Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2016

Further Mathematics

Unit 2
Mechanics and Statistics

[GMF21]

GMF21

THURSDAY 23 JUNE, MORNING

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in blue or black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all fifteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

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Formula Sheet

PURE MATHEMATICS

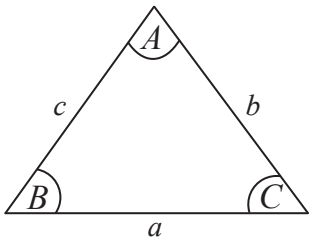
Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \dfrac{1}{2} ab \sin C$



Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: $\text{Mean} = \frac{\Sigma fx}{\Sigma f}$ $\quad \text{Median} = L_1 + \frac{\left\{\frac{N}{2} - (\Sigma f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
N is total frequency
$(\Sigma f)_1$ is the sum of the frequencies up to but not including the median class
f_{median} is the frequency of the median class
c is the width of the median class

$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

[Turn over

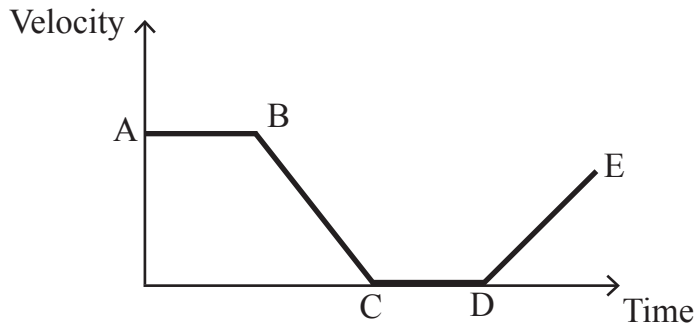
Section A

Mechanics

You should spend approximately **one hour** on this section.

Take $\mathbf{g = 10\,m/s^2}$ when required.

1 **(a)** The velocity/time graph for a car travelling along a straight road is given below.

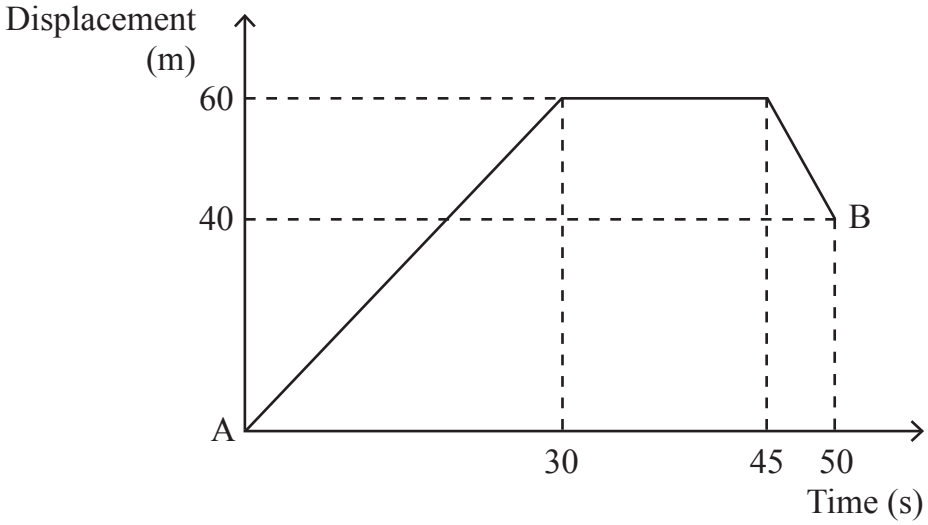


Between which points is the car stationary?

Answer _____ [1]

(b) A particle starts from a point A and travels along a straight line. After 50 seconds it is at a point B on the line.

The displacement/time graph for its motion is shown in the diagram below.



Determine

(i) the speed of the particle in the first 30 seconds,

Answer _____ m/s [1]

(ii) the speed of the particle between 30 seconds and 45 seconds,

Answer _____ m/s [1]

(iii) the total distance travelled by the particle.

Answer _____ m [1]

[Turn over

2 (Throughout this question **i** and **j** denote unit vectors parallel to a set of standard x-y axes.)

A body is initially at an origin O and is travelling with an initial velocity of $(4\mathbf{i} + 5\mathbf{j})$ m/s.

The body accelerates uniformly for 4 seconds to a velocity of $(-4\mathbf{i} - \mathbf{j})$ m/s.

Calculate

(i) the acceleration of the body,

Answer _____ m/s^2 [2]

(ii) the position vector of the body relative to O after the 4 seconds.

Answer _____ m [2]

[Turn over

3 A body leaves a point O with initial velocity u m/s. It moves along a straight line with constant acceleration a m/s^2.

Three seconds after leaving O the body is at a point A with velocity 12 m/s.

Eight seconds after leaving O the body is at a point B with velocity 16 m/s.

(i) Using the above information, write down two equations satisfied by u and a.

Answer _____ [1]

_____ [1]

Calculate

(ii) the value of a,

Answer _____ [1]

(iii) the value of u,

Answer _____ [1]

(iv) the distance between A and B.

Answer _____ m [2]

[Turn over

4 Laura is jogging at a constant velocity of 3 m/s when she passes a post office.

A cyclist, Emma, leaves the post office **one minute** after Laura had passed it.

Emma starts from rest and accelerates uniformly at 1.5 m/s^2 for 10 seconds to a maximum velocity of 15 m/s. Emma continues at this maximum velocity until she catches up with Laura.

(i) Sketch the velocity/time graph for Emma's journey on the diagram below.

[2]

Emma catches up with Laura T seconds after Laura jogged past the post office.

Find expressions, in terms of T, for

(ii) the distance travelled by Laura,

Answer _____ m [1]

(iii) the distance travelled by Emma.

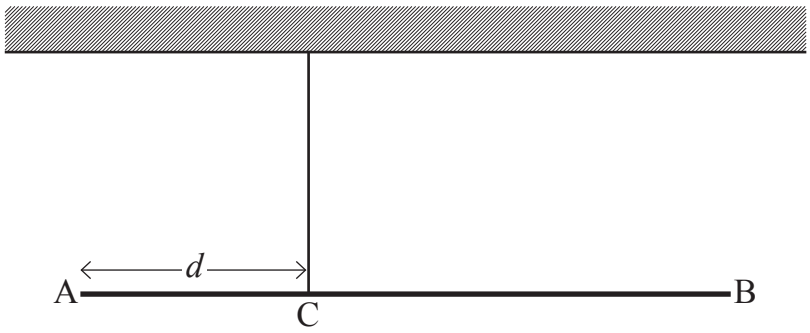
Answer _____ m [2]

(iv) Calculate the time for which Emma cycles before she catches up with Laura.

Answer _____ s [2]

[Turn over

5 A uniform rod AB, of length 6 m and mass 2 kg, is connected to a fixed point on a ceiling by a light inextensible string. The string is connected to the rod at a point C, such that the distance AC is d m, as shown in the diagram below.



A mass of 5 kg is attached at A and a mass of 3 kg is attached at B.

The rod remains horizontal and in equilibrium.

(i) Mark on the diagram above all the forces acting on the rod. [2]

Calculate

(ii) the tension in the string,

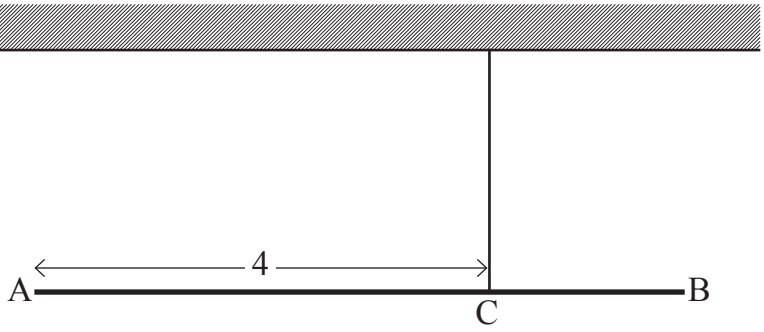
Answer _____ N [1]

(iii) the value of d.

Answer _____ m [3]

Question 5 continues overleaf

The point C is repositioned to a point 4 m from A, as shown in the diagram below.



The mass of 3 kg is removed from B and replaced by a mass of M kg.

The mass of 5 kg remains at A.

(iv) Calculate the value of M if the rod remains horizontal and in equilibrium.

Answer _____ [3]

6 A block of mass 8 kg lies at rest on a rough horizontal table and is acted upon by a horizontal force of 40 N, as shown in the diagram below.

The block is on the point of moving.

(i) Mark on the diagram above all the forces acting on the block. [1]

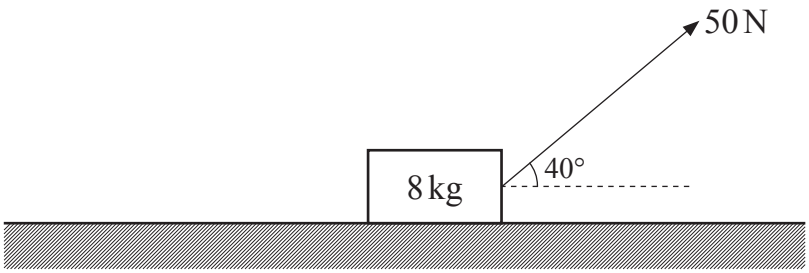
(ii) Calculate the coefficient of friction between the block and the table.

Answer _____ [2]

Question 6 continues overleaf

[Turn over

The force of 40 N is removed and replaced by a force of 50 N which acts at an angle of 40º to the horizontal, as shown in the diagram below.



Calculate

(iii) the normal reaction between the block and the table,

Answer _____ N [2]

(iv) the acceleration of the block.

Answer _____ m/s^2 [4]

[Turn over

7 A car of mass 1200 kg is towing a trailer of mass 500 kg by means of a light horizontal tow bar. The tractive force produced by the car's engine is 5330 N.

The car and trailer are travelling along a straight horizontal road, as shown below.

The resistance to the motion of the car is 1.6 N per kg of mass and the resistance to the motion of the trailer is 0.7 N per kg of mass.

The car and trailer accelerate uniformly from rest.

Calculate

(i) the **total** resistance to the motion of the car and trailer,

Answer _____ N [2]

(ii) the acceleration of the car and trailer,

Answer _____ m/s^2 [2]

(iii) the tension in the tow bar.

Answer _____ N [2]

Question 7 continues overleaf

After travelling for 10 seconds the tow bar breaks.

(iv) Calculate the speed of the car and trailer when the tow bar breaks.

Answer _____ m/s [1]

(v) Assuming that the resistance to the motion of the trailer remains the same, calculate the additional distance travelled by the trailer before it comes to rest.

Answer _____ m [4]

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(Questions continue overleaf)

Section B

Statistics

You should spend approximately **one hour** on this section.

8 The lengths, in centimetres, of 50 objects were measured and recorded to 1 decimal place. The results are summarised in the table below.

Length (cm)	10.4–11.3	11.4–12.3	12.4–13.3	13.4–14.3
Number of objects	17	12	8	13

Write down

(i) the limits of the median class,

Answer _____ [2]

(ii) the boundaries of the modal class.

Answer _____ [2]

Each class has the same width.

(iii) Write down the class width.

Answer _____ cm [1]

[Turn over

9 The masses, in kilograms, of 37 bags of fruit are summarised in the table below.

Mass M (kg)	Number of bags
$1.4 < M \leqslant 2.2$	6
$2.2 < M \leqslant 3.0$	8
$3.0 < M \leqslant 3.8$	7
$3.8 < M \leqslant 4.6$	9
$4.6 < M \leqslant 5.4$	7

(i) Calculate an estimate for the median mass.

Answer _____ kg [4]

Two more bags, with masses in the range $3.0 < M \leqslant 3.8$, were then included.

(ii) Identify from the list below which statement can be deduced from this information.

A The **actual** median will be lowered

B The **actual** median will be unchanged

C The **actual** median will be raised

D It is impossible to say what the effect will be on the **actual** median

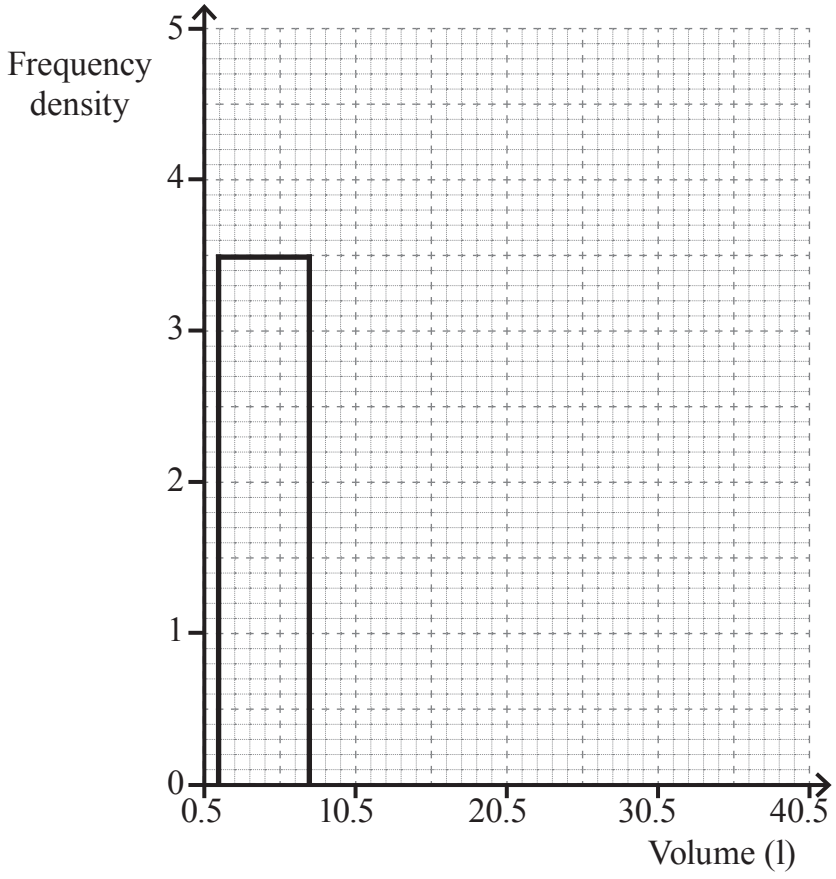
Answer _____ [1]

[Turn over

10 The volumes, in litres, of different containers were measured and rounded to the nearest litre. The results are summarised in the incomplete table below.

Volume (l)	Frequency
2–7	
8–12	18
13–22	42
23–37	36

A histogram to show the information has been started below.



(i) Calculate the frequency of containers in the class 2–7 litres.

Answer _____ [2]

(ii) Complete the histogram on the page opposite for the other three classes. [4]

[Turn over

11 Five candidates sat a History examination. Their actual marks are converted to uniform marks by doubling each mark and then adding 10

(i) Complete the table below for the mean and standard deviation of the uniform marks.

	Actual marks	**Uniform marks**
Mean	31.0	
Standard deviation	4.6	

[2]

A sixth candidate had an actual mark of 43

(ii) Find the mean of the **actual** marks for all six candidates.

Answer _____ [2]

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12 The average speeds, in km/h, and the times taken, in minutes, for nine people to travel to work are recorded in the table below.

	Conall	**Jill**	**Frank**	**Paddy**	**Arthur**	**Declan**	**Faye**	**Hope**	**Thomas**
Average speed (km/h)	48	54	88	72	64	92	42	58	76
Time (mins)	96	80	92	70	68	51	96	96	62
Ranks (Average speed)									
Ranks (Time)									

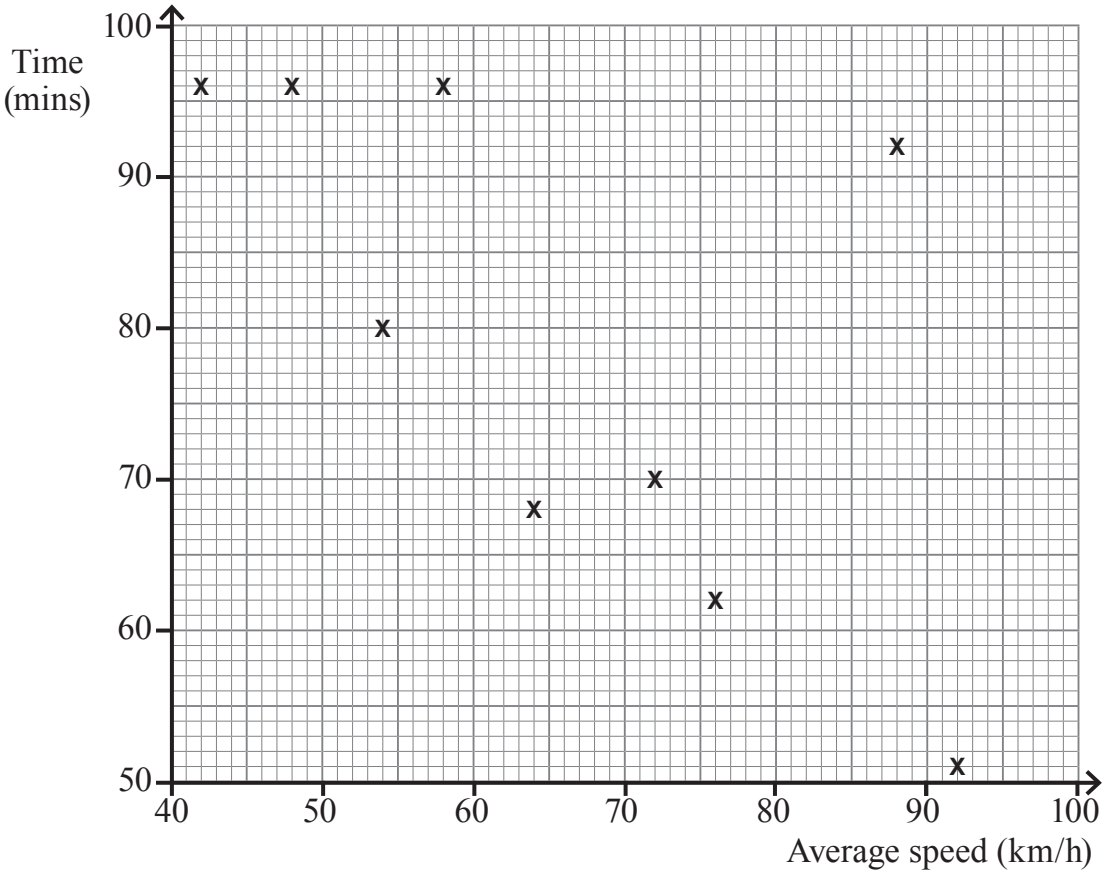
(i) By finding the rank orders for the average speeds and times, calculate Spearman's coefficient of rank correlation.

Answer _____ [6]

(ii) What significance, if any, do you attach to the value you obtained in part **(i)**?

Answer _____ [1]

The data from the table are plotted on the graph below.



(iii) Calculate the mean average speed and the mean time.

Answer _____ km/h

Answer _____ mins [1]

(iv) Draw your line of best fit on the graph. [2]

Question 12 continues overleaf

[Turn over

(v) The average speed and time for one person does not fit the pattern of the others. Which person is this? Give a possible reason.

Answer _____

_____ [1]

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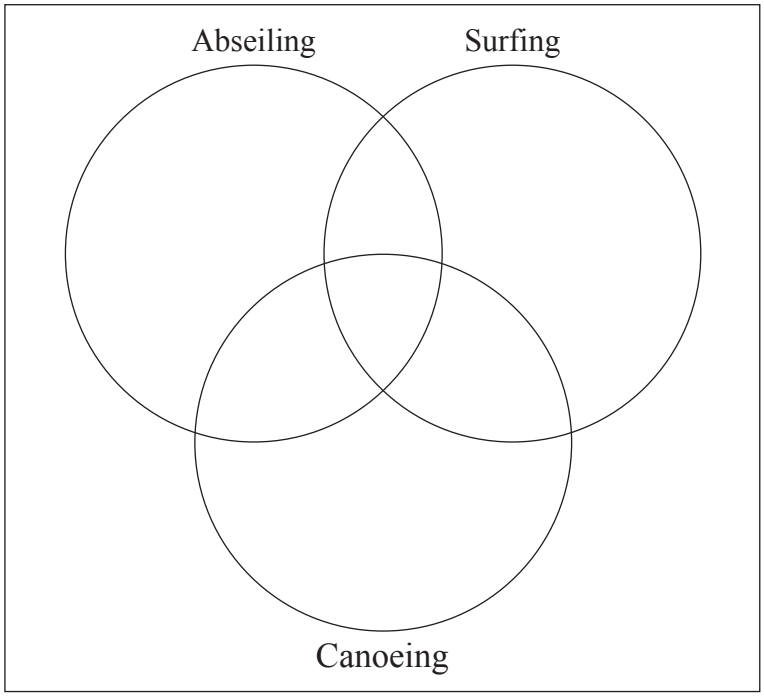
(Questions continue overleaf)

13 Ciaran went on an activity holiday over the summer.

The three different activities that he could take part in were abseiling, surfing and canoeing.

On a day chosen at random, the probability that he took part in
all three activities was 0.07
abseiling and surfing was 0.17
abseiling and canoeing was 0.21
surfing and canoeing was 0.27
abseiling was 0.47
surfing was 0.5
none of these activities was 0.02

(i) Illustrate this information on the Venn diagram below.



[3]

(ii) On a day chosen at random, find the probability that Ciaran only took part in canoeing.

Answer _____ [2]

On a particular day Ciaran did not take part in abseiling.

(iii) Find the probability that he took part in surfing on that day.

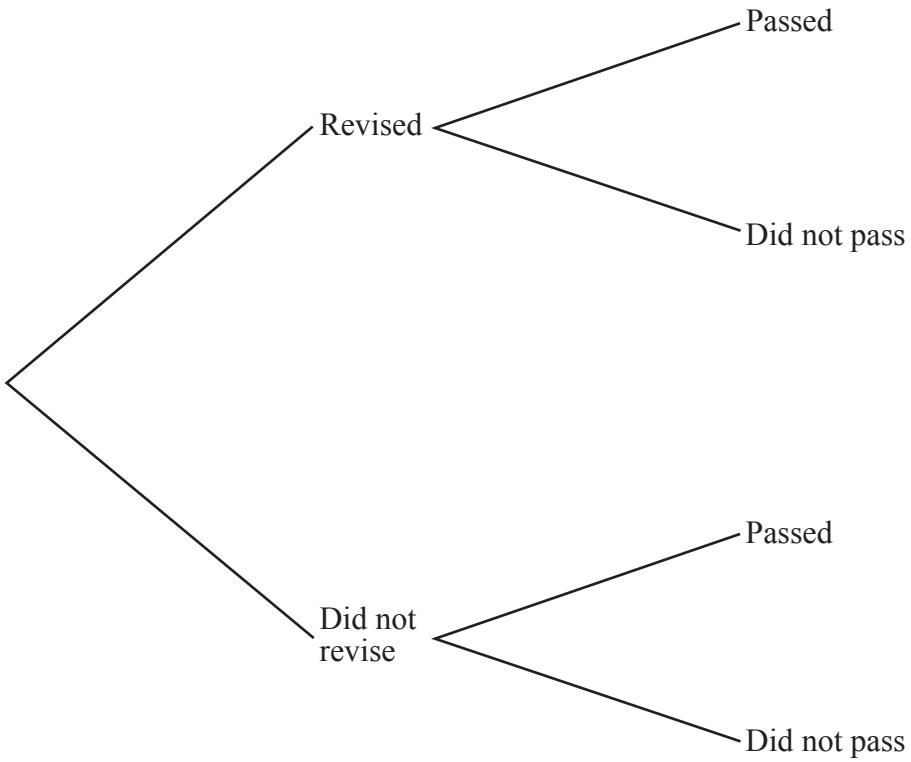
Answer _____ [2]

[Turn over

14 Of all the candidates who sat a Maths exam, 80% revised for the exam.

Of those who revised, 85% passed the exam.

The percentage of candidates who did not revise and who did not pass the exam was 15.2%.



A candidate is chosen at random.

Using the tree diagram above,

(i) calculate the probability that the candidate revised and passed the exam.

Answer _____ [1]

(ii) John did not revise. Calculate the probability that he did not pass.

Answer _____ [2]

125 candidates sat the exam.

(iii) How many candidates passed the exam?

Answer _____ [3]

[Turn over

15 On her way home from school each day, Lily either goes to the shop, visits her granny, does both or does neither.

The probability that Lily visits her granny is 0.55

The probability that Lily goes to the shop if she visits her granny is 0.4

The probability that Lily visits her granny if she goes to the shop is 0.88

Calculate the probability that

(i) Lily goes to the shop **and** visits her granny,

Answer _____ [2]

(ii) Lily goes to the shop,

Answer _____ [2]

Question 15 continues overleaf

(iii) Lily goes to the shop **or** visits her granny.

Answer _____ [2]

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