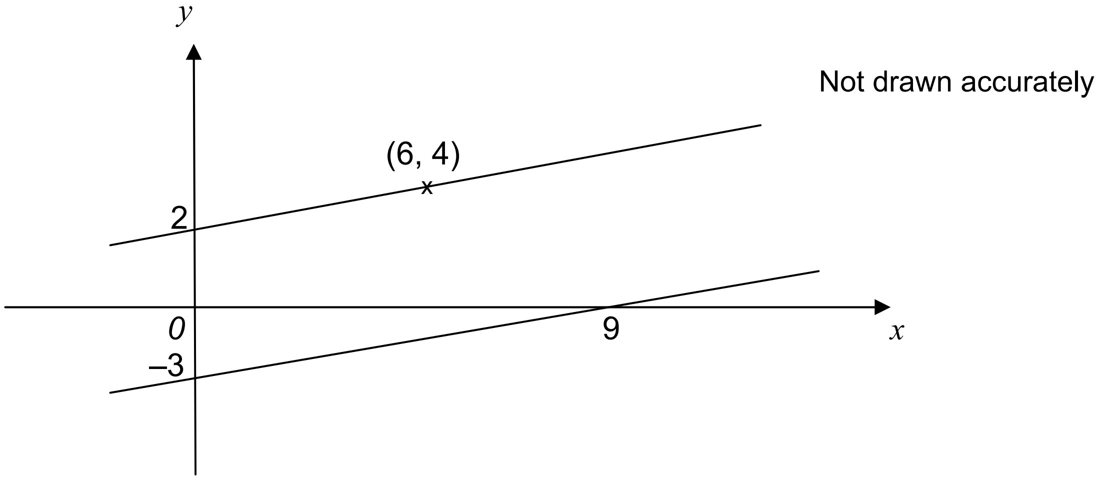
When working out how many tins need to be bought at CostCut, they make an error. They interpret ‘pay for 3, get 1 free’ as ‘pay for 12, get 4 free’ and wrongly assume that 8 tins need to be bought.
This was an error made by a significant number of candidates.

It is an example of incorrect reasoning, so no marks can be awarded for the CostCut calculation, and consequently the Q mark cannot be awarded.

Mark awarded = 2

Question 10

10 Two straight lines are shown.



Prove that the lines never meet. *(3 marks)*

Mark scheme:

10	Right-angled triangle drawn above or below either line, with lengths indicated or Either 2 and 6 or 3 and 9 used as a ratio or fraction	M1	Correct substitution into gradient formula $\frac{y_2 - y_1}{x_2 - x_1}$ … or inverted Award for $^1/_3$ seen with no working
	$^2/_6$ and $^3/_9$	A1	
	Both simplify to $^1/_3$ so lines parallel or have same gradient or Equations are $y = {}^1/_3 x + 2$ and $y = {}^1/_3 x - 3$ hence lines are parallel or lines have same gradient	A1	

This question was targeted at grade B and assessed AO3.

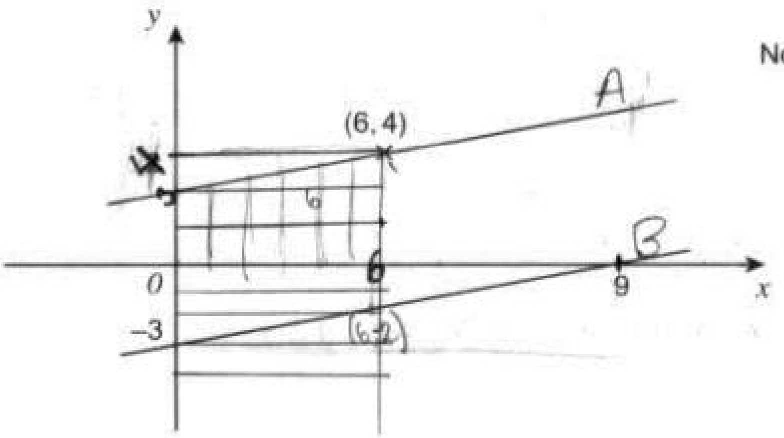
The mean mark was 0.36 and only a very small number of candidates realised that calculating the gradient was all that was needed to effect a proof.

There were many vague descriptions such as ‘they are parallel so they will never meet’ and even instances of measurement, for example ‘they are 2.1cm apart at both ends so they must be parallel’.

Of the candidates who attempted a gradient calculation, some managed to invert the answer and some were unsure as to whether the gradient was positive or negative.

Candidate E

Not drawn accurately



Prove that the lines never meet.

They are parellel - have same gradient

Line B → $\frac{\Delta y}{\Delta x} = \frac{3}{9} = \frac{1}{3} = 0.\dot{3}$

Line A → $\frac{\Delta y}{\Delta x} = \frac{1}{3} = 0.\dot{3}$

A ⇒ $y = 1x - 2$ B ⇒ $y = 1x + 3$

(3 marks)

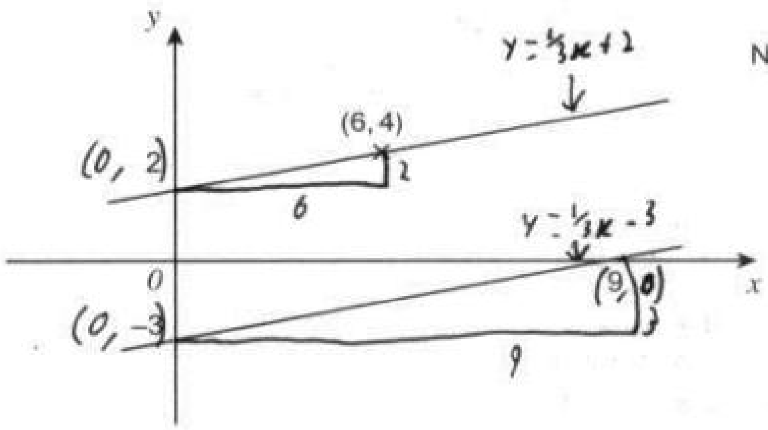
The gradient has been calculated for line B with the steps being clearly shown.

Knowing that the lines need to be shown to be parallel, the candidate then states the same gradient for line A, but does not show their calculation.

In a proof, all the working must be shown. It is insufficient, in this case, to state that both gradients are $^1/_3$ without showing both sets of working.

Mark awarded = 1

Candidate F



Not drawn accurately

Prove that the lines never meet.

$\frac{2}{6} = \frac{1}{3}$

$\frac{3}{9} = \frac{1}{3}$

They both have the same gradient of $\frac{1}{3}$

(3 marks)

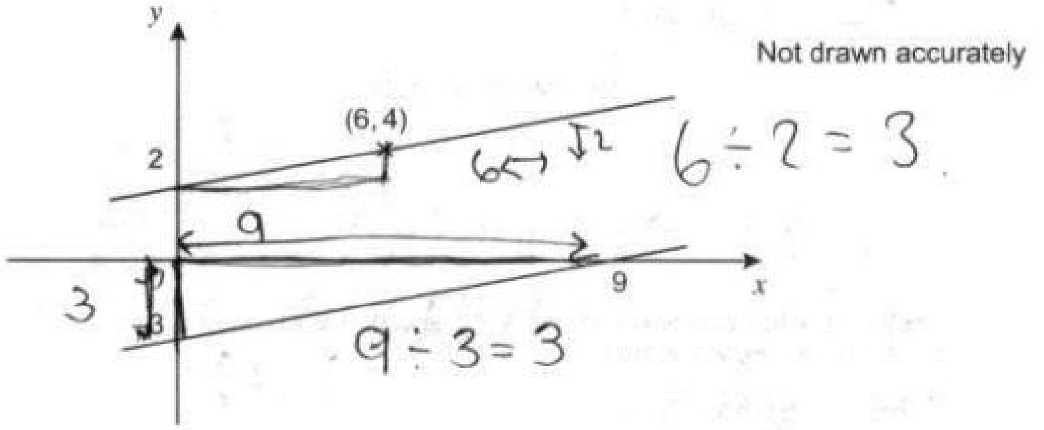
This candidate gives all the information needed.

Right-angled triangles are drawn, with lengths marked, gradients are shown as $^2/_6$ and $^3/_9$ before being simplified to $^1/_3$ and both of the straight line equations are seen.

The final statement is clear evidence of a complete proof.

Mark awarded = 3

Candidate G



Prove that the lines never meet.

the gradient of both lines is 3 therefore, the lines are parrallell. paralel lines never meet

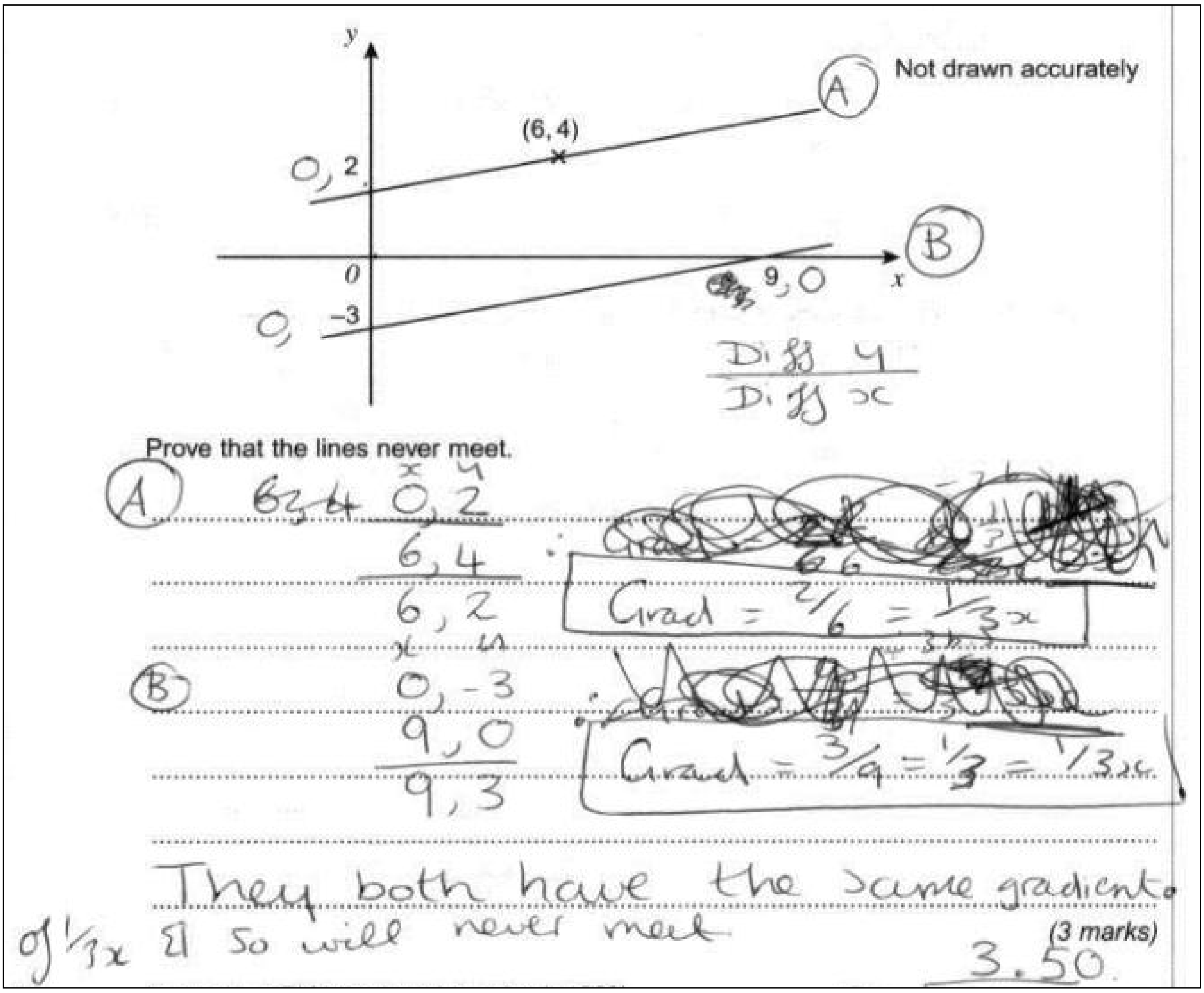
(3 marks)

The candidate knows to try to find the gradient by firstly drawing right-angled triangles on the diagram and marking the lengths on both of them. Unfortunately, the gradient calculations are inverted, something that occurred on several candidates' attempts.

The first method mark was awarded, but nothing thereafter.

Mark awarded = 1

Candidate H



Although the presentation is a little untidy, this candidate understands what is required for the proof.

Diff y / Diff x is followed by working showing where the fractions $^2/_6$ and $^3/_9$ come from and that both of them simplify to $^1/_3$.

Answers of $^1/_3 x$ for the gradient were condoned.

The candidate clearly states a conclusion.

Mark awarded = 3

Question 14

14a Show clearly that $4^{3/2} = 8$ *(2 marks)*

14b Hence, or otherwise, work out the value of y if $4^y = 8^6$ *(2 marks)*

Mark scheme:

14a	Sight of $\sqrt{4} = 2$ followed by 2^3 or 4^3 followed by $\sqrt{64}$	B2	B1 for partial solution but incomplete eg for $\sqrt{4} = 2$ seen or 64 seen
14b	$(4^y =)\ (4^{1.5})^6$ or $(2^2)^y = (2^3)^6$	M1	Allow 1.5×6 or $2 \times y = 3 \times 6$
	9	A1	Allow $^{18}/_2$ and 4^9

This question was targeted at grade A and assessed AO1 (part a) and AO2 (part b).

The mean mark for part (a) was 0.74 and for part (b) was 0.2

There were many good answers for part (a), although the explanation was not always clear. Part (b) proved to be too challenging for the majority of candidates as was expected with a grade A question.

Replacing 8^6, using the answer from part (a) was the expected first step, which would have earned a mark, but this was rare.

There were other elegant answers, such as starting with the fact that $8^2 = 4^3$ and using powers of this result to give firstly $8^4 = 4^6$ and then $8^6 = 4^9$.

Candidate I

(a) Show clearly that $4^{\frac{3}{2}} = 8$

$4^{3/2} = (\sqrt{4})^3 = (2)^3 = 2 \times 2 \times 2 = \underline{\underline{8}}$

(2 marks)

(b) Hence, or otherwise, work out the value of y if $4^y = 8^6$

$(4^{3/2} = 8)$ $\quad 3/2 \times 6 = \cancel{18/2}\ 9/2$

$2^3 = 8$ $\quad 2^{9} = 8^6$ $\quad (\sqrt{4})^9 = 8^6$

$2^{18} = 8^6$ $\quad 4^{9/2} = 8^6$

Answer y = 9/2 *(2 marks)*

The candidate offers a perfect solution to part (a).

Mark awarded = 2

In part (b) they realise the need to use the result from part (a) and correctly decide to multiply the power of $^3/_2$ by 6, firstly writing $^{18}/_2$ (correct, but crossed out) and then $^9/_2$ (incorrect) which is given as the final answer.

The candidate was so nearly there.

Mark awarded = 1

Candidate J

(a) Show clearly that $4^{\frac{3}{2}} = 8$

$4^{3/2} = (\sqrt{4})^3$

$\sqrt{4} = +/- 2$

$2^3 = 8$

(2 marks)

(b) Hence, or otherwise, work out the value of y if $4^y = 8^6$

$4^y = 8^6$ $\quad 2^3 = 8^1 \quad 8^6 = 8^1 \times 8^5$

$2^{15} \times 2^3 = 8^1 \times 8^5$

$2^{18} = 8^6$

Answer y = $18/2 = 9$ *(2 marks)*

This candidate answers part (a) correctly.

Mark awarded = 2

Part (b) is also correct.

They first of all try to express 8^6 as a power of 2, correctly deducing 2^{18}, before dividing the power by 2 to take into account the fact that $2^2 = 4$.

Mark awarded = 2

__Candidate K__

(a) Show clearly that $4^{\frac{3}{2}} = 8$

$\sqrt{4} = 2 \quad 2^3 = 8$

(2 marks)

(b) Hence, or otherwise, work out the value of y if $4^y = 8^6$

$8^6 = 4^{12}$

$8^2 = 64 \quad 4 \quad 4^4 = 64 \quad 4^3 = 16$

$8^2 = 4^3 \quad 8^4 = 4^6 \quad 8^6 = 4^9$

Answer y = ~~12~~ 9 *(2 marks)*

This candidate gave an excellent answer to both parts of this question.

In part (a) they write the minimum, but it is sufficient.

Mark awarded = 2

In part (b) the candidate starts with the equivalence of 8^2 and 4^3 and goes on to give a very neat solution.

Mark awarded = 2

Candidate L

(a) Show clearly that $4^{\frac{3}{2}} = 8$

$\left(\sqrt[2]{4}\right)^3$ $\quad \sqrt[2]{4} = 2 \quad 2^3 = 8$

(2 marks)

(b) Hence, or otherwise, work out the value of y if $4^y = 8^6$

$8^6 \quad 8 \times 8 = 64^{\,2} \quad 64 \times 8 = 512^{\,3} \quad = 4096^{\,4}$

$4096 \times 8 = 32768^{\,5} \quad = 262{,}144$

$8^6 = 262{,}144 \quad 262{,}144$

Answer y = ... *(2 marks)*

There was no problem in part (a) for this candidate.

Mark awarded = 2

In part (b) there was a valiant, and correct, attempt to work out 8^6 (262144) but the candidate did not work out $4 \times 4 \times 4 \times$ ... ... to see how many are needed to give 262144.

An attempt at this might have earned a mark and although it is not the best method, it would have been acceptable.

Mark awarded = 0

Question 16

16 A bag contains only blue and green counters.

If there were three times as many blue counters and the original number of green counters, the total number of counters in the bag would be 62.

If there were twice as many green counters and the original number of blue counters, the total number of counters in the bag would be 59.

How many of each colour are in the bag?
Do **not** use trial and improvement.
You **must** show your working. *(4 marks)*

Mark scheme:

16	$3b + g = 62$ or $b + 2g = 59$	B1	
	$3b + g = 62$ and $3b + 6g = 177$ or $6b + 2g = 124$ and $b + 2g = 59$ or $3b + g = 62$ and $2b - g = 3$	M1	oe Correct attempt at elimination … Allow one error in the two elimination steps If substitution method used then allow one error in the substitution or simplification
	$5g = 115$ or $5b = 65$	M1 dep	oe
	$b = 13$ and $g = 23$	A1	SC2 for correct solution by trial and improvement

This question was targeted at grade A and assessed AO3.

The mean mark was 1.36 and although half of the candidates were unsuccessful (scoring zero or making no attempt) over 23% of candidates scored full marks, successfully setting up simultaneous equations and solving them correctly.

It was encouraging to see that a significant number of candidates did try to use simultaneous equations rather than trial and improvement. The latter will yield a solution, although 13 and 23 are not the easiest numbers to spot, but it ought not to be the method of choice for good candidates.

A correct trial and improvement solution earned 2 marks as a special case, even though the question stated that this method ought not to be used.

Candidate M

How many of each colour are in the bag?
Do **not** use trial and improvement.
You **must** show your working.

$3b+g=62$ − ①
$b+2g=59$ − ②
① x2 =
$6b+2g=124$ − ③
③ − ② = $6b+2g=124$
$6b+2g=59$
$5b+0g=65$

$5b=65$
$b=13$
apply '$b=13$' to ~~b+2g~~ ②:
$1(13)+2g=59$
$13+2g=59$
(-13) $2g=46$
$(\div 2)$ $g=23$

Answer blue 13 green 23 *(4 marks)*

$3(13)+1(23)=62$
$39+23=62$ ✓

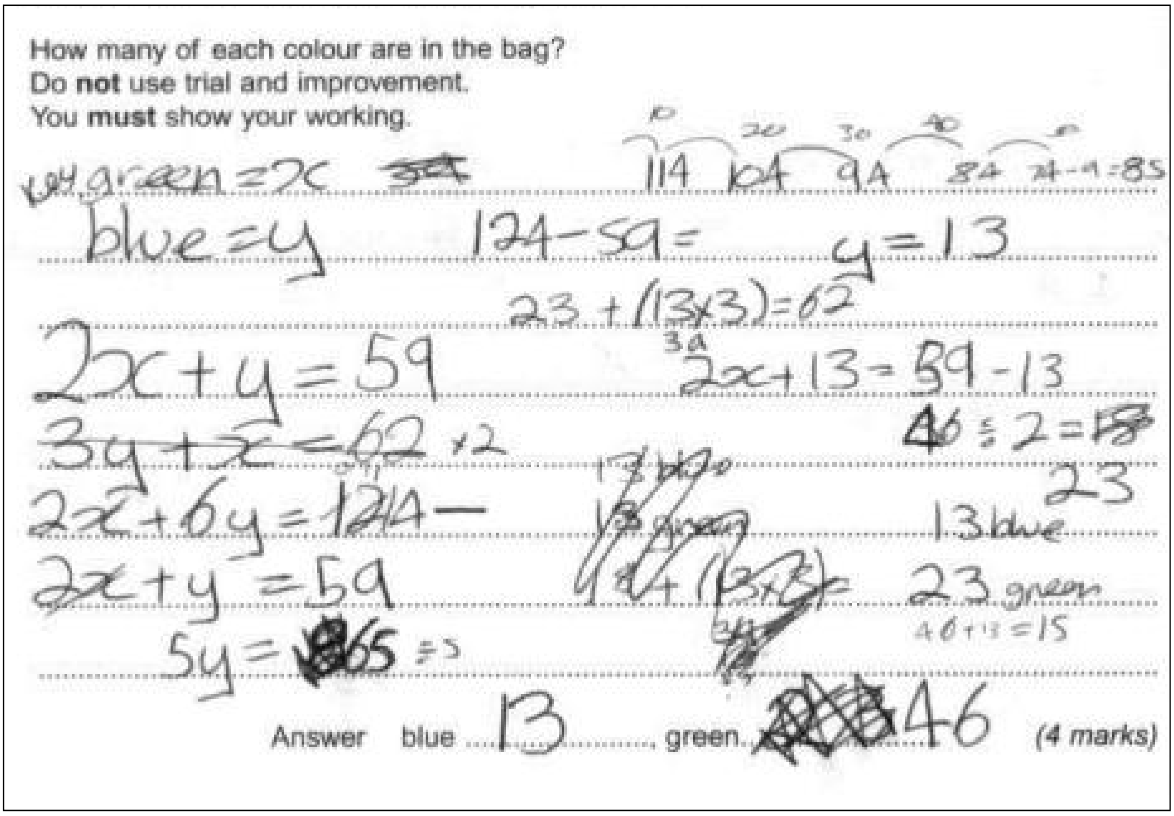
$1(13)+2(23)=59$
$13+46=59$ ✓

This is an exemplary solution.

Simultaneous equations set up and used correctly, with the answers checked.

Mark awarded = 4

Candidate N


How many of each colour are in the bag?
Do **not** use trial and improvement.
You **must** show your working.

green = x
blue = y

$2x + y = 59$
$3y + x = 62$ ×2
$2x + 6y = 124$ −
$2x + y = 59$
$5y = 65$ ÷5

124 − 59 =
$y = 13$
$23 + (13 \times 3) = 62$
$2x + 13 = 59 - 13$
$46 \div 2 = 23$
13 blue
23 green

Answer blue 13, green 46 (4 marks)

This candidate does everything correctly until the very final step. Having decided that 13 blue and 23 green is the answer, they cross out 23 and replace it with 46 on the answer line.

This has to be interpreted as 'choice', so the final accuracy mark is not awarded.

Mark awarded = 3

Candidate O

How many of each colour are in the bag?
Do **not** use trial and improvement.
You **must** show your working.

23 green · 46 : 13 blue

less

$13 \times 3 = 39$

$+ 23$

62

Answer blue 13, green 23 *(4 marks)*

No equations or working can be seen. Presumably the candidate's working for trial and improvement was done on an extra sheet of paper that was not submitted or perhaps this was their first guess?

Mark awarded = 2

Candidate P

How many of each colour are in the bag?
Do **not** use trial and improvement.
You **must** show your working.

$3b+g=62 \quad \times 2 \quad 6b+2g=124 \ldots (1)$

$b+2g=59 \quad (2)$

$(1)-(2) = 5b = 75$

$b=15$

Subhitute back in ... (2)

$15+2g=59$

$2g=44$

$g=22$

Answer blue ......15......, green......22...... *(4 marks)*

The correct equations have been set up, but the elimination of '$2g$' from the equations $6b + 2g = 124$ and $b + 2g = 59$ is accompanied by an arithmetic error, resulting in $5b = 75$, and hence $b = 15$.

This is the candidate's only error and the mark scheme permits one error in either stage of the elimination process, so all the method marks can be awarded.

Mark awarded = 3

Question 18*

18* The sum of the squares of two consecutive integers is one greater than twice the product of the integers.

For example $9^2 + 10^2 = 81 + 100$ and $2 \times 9 \times 10 = 180$
$= 181$

Prove this result algebraically. *(5 marks)*

Mark scheme:

18	$n^2 + (n + 1)^2$	M1	Condone missing brackets if recovered
	$n^2 + n^2 + 2n + 1$	M1 dep	
	$2n^2 + 2n + 1$	A1	
	$2n(n + 1) + 1$	A1	Accept $2n(n + 1) + 1 = 2n^2 + 2n + 1$ or $2n(n + 1) = 2n^2 + 2n$ for this mark provided the first 3 marks have been earned
	Complete solution with all stages clearly shown	Q1	Strand (ii) Clear explanation Do not award if first line assumes answer with use of = sign eg $n^2 + (n + 1)^2 = 2n(n + 1) + 1$
	Alternative method		
	$n^2 + (n + 1)^2 – 2n(n + 1)$	M1	Condone missing brackets if recovered
	$n^2 + n^2 + 2n + 1 – 2n(n + 1)$	M1 dep	
	$2n^2 + 2n + 1 – 2n(n + 1)$	A1	
	$2n^2 + 2n + 1 – 2n^2 – 2n$	A1	Allow $2n^2 + 2n + 1 – (2n^2 + 2n)$
	Complete solution with all stages clearly shown	Q1	Strand (ii) Clear explanation Do not award if first line assumes answer with use of = sign eg $n^2 + (n + 1)^2 – 2n(n + 1) = 1$

This question was targeted at grade A* and assessed AO2 and strand (ii) QWC.

The mean mark was 0.37, which is very low but not surprising since this question is likely to be too challenging for all but the most able candidates. Only 15% scored any marks at all, although 7% scored at least 3 marks.

The Q mark was only awarded to those candidates who were rigorous in their methods, starting with one side of the algebraic statement and working their way through to a point where they could make a comparison, deducing the required result.

Candidate Q

Prove this result algebraically.

$2(n+1)(n+2) = n^2+2n+1n+2$

$= n^2+3n+2 \quad \times 2$

$= 2n^2+6n+4$

$(n+1)(n+1) + (n+2)(n+2)$

$= n^2+1n+1n+1 + n^2+2n+2n+4$

$= n^2+2n+1 + n^2+4n+4$

$= 2n^2+6n+5$

$5-4 = \underline{1}$

(5 marks)

This candidate gave themselves more work to do by selecting (n + 1) and (n + 2) as their consecutive numbers, but this caused no problems.

The algebra is well handled and clearly set out, and the final statement of 5 – 4 = 1 is sufficient to illustrate the result to be proved.

Mark awarded = 5

Candidate R

Prove this result algebraically.

$a^2+(a+1)^2 = a^2+(a+1)(a+1) = a^2+a^2+a+a+1$

$= 2a^2+2a+1$

$2 \times a \times (a+1) = 2a(a+1)$

$= 2a^2+2a$

$2a^2+2a+1$ is the sum of two consecutive numbers, it is one greater than twice the product of the intergers which is $2a^2+2a$

$(2a^2+2a+1)-1 = 2a^2+2a$

(5 marks)

The candidate tackles both statements separately and clearly demonstrates that the two algebraic expressions differ by 1.

A well constructed and fully correct argument.

Mark awarded = 5

Candidate S

Prove this result algebraically.

$n^2 \quad n+1^2$

$n^2 + (n+1)^2 = n^2 + (n+1)^2$

$= n^2 + (n+1)(n+1)$

$n^2 + n^2 + 2n + 2$

×	n	+1
n	n^2	$1n$
+1	n	2

(5 marks)

This candidate knows how to start the proof and attempts to expand the bracket.

Unfortunately there is an error in the expansion and the expression is not simplified fully.

If there had not been an error in the expansion, and the expression had been simplified fully, the candidate would have scored 3 of the possible 5 marks.

Mark awarded = 1

Candidate T

Prove this result algebraically.

$x^2 + (x+1)^2 = x^2 \; 2x(x+1) + 1$

$x^2 + x^2 + 2x + 1 = 2x(x+1) + 1$

$2x^2 + 2x = 2x(x+1)$

$2x(x+1) = 2x(x+1)$

(5 marks)

This candidate starts by making an algebraic statement that assumes the result to be proved.

Although there are no mistakes in the simplification, approaching a proof in this way is insufficiently rigorous for full marks to be awarded.

The strand (ii) Q mark for a completely correct, clearly set out and fully explained solution, was not awarded in this case.

Mark awarded = 4