Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2015

Further Mathematics

Unit 1
Pure Mathematics

[GMF11]

GMF11

MONDAY 8 JUNE, MORNING

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in blue or black ink only. Do not write with a gel pen.
All working should be clearly shown since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all sixteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

9333

Formula Sheet

PURE MATHEMATICS

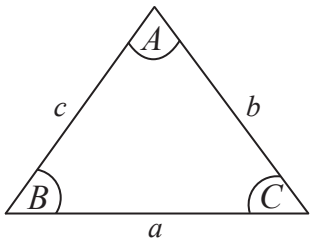
Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



Differentiation: If $y = ax^n$ then $\dfrac{dy}{dx} = nax^{n-1}$

Integration: $\int ax^n \, dx = \dfrac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: $\text{Mean} = \frac{\Sigma fx}{\Sigma f}$ $\quad \text{Median} = L_1 + \frac{\left\{\frac{N}{2} - (\Sigma f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
N is total frequency
$(\Sigma f)_1$ is the sum of the frequencies up to but not including the median class
f_{median} is the frequency of the median class
c is the width of the median class

$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$

[Turn over

1 Evaluate

(a) $\begin{bmatrix} p \\ 3 \end{bmatrix} + \begin{bmatrix} 2p \\ r \end{bmatrix}$

Answer _____ [1]

(b) $\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

Answer _____ [1]

(c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \end{bmatrix}$

Answer _____ [2]

2 Solve the equation $x^2 - 10x + 2 = 0$ by completing the square.

Give your answer in the form $a \pm \sqrt{b}$, where a and b are whole numbers.

Answer _____ [4]

[Turn over

3 A wooden block is x cm long, $\frac{1}{2}x$ cm wide and $\frac{1}{3}x$ cm high.

The total surface area of the block is 72 cm^2.

Form an equation in x and hence find the value of x.

Answer _____ [5]

4 If $y = \frac{3}{4}x^8 + 2x + \frac{3}{x^3}$

(i) find $\frac{dy}{dx}$

Answer _____ [3]

(ii) Hence find $\frac{d^2y}{dx^2}$

Answer _____ [2]

[Turn over

5 Find $\int\left(4x^2 - \frac{3}{x^3} + 1\right)\mathrm{d}x$

Answer _____ [4]

6 **(i)** Solve the equation

$$\tan \theta = 1.8$$

for $0° \leqslant \theta < 360°$

Answer _____ [2]

(ii) **Hence** solve the equation

$$\tan (0.9x - 40°) = 1.8$$

for $0° \leqslant x < 360°$

Answer _____ [3]

[Turn over

7 The inverse $\mathbf{A}^{-1}$ of a matrix $\mathbf{A}$ is given by

$$\mathbf{A}^{-1} = \frac{1}{5}\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}$$

Find the matrix $\mathbf{A}$

Answer _____ [2]

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(Questions continue overleaf)

[Turn over

8 **(a)** Simplify **fully**

$$\frac{1}{4x} \div \frac{x-3}{6x^2-2x}$$

Answer _____ [3]

(b) (i) Simplify **fully** $\dfrac{x^2 - 2x}{x^2 - x - 2}$

Answer _____ [2]

(ii) Hence simplify fully $\dfrac{2x}{x^2 - 1} + \dfrac{x^2 - 2x}{x^2 - x - 2}$

Answer _____ [3]

[Turn over

9 The owners of Wee Dotes Day Nursery plan to build a rectangular play area. They have enough edging bricks to make the perimeter 42 m and enough paving stones to cover an area of $108\,m^2$.

Let x and y represent the length and width, in metres, of the planned play area.

(i) Write down two equations connecting x and y.

Answer _____

Answer _____ [2]

(ii) **Hence** find the length and width of the play area.

Answer _____ m

Answer _____ m [4]

[Turn over

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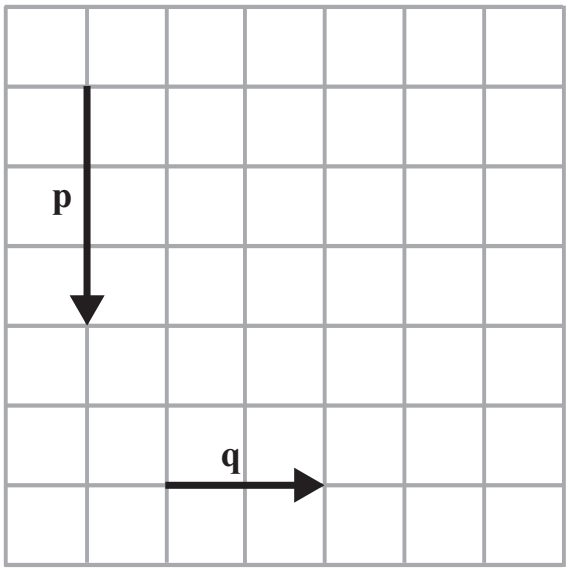
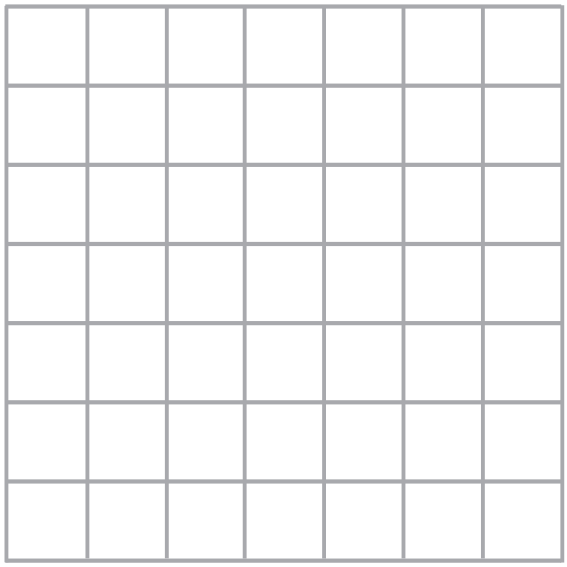
10 (a) The vectors **p** and **q** are shown below.

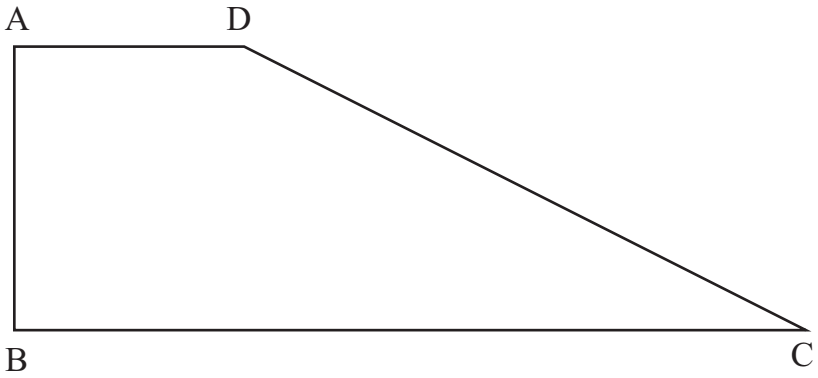


On the grid below show the vector $\mathbf{q} - 2\mathbf{p}$



[1]

(b) In the trapezium ABCD below, AD is parallel to BC and AD = $\frac{1}{3}$ BC.
$\overrightarrow{AB}$ represents the vector **r** and $\overrightarrow{BD}$ represents the vector **s**



(i) Express $\overrightarrow{AD}$ in terms of **r** and **s**

Answer _____ [1]

(ii) Express $\overrightarrow{BC}$ in terms of **r** and **s**

Answer _____ [1]

(iii) Express $\overrightarrow{DC}$ in terms of **r** and **s**, simplifying your answer as far as possible.

Answer _____ [2]

E is the point on BD such that BE = 2 ED.

(iv) Express $\overrightarrow{AE}$ in terms of **r** and **s**

Answer _____ [2]

(v) Show that AE is parallel to DC.

[1]

[Turn over

11 **(a)** Solve the equation

$$6^{3x+1} = 3^{x-2}$$

Answer _____ [5]

(b) Write $\log\left(\frac{ab}{c}\right)$ in terms of $\log a$, $\log b$ and $\log c$.

Answer _____ [2]

(c) Given that $\log 2 = p$, express $\log \sqrt{8}$ in terms of p.

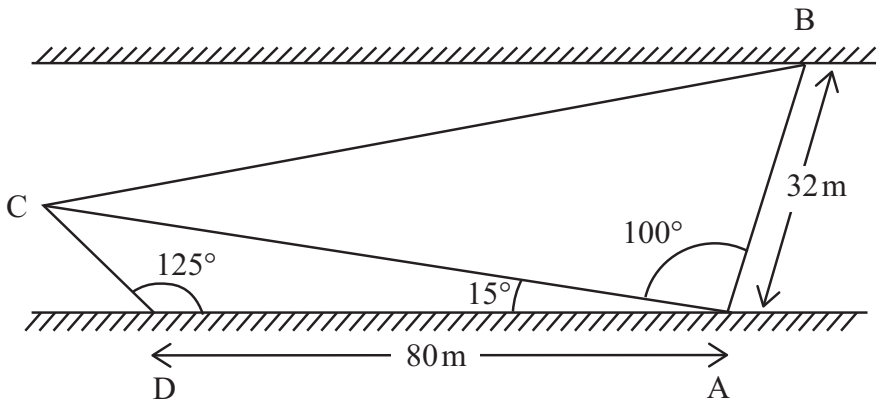
Answer _____ [3]

[Turn over

12 On its migration from Tanzania to Kenya, a wildebeest had to cross the Mara River. It entered the river at a point A and swam in a straight line towards a point B, 32 m from A, on the other side of the river where it could climb up the bank.

At the moment when it entered the river an observer at a point D, 80 m from A, noticed a crocodile in the river at a point C, and measured the angle $C\hat{D}A$ as 125°.

Another observer at the point A also spotted the crocodile and measured the angles $C\hat{A}D$ and $C\hat{A}B$ as 15° and 100° respectively, as shown in the diagram below.



Calculate

(i) the size of the angle $A\hat{C}D$,

Answer _____ ° [1]

(ii) the distance AC,

Answer _____ m [2]

(iii) the distance CB.

Answer _____ m [2]

[Turn over

The wildebeest swam at a constant speed of 0.8 m/s. The crocodile swam at a constant speed of 2.7 m/s in a straight line towards B.

(iv) Calculate whether or not the wildebeest made it safely to the other side or was caught by the crocodile.

[2]

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13 (i) Find the equation of the **normal** to the curve $y = x^2 - \frac{5}{2}x + \frac{1}{2}$ at the point $(1, -1)$.

Answer _____ [4]

This normal cuts the x-axis at the point P and the y-axis at the point Q.

(ii) Find the coordinates of the point P.

Answer _____ [1]

(iii) Find the coordinates of the point Q.

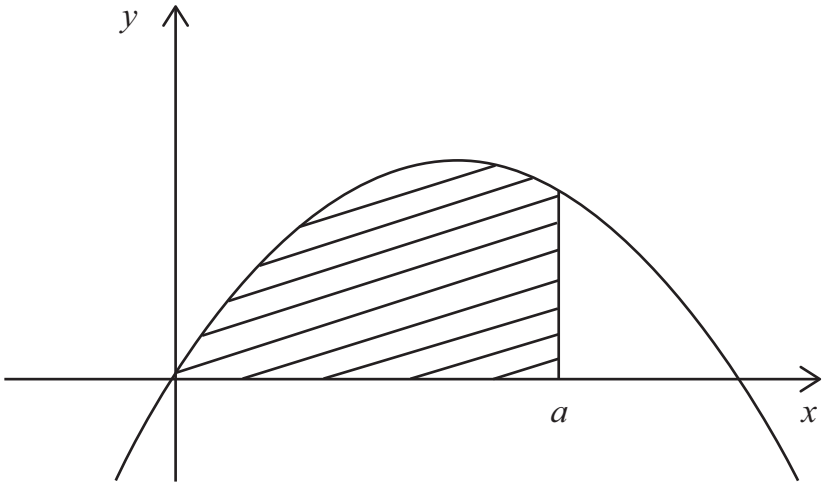
Answer _____ [1]

(iv) Hence find the area of the triangle OPQ, where O is the origin.

Answer _____ [1]

[Turn over

14 A sketch of the curve $y = -3x^2 + 8x$ is shown below.



The area between the curve, the x-axis and the line $x = a$ is shaded.

The shaded area is a^3.

Find the value of a.

Answer _____ [5]

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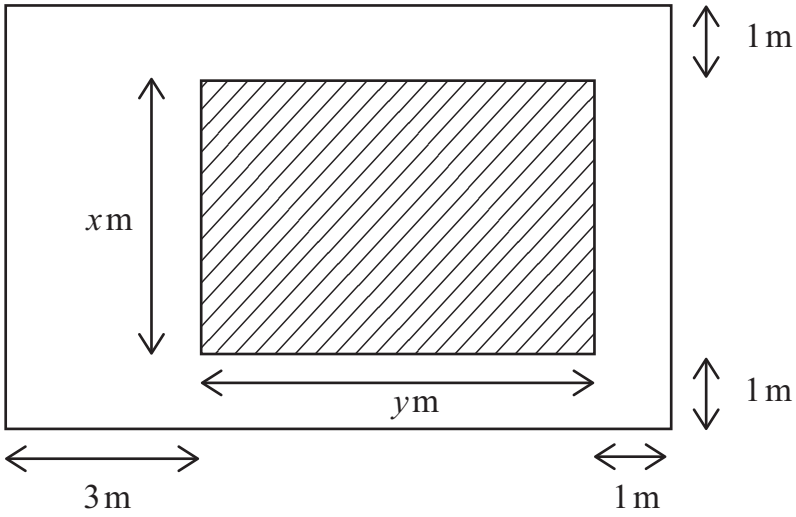
[Turn over

15 The owner of a stately home wishes to build a rectangular ornamental pond in his grounds. He wishes to surround the pond with a tarmac path, 1 m wide on three sides, and 3 m wide at one end to allow for seating.



Let x and y be the lengths of the sides of the pond in metres.

Express in terms of x and y,

(i) the surface area of the pond,

Answer _____ [1]

(ii) the **total** surface area A of the tarmac.

Answer _____ [3]

The surface area of the pond is to be $100\,m^2$.

(iii) Show that the total area of the tarmac is given by

$$A = 4x + \frac{200}{x} + 8$$

[2]

[Turn over

The owner wishes to minimise the amount of tarmac necessary.

(iv) Find the value of x which will minimise the area of the tarmac, proving that it is a minimum.

Answer _____ [4]

(v) Hence determine the corresponding dimensions of the pond.

Answer _____ [1]

[Turn over

16 A curve is defined by the equation $y = 15 - 4x - 3x^2$

(i) Find the coordinates of the points where the curve crosses the x-axis.

Answer _____ [2]

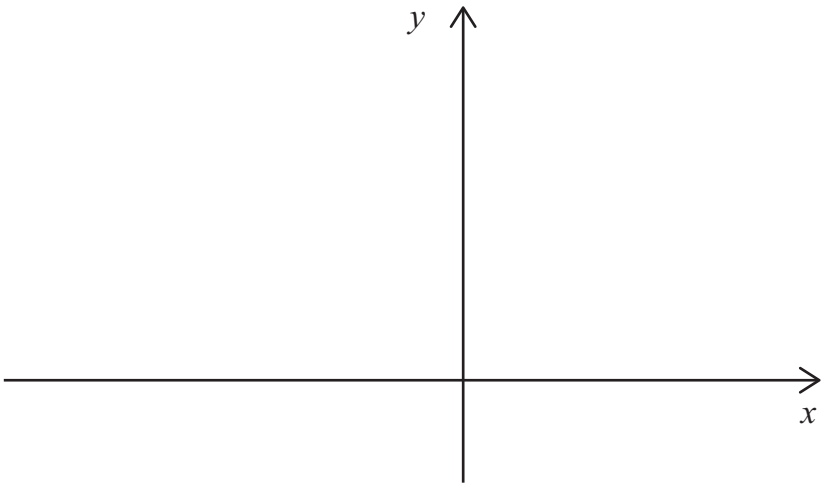
(ii) Find the coordinates of the turning point of the curve.

Answer _____ [4]

(iii) Identify the turning point as either a maximum or a minimum point. You **must** show working to justify your answer.

Answer _____ [1]

(iv) Sketch the curve on the axes below. Your sketch must show the turning point and where the curve crosses the x-axis.



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For Examiner's use only	
Question Number	**Marks**
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