



General Certificate of Secondary Education
2015

Centre Number

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Candidate Number

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Further Mathematics

Unit 2
Mechanics and
Statistics



[GMF21]

GMF21

THURSDAY 11 JUNE, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

You must answer the questions in the spaces provided.

Do not write outside the boxed area on each page, on blank pages or tracing paper.

Complete in blue or black ink only. **Do not write with a gel pen.**

All working should be clearly shown since marks may be awarded for partially correct solutions.

Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.

Answer **all fourteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

You may use a calculator.

The Formula Sheet is on pages 2 and 3.

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Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ ($a \neq 0$)

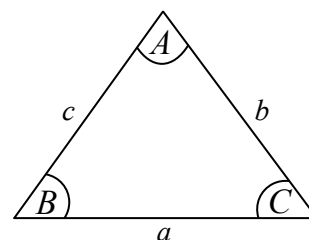
$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometry:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$



Differentiation:

$$\text{If } y = ax^n \quad \text{then} \quad \frac{dy}{dx} = nax^{n-1}$$

Integration:

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c \quad (n \neq -1)$$

Logarithms:

$$\text{If } a^x = n \quad \text{then} \quad x = \log_a n$$

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

Matrices:

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{then } \det \mathbf{A} = ad - bc$$

$$\text{and } \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (ad - bc \neq 0)$$



MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and \mathbf{i} is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $s = \frac{1}{2}(u + v)t$
 $v^2 = u^2 + 2as$ $s = ut + \frac{1}{2}at^2$

where u is initial velocity t is time
 v is final velocity s is change in displacement
 a is acceleration

Newton's Second Law: $F = ma$

where F is resultant force m is mass
 a is acceleration

STATISTICS

Statistical measures: Mean = $\frac{\sum fx}{\sum f}$ Median = $L_1 + \frac{\left\{\frac{N}{2} - (\sum f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
 N is total frequency
 $(\sum f)_1$ is the sum of the frequencies up to but not including the median class
 f_{median} is the frequency of the median class
 c is the width of the median class

Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$ where \bar{x} is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$



Section A

Mechanics

You should spend approximately **one hour** on this section.

Take $g = 10 \text{ m/s}^2$

- 1 (Throughout this question \mathbf{i} and \mathbf{j} denote unit vectors parallel to a set of standard x - y axes.)

A body of mass 4 kg is acted upon by three forces \mathbf{a} , \mathbf{b} and \mathbf{c} , where

$$\mathbf{a} = (3\mathbf{i} - \mathbf{j}) \text{ N} \quad \mathbf{b} = (p\mathbf{i} + q\mathbf{j}) \text{ N} \quad \mathbf{c} = (-\mathbf{i} + 4\mathbf{j}) \text{ N}$$

- (i) Given that the body is in equilibrium, calculate the values of p and q .

Answer $p =$ _____

$q =$ _____ [3]



The force \mathbf{b} is now replaced with the force $(6\mathbf{i} - 9\mathbf{j})$ N.

Calculate

(ii) the resultant force now acting on the body,

Answer _____ N [2]

(iii) the **magnitude** of the acceleration of the body.

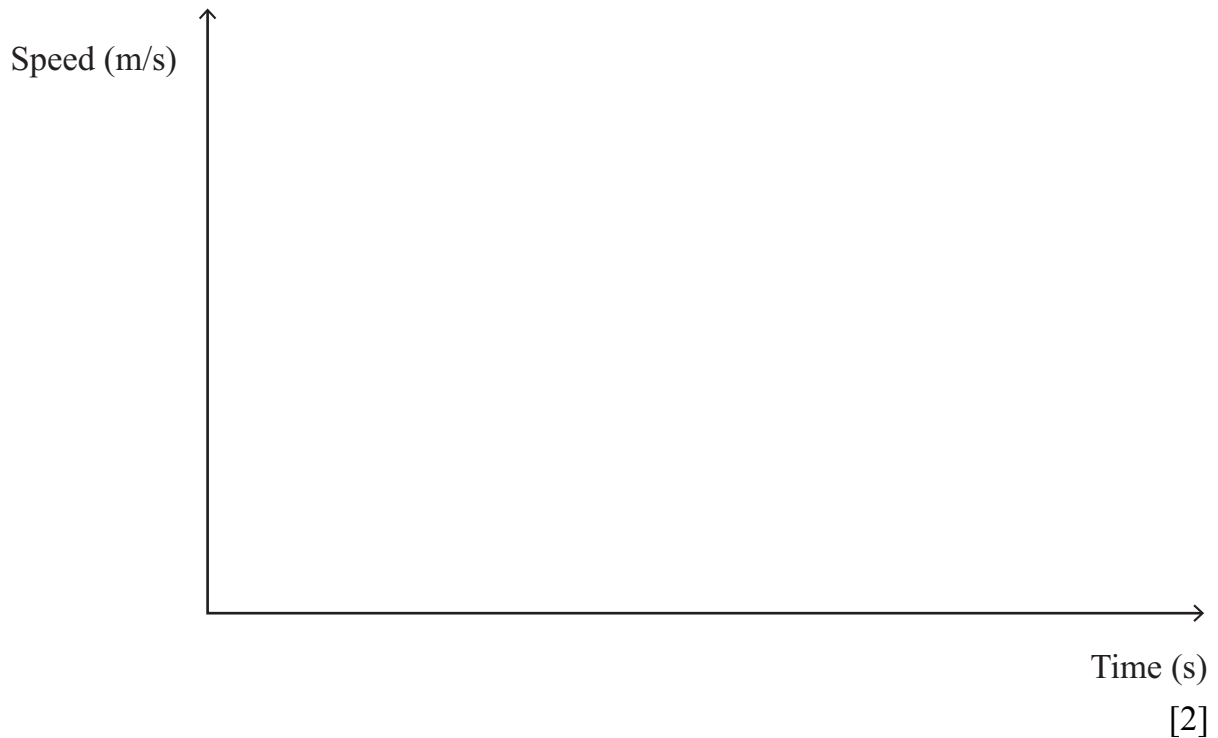
Answer _____ m/s^2 [3]

[Turn over



- 2 Eleanore is driving her car on a straight horizontal road.
When the car is at a point P, 250 m from road-works R, its speed is 20 m/s.
The car continues at this constant speed for 4 seconds after passing P.
Eleanore then applies the brakes so that the car has constant deceleration and comes to rest at R.

- (i) On the axes below, sketch the speed-time graph to illustrate the motion of the car in travelling from P to R.



(ii) Find the time taken by the car to travel from P to R.

Answer _____ s [4]



- 3 A stone is thrown vertically upwards at a speed of 15 m/s from the top of a vertical cliff.

Calculate

- (i) the maximum height above the cliff reached by the stone,

Answer _____ m [2]

- (ii) the time taken to reach the maximum height.

Answer _____ s [1]



The stone reaches the bottom of the cliff 7 seconds after being thrown upwards.

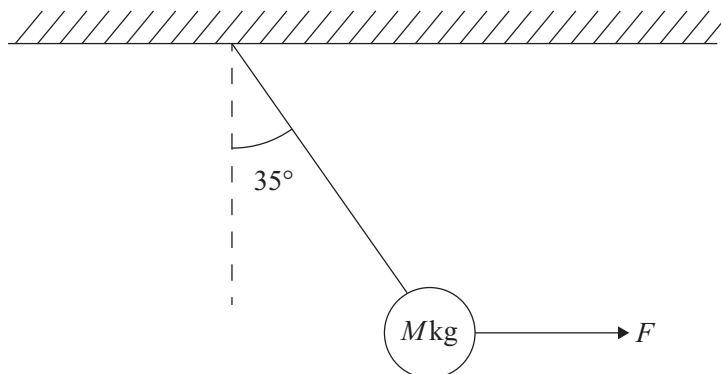
(iii) Calculate the height of the cliff.

Answer _____ m [3]



- 4 A solid ball of mass M kg hangs from a light inextensible string attached to a ceiling. The ball is pulled by a horizontal force F so that the ball is held in equilibrium, with the string at an angle of 35° to the vertical, as shown in the diagram below.

The tension in the string is 40 N.



- (i) On the diagram above, mark all the forces acting on the ball. [2]

Calculate

- (ii) the magnitude of the force F ,

Answer _____ N [2]



(iii) the value of M .

Answer _____ [2]



- 5 In a game of see-saw, Mary and John sit on either end of a uniform plank AB, of length 8 m and mass 5 kg.
Mary sits at the end A and her mass is 70 kg.
John sits at the end B and his mass is 25 kg.
The plank rests on a smooth pivot at the point C, where C is x m from A, as shown in the diagram below.

When Mary and John sit on the plank, the plank remains horizontal and is in equilibrium.



Calculate

- (i) the value of the normal reaction at C,

Answer _____ N [2]

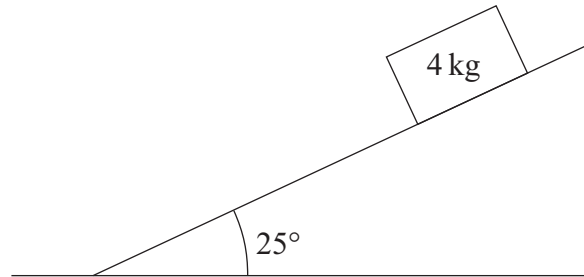


(ii) the value of x .

Answer _____ [3]



- 6 A block of mass 4 kg is at rest on a rough slope. The slope makes an angle of 25° with the horizontal, as shown in the diagram below.



- (i) Mark on the diagram all the forces acting on the block. [2]

The block is on the point of sliding down the slope.

- (ii) Calculate the coefficient of friction between the block and the slope.

Answer _____ [3]



The angle of the slope is now increased to 30° and the block starts to slide down the slope.

(iii) Calculate the acceleration of the block down the slope.

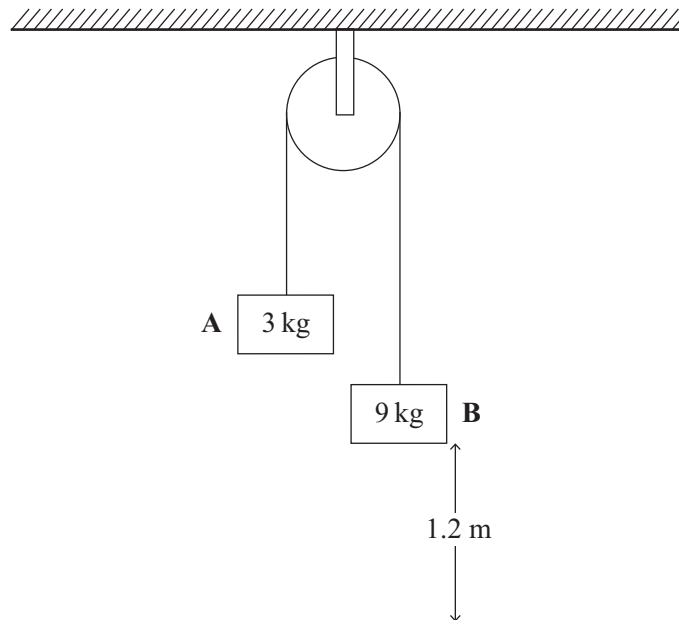
Answer _____ m/s^2 [5]

[Turn over



- 7 Two blocks, **A** and **B**, of masses 3 kg and 9 kg respectively, are connected by a light inextensible string that passes over a smooth fixed pulley which is attached to a ceiling.

Block **B** is held at a distance of 1.2 m above horizontal ground, as shown in the diagram below.



The blocks are released from rest.

- (i) Mark on the diagram all the forces acting on the blocks.

[2]

Calculate

- (ii) the acceleration of the blocks,

Answer _____ m/s² [3]



(iii) the tension in the string.

Answer _____ N [2]

After the blocks are released, block **B** hits the ground.

(iv) Assuming that block **A** does not reach the pulley, calculate the speed at which block **B** hits the ground.

Answer _____ m/s [2]

[Turn over



Section B

Statistics

You should spend approximately **one hour** on this section.

- 8 The table below shows the mean and standard deviation of the percentage marks for each modern language in the year 12 mock exams, and also the number of students who took each exam.

	French	German	Spanish
Mean mark	63	48	72
Standard deviation	5.3	8.4	6.8
Number of students	47	21	49

- (i) Calculate the overall mean mark for modern languages.

Answer _____ [2]

- (ii) In which modern language did the students show the least variation in ability?
Give a reason to justify your answer.

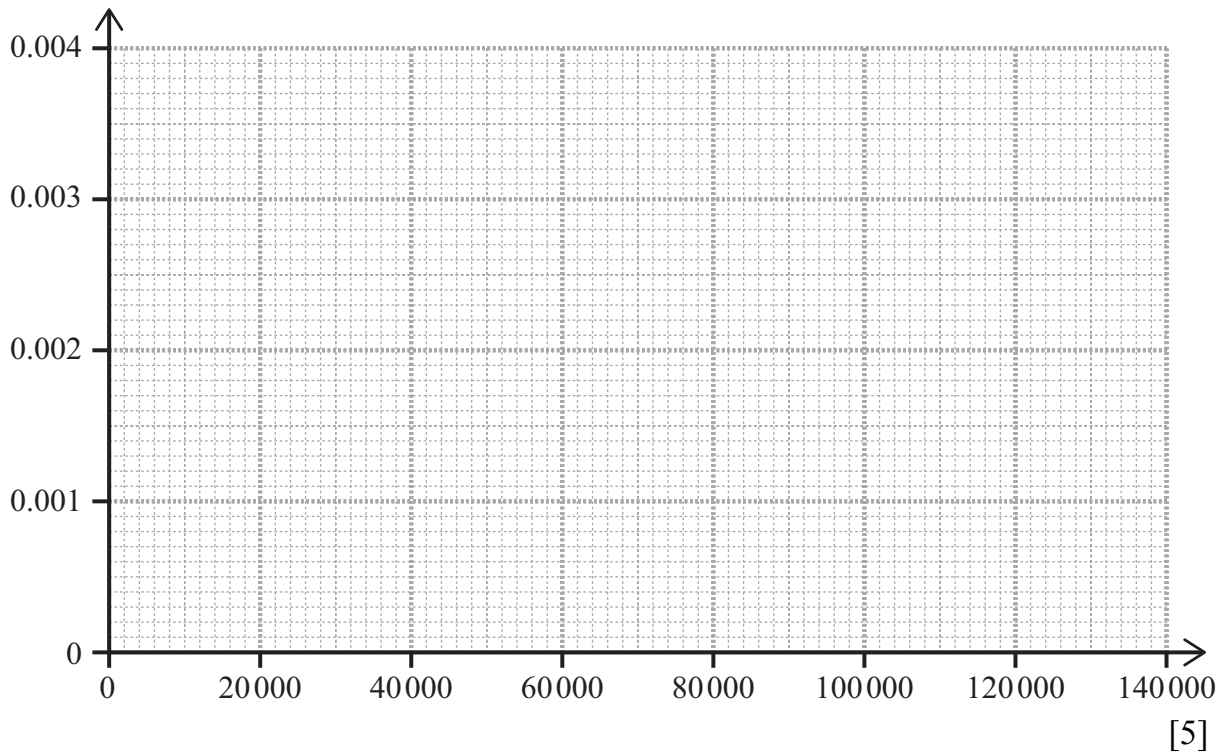
Answer _____
_____ [1]



- 9 At a local MOT centre the mileage of each car is recorded.
The table below summarises the mileages recorded in one day.

Mileage (M)	$5000 \leq M < 30000$	$30000 \leq M < 50000$	$50000 \leq M < 70000$	$70000 \leq M < 100000$	$100000 \leq M < 140000$
Numbers of cars	10	48	66	81	20

Using the axes below, draw a histogram to represent this information.
Label each axis clearly.



[Turn over



- 10 The finishing times for the runners in a marathon were recorded. The results are summarised in the table below.

Finishing time, t (minutes)	Number of runners
$120 \leq t < 150$	5
$150 \leq t < 180$	37
$180 \leq t < 210$	226
$210 \leq t < 240$	555
$240 \leq t < 270$	435
$270 \leq t < 300$	527
$300 \leq t < 330$	338
$330 \leq t < 360$	176

- (i) State the modal class.

Answer _____ [1]

- (ii) Calculate an estimate for the median time.

Answer _____ minutes [4]



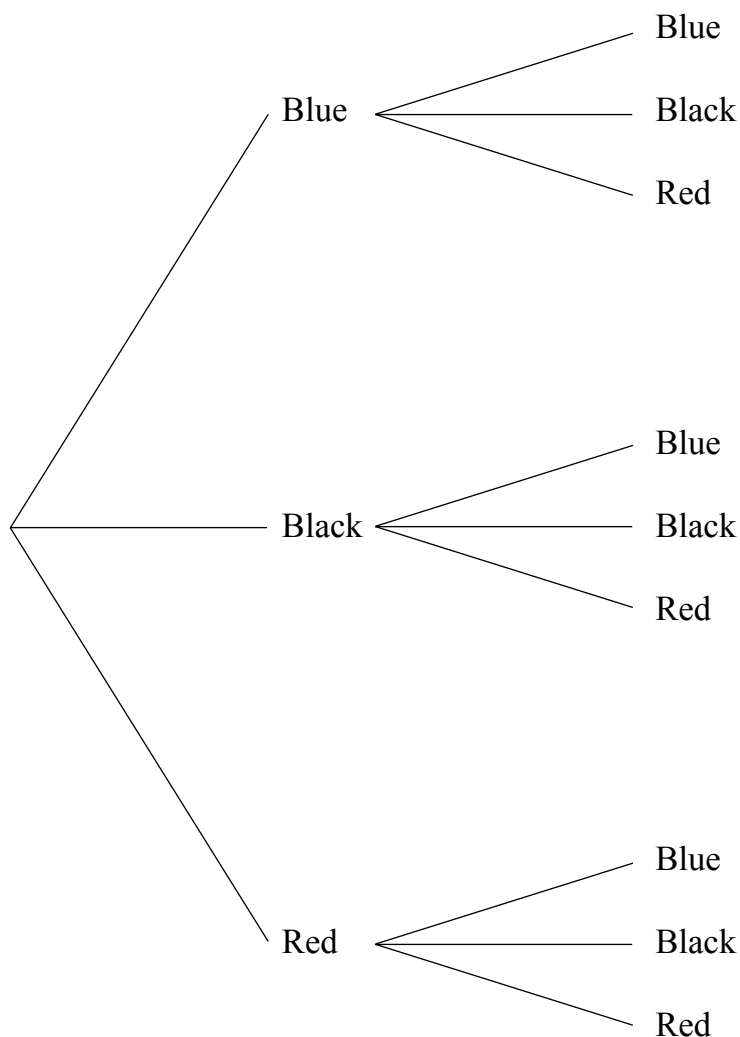
(iii) Calculate an estimate for the mean time.

Answer _____ minutes [3]



11 There are 6 blue pens, 5 black pens and 2 red pens in a drawer.

George selects two pens at random.



By completing the tree diagram above or otherwise, answer the following questions.

- (i) If the first pen that George selects is blue, what is the probability that the second pen will be black?

Answer _____ [1]



(ii) Calculate the probability that the two pens which George will select are

(a) the same colour,

Answer _____ [2]

(b) different colours.

Answer _____ [2]

(iii) Given that both pens that George selects are the same colour, what is the probability that they are both red?

Answer _____ [3]

[Turn over



12 A two digit number is chosen at random. It is known that the 'tens' digit is odd and the 'units' digit is less than 7

(i) Show that there are 35 possibilities for this number.

[2]

(ii) Calculate the probability that the number is greater than 55

Answer _____ [2]





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- 13 A group of friends recorded their heights and shoe sizes. The results are recorded in the following table.

Height (cm)	161	170	156	164	172	158	160	168	163
Shoe size	4	6	4	5.5	6.5	4	5	7	4.5

- (i) Find the rank orders for the heights and for the shoe sizes.

[2]

- (ii) Calculate Spearman's coefficient of rank correlation.

Answer _____ [4]



(iii) Interpret your answer to part **(ii)**.

Answer _____ [1]

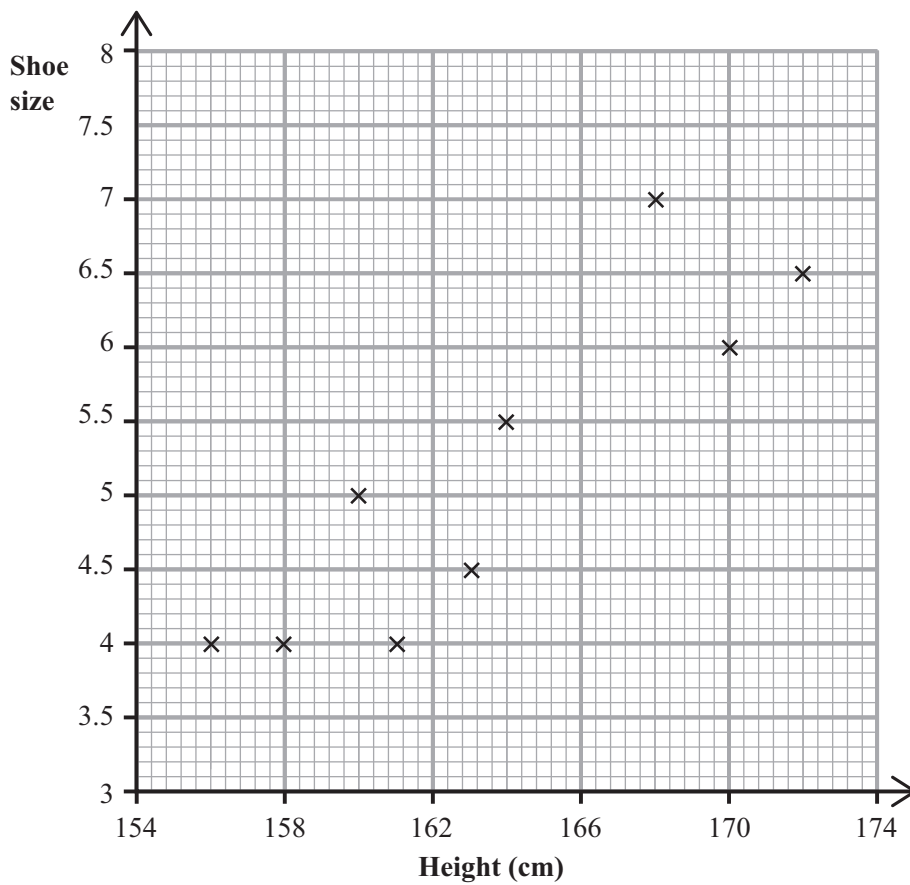
(iv) Calculate the mean height and the mean shoe size.

Answer Mean height = _____ cm

Mean shoe size = _____ [1]



The data from the table are plotted on the graph below.



(v) Draw your line of best fit on the graph above.

[2]



(vi) Determine the equation of the line of best fit which you have drawn.

Answer _____ [3]

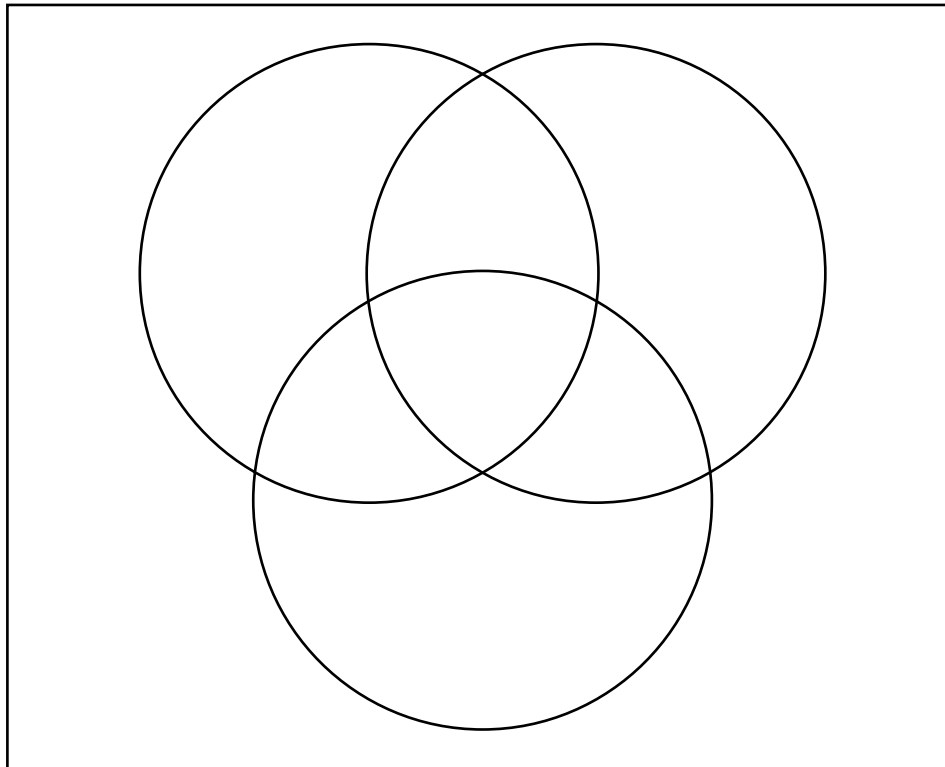


14 As an opening promotion a new pizza company offered the first 100 customers up to three extra toppings free.

The available toppings were mushrooms, peppers and ham.

Each of these 100 customers chose at least one extra topping:

- 6 chose mushrooms only,
- 3 chose peppers only,
- 9 chose ham only,
- 47 chose all three toppings,
- 23 did not choose ham.



- (i) Using a Venn diagram, find the probability that a customer selected at random chose both mushrooms and peppers.

Answer _____ [5]

- (ii) Calculate the probability that a customer selected at random chose exactly two extra toppings.

Answer _____ [4]

THIS IS THE END OF THE QUESTION PAPER



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Question Number	Marks
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Examiner Number

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