Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2016

Further Mathematics

Unit 1
Pure Mathematics

[GMF11]

GMF11

THURSDAY 16 JUNE, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in blue or black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all sixteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

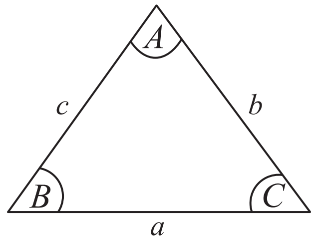
Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$

Differentiation: If $y = ax^n$ then $\dfrac{dy}{dx} = nax^{n-1}$

Integration: $\int ax^n \, dx = \dfrac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

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MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: Mean $= \dfrac{\Sigma fx}{\Sigma f}$ $\qquad$ Median $= L_1 + \dfrac{\left\{\frac{N}{2} - (\Sigma f)_1\right\}c}{f_{median}}$

where

L_1	is lower class boundary of the median class
N	is total frequency
$(\Sigma f)_1$	is the sum of the frequencies up to but not including the median class
f_{median}	is the frequency of the median class
c	is the width of the median class

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ $\quad$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

[Turn over

1 Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{B} = [-1 \quad 2] \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Express as a single matrix:

(i) $\mathbf{A} - 3\mathbf{C}$

Answer ______________________________ [2]

(ii) $\mathbf{AB}$

Answer ______________________________ [2]

2 A function f(x) is defined by $f(x) = x^2 - 4x - 5$

(i) Use the method of completing the square to rewrite f(x) in the form $(x + a)^2 + b$.

Answer ______________________________ [2]

(ii) **Hence** find

(a) the minimum value of f(x),

Answer ______________________________ [1]

(b) the value of x for which this minimum occurs.

Answer ______________________________ [1]

[Turn over

3 If $y = \frac{2}{3}x^3 - \frac{7}{x}$

(i) find $\frac{dy}{dx}$

Answer ______________________________ [2]

(ii) **Hence** find $\frac{d^2y}{dx^2}$

Answer ______________________________ [2]

4 Find $\int_1^2 \left(x^3 - \frac{3}{2x^2}\right) \mathrm{d}x$

Answer ______________________________ [4]

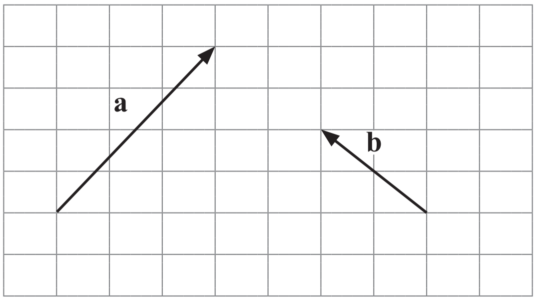
[Turn over

5 **(a)** Find the values of p and q if

$$p\begin{bmatrix}5\\2\end{bmatrix}+3\begin{bmatrix}-1\\q\end{bmatrix}=\begin{bmatrix}7\\-5\end{bmatrix}$$

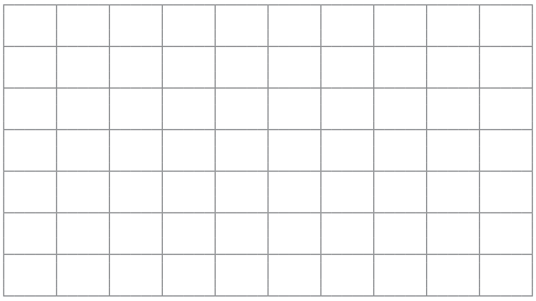
Answer $p =$ ________________ , $q =$ ________________ [2]

(b) The vectors **a** and **b** are shown below.



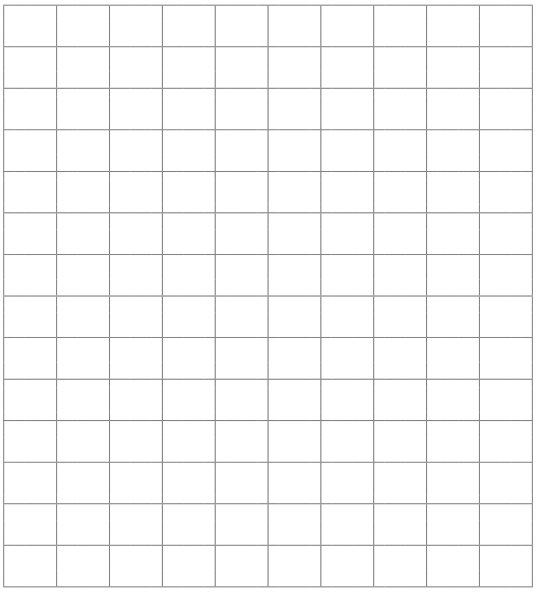
On the grids below, draw diagrams to show the vectors

(i) **b** – **a**



[1]

(ii) 2**a** + **b**



[1]

[Turn over

6 **(i)** Solve the equation

$$\cos\theta = -0.65$$

for $0° \leq \theta \leq 360°$

Answer ______________________________ [2]

(ii) **Hence** solve the equation

$$\cos(2x + 20°) = -0.65$$

for $0° \leq x \leq 180°$

Answer ______________________________ [3]

7 Solve the equation

$$3^{2x+1} = 8^{4-x}$$

Answer ______________________________________ [5]

[Turn over

8 The matrix **P** is defined by

$$\mathbf{P} = \begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}$$

(i) Find the matrix $\mathbf{P}^{-1}$, the inverse of **P**.

Answer ______________________________ [2]

(ii) Hence, **using a matrix method**, solve the following simultaneous equations for x and y.

$$4x + 3y = 34$$
$$x - 2y = 3$$

Answer $x =$ ______________ , $y =$ ______________ [4]

[Turn over

9 Simplify **fully** the following algebraic expressions:

(i) $$\frac{x}{x^2 + 6x + 8} + \frac{1}{x + 2}$$

Answer ______________________________ [3]

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(ii) $$\frac{x^2 - 9}{4x + 12} \div \frac{x^2 + x - 12}{6}$$

Answer ______________________________ [4]

[Turn over

10 A curve is defined by the equation $y = (x + 3)(x - 4)$

(i) Write down the coordinates of the points where the curve crosses the x-axis.

Answer ______________________________ [2]

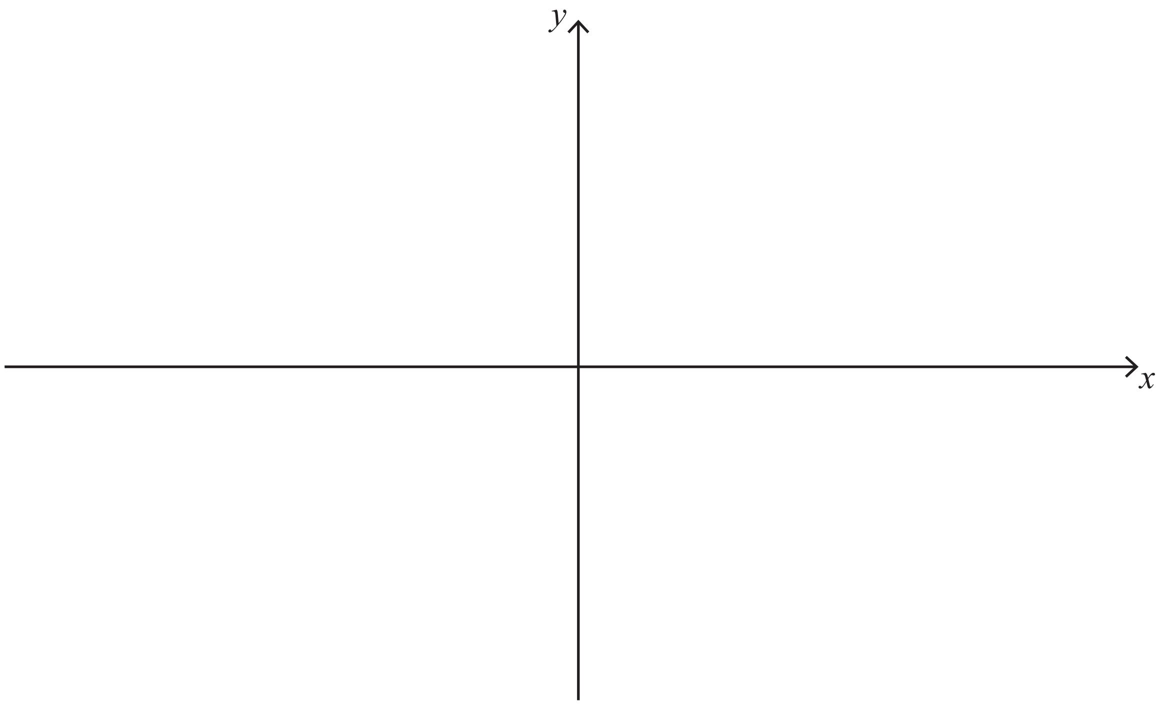
(ii) Find the coordinates of the turning point of the curve.

Answer ______________________________ [4]

(iii) Identify the turning point as either a maximum or a minimum point. You **must** show working to justify your answer.

Answer _______________________________ [1]

(iv) Using your results from parts **(i)** to **(iii)**, sketch the curve on the axes below.



[2]

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[Turn over

11 A local theatre company is putting on a weekend performance of a musical.

Tickets cost £x for seats in the stalls, £y for seats in the main circle and £z for seats in the balcony.

For the Friday evening performance they sold 60 tickets for the stalls, 84 tickets for the main circle and 48 tickets for the balcony. The total income from ticket sales was £3696

(i) Show that x, y and z satisfy the equation

$$5x + 7y + 4z = 308$$

[1]

For the Saturday evening performance they sold 56 tickets for the stalls, 63 tickets for the main circle and 42 tickets for the balcony. The total income from ticket sales was £3045

(ii) Show that x, y and z also satisfy the equation

$$8x + 9y + 6z = 435$$

[1]

For the Saturday matinee performance the price of **all** tickets is reduced by £5

For the matinee performance they sold 45 tickets for the stalls, 54 tickets for the main circle and 18 tickets for the balcony. The total income from ticket sales was £1746

(iii) Show that x, y, and z also satisfy the equation

$$5x + 6y + 2z = 259$$

[2]

Question 11 continues overleaf

(iv) Solve the equations

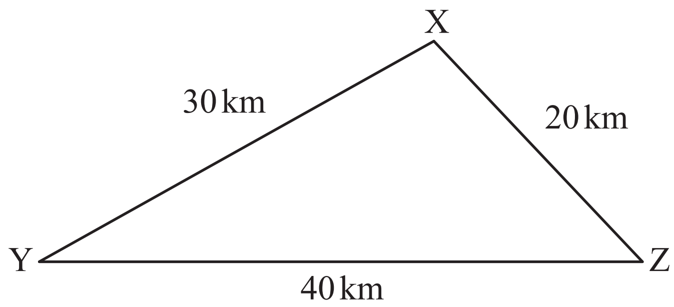
$$5x + 7y + 4z = 308$$
$$8x + 9y + 6z = 435$$
$$5x + 6y + 2z = 259$$

to find the **original** price of each type of ticket, showing clearly each stage of your solution.

Answer Stalls £ _________ , Main circle £ _________ , Balcony £ _________ [8]

You may use this page if needed.
(Questions continue overleaf.)

12 A search team wishes to look for an old galleon which was carrying gold and sank somewhere in the triangle between three points X, Y and Z in the ocean. The distances XY, YZ and XZ are 30 km, 40 km and 20 km respectively, as shown in the diagram below.



Calculate

(i) the size of the angle $X\hat{Y}Z$,

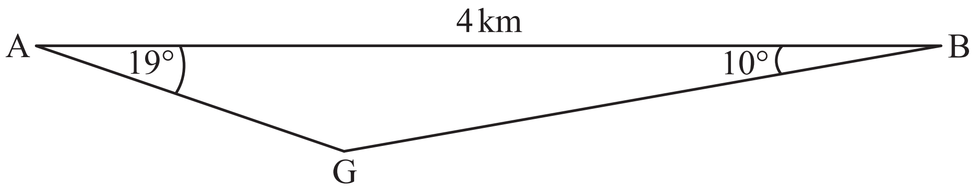
Answer ______________________________ ° [3]

(ii) the area of the search region XYZ.

Answer ______________________________ km^2 [2]

Using sonar signals, two ships, A and B, 4 km apart, detected the galleon G on the bottom of the ocean, where A, B and G were in the same vertical plane.

The angles $B\hat{A}G$ and $A\hat{B}G$ were measured as 19º and 10º respectively.



(iii) Write down the size of the angle $A\hat{G}B$.

Answer ______________________________ ° [1]

(iv) Calculate the distance AG.

Answer ______________________ km [2]

Ship A has a probe which can be lowered vertically downwards to inspect the galleon.

(v) Calculate the distance ship A needs to travel towards ship B to be vertically above the galleon.

Answer ______________________ km [2]

13 A curve is defined by the equation

$$y = x^2 + \frac{3}{2}x + \frac{5}{2}$$

Find the equation of the **normal** to the curve at the point A(–1, 2).

Answer ________________________________ [4]

14 Mark drove to his cousin's wedding in Roscommon. The journey had two stages.

For the first stage of his journey he travelled 140 km at a speed of x km/h.

For the second stage of his journey he travelled x km at 64 km/h.

(i) Write down expressions in terms of x for the times taken in each stage.

Answer First stage ____________________________ h [1]

Second stage ____________________________ h [1]

The **total** time for both stages of his journey was 3 hours.

(ii) Show that x satisfies the quadratic equation

$$x^2 - 192x + 8960 = 0$$

[2]

(iii) Solve this equation to find x, given that Mark did not break the speed limit of 96 km/h at any time.

Answer ______________________________ [2]

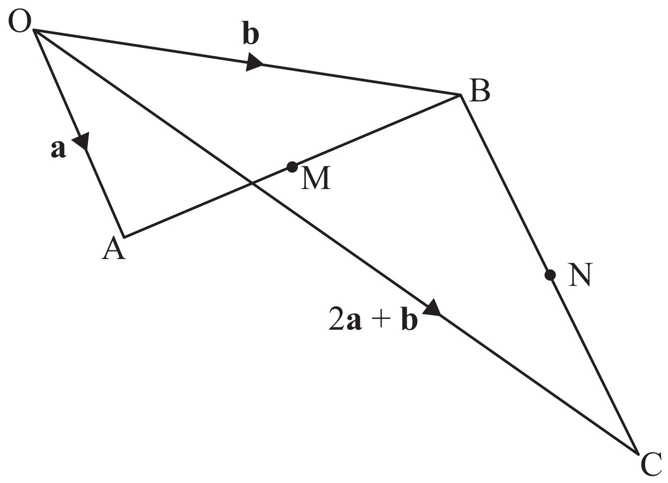
(iv) Find the total distance travelled by Mark.

Answer ____________________________ km [1]

15 In the diagram below, A, B and C are points such that

$$\overrightarrow{OA} = \mathbf{a}, \ \overrightarrow{OB} = \mathbf{b} \ \text{ and } \ \overrightarrow{OC} = 2\mathbf{a} + \mathbf{b}.$$

The midpoint of AB is M and the midpoint of BC is N.



(i) Find in terms of **a** and **b**, simplifying your answers as far as possible:

(a) $\overrightarrow{OM}$

Answer ______________________________ [2]

(b) $\overrightarrow{ON}$

Answer ______________________________ [2]

(ii) Prove that OANB is a parallelogram.

[2]

16 A tour company organises package holidays for walking groups to the Alps.

If there are x people in the group the company will charge £$(1000 - 2x)$ per person for the holiday.

(i) Write down, in terms of x, the total income the tour company will receive if a group of x people travel.

Answer ______________________________ [1]

To run the holiday the company has to pay a fixed cost of £20 000 to the airline operating the flight, as well as an additional cost of £400 per person for accommodation.

(ii) Show that when a group of x people travel the profit, £P, for the tour company is given by

$$P = 600x - 2x^2 - 20\,000$$

[3]

(iii) Find the number of people in a group which will maximise the profit for the tour company, showing that it is a maximum.

Answer ____________________________________ [4]

(iv) Find the corresponding cost of the holiday for each member of the walking group.

Answer £ ______________________________ [1]

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Question Number	Marks
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