



# General Certificate of Secondary Education

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## Mathematics 4307 *Specification B (Modular)* 2009

Material accompanying this Specification

- Specimen Assessment Materials
- Teachers' Guide

# SPECIFICATION

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# Background Information

## 1

### Introduction

Following a review of the National Curriculum requirements, and the establishment of the National Qualifications Framework, all the unitary awarding bodies revised their GCSE syllabuses for first examination in 2003. These specifications were further revised for examination in 2008 to incorporate necessary changes in moving to two tiers of assessment. These specifications have been further revised following the withdrawal of coursework in GCSE Mathematics. This new specification will be used by schools and colleges for two-year courses starting in September 2007.

#### 1.1 National Qualifications Framework

GCSE qualifications have the following equivalence in the National Qualifications Framework.

Grades A\* – C = Level 2  
Grades D – G = Level 1

#### 1.2 Requirements at GCSE

##### ICT

The subject content of all GCSEs must require candidates to make effective use of ICT and provide, where appropriate, assessment opportunities for ICT. Details of how the teaching of this specification can encourage the application and development of ICT skills are given in Section 13. However, ICT skills are not assessed by any component of this specification.

##### Key Skills

All GCSE specifications must identify, as appropriate, opportunities for generating evidence on which candidates may be assessed in the “main” Key Skills of *Communication*, *Application of Number* and *Information and Communication Technology* at the appropriate level(s). Also, where appropriate, they must identify opportunities for developing and generating evidence for addressing the ‘wider’ Key Skills of *Working with Others*, *Improving own Learning and Performance* and *Problem Solving*.

Communication	<p>All GCSE specifications must ensure that the assessment arrangements require that, when they produce extended written material, candidates have to:</p> <ul style="list-style-type: none"><li>• ensure that text is legible and that spelling, punctuation and grammar are accurate so that meaning is clear</li><li>• present information in a form that suits its purpose</li><li>• use a suitable structure and style of writing.</li></ul> <p>Further details for this specification are given in Section 6.2.</p>
Tiering	<p>In GCSE Mathematics the scheme of assessment must include question papers targeted at two tiers of grades: A* – D (Higher), and C – G (Foundation).</p> <p>Candidates should be entered at the tier appropriate to their attainment. In GCSE Mathematics (Modular) each candidate may enter for each individual module at a different tier of entry. However, the final range of grades available to a candidate is determined by the tier of entry for Module 5. A safety net of an allowed grade E is provided for Higher Tier candidates who just fail to achieve grade D.</p>
Citizenship	<p>From 2002, students in England have been required to study Citizenship as a national curriculum subject. Each GCSE specification must signpost, where appropriate, opportunities for developing citizenship knowledge, skills and understanding. Further details for this specification are given in Section 13.</p>
Other Issues	<p>All specifications must identify ways in which the study of the subject can contribute to developing understanding of spiritual, moral, ethical, social and cultural issues, European developments, environmental issues, and health and safety. Further details for this specification are given in Section 13.</p>
Wales and Northern Ireland	<p>Centres in Northern Ireland/Wales must refer to the Statement in Section 8.1 of this specification.</p>

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1.3 The Mathematics Criteria All assessment is external.

## 2

## Specification at a Glance


### *Mathematics B (Modular)*

- This is one of two Mathematics specifications offered by AQA. Specification A is a traditional linear scheme; Specification B is modular and is suitable for both pre-16 and post-16 candidates.
- There are two tiers of assessment, Foundation (C – G) and Higher (A\* – D).

GCSE Mathematics B (4307)	
Module 1	
Written Paper	18% of total assessment
2 x 30 minutes (Both tiers)	
Section A – Calculator	
Section B – Non-calculator	
Module 3	
Written Paper	27% of total assessment
2 x 45 minutes (Both tiers)	
Section A – Calculator	
Section B – Non-calculator	
Module 5	
Written Paper	55% of total assessment
Paper 1 Non-calculator	
Foundation Tier	1 hour 15 minutes
Higher Tier	1 hour 15 minutes
Paper 2 Calculator	
Foundation Tier	1 hour 15 minutes
Higher Tier	1 hour 15 minutes

Foundation Tier
Higher Tier
Modules 1, 3 and 5 are available in both tiers
See entry code information in Section 3.2



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## Availability of Assessment Units and Entry Details

### 3.1 Availability of Assessment Units

Specification B is a modular assessment of GCSE Mathematics designed to be taken over a one- or two-year course of study. To offer maximum flexibility to centres and to suit different teaching programmes, Modules 1 and 3 can be taken in either order and candidates can enter at different tiers for the different modules. Module 5 is the certifying module and must be taken in the final examination series. This is to meet the QCA requirement that at least 50 per cent of the qualification is externally examined at the end of the course. The first examination series in which Module 5 and certification is available will be June 2009.

Examinations based on this specification will be available as follows.

Series	Availability of Modules		
	Module 1	Module 3	Module 5
<b>November 2007</b>	Both tiers	Both tiers	–
<b>March 2008</b>	Both tiers	Both tiers	–
<b>June 2008</b>	Both tiers	Both tiers	–
<b>November 2008</b>	Both tiers	Both tiers	–
<b>March 2009</b>	Both tiers	Both tiers	–
<b>*June 2009</b>	Both tiers	Both tiers	Both tiers
<b>*November 2009</b>	Both tiers	Both tiers	Both tiers
<b>March 2010</b>	Both tiers	Both tiers	–

\*Certification is available in this series.



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3.2	Entry Codes	Normal entry requirements apply, but the following information should be noted.  The <b>Subject Code</b> for entry to the GCSE award is 4307.  A separate entry is needed for each of the three modules. In addition, an entry for the overall subject award, 4307, must be submitted, together with module 5 entry, by 21 February for the June examination or with the entries for the November examination.  Candidates may enter for only one tier in each module in a particular examination series.  More detailed information, including component codes, will be issued to examination centres in a separate document.  Information about re-taking Modules is given in Section 14.5.
3.3	Classification Codes	Each specification is assigned to a national classification code, indicating the subject area to which it belongs.  Centres should be aware that candidates who enter for more than one GCSE qualification with the same classification code, will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.  The classification code for this specification is 2210.
3.4	Private Candidates	Private candidates should write to AQA for a copy of <i>Supplementary Guidance for Private candidates</i> .
3.5	Special Consideration	Special consideration may be requested for candidates whose work has been affected by illness or other exceptional circumstances. The appropriate form and all relevant information should be forwarded to the AQA office which deals with such matters for the centre concerned. Special arrangements may be provided for candidates with special needs.  Details are available from the AQA Website ( <a href="http://www.aqa.org.uk">www.aqa.org.uk</a> ). Centres contacting AQA should ask for a copy of <i>Regulations and Guidance relating to Candidates who are Eligible for Adjustments in Examinations</i> .
3.6	Access Arrangements	Special arrangements may be provided for candidates with special needs.  Details are available from the AQA Website ( <a href="http://www.aqa.org.uk">www.aqa.org.uk</a> ). Centres contacting AQA should ask for the Candidate Services Departments and a copy of <i>Regulations and Guidance relating to Candidates who are Eligible for Adjustments in Examinations</i> .
3.7	Language of Examinations	All components are provided in English only.

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# Scheme of Assessment

## 4

## Introduction

### 4.1 National Criteria

This GCSE Mathematics (Modular) Specification complies with the following:

the GCSE Subject Criteria for Mathematics

the GCSE and GCE A/AS Code of Practice

the GCSE Qualification Specific Criteria

the Arrangements for the Statutory Regulation of External Qualifications in England, Wales and Northern Ireland: Common Criteria.

### 4.2 Rationale

AQA offers a suite of qualifications for GCSE Mathematics. Specification A is a traditional scheme. Specification B is a modular scheme suitable for both pre-16 and post-16 candidates.

Mathematics is essentially a holistic subject, and as such should be taught in this way with appropriate connections being made between the sections on *Number and algebra*, *Shape, space and measures*, and *Handling data*, as required in the National Curriculum. For example, Number underpins the whole of Mathematics. Modular Mathematics Specification B is designed to be more reflective of the way in which candidates are likely to revise for examinations when they tend to cover just one area of Mathematics at a time. Specification B also allows candidates to take modules based on *Handling data* (AO4) and the mainly number part of *Number and algebra* (AO2) early in the course.

The final module comprises 55 per cent of the assessment and concentrates on the mainly algebraic part of *Number and algebra* (AO2) and the whole of *Space, shape and measures* (AO3).

The division into discrete topic areas gives candidates much more insight into their strengths and weaknesses. Specification B provides a natural link between KS3/KS4 (which are taught holistically) and A-level where Mathematics is examined in discrete topic areas, but not necessarily taught as such. The modular nature of the specification can allow candidates who fail to obtain a GCSE Grade C at KS4 to carry forward some of their module results into post-16 education.

Specification B used in a post-16 centre gives links to Free-Standing Mathematics Qualifications (FSMQs) and the Key Skill of *Application of Number*, and in some cases this could lead to co-teaching opportunities.

**4.3 Prior Level of Attainment and Recommended Prior Learning**

There is progression of material through all levels at which the subject is studied. This specification therefore builds on the Key Stage 3 Programme of Study.

It is also expected that candidates will have reached the required level of literacy through study at Key Stage 3.

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**4.4 Progression**

This qualification is a recognised part of the National Qualifications Framework. As such, GCSE Mathematics provides progression from Key Stage 3 to GCE A/AS Mathematics or further study at Advanced or Advanced Subsidiary level in other subjects or further study at GNVQ level, or directly into employment.

## Aims

The aims set out below are consistent with the 1999 National Curriculum Order for Mathematics and the GCSE Criteria for Mathematics. Most of the aims are reflected in the Assessment Objectives; others are not because they cannot be readily translated into assessment objectives.

A course based on this specification should encourage candidates to:

- a consolidate their understanding of mathematics
- b be confident in their use of mathematics
- c extend their use of mathematical vocabulary, definitions and formal reasoning
- d develop the confidence to use mathematics to tackle problems in the work place and everyday life
- e take increasing responsibility for the planning and execution of their work
- f develop an ability to think and reason mathematically
- g learn the importance of precision and rigour in mathematics
- h make connections between different areas of mathematics
- i realise the application of mathematics in the world around them
- j use ICT appropriately
- k develop a firm foundation for appropriate further study.

## 6

## Assessment Objectives

6.1 A course based on this specification requires candidates to demonstrate their knowledge, understanding and skills in the following assessment objectives. These relate to the knowledge, skills and understanding in the National Curriculum Programme of Study.

AO1 *Using and applying mathematics*

AO2 *Number and algebra*

AO3 *Shape, space and measures*

AO4 *Handling data*

The Assessment Objective AO1, *Using and applying mathematics*, will be assessed in contexts provided by the other assessment objectives.

6.2 **Quality of Written Communication** This specification does not formally assess the quality of written communication.

## 7

## Scheme of Assessment

7.1 **Assessment Units** The Scheme of Assessment comprises three separate modules as detailed below:

Module 1

Written Paper (Section A – Calculator)  
(Section B – Non-calculator)

Foundation Tier 2 × 30 minutes

Higher Tier 2 × 30 minutes

18% of the total assessment 2 × 23 marks

This written paper assesses AO4 (Handling data). All questions are compulsory. A question and answer booklet will be provided.

Module 3

Written Paper	(Section A – Calculator) (Section B – Non-calculator)
Foundation Tier	2 × 45 minutes
Higher Tier	2 × 45 minutes
27% of the total assessment	2 × 35 marks

This written paper assesses mainly the number part of AO2 (Number and algebra).  
All questions are compulsory. A question and answer booklet will be provided.

Module 5  
(Terminal Module)

Written Paper Paper 1 (Non-calculator)		
Foundation Tier	70 marks	1 hour 15 mins
Higher Tier	70 marks	1 hour 15 mins
27.5% of the total assessment		

Written Paper Paper 2 (Calculator)		
Foundation Tier	70 marks	1 hour 15 mins
Higher Tier	70 marks	1 hour 15 mins
27.5% of the total assessment		

Both papers assess the mainly algebra part of AO2 (Number and algebra) and the whole of AO3 (Shape, space and measures).  
All questions are compulsory. A question and answer booklet will be provided.

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## 7.2 Written Papers

On the written papers the assessment of AO1 is subsumed within the other Assessment Objectives covered by the Module. Thus 20% of the total written paper assessment will also assess *Using and Applying Mathematics* within the contexts of the questions.

The written papers at both tiers are designed so that 50% of the marks are focussed on the lowest two grades available. 25 – 30% of marks will focus on the highest two grades.

Common questions will be set on papers across the two tiers. Some questions will be designed to assess the unprompted solution of multi-step problems. On the calculator papers, candidates will be required to demonstrate the effective use of a calculator.

In Modules 1 and 3, the written papers are divided into two separate sections. Section A is the calculator section and is issued to candidates at the beginning of the examination. After the time for Section A has been completed (30 minutes for Module 1 and 45 minutes for Module 3) candidates are instructed to stop working and place their calculators beneath their seats. Section B (the non-calculator section) is then issued. At the end of the examination, the two sections are tagged together and the papers are collected.

The written papers for Module 5 are taken on two separate days. The use of a calculator is not permitted with Paper 1. Slide rules, logarithmic tables and all other calculating aids are also forbidden. On Paper 2 candidates will be required to demonstrate the effective use of a calculator.

Formulae sheets for Module 5 Foundation and Higher tier papers are provided in Appendix B and on page 2 of each examination paper.

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## 7.3 Calculators

Candidates will be expected to have a suitable electronic calculator for use with the calculator papers. The calculator should possess the following as a minimum requirement:

four rules and a square, square root, reciprocal and power function, brackets, a memory facility and appropriate exponential, trigonometric and statistical functions.

Further guidance on regulations relating to calculators can be obtained from *Instructions for the Conduct of Examinations*.

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## 7.4 Entry Policy

Centres are encouraged to enter candidates aiming to achieve grades D, E, F and G for the Foundation tier, and grades A\*, A, B and C for the Higher tier.

# Subject Content

## 8

## Summary of Subject Content

### 8.1 Introduction

There are two tiers of entry for GCSE Mathematics candidates: Foundation and Higher. In the National Curriculum, published in 1999, the Key Stage 4 Programme of Study was divided into two tiers. The division of the Programme of Study into two tiers in the subject content of this specification is common to all Awarding Bodies. Thus:

- the subject content unique to the Foundation tier is based on the Foundation Programme of Study
- the subject content common to both tiers and of the Higher tier only is based on the Higher Programme of Study
- in general, the Higher tier content of the specification subsumes the Foundation tier content.

This GCSE Specification has been written against the Key Stage 4 Programme of Study for England. Candidates entering for this GCSE in Northern Ireland and Wales must be taught all the material required by the National Curriculum in their own country.

### 8.2 Assessment Objectives

Within the modules of this specification the subject content is presented under the following Assessment Objectives.

The Assessment Objective AO1 (*Using and applying mathematics*) is assessed in contexts provided by the other Assessment Objectives.

#### A02 Number and algebra

- 1 Using and applying number and algebra
- 2 Numbers and the number system
- 3 Calculations
- 4 Solving numerical problems
- 5 Equations, formulae and identities
- 6 Sequences, functions and graphs

#### A03 Shape, space and measures

- 1 Using and applying shape, space and measures
- 2 Geometrical reasoning
- 3 Transformations and coordinates
- 4 Measures and construction



**AO4 Handling data**

- 1 Using and applying handling data
- 2 Specifying the problem and planning
- 3 Collecting data
- 4 Processing and representing data
- 5 Interpreting and discussing results

**8.3 Modules****Module 1**

This includes all of the subject content from AO4 (*Handling data*) of the National Curriculum for Mathematics, divided into two tiers of entry.

**Module 3**

This includes the mainly number subject content from AO2 (*Number and algebra*) of the National Curriculum. At the Higher tier selected algebra topics from AO2 are also assessed.

**Module 5**

This includes the mainly algebra subject content from AO2 (*Number and Algebra*) and all of the subject content from AO3 (*Shape, Space and Measures*). At the Foundation tier selected number topics from AO2 (*Number and algebra*) are also assessed. At the Higher tier only the algebra topics from AO2 (*Number and algebra*) are assessed in this module.

**8.4 Breadth of Study**

In addition to the required knowledge, skills and understanding, the National Curriculum Programme of Study also specifies the Breadth of Study expected.

**Foundation Tier**

Pupils should be taught the knowledge, skills and understanding through:

- a extending mental and written calculation strategies and using efficient procedures confidently to calculate with integers, fractions, decimals, percentages, ratio and proportion
- b solving a range of familiar and unfamiliar problems, including those drawn from real-life contexts and other areas of the curriculum
- c activities that provide frequent opportunities to discuss their work, to develop reasoning and understanding and to explain their reasoning and strategies
- d activities focused on developing short chains of deductive reasoning and the correct use of the '=' sign
- e activities in which they do practical work with geometrical objects, visualise them and work with them mentally

- f practical work in which they draw inferences from data, consider how statistics are used in real life to make informed decisions, and recognise the difference between meaningful and misleading representations of data
- g activities focused on the major ideas of statistics, including using appropriate populations and representative samples, using different measurement scales, using probability as a measure of uncertainty, using randomness and variability, reducing bias in sampling and measuring, and using inference to make decisions
- h substantial use of tasks focused on using appropriate ICT (for example, spreadsheets, databases, geometry or graphic packages), using calculators correctly and efficiently, and knowing when not to use a calculator.

Higher Tier

Pupils should be taught the knowledge, skills and understanding through:

- a activities that ensure they become familiar with and confident using standard procedures for the range of calculations appropriate to this level of study
- b solving familiar and unfamiliar problems in a range of numerical, algebraic and graphical contexts and in open-ended and closed form
- c using standard notations for decimals, fractions, percentages, ratio and indices
- d activities that show how algebra, as an extension of number using symbols, gives precise form to mathematical relationships and calculations
- e activities in which they progress from using definitions and short chains of reasoning to understanding and formulating proofs in algebra and geometry
- f a sequence of practical activities that address increasingly demanding statistical problems in which they draw inferences from data and consider the uses of statistics in society
- g choosing appropriate ICT tools and using these to solve numerical and graphical problems, to represent and manipulate geometrical configurations and to present and analyse data.

**8.5 Subject Content  
Presentation**

The subject content, Section 9, is shown in two columns. Column 1 shows the content which will be explicitly assessed on the Foundation tier only, although a thorough knowledge of this content is assumed in the Higher tier assessment.

Column 2 shows the content which will be assessed on the Higher tier only.

The subject content is taken directly from the Statutory Orders for Mathematics, 1999.

In Module 3 Foundation the statements for some number topics are shown in Module 3 but are shaded to show that they are **not examined** until Module 5. The statements are then repeated in Module 5.

For each of Modules 1, 3 and 5, the using and applying statements are given at the beginning.

Each statement is referenced to the appropriate statement in the Foundation or Higher Programme of Study. Thus, F2.1a refers to AO2, Foundation Programme of Study statement 1a.

9

Module 1

A04 Handling data

1 Using and applying handling data

Pupils should be taught to:

	Higher Tier
Foundation Tier	

**Problem solving**

F4.1a	<p>carry out each of the four aspects of the handling data cycle to solve problems:</p> <ul style="list-style-type: none"> <li>(i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed</li> <li>(ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources</li> <li>(iii) process and represent the data: turn the raw data into usable information that gives insight into the problem</li> <li>(iv) interpret and discuss the data: answer the initial question by drawing conclusions from the data</li> </ul>	H4.1a	<p>carry out each of the four aspects of the handling data cycle to solve problems:</p> <ul style="list-style-type: none"> <li>(i) specify the problem and plan: formulate questions in terms of the data needed, and consider what inferences can be drawn from the data; decide what data to collect (including sample size and data format) and what statistical analysis is needed</li> <li>(ii) collect data from a variety of suitable sources, including experiments and surveys, and primary and secondary sources</li> <li>(iii) process and represent the data: turn the raw data into usable information that gives insight into the problem</li> <li>(iv) interpret and discuss the data: answer the initial question by drawing conclusions from the data</li> </ul>
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Foundation Tier		Higher Tier
F4.1b H4.1b	identify what further information is needed to pursue a particular line of enquiry; select the problem-solving strategies to use in statistical work, and monitor their effectiveness (these strategies should address the scale and manageability of the tasks, and should consider whether the mathematics and approach used are delivering the most appropriate solutions)	H4.1b select the problem-solving strategies to use in statistical work, and monitor their effectiveness (these strategies should address the scale and manageability of the tasks, and should consider whether the mathematics and approach used are delivering the most appropriate solutions)
F4.1c	select and organise the appropriate mathematics and resources to use for a task	
F4.1d	review progress while working; check and evaluate solutions	
<b>Communicating</b>		
F4.1e	interpret, discuss and synthesise information presented in a variety of forms	
F4.1f	communicate mathematically, including using ICT, making use of diagrams and related explanatory text	H4.1c communicate mathematically, with emphasis on the use of an increasing range of diagrams and related explanatory text, on the selection of their mathematical presentation, explaining its purpose and approach, and on the use of symbols to convey statistical meaning
F4.1g	examine critically, and justify, their choices of mathematical presentation of problems involving data	

Foundation Tier		Higher Tier
<b>Reasoning</b>		
F4.1h	apply mathematical reasoning, explaining and justifying inferences and deductions	apply mathematical reasoning, explaining and justifying inferences and deductions, justifying arguments and solutions
H4.1e	identify exceptional or unexpected cases when solving statistical problems	identify exceptional or unexpected cases when solving statistical problems
F4.1i H4.1f	explore connections in mathematics and look for relationships between variables when analysing data	explore connections in mathematics and look for relationships between variables when analysing data
F4.1j	recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis	recognise the limitations of any assumptions and the effects that varying the assumptions could have on the conclusions drawn from data analysis

## 2 Specifying the problem and planning

Pupils should be taught to:

Foundation Tier		Higher Tier
F4.2a	see that random processes are unpredictable	see that random processes are unpredictable
F4.2b H4.2b	identify key questions that can be addressed by statistical methods	identify key questions that can be addressed by statistical methods
F4.2c	discuss how data relate to a problem, identify possible sources of bias and plan to minimise it	discuss how data relate to a problem, identify possible sources of bias and plan to minimise it

Foundation Tier		Higher Tier
F4.2d	identify which primary data they need to collect and in what format, including grouped data, considering appropriate equal class intervals	H4.2d identify which primary data they need to collect and in what format, including grouped data, considering appropriate equal class intervals; select and justify a sampling scheme and a method to investigate a population, including random and stratified sampling
F4.2e H4.2e	design an experiment or survey; decide what primary and secondary data to use	H4.2e design an experiment or survey; decide what primary and secondary data to use

### 3 Collecting data

Pupils should be taught to:

Foundation Tier		Higher Tier
F4.3a	design and use data-collection sheets for grouped, discrete and continuous data; collect data using various methods, including observation, controlled experiment, data logging, questionnaires and surveys	H4.3a collect data using various methods, including observation, controlled experiment, data logging, questionnaires and surveys
F4.3b	gather data from secondary sources, including printed tables and lists from ICT-based sources	H4.3b gather data from secondary sources, including printed tables and lists from ICT-based sources
F4.3c	design and use two-way tables for discrete and grouped data	H4.3c design and use two-way tables for discrete and grouped data
		H4.3d deal with practical problems such as non-response or missing data

## 4 Processing and representing data

Pupils should be taught to:

Foundation Tier		Higher Tier
F4.4a	draw and produce, using paper and ICT, pie charts for categorical data, and diagrams for continuous data, including line graphs for time series, scatter graphs, frequency diagrams and stem-and-leaf diagrams	H4.4a draw and produce, using paper and ICT, pie charts for categorical data, and diagrams for continuous data, including line graphs (time series), scatter graphs, frequency diagrams, stem-and-leaf diagrams, cumulative frequency tables and diagrams, box plots and histograms for grouped continuous data
F4.4b	calculate mean, range and median of small data sets with discrete then continuous data; identify the modal class for grouped data	
F4.4c	understand and use the probability scale	
F4.4d	understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency	H4.4b understand and use estimates or measures of probability from theoretical models, or from relative frequency
F4.4e	list all outcomes for single events, and for two successive events, in a systematic way	H4.4c list all outcomes for single events, and for two successive events, in a systematic way
F4.4f	identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1	H4.4d identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1
F4.4g	find the median for large data sets and calculate an estimate of the mean for large data sets with grouped data	H4.4e find the median, quartiles and interquartile range for large data sets and calculate the mean for large data sets with grouped data
		H4.4f calculate an appropriate moving average



Foundation Tier		Higher Tier
		H4.4g know when to add or multiply two probabilities: if $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$ , whereas if $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$
		H4.4h use tree diagrams to represent outcomes of compound events, recognising when events are independent
F4.4h	draw lines of best fit by eye, understanding what these represent	H4.4i draw lines of best fit by eye, understanding what these represent
H4.4j	use relevant statistical functions on a calculator or spreadsheet	H4.4j use relevant statistical functions on a calculator or spreadsheet

## 5 Interpreting and Discussing Results

Pupils should be taught to:

Foundation Tier		Higher Tier
F4.5a	relate summarised data to the initial questions	H4.5a relate summarised data to the initial questions
F4.5b	interpret a wide range of graphs and diagrams and draw conclusions	H4.5b interpret a wide range of graphs and diagrams and draw conclusions; identify seasonality and trends in time series
F4.5c	look at data to find patterns and exceptions	H4.5c look at data to find patterns and exceptions
F4.5d	compare distributions and make inferences, using the shapes of distributions and measures of average and range	H4.5d compare distributions and make inferences, using shapes of distributions and measures of average and spread, including median and quartiles; understand frequency density
F4.5e	consider and check results and modify their approach if necessary	H4.5e consider and check results, and modify their approach if necessary

Foundation Tier		Higher Tier
F4.5f H4.5f	appreciate that correlation is a measure of the strength of the association between two variables; distinguish between positive, negative and zero correlation using lines of best fit; appreciate that zero correlation does not necessarily imply ‘no relationship’ but merely ‘no linear relationship’	H4.5f appreciate that correlation is a measure of the strength of the association between two variables; distinguish between positive, negative and zero correlation using lines of best fit; appreciate that zero correlation does not necessarily imply ‘no relationship’ but merely ‘no linear relationship’
F4.5g	use the vocabulary of probability to interpret results involving uncertainty and prediction	H4.5g use the vocabulary of probability to interpret results involving uncertainty and prediction
F4.5h	compare experimental data and theoretical probabilities	H4.5h compare experimental data and theoretical probabilities
F4.5i	understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics	H4.5i understand that if they repeat an experiment, they may – and usually will – get different outcomes, and that increasing sample size generally leads to better estimates of probability and population parameters
F4.5j	discuss implications of findings in the context of the problem	
F4.5k	interpret social statistics including index numbers; time series; and survey data	F4.5k interpret social statistics including index numbers; time series; and survey data

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Module 3

A02 Number and algebra

1 Using and applying number and algebra

Pupils should be taught to:

		Foundation Tier	Higher Tier
<b>Problem solving</b>			
F2.1a	select and use suitable problem-solving strategies and efficient techniques to solve numerical and algebraic problems	H2.1a	select and use appropriate and efficient techniques and strategies to solve problems of increasing complexity, involving numerical and algebraic manipulation
F2.1b	break down a complex calculation into simpler steps before attempting to solve it	H2.1d	break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods
F2.1c	use algebra to formulate and solve a simple problem – identifying the variable, setting up an equation, solving the equation and interpreting the solution in the context of the problem	H2.1c	make mental estimates of the answers to calculations; present answers to sensible levels of accuracy; understand how errors are compounded in certain calculations
F2.1d	make mental estimates of the answers to calculations; use checking procedures, including use of inverse operations; work to stated levels of accuracy		
	Assessed in Module 5	H2.1b	identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches

Foundation Tier		Higher Tier
<b>Communicating</b>		
F2.1e	interpret and discuss numerical and algebraic information presented in a variety of forms	H2.1e discuss their work and explain their reasoning using an increasing range of mathematical language and notation
F2.1f	use notation and symbols correctly and consistently within a given problem	H2.1h use notation and symbols correctly and consistently within a given problem
F2.1g	use a range of strategies to create numerical, algebraic or graphical representations of a problem and its solution	H2.1f use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem
F2.1h	present and interpret solutions in the context of the original problem	H2.1g present and interpret solutions in the context of the original problem
F2.1i	review and justify their choice of mathematical presentation	H2.1i examine critically, improve, then justify their choice of mathematical presentation, present a concise, reasoned argument

Assessed in Module 5

Foundation Tier		Higher Tier
<b>Reasoning</b>		
F2.1j	explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems	H2.1j explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems
F2.1k	show step-by-step deduction in solving a problem	H2.1k understand the difference between a practical demonstration and a proof H2.1l show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning H2.1m recognise the significance of stating constraints and assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem

Assessed in Module 5

## 2 Numbers and the number system

Pupils should be taught to:

Foundation Tier		Higher Tier
<b>Integers</b>		
F2.2a H2.2a	use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use positive numbers and negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition	H2.2a  use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition
<b>Powers and roots</b>		
F2.2b H2.2b	use the terms square, positive and negative square root, cube and cube root; use index notation for squares, cubes and powers of 10; use index laws for multiplication and division of integer powers; express standard index form both in conventional notation and on a calculator display	H2.2b  use the terms square, positive square root, negative square root, cube and cube root; use index notation and index laws for multiplication and division of integer powers; use standard index form, expressed in conventional notation and on a calculator display

Foundation Tier		Higher Tier
<b>Fractions</b>		
F2.2c	understand equivalent fractions, simplify a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator	H2.2c understand equivalent fractions, simplify a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator
<b>Decimals</b>		
F2.2d H2.2d	use decimal notation and recognise that each terminating decimal is a fraction; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals	H2.2d recognise that each terminating decimal is a fraction; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals
<b>Percentages</b>		
F2.2e	understand that ‘percentage’ means ‘number of parts per 100’ and use this to compare proportions; interpret percentage as the operator ‘so many hundredths of’; use percentage in real-life situations	F2.2e H2.2e understand that ‘percentage’ means ‘number of parts per 100’ and use this to compare proportions; interpret percentage as the operator ‘so many hundredths of’; use percentage in real-life situations
<b>Ratio</b>		
F2.2f	use ratio notation, including reduction to its simplest form and its various links to fraction notation	H2.2f use ratio notation, including reduction to its simplest form and its various links to fraction notation



### 3 Calculations

Pupils should be taught to:

Foundation Tier		Higher Tier
<b>Number operations and the relationships between them</b>		
F2.3a H2.3a	add, subtract, multiply and divide integers and then any number; multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand ‘reciprocal’ as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers; use inverse operations	H2.3a  multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand ‘reciprocal’ as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer, fractional and negative powers; use inverse operations, understanding that the inverse operation of raising a positive number to power $n$ is raising the result of this operation to power $\frac{1}{n}$
F2.3b	use brackets and the hierarchy of operations	H2.3b  use brackets and the hierarchy of operations
F2.3c	calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal	H2.3c  calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal; distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals); convert a recurring decimal to a fraction



Foundation Tier		Higher Tier
F2.3d H2.3d	understand and use unit fractions as multiplicative inverses; multiply and divide a fraction by an integer, by a unit fraction and by a general fraction	H2.3d understand and use unit fractions as multiplicative inverses; multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction
F2.3e	convert simple fractions of a whole to percentages of the whole and vice versa, then understand the multiplicative nature of percentages as operators	H2.3e convert simple fractions of a whole to percentages of the whole and vice versa; then understand the multiplicative nature of percentages as operators; calculate an original amount when given the transformed amount after a percentage change; reverse percentage problems
F2.3f	divide a quantity in a given ratio	H2.3f divide a quantity in a given ratio

**Mental methods**

F2.3g H2.3g	recall all positive integer complements to 100; recall all multiplication facts to $10 \times 10$ , and use them to derive quickly the corresponding division facts; recall integer squares from $11 \times 11$ to $15 \times 15$ and the corresponding square roots, recall the cubes of 2, 3, 4, 5 and 10, and the fraction-to-decimal conversion of familiar simple fractions	H2.3g recall integer squares from $2 \times 2$ to $15 \times 15$ and the corresponding square roots, the cubes of 2, 3, 4, 5 and 10, the fact that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers $n$ , the corresponding rule for negative numbers, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number $n$
F2.3h	round to the nearest integer and to one significant figure; estimate answers to problems involving decimals	H2.3h round to a given number of significant figures; develop a range of strategies for mental calculation; derive unknown facts from those they know; convert between ordinary and standard index form representations, converting to standard index form to make sensible estimates for calculations involving multiplication and/or division

Foundation Tier		Higher Tier
F2.3i	develop a range of strategies for mental calculation; add and subtract mentally numbers with up to two decimal places; derive unknown facts from those they know, multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments	F2.3i develop a range of strategies for mental calculation; add and subtract mentally numbers with up to two decimal places; multiply and divide numbers with no more than one decimal digit, using the commutative, associative, and distributive laws and factorisation where possible, or place value adjustments

**Written methods**

F2.3j	use standard column procedures for addition and subtraction of integers and decimals	
F2.3k	use standard column procedures for multiplication of integers and decimals, understanding where to position the decimal point by considering what happens if they multiply equivalent fractions; solve a problem involving division by a decimal (up to 2 d.p.) by transforming it to a problem involving division by an integer	F2.3k division by decimal (up to 2 d.p.) by division using an integer; understand where to position the decimal point by considering what happens if they multiply equivalent fractions, e.g. given that...work out...
F2.3l	use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer	H2.3i use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that, in many cases, only a fraction can express the exact answer
F2.3m	solve simple percentage problems, including increase and decrease	H2.3j solve percentage problems, including percentage increase and decrease; and reverse percentages
F2.3n	solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution	F2.3n solve word problems about ratio and proportion, including using informal strategies and the unitary method of solution
		H2.3k represent repeated proportional change using a multiplier raised to a power

Foundation Tier		Higher Tier
		calculate an unknown quantity from quantities that vary in direct or inverse proportion
		calculate with standard index form
H2.3n	use $\pi$ in exact calculations, without a calculator	use surds and $\pi$ in exact calculations, without a calculator; rationalise a denominator such as $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

**Calculator methods**

F2.3o	use calculators effectively and efficiently; know how to enter complex calculations and use function keys for reciprocals, squares and powers	H2.3o	use calculators effectively and efficiently, knowing how to enter complex calculations; use an extended range of function keys, including trigonometrical and statistical functions relevant across this programme of study
F2.3p	enter a range of calculations, including those involving standard index form and measures	F2.3p	enter a range of calculations, including those involving measures
F2.3q H2.3p	understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation	H2.3p	understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation
		H2.3q	use calculators, or written methods, to calculate the upper and lower bounds of calculations, particularly when working with measurements

Foundation Tier		Higher Tier	
	H2.3r		use standard index form display and know how to enter numbers in standard index form
	H2.3s		use calculators for reverse percentage calculations by doing an appropriate division
	H2.3t		use calculators to explore exponential growth and decay, using a multiplier and the power key

## 4 Solving numerical problems

Pupils should be taught to:

Foundation Tier		Higher Tier
F2.4a H2.4a	draw on their knowledge of operations, inverse operations and the relationships between them, and of simple integer powers and their corresponding roots, and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, a range of measures and compound measures, metric units and conversion between metric and common imperial units, set in a variety of contexts	H2.4a  draw on their knowledge of operations and inverse operations (including powers and roots), and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, repeated proportional change, fractions, percentages and reverse percentages, inverse proportion, surds, measures and conversion between measures, and compound measures defined within a particular situation
F2.4b	select appropriate operations, methods and strategies to solve number problems, including trial and improvement where a more efficient method to find the solution is not obvious	
F2.4c H2.4b	estimate answers to problems; use a variety of checking procedures, including working the problem backwards, and considering whether a result is of the right order of magnitude	H2.4b  check and estimate answers to problems; select and justify appropriate degrees of accuracy for answers to problems; recognise limitations on the accuracy of data and measurements
F2.4d	give solutions in the context of the problem to an appropriate degree of accuracy, interpreting the solution shown on a calculator display, and recognising limitations on the accuracy of data and measurements	

## 5 Equations, formulae and identities

Pupils should be taught to:

Foundation Tier		Higher Tier
<b>Use of symbols</b>		
F2.5a	Assessed in Module 5	H2.5a distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations, defined quantities or variables in formulae, general, unspecified and independent numbers in identities, and in functions they define new expressions or quantities by referring to known quantities
F2.5b	Assessed in Module 5	H2.5b understand that the transformation of algebraic entities obeys and generalises the well-defined rules of generalised arithmetic; expand the product of two linear expressions; manipulate algebraic expressions by collecting like terms, multiplying a single term over a bracket, taking out common factors, multiplying quadratic expressions including the difference of two squares and cancelling common factors in rational expressions
		H2.5c know the meaning of and use the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’

### Direct and inverse proportion

H2.5h	set up and use equations to solve word and other problems involving direct proportion or inverse proportion and relate algebraic solutions to graphical representation of the equations
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## 6 Sequences, functions and graphs

Pupils should be taught to:

Foundation Tier		Higher Tier
<b>Quadratic functions</b>		
H2.6e	Assessed in Module 5	H2.6e generate points and plot graphs of simple quadratic functions, then more general quadratic functions; find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function; find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions

## Module 5

### A02 Number and algebra

#### 1 Using and applying number and algebra

Pupils should be taught to:

		Foundation Tier	Higher Tier
<b>Problem solving</b>			
F2.1a	select and use suitable problem-solving strategies and efficient techniques to solve numerical and algebraic problems	H2.1a	select and use appropriate and efficient techniques and strategies to solve problems of increasing complexity, involving numerical and algebraic manipulation
H2.1b	identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches	H2.1b	identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches
F2.1b H2.1c	break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods	H2.1c	break down a complex calculation into simpler steps before attempting to solve it and justify their choice of methods
F2.1d	make mental estimates of the answers to calculations; use checking procedures, including use of inverse operations; work to stated levels of accuracy	H2.1d	make mental estimates of the answers to calculations; present answers to sensible levels of accuracy; understand how errors are compounded in certain calculations
F2.1c	use algebra to formulate and solve a simple problem – identifying the variable, setting up an equation, solving the equation and interpreting the solution in the context of the problem		



Foundation Tier		Higher Tier
<b>Communicating</b>		
F2.1e	interpret and discuss numerical and algebraic information presented in a variety of forms	H2.1e discuss their work and explain their reasoning using an increasing range of mathematical language and notation
F2.1f	use notation and symbols correctly and consistently within a given problem	H2.1h use notation and symbols correctly and consistently within a given problem
F2.1g	use a range of strategies to create numerical, algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem	H2.1f use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem
F2.1h	present and interpret solutions in the context of the original problem	H2.1g present and interpret solutions in the context of the original problem
F2.1i	review and justify their choice of mathematical presentation	H2.1i examine critically, improve, then justify their choice of mathematical presentation, present a concise, reasoned argument
<b>Reasoning</b>		
F2.1j	explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems	H2.1j explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether particular cases can be generalised further, and understanding the importance of a counter-example, identify exceptional cases when solving problems
F2.1k	show step-by-step deduction in solving a problem	H2.1l show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning

Foundation Tier		Higher Tier
F2.1l	understand the difference between a practical demonstration and a proof	H2.1k understand the difference between a practical demonstration and a proof
F2.1m	recognise the importance of assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem	H2.1m recognise the significance of stating constraints and assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem

## 2 Numbers and the number system

Pupils should be taught to:

Foundation Tier	Higher Tier
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### Integers

F2.2a H2.2a	use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use positive numbers and negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple and common factor	H2.2a use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition
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Foundation Tier		Higher Tier
<b>Powers and roots</b>		
F2.2b H2.2b	use the terms square, positive and negative square root, cube and cube root; use index notation for squares, cubes and powers of 10; use index laws for multiplication and division of integer powers; express standard index form both in conventional notation and on a calculator display	Assessed in Module 3
<b>Fractions</b>		
F2.2c	understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator	understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator
<b>Decimals</b>		
F2.2d H2.2d	recognise that each terminating decimal is a fraction; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals	recognise that each terminating decimal is a fraction; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals; order decimals
<b>Percentages</b>		
F2.2e	understand that ‘percentage’ means ‘number of parts per 100’ and use this to compare proportions; interpret percentage as the operator ‘so many hundredths of’; use percentage in real-life situations	understand that ‘percentage’ means ‘number of parts per 100’ and use this to compare proportions; interpret percentage as the operator ‘so many hundredths of’; use percentage in real-life situations

### 3 Calculations

Pupils should be taught to:

Foundation Tier	Higher Tier
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#### Number operations and the relationships between them

F2.3a H2.3a	add, subtract, multiply and divide integers and then any number; multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers; use inverse operations	H2.3a	Assessed in Module 3
F2.3b	use brackets and the hierarchy of operations	H2.3b	Assessed in Module 3

#### Mental methods

F2.3g H2.3g	recall all positive integer complements to 100; recall all multiplication facts to $10 \times 10$ , and use them to derive quickly the corresponding division facts; recall integer squares from $11 \times 11$ to $15 \times 15$ and the corresponding square roots, recall the cubes of 2, 3, 4, 5 and 10, and the fraction-to-decimal conversion of familiar simple fractions	H2.3g	Assessed in Module 3
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Foundation Tier	Higher Tier
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**Calculator methods**

F2.3o	use calculators effectively and efficiently; know how to enter complex calculations and use function keys for reciprocals, squares and powers	H2.3o	Assessed in Module 3
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**4 Solving numerical problems**

Pupils should be taught to:

Foundation Tier		Higher Tier	
F2.4a H2.4a	draw on their knowledge of operations, inverse operations and the relationships between them, and of simple integer powers and their corresponding roots, and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, fractions, percentages and measures and conversion between measures, and compound measures defined within a particular situation	H2.4a	Assessed in Module 3

## 5 Equations, formulae and identities

Pupils should be taught to:

Foundation Tier		Higher Tier
<b>Use of symbols</b>		
F2.5a H2.5a	distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations, defined quantities or variables in formulae, general, unspecified and independent numbers in identities, and in functions they define new expressions or quantities by referring to known quantities	H2.5a distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations, defined quantities or variables in formulae, general, unspecified and independent numbers in identities, and in functions they define new expressions or quantities by referring to known quantities
<b>Foundation Tier</b>		
F2.5b H2.5b	understand that the transformation of algebraic expressions obeys and generalises the rules of generalised arithmetic, expand the product of two linear expressions; manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors; distinguish in meaning between the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’	H2.5b understand that the transformation of algebraic entities obeys and generalises the well-defined rules of generalised arithmetic; expand the product of two linear expressions; manipulate algebraic expressions by collecting like terms, multiplying a single term over a bracket, taking out common factors, factorising quadratic expressions including the difference of two squares and cancelling common factors in rational expressions
<b>Higher Tier</b>		
H2.5c		know the meaning of and use the words ‘equation’, ‘formula’, ‘identity’ and ‘expression’

Foundation Tier		Higher Tier
<b>Index notation</b>		
F2.5c	use index notation for simple integer powers, and simple instances of index laws; substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$	H2.5d use index notation for simple integer powers, and simple instances of index laws; substitute positive and negative numbers into expressions such as $3x^2 + 4$ and $2x^3$
<b>Inequalities</b>		
F2.5d	solve simple linear inequalities in one variable, and represent the solution set on a number line	H2.5j solve linear inequalities in one variable, and represent the solution set on a number line; solve several linear inequalities in two variables and find the solution set
<b>Equations</b>		
H2.5e	set up simple equations; solve simple equations by using inverse operations or by transforming both sides in the same way	H2.5e set up simple equations; solve simple equations by using inverse operations or by transforming both sides in the same way
<b>Linear Equations</b>		
F2.5e	solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution	H2.5f solve linear equations in one unknown, with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution



	Foundation Tier		Higher Tier
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**Formulae**

F2.5f	use formulae from mathematics and other subjects expressed initially in words and then using letters and symbols; substitute numbers into a formula; derive a formula and change its subject	H2.5g	use formulae from mathematics and other subjects; substitute numbers into a formula; change the subject of a formula including cases where the subject occurs twice, or where a power of the subject appears; generate a formula
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**Simultaneous linear equations**

H2.5i	find the exact solutions of two simultaneous equations in two unknowns by eliminating a variable and interpret the equations as lines and their common solution as the point of intersection
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**Quadratic equations**

H2.5k	solve quadratic equations by factorisation, completing the square and using the quadratic formula
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**Simultaneous linear and quadratic equations**

H2.5l	solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other, or where the second is of the form $x^2 + y^2 = r^2$
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**Numerical Methods**

H2.5m	use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
H2.5m	use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them



## 6 Sequences, functions and graphs

Pupils should be taught to:

Foundation Tier		Higher Tier
<b>Sequences</b>		
F2.6a H2.6a	generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); use linear expressions to describe the $n$ th term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated	H2.6a generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the $n$ th term of an arithmetic sequence, justifying its form by reference to the activity or context from which it was generated
<b>Graphs of linear functions</b>		
F2.6b	use the conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for $m$ and $c$ ) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which $y$ is given explicitly in terms of $x$ , or implicitly	H2.6b use conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for $m$ and $c$ ) that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which $y$ is given explicitly in terms of $x$ , or implicitly; no table or axes given
F2.6c	construct linear functions from real-life problems and plot their corresponding graphs; discuss and interpret graphs modelling real situations; understand that the point of intersection of two different lines in the same two variables that simultaneously describe a real situation is the solution to the simultaneous equations represented by the lines; draw line of best fit through a set of linearly related points and find its equation	H2.6c find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for $m$ and $c$ ); understand that the form $y = mx + c$ represents a straight line and that $m$ is the gradient of the line and $c$ is the value of the $y$ – intercept; explore the gradients of parallel lines and lines perpendicular to each other

Foundation Tier		Higher Tier
<b>Gradients</b>		
F2.6d	Find the gradient of lines given by equations of the form $y = mx + c$ (when values are given for $m$ and $c$ ), investigate the gradients of parallel lines	
<b>Interpreting graphical information</b>		
F2.6e	interpret information presented in a range of linear and non-linear graphs	construct linear functions and plot the corresponding graphs arising from real-life problems; discuss and interpret graphs modelling real situations
<b>Quadratic functions</b>		
H2.6e	generate points and plot graphs of simple quadratic functions, then more general quadratic functions; find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function	generate points and plot graphs of simple quadratic functions, then more general quadratic functions; find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function; find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions
<b>Other functions</b>		
H2.6f		plot graphs of simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$ , the exponential function $y = k^x$ for integer values of $x$ and simple positive values of $k$ , the circular functions $y = \sin x$ and $y = \cos x$ , using a spreadsheet or graph plotter as well as pencil and paper; recognise the characteristic shapes of all these functions

Foundation Tier	Higher Tier
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**Transformation of functions**

H2.6g	<p>apply to the graph of <math>y = f(x)</math> the transformations <math>y = f(x) + a</math>, <math>y = f(ax)</math>, <math>y = f(x + a)</math>, <math>y = af(x)</math> for linear, quadratic, sine and cosine functions <math>f(x)</math></p>
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**Loci**

H2.6h	<p>construct the graphs of simple loci including the circle <math>x^2 + y^2 = r^2</math> for a circle of radius <math>r</math> centred at the origin of coordinates; find graphically the intersection points of a given straight line with this circle and know that this corresponds to solving the two simultaneous equations representing the line and the circle</p>
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## A03 Shape, space and measures

### 1 Using and applying shape, space and measures

Pupils should be taught to:

Foundation Tier	Higher Tier
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#### Problem solving

F3.1a H3.1a	select problem-solving strategies and resources, including ICT tools, to use in geometrical work, and monitor their effectiveness; consider and explain the extent to which the selections they made were appropriate	H3.1a	select the problem-solving strategies to use in geometrical work, and consider and explain the extent to which the selections they made were appropriate
F3.1b	select and combine known facts and problem-solving strategies to solve complex problems	H3.1b	select and combine known facts and problem-solving strategies to solve more complex geometrical problems
F3.1c H3.1c	identify what further information is needed to solve a geometrical problem; break complex problems down into a series of tasks; develop and follow alternative lines of enquiry	H3.1c	develop and follow alternative lines of enquiry, justifying their decisions to follow or reject particular approaches

#### Communicating

F3.1d	interpret, discuss and synthesise geometrical information presented in a variety of forms		
F3.1e H3.1d	communicate mathematically with emphasis on a critical examination of the presentation and organisation of results, and on effective use of symbols and geometrical diagrams	H3.1d	communicate mathematically, with emphasis on a critical examination of the presentation and organisation of results, and on effective use of symbols and geometrical diagrams

Foundation Tier		Higher Tier
F3.1f	use geometrical language appropriately	H3.1e use precise formal language and exact methods for analysing geometrical configurations
F3.1g	review and justify their choices of mathematics presentation	F3.1g review and justify their choices of mathematics presentation

**Reasoning**

F3.1h	distinguish between practical demonstrations and proofs	F3.1h distinguish between practical demonstrations and proofs
F3.1i	apply mathematical reasoning, explaining and justifying inferences and deductions	H3.1f apply mathematical reasoning, progressing from brief mathematical explanations towards full justifications in more complex contexts
F3.1j	show step-by-step deduction in solving a geometrical problem	H3.1h show step-by-step deduction in solving a geometrical problem
F3.1k	state constraints and give starting points when making deductions	H3.1i state constraints and give starting points when making deductions
F3.1l	recognise the limitations of any assumptions that are made; understand the effects that varying the assumptions may have on the solution	
F3.1m	identify exceptional cases when solving geometrical problems	H3.1j understand the necessary and sufficient conditions under which generalisations, inferences and solutions to geometrical problems remain valid
		H3.1g explore connections in geometry; pose conditional constraints of the type ‘If... then...’; and ask questions ‘What if...?’ or ‘Why?’

## 2 Geometrical reasoning

Pupils should be taught to:

Foundation Tier	Higher Tier
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### Angles

F3.2a	recall and use properties of angles at a point, angles on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
F3.2b	distinguish between acute, obtuse, reflex and right angles; estimate the size of an angle in degrees

### Properties of triangles and other rectilinear shapes

F3.2c	distinguish between lines and line segments; use parallel lines, alternate angles and corresponding angles; understand the consequent properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices	H3.2a	distinguish between lines and line segments; use parallel lines, alternate angles and corresponding angles; understand the consequent properties of parallelograms and a proof that the angle sum of a triangle is 180 degrees; understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices
F3.2d	use angle properties of equilateral, isosceles and right-angled triangles; understand congruence; explain why the angle sum of a quadrilateral is 360 degrees	H3.2b	use angle properties of equilateral, isosceles and right-angled triangles; explain why the angle sum of a quadrilateral is 360 degrees
F3.2e	use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle	F3.2e	use their knowledge of rectangles, parallelograms and triangles to deduce formulae for the area of a parallelogram, and a triangle, from the formula for the area of a rectangle

Foundation Tier		Higher Tier
F3.2f H3.2c	recall the essential properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties	H3.2c recall the definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium and rhombus; classify quadrilaterals by their geometric properties
F3.2g	calculate and use the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons; calculate and use the angles of regular polygons	H3.2d calculate and use the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons; calculate and use the angles of regular polygons.
		H3.2e understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and compass constructions
F3.2h	understand, recall and use Pythagoras' theorem	H3.2f understand, recall and use Pythagoras' theorem in 2-D, then 3-D problems; investigate the geometry of cuboids including cubes, and shapes made from cuboids, including the use of Pythagoras' theorem to calculate lengths in three dimensions
		H3.2g understand similarity of triangles and of other plane figures, and use this to make geometric inferences; understand, recall and use trigonometrical relationships in right-angled triangles, and use these to solve problems, including those involving bearings, then use these relationships in 3-D contexts, including finding the angles between a line and a plane (but not the angle between two planes or between two skew lines); calculate the area of a triangle using $\frac{1}{2} ab \sin C$ ; draw, sketch and describe the graphs of trigonometric functions for angles of any size, including transformations involving scalings in either or both the $x$ and $y$ directions; use the sine and cosine rules to solve 2-D and 3-D problems



Foundation Tier		Higher Tier
<b>Properties of circles</b>		
F3.2i	recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that inscribed regular polygons can be constructed by equal division of a circle	H3.2h recall the definition of a circle and the meaning of related terms, including centre, radius, chord, diameter, circumference, tangent, arc, sector and segment; understand that the tangent at any point on a circle is perpendicular to the radius at that point; understand and use the fact that tangents from an external point are equal in length; explain why the perpendicular from the centre to a chord bisects the chord; understand that inscribed regular polygons can be constructed by equal division of a circle; prove and use the facts that the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference, the angle subtended at the circumference by a semicircle is a right angle, that angles in the same segment are equal, and that opposite angles of a cyclic quadrilateral sum to 180 degrees; prove and use the alternate segment theorem
<b>3-D shapes</b>		
F3.2j	explore the geometry of cuboids (including cubes), and shapes made from cuboids	
F3.2k H3.2i	use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms	use 2-D representations of 3-D shapes and analyse 3-D shapes through 2-D projections and cross-sections, including plan and elevation; solve problems involving surface areas and volumes of prisms, pyramids, cylinders, cones and spheres; solve problems involving more complex shapes and solids, including segments of circles and frustums of cones



### 3 Transformations and coordinates

Pupils should be taught to:

Foundation Tier		Higher Tier
<b>Specifying transformations</b>		
F3.3a H3.3a	understand that rotations are specified by a centre and an (anticlockwise) angle; rotate a shape about the origin, or any other point; measure the angle of rotation using right angles, simple fractions of a turn or degrees; understand that reflections are specified by a mirror line, at first using a line parallel to an axis, then a mirror line such as $y = x$ or $y = -x$ ; understand that translations are specified by a distance and direction (or a vector), and enlargements by a centre and positive scale factor	H3.3a understand that rotations are specified by a centre and an (anticlockwise) angle; use any point as the centre of rotation; measure the angle of rotation, using right angles, fractions of a turn or degrees; understand that reflections are specified by a (mirror) line; understand that translations are specified by giving a distance and direction (or a vector), and enlargements by a centre and a positive scale factor

#### Properties of transformations

F3.3b H3.3b	recognise and visualise rotations, reflections and translations, including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations, recognising that these transformations preserve length and angle, so that any figure is congruent to its image under any of these transformations; distinguish properties that are preserved under particular transformations	H3.3b recognise and visualise rotations, reflections and translations including reflection symmetry of 2-D and 3-D shapes, and rotation symmetry of 2-D shapes; transform triangles and other 2-D shapes by translation, rotation and reflection and combinations of these transformations; use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations; distinguish properties that are preserved under particular transformations
F3.3c	recognise, visualise and construct enlargements of objects using positive scale factors greater than one, then positive scale factors less than one; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not	H3.3c recognise, visualise and construct enlargements of objects; understand from this that any two circles and any two squares are mathematically similar, while, in general, two rectangles are not, then use positive fractional and negative scale factors

Foundation Tier		Higher Tier
F3.3d	recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments and apply this to triangles; understand the implications of enlargement for perimeter; use and interpret maps and scale drawings; understand the implications of formulae for perimeter, area and volume by considering dimensions; distinguish between formulae for perimeter, area and volume by considering dimensions; understand and use simple examples of the relationship between enlargement and areas and volumes of shapes and solids	H3.3d recognise that enlargements preserve angle but not length; identify the scale factor of an enlargement as the ratio of the lengths of any two corresponding line segments; understand the implications of enlargement for perimeter; use and interpret maps and scale drawings; understand the difference between formulae for perimeter, area and volume by considering dimensions; understand and use the effect of enlargement on areas and volumes of shapes and solids

**Coordinates**

F3.3e	understand that one coordinate identifies a point on a number line, two coordinates identify a point in a plane and three coordinates identify a point in space, using the terms ‘1-D’, ‘2-D’ and ‘3-D’; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; find the coordinates of points identified by geometrical information; find the coordinates of the midpoint of the line segment AB, given points A and B, then calculate the length AB	H3.3e understand that one coordinate identifies a point on a number line, that two coordinates identify a point in a plane and three coordinates identify a point in space, using the terms ‘1-D’, ‘2-D’ and ‘3-D’; use axes and coordinates to specify points in all four quadrants; locate points with given coordinates; find the coordinates of points identified by geometrical information; find the coordinates of the midpoint of the line segment AB, given the points A and B, then calculate the length AB
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**Vectors**

H3.3f	understand and use vector notation for translations	H3.3f understand and use vector notation; calculate, and represent graphically the sum of two vectors, the difference of two vectors and a scalar multiple of a vector; calculate the resultant of two vectors; understand and use the commutative and associative properties of vector addition; solve simple geometrical problems in 2-D using vector methods
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## 4 Measures and construction

Pupils should be taught to:

Foundation Tier		Higher Tier
<b>Measures</b>		
F3.4a	interpret scales on a range of measuring instruments, including those for time and mass; know that measurements using real numbers depend on the choice of unit; recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction; convert measurements from one unit to another; know rough metric equivalents of pounds, feet, miles, pints and gallons; make sensible estimates of a range of measures in everyday settings	H3.4a use angle measure; know that measurements using real numbers depend on the choice of unit; recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction; convert measurements from one unit to another; understand and use compound measures, including speed and density
F3.4b	understand angle measure using the associated language	
F3.4c	understand and use compound measures, including speed and density	
H3.4a		
<b>Construction</b>		
F3.4d	measure and draw lines to the nearest millimetre, and angles to the nearest degree; draw triangles and other 2-D shapes using a ruler and protractor, given information about their side lengths and angles; understand, from their experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; construct cubes, regular tetrahedra, square-based pyramids and other 3-D shapes from given information	F3.4d H3.4b draw approximate constructions of triangles and other 2-D shapes, using a ruler and protractor, given information about side lengths and angles; understand, from their experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not; construct specified cubes, regular tetrahedra, square-based pyramids and other 3-D shapes

Foundation Tier		Higher Tier
F3.4e	use straight edge and compasses to do standard constructions, including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle	H3.4c use straight edge and compasses to do standard constructions including an equilateral triangle with a given side, the midpoint and perpendicular bisector of a line segment, the perpendicular from a point to a line, the perpendicular from a point on a line, and the bisector of an angle

**Mensuration**

F3.4f	find areas of rectangles, recalling the formula, understanding the connection to counting squares and how it extends this approach; recall and use the formulae for the area of a parallelogram and a triangle; find the surface area of simple shapes using the area formulae for triangles and rectangles; calculate perimeters and areas of shapes made from triangles and rectangles	F3.4f F3.4i H3.4d calculate perimeters and areas of shapes made from triangles and rectangles; find the surface area of simple shapes by using the formulae for the areas of triangles and rectangles; find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids; convert between area measures, including square centimetres and square metres, and volume measures, including cubic centimetres and cubic metres; find circumferences of circles and areas enclosed by circles, recalling relevant formulae; calculate the lengths of arcs and the areas of sectors of circles
F3.4g	find volumes of cuboids, recalling the formula and understanding the connection to counting cubes and how it extends this approach; calculate volumes of right prisms and of shapes made from cubes and cuboids	
F3.4h	find circumferences of circles and areas enclosed by circles, recalling relevant formulae	
F3.4i	convert between area measures, including square centimetres and square metres, and volume measures, including cubic centimetres and cubic metres	

Foundation Tier		Higher Tier
<b>Loci</b>		
F3.4j	find loci, both by reasoning and by using ICT to produce shapes and paths	H3.4e find loci, both by reasoning and by using ICT to produce shapes and paths

## Key Skills and Other Issues

### 12

## Key Skills – Teaching, Developing and Providing Opportunities for Generating Evidence

### 12.1 Introduction

The Key Skills Qualification requires candidates to demonstrate levels of achievement in the Key Skills of *Communication*, *Application of Number* and *Information and Communication Technology*.

The units for the ‘wider’ Key Skills of *Working with Others*, *Improving own Learning and Performance* and *Problem Solving* are also available. The acquisition and demonstration of ability in these ‘wider’ Key Skills is deemed highly desirable for all candidates, but they do not form part of the Key Skills Qualification.

Centres intending to use this specification to meet the Key Skills requirements are advised to cross-check the requirement with the QCA documentation.

Copies of the Key Skills Units may be downloaded from the QCA Website (<http://www.qca.org.uk/keyskills>).

The units for each Key Skill comprise three sections:

- A What you need to know
- B What you must do
- C Guidance

Candidates following a course of study based on this Specification for GCSE Mathematics (Modular) can be offered opportunities to develop and generate evidence of attainment in aspects of the Key Skills of *Communication*, *Application of Number*, *Information and Communication Technology*, *Working with Others*, *Improving own Learning and Performance* and *Problem Solving*. Areas of study and learning that can be used to encourage the acquisition and use of Key Skills, and to provide opportunities to generate evidence for Part B of the units, are signposted below.

### 12.2 Key Skills Opportunities in Mathematics (Modular)

The signposting which follows indicates the opportunities to acquire and produce evidence of the Key Skills in AO2 – 4. AO1, *Using and applying mathematics* which is assessed in the context of AO2 – 3, also provides opportunities.

**Communication Level 1**

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>C1.1</b> Take part in either a <b>one-to-one</b> discussion or a <b>group</b> discussion.	✓	✓	✓
<b>C1.2</b> Read and obtain information from at least <b>one</b> document.	✓	✓	✓
<b>C1.3</b> Write <b>two</b> different types of documents.	–	–	–

**Communication Level 2**

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>C2.1a</b> Take part in a group discussion.	✓	✓	✓
<b>C2.1b</b> Give a talk of at least four minutes.	✓	✓	✓
<b>C2.2</b> Read and summarise information from at least <b>two</b> documents about the same subject. Each document must be a minimum of 500 words long.	✓	✓	✓
<b>C2.3</b> Write <b>two</b> different types of documents each one giving different information. One document must be at least 500 words long.	–	–	–



### Application of Number Level 1

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>N1.1</b> Interpret information from <b>two</b> different sources. At least <b>one</b> source must include a table, chart, graph or diagram.	✓	✓	✓
<b>N1.2</b> Carry out and check calculations to do with: a amounts or sizes b scales or proportion c handling statistics.	✓	✓	✓
<b>N1.3</b> Interpret the results of your calculations and present your findings – in two different ways using charts or diagrams.	✓	✓	✓

### Application of Number Level 2

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>N2.1</b> Interpret information from a suitable source.	✓	✓	✓
<b>N2.2</b> Use your information to carry out calculations to do with: a amounts or sizes b scales or proportion c handling statistics d using formulae.	✓	✓	✓
<b>N2.3</b> Interpret the results of your calculations and present your findings.	✓	✓	✓



**Information and Communication Technology Level 1**

<b>What you must do ...</b>	<b>Signposting of Opportunities for Generating Evidence in Subject Content</b>		
<b>ICT1.1</b> Find and select relevant information.	✓	✓	✓
<b>ICT1.2</b> Enter and develop information to suit the task.	✓	✓	✓
<b>ICT1.3</b> Develop the presentation so that the final output is accurate and fit for purpose.	–	–	–

**Information and Communication Technology Level 2**

<b>What you must do ...</b>	<b>Signposting of Opportunities for Generating Evidence in Subject Content</b>		
	<b>AO2</b>	<b>AO3</b>	<b>AO4</b>
<b>ICT2.1</b> Search for and select information to meet your needs. Use different information sources for each task and multiple search criteria in at least one case.	✓	✓	✓
<b>ICT2.2</b> Enter and develop the information to suit the task and derive new information.	✓	✓	✓
<b>ICT2.3</b> Present combined information such as text with image, text with number, image with number.	✓	✓	✓

**Working with Others Level 1**

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>WO1.1</b> Confirm you understand the given objectives, and plan for working together.	✓	✓	✓
<b>WO1.2</b> Work with others towards achieving the given objectives.	✓	✓	✓
<b>WO1.3</b> Identify ways you helped to achieve things and how to improve your work with others.	✓	✓	✓

**Working with Others Level 2**

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>WO2.1</b> Plan work with others.	✓	✓	✓
<b>WO2.2</b> Work co-operatively towards achieving the identified objectives.	✓	✓	✓
<b>WO2.3</b> Review your contributions and agree ways to improve work with others.	✓	✓	✓

**Improving Own Learning and Performance Level 1**

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>LP1.1</b> Confirm your targets and plan how to meet these with the person setting them.	✓	✓	✓
<b>LP1.2</b> Follow your plan, to help meet targets and improve your performance.	✓	✓	✓
<b>LP1.3</b> Review your progress and achievements in meeting targets, with an appropriate person.	✓	✓	✓

**Improving Own Learning and Performance Level 2**

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>LP2.1</b> Help set targets with an appropriate person and plan how these will be met.	✓	✓	✓
<b>LP2.2</b> Take responsibility for some decisions about your learning, using your plan to help meet targets and improve your performance.	✓	✓	✓
<b>LP2.3</b> Review progress with an appropriate person and provide evidence of your achievements.	✓	✓	✓

**Problem Solving Level 1**

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>PS1.1</b> Confirm with an appropriate person that you understand the given problem and identify different ways of tackling it.	✓	✓	✓
<b>PS1.2</b> Confirm with an appropriate person what you will do and follow your plan for solving the problem.	✓	✓	✓
<b>PS1.3</b> Check with an appropriate person if the problem has been solved and how to improve your problem solving skills.	✓	✓	✓

**Problem Solving Level 2**

What you must do ...	Signposting of Opportunities for Generating Evidence in Subject Content		
	AO2	AO3	AO4
<b>PS2.1</b> Identify a problem, with help from an appropriate person, and identify different ways of tackling it.	✓	✓	✓
<b>PS2.2</b> Plan and try out at least one way of solving the problem.	✓	✓	✓
<b>PS2.3</b> Check if the problem has been solved and identify ways to improve problem solving skills.	✓	✓	✓

12.3	Exemptions for the Key Skills Qualification	GCSE A* – C examination performance on this specification provides exemptions for the external test in Application of Number at Level 2.
12.4	Further Guidance	More specific guidance and examples of tasks that can provide evidence of single Key Skills, or composite tasks that can provide evidence of more than one Key Skill are given in the AQA specification support material, particularly the document <i>A Teacher's Guide</i> .

## 13

# Spiritual, Moral, Ethical, Social, Cultural and Other Issues

13.1	Spiritual, Moral, Ethical, Social, Cultural and Other Issues	<p>Mathematics provides opportunities to promote:</p> <ul style="list-style-type: none"> <li>• <i>spiritual development</i>, through explaining the underlying mathematical principles behind some of the natural forms and patterns in the world around us</li> <li>• <i>moral development</i>, helping pupils recognise how logical reasoning can be used to consider the consequences of particular decisions and choices helping them learn the value of mathematical truth</li> <li>• <i>social development</i>, through helping pupils work together productively on complex mathematical tasks and helping them see that the result is often better than could be achieved separately</li> <li>• <i>cultural development</i>, through helping pupils appreciate that mathematical thought contributes to the development of our culture and is becoming increasingly central to our highly technological future, and through recognising that mathematicians from many cultures have contributed to the development of modern day mathematics.</li> </ul>
13.2	European Dimension	AQA has taken account of the 1988 Resolution of the Council of the European Community in preparing this specification and associated specimen papers.
13.3	Environmental Issues	AQA has taken account of the 1988 Resolution of the Council of the European Community and the Report <i>Environmental Responsibility: An Agenda for Further and Higher Education</i> 1993 in preparing this specification and associated specimen papers.

13.4	Citizenship	A number of specification references, particularly those for AO4 <i>Handling data</i> , promote the skills of enquiry and communication. They also encourage the skill of participation and responsible action in the educational establishment and/or communication.
13.5	Avoidance of Bias	AQA has taken great care in the preparation of this specification and associated specimen papers to avoid bias of any kind.
13.6	Health and Safety	Aspects of the specification, particularly within AO4 <i>Handling data</i> provide opportunities to promote Health and Safety issues.
13.7	ICT	<p>(a) Pupils should be given opportunities to apply and develop their ICT capability through the use of ICT tools to support their learning in mathematics.</p> <p>(b) Pupils should be given opportunities to support their work by being taught to:</p> <ul style="list-style-type: none"><li>(i) find things out from a variety of sources, selecting and synthesising the information to meet their needs and developing an ability to question its accuracy, bias and plausibility</li><li>(ii) develop their ideas using ICT tools to amend and refine their work and enhance its quality and accuracy</li><li>(iii) exchange and share information, both directly and through electronic media</li><li>(iv) review, modify and evaluate their work, reflecting critically on its quality, as it progresses.</li></ul>
13.8	Other issues	<p>Mathematics provides opportunities to promote:</p> <ul style="list-style-type: none"><li>• <i>thinking skills</i>, through developing pupils' problem-solving skills and deductive reasoning</li><li>• <i>financial capability</i>, through applying mathematics to problems set in financial contexts</li><li>• <i>enterprise and entrepreneurial skills</i>, through developing pupils' abilities to apply mathematics in science and technology, in economics and in risk assessment;</li><li>• <i>work related learning</i>, through developing pupils' abilities to use and apply mathematics in workplace situations and in solving real-life problems.</li></ul>

# Awarding and Reporting

## 14

### Grading, Shelf-life and Re-sits

#### 14.1 Qualification Titles

The qualification based on this specification has the following title:  
AQA General Certificate of Secondary Education in Mathematics.

#### 14.2 Grading System

The qualification will be graded on an 8 point grade Scale A\*, A, B, C, D, E, F, G. Candidates who fail to reach the minimum standard for grade G will be recorded as U (unclassified) and will not receive a qualification certificate.

The written paper modules are offered at two tiers of entry: Foundation tier and Higher tier. For candidates entered for the Foundation tier, grades C – G are available. For candidates entered for the Higher tier, grades A\* – D are available. There is a safety net for candidates entered for the Higher tier, where an allowed grade E will be awarded where candidates just fail to achieve grade D. Candidates may enter for each individual module at a different tier of entry. However, the final range of grades available to a candidate is determined by the tier of entry of Module 5. Candidates who fail to achieve grade E on the Higher tier or grade G on the Foundation tier will be reported as U (unclassified).

#### 14.3 The Determination of Candidates' Final Grades

For each module, candidates' results are reported on a *Uniform Mark Scale* which is related to grades by means of the following correspondence.

#### Module 1 (Maximum Uniform Mark = 108)

Mark Range	Grade
97 – 108	A*
86 – 96	A
76 – 85	B
65 – 75	C
54 – 64	D
43 – 53	E
32 – 42	F
22 – 31	G
0 – 21	U

**Module 3 (Maximum Uniform Mark = 162)**

Mark Range	Grade
146 – 162	A*
130 – 145	A
113 – 129	B
97 – 112	C
81 – 96	D
65 – 80	E
49 – 64	F
32 – 48	G
0 – 31	U

**Module 5 (Maximum Uniform Mark = 330)**

Mark Range	Grade
297 – 330	A*
264 – 296	A
231 – 263	B
198 – 230	C
165 – 197	D
132 – 164	E
99 – 131	F
66 – 98	G
0 – 65	U

A candidate's uniform mark is calculated from his/her raw mark for the module by using the grade boundaries set by the Awarding Committee. For example, a candidate who achieved the minimum raw mark required for grade B on Module 1 receives a uniform mark of 76.

A candidate cannot obtain a uniform mark corresponding to a grade which is above the range for the tier. For example, on Module 1 a candidate entered for the Foundation tier (grade range C – G) cannot obtain a uniform mark higher than 75, even if he/she achieves the maximum (raw) marks for the paper.

On individual modules there is a small 'safety net' for candidates who fail to reach the minimum mark required for the lowest grade available in the tier.



For example, on Module 1 a candidate entered for the Higher tier (grade range A\* – D) who just fails to reach the standard required for grade D does not obtain zero uniform marks. However, centres should note that such a candidate will normally be awarded fewer uniform marks than a Foundation tier candidate who reaches the same standard.

A candidate's overall uniform mark is obtained by adding together the uniform marks for the three modules. This overall mark is then converted to a grade by means of the following correspondence.

**Overall (Maximum Uniform Mark = 600)**

Mark Range	Grade
540 – 600	A*
480 – 539	A
420 – 479	B
360 – 419	C
300 – 359	D
240 – 299	E
180 – 239	F
120 – 179	G

The final grade must be in a range which is available for the candidate's tier of entry for Module 5.

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**14.4 Shelf-life of Module Results**

The shelf-life of individual module results, prior to the award of the qualification, is limited only by the shelf-life of the specification.

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**14.5 Re-taking Modules and Carrying Forward of Module Results**

Each tier of Modules 1 and 3 may be re-taken once before certification of the qualification. The best result for each module will count towards the final award.

Candidates who wish to re-take the qualification after first certification may re-use results from Modules 1 and 3, but Module 5 must be taken again. For Modules 1 and 3 the two most recent results from each tier will be considered, and the best of these results will count towards the final award. For example, if a candidate attempts Module 1 once at the Higher tier and twice at the Foundation tier before first certification, then once more at the Foundation tier before certifying again, the Higher tier attempt and the second and third Foundation tier attempts are eligible to count towards the final award.

Results for Modules 1 and 3 from specifications 3302 and 4302 cannot be carried forward to this specification.

In the case of Module 5 the most recent attempt will always be the one that counts. Candidates may take the whole qualification an unlimited number of times. There is no limit to the number of times a result for Modules 1 and 3 may be re-used.

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14.6 Minimum Requirements

Candidates will be graded on the basis of work submitted for assessment.

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14.7 Awarding and Reporting

The procedures for Awarding Grades and Reporting Results to centres comply with the GCSE Code of Practice issued by the Regulatory Authorities.

# Appendices

## A

### Grade Descriptions

The following grade descriptors indicate the level of attainment characteristic of the given grade at GCSE. They give a general indication of the required learning outcomes at each specific grade. The descriptors should be interpreted in relation to the content outlined in the specification; they are not designed to define that content.

The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives (as in Section 6) overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

**Grade A** Candidates give reasons for the choices they make when investigating within mathematics itself or when using mathematics to analyse tasks: these reasons explain why particular lines of enquiry or procedures are followed and others rejected. Candidates apply the mathematics they know in familiar and unfamiliar contexts. Candidates use mathematical language and symbols effectively in presenting a convincing reasoned argument. Their reports include mathematical justifications, explaining their solutions to problems involving a number of features or variables.

Candidates manipulate simple surds. They determine the bounds of intervals. Candidates understand and use direct and inverse proportion. They manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions. In simplifying algebraic expressions, they use rules of indices for negative and fractional values. In finding formulae that approximately connect data, candidates express general laws in symbolic form. They solve problems using intersections and gradients of graphs.

Candidates sketch the graphs of sine, cosine and tangent functions for any angle, and generate and interpret graphs based on these functions. Candidates use sine, cosine and tangent of angles of any size, and Pythagoras' theorem when solving problems in two and three dimensions. They use the conditions for congruent triangles in formal geometric proofs. They calculate lengths of circular arcs and areas of sectors, and calculate the surface area of cylinders and volumes of cones and spheres. They understand and use the effect of enlargement on areas and volumes of shapes and solids.

Candidates interpret and construct histograms. They understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn; they select and justify a sample and method to investigate a population. They recognise when and how to work with probabilities associated with independent and mutually exclusive events.

Grade C Starting from problems or contexts that have been presented to them, candidates refine or extend the mathematics used to generate fuller solutions. They give a reason for their choice of mathematical presentation, explaining features they have selected. Candidates justify their generalisations, arguments or solutions, showing some insight into the mathematical structure of the problem. They appreciate the difference between mathematical explanation and experimental evidence

In making estimates candidates use appropriate techniques and multiply and divide mentally. They solve numerical problems involving multiplication and division with numbers of any size using a calculator efficiently and appropriately. They understand the effects of multiplying and dividing by numbers between 0 and 1. They use ratios in appropriate situations. They understand and use proportional changes. Candidates find and describe in symbols the next term or the  $n$ th term of a sequence, where the rule is linear. They multiply two expressions of the form  $(x + n)$ ; they simplify the corresponding quadratic expressions. They solve simple polynomial equations by trial and improvement and represent inequalities using a number line. They formulate and solve linear equations with whole number coefficients. They manipulate simple algebraic formulae, equations and expressions. Candidates draw and use the graphs of quadratic functions.

Candidates solve problems using angle and symmetry properties of polygons and properties of intersecting and parallel lines. They understand and apply Pythagoras' theorem when solving problems in two-dimensions. Candidates solve problems involving areas and circumferences of circles. They calculate lengths, areas and volumes in plane shapes and right prisms. Candidates enlarge shapes by a positive whole number or fractional scale factor. They appreciate the imprecision of measurement and recognise that a measurement given to the nearest whole number may be inaccurate by up to one half in either direction. They understand and use compound measures such as speed. Candidates use mathematical instruments to carry out accurate constructions of loci.

Candidates construct and interpret frequency diagrams. They specify hypotheses and test them. They determine the modal class and estimate the mean, median and range of a set of grouped data, selecting the statistic most appropriate to their line of enquiry. They use measures of average and range with associated frequency polygons, as appropriate, to compare distributions and make inferences. They draw a line of best fit on a scatter diagram by inspection. Candidates understand relative frequency as an estimate of probability and use this to compare outcomes of experiments.

Grade F In order to carry through tasks and solve mathematical problems, candidates identify and obtain necessary information; they check their results, considering whether these are sensible. Candidates show understanding of situations by describing them mathematically using symbols, words and diagrams. They draw simple conclusions of their own and give an explanation of their reasoning.

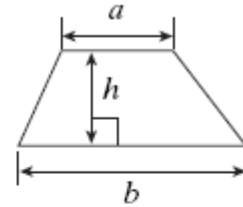
Candidates use their understanding of place value to multiply and divide whole numbers and decimals by 10, 100 and 1000. They order, add and subtract negative numbers in context. They use all four operations with decimals to two places. They reduce a fraction to its simplest form by cancelling common factors and solve simple problems involving ratio and direct proportion. They calculate fractional or percentage parts of quantities and measurements, using a calculator where necessary. Candidates understand and use an appropriate non-calculator method for solving problems involving multiplying and dividing any three-digit by any two-digit number. In solving problems with or without a calculator, candidates check the reasonableness of their results by reference to their knowledge of the context or to the size of the numbers, by applying inverse operations or by estimating using approximations. Candidates explore and describe number patterns and relationships including multiple, factor and square. They construct, express in symbolic form, and use simple formulae involving one or two operations.

When constructing models and when drawing, or using shapes, candidates measure and draw angles as accurately as practicable and use language associated with angle. They know the angle sum of a triangle and that of angles at a point. They identify all the symmetries of 2-D shapes. They know the rough metric equivalents of imperial units still in daily use and convert one metric unit to another. They make sensible estimates of a range of measures in relation to everyday situations. Candidates calculate areas of rectangles. Candidates use coordinates in all four quadrants to locate and specify points.

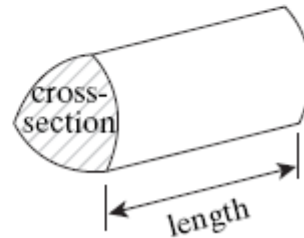
Candidates understand and use the mean of discrete data. They compare two simple distributions using the range and one of the mode, median or mean. They interpret graphs and diagrams, including pie charts, and draw conclusions. They understand and use the probability scale from 0 to 1. Candidates make and justify estimates of probability by selecting and using a method based on equally likely outcomes or on experimental evidence as appropriate. They understand that different outcomes may result from repeating an experiment.

**B****Formulae Sheets for Module 5****Foundation Tier**

**Area of trapezium** =  $\frac{1}{2}(a+b)h$

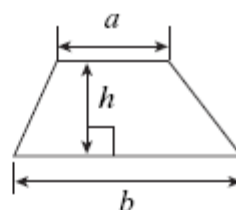


**Volume of prism** = area of cross-section  $\times$  length

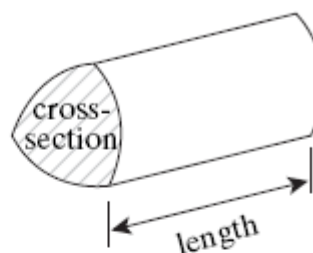


# Higher Tier

**Area of trapezium** =  $\frac{1}{2}(a+b)h$

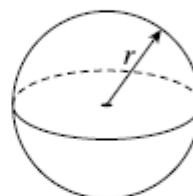


**Volume of prism** = area of cross-section  $\times$  length



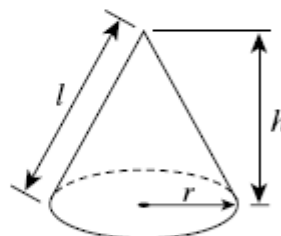
**Volume of sphere** =  $\frac{4}{3} \pi r^3$

**Surface area of sphere** =  $4 \pi r^2$



**Volume of cone** =  $\frac{1}{3} \pi r^2 h$

**Curved surface area of cone** =  $\pi r l$

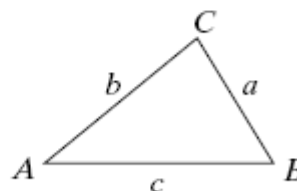


**In any triangle ABC**

**Area of triangle** =  $\frac{1}{2} ab \sin C$

**Sine rule**  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

**Cosine rule**  $a^2 = b^2 + c^2 - 2bc \cos A$



## The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$



## C

## Overlaps with other Qualifications

The subject content of this Specification is identical, though differently structured, to that of AQA GCSE Mathematics Specification A.

There is some overlap between Module 1 of this specification and GCSE Statistics.

There is a considerable overlap of skills and content between the modules of GCSE Mathematics (Modular) Specification B, Free-Standing Mathematics Qualifications (FSMQs) and the Key Skill of *Application of Number*. In some post-16 centres candidates on the different courses may be grouped together.

Further information about the links between these subjects can be obtained from AQA.