Rewarding Learning

General Certificate of Secondary Education
2015

Further Mathematics

Unit 2
Mechanics and Statistics

[GMF21]

THURSDAY 11 JUNE, AFTERNOON

MARK SCHEME

GCSE Further Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW indicates marks for combined method and accurate working.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to regard candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

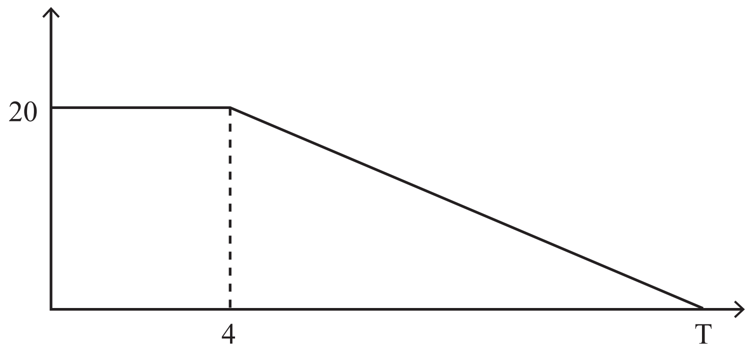
(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

				AVAILABLE MARKS
1	**(i)**	$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$		
		$(3\mathbf{i} - \mathbf{j}) + (p\mathbf{i} + q\mathbf{j}) + (-\mathbf{i} + 4\mathbf{j}) = (0\mathbf{i} + 0\mathbf{j})$	M1	
		$\therefore 3 + p - 1 = 0$		
		$p = -2$	MW1	
		$-1 + q + 4 = 0$		
		$q = -3$	MW1	
	(ii)	Resultant $= (3\mathbf{i} - \mathbf{j}) + (6\mathbf{i} - 9\mathbf{j}) + (-\mathbf{i} + 4\mathbf{j})$	M1	
		$= (8\mathbf{i} - 6\mathbf{j})$N	W1	
	(iii)	$\mathbf{F} = m\mathbf{a}$		
		$8\mathbf{i} - 6\mathbf{j} = 4\mathbf{a}$		
		$\mathbf{a} = 2\mathbf{i} - 1.5\mathbf{j}$	MW1	
		$\therefore \lvert \mathbf{a} \rvert = \sqrt{2^2 + 1.5^2}$	M1	
		$= 2.5\,\text{m/s}^2$	W1	8

2 **(i)**



MW1 (0–4s)
MW1 (4–Ts)

(ii)	Distance at constant speed = 4 × 20 = 80 m	MW1	
	Total distance = 250 m		
	∴ distance while decelerating = 250 – 80 = 170 m	W1	
	So $\frac{1}{2} \times 20 \times (T - 4) = 170$	MW1	
	$T = 21$ s	W1	6

Alternative solution:

Area of trapezium = $\frac{1}{2}(4 + T)20$	M1, W1
$\frac{1}{2}(4 + T)20 = 250$	MW1
$T = 21$ s	W1

			AVAILABLE MARKS
3 **(i)**	$u = 15$, $a = -10$, $v = 0$		
	$v^2 = u^2 + 2as$		
	$0 = 15^2 + 2(-10)s$	MW1	
	$s = 11.25\,\text{m}$	W1	
(ii)	$v = u + at$		
	$0 = 15 - 10t$		
	$t = 1.5\,\text{s}$	MW1	
(iii)	$u = 15$, $a = -10$, $t = 7$	M1	
	$s = ut + \frac{1}{2}at^2$		
	$s = 15 \times 7 + \frac{1}{2}(-10)\ 49$	MW1	
	$s = -140$		
	$\therefore$ height of cliff = 140 m	W1	6

Alternative solution 1:

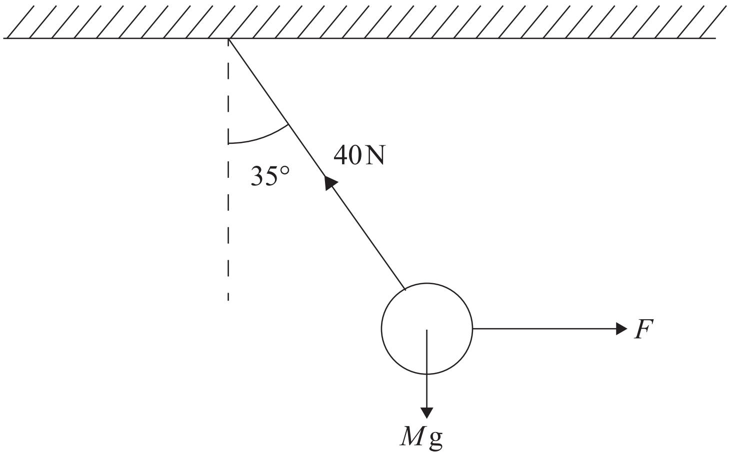
$u = 0$, $a = 10$, $t = 5.5$	M1	
$s = ut + \frac{1}{2}at^2$		
$s = 0 + \frac{1}{2} \times 10 \times 5.5^2$	MW1	
$s = 151.25$		
$\therefore$ height of cliff = 151.25 – 11.25 = 140 m	W1	

Alternative solution 2:

$u = 15$, $a = 10$, $t = 4$	M1	
$s = ut + \frac{1}{2}at^2$		
$s = 15 \times 4 + \frac{1}{2} \times 10 \times 4^2$	MW1	
$s = 140\,\text{m}$	W1	

AVAILABLE MARKS

4 **(i)**



MW1 (40N)
MW1 (Mg)

(ii) Resolve horizontally:

$F = 40 \sin 35°$ — MW1

$= 22.94\text{N}$ — W1

(iii) Resolve vertically:

$Mg = 40 \cos 35°$ — MW1

$M = 3.28$ — W1 — 6

5

(i) Resolve vertically:

$R = 70g + 5g + 25g$ — MW1

$= 100g \quad = 1000\text{N}$ — W1

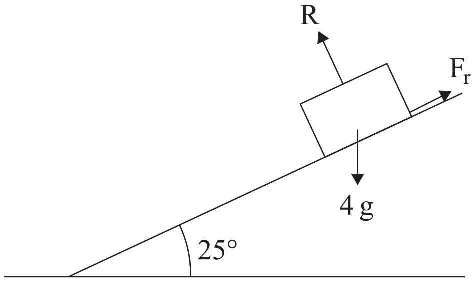
(ii) Take moments about A:

$Rx = 5g \times 4 + 25g \times 8$ — MW1, MW1

$1000x = 2200$

$x = 2.2$ — W1 — 5

6 (i)



MW2
(all 3 forces correctly labelled)
[MW1 for 2 correct forces]

(ii) Resolve perpendicular to slope

R = 4 g cos 25° MW1

R = 36.252 N

Resolve parallel to slope

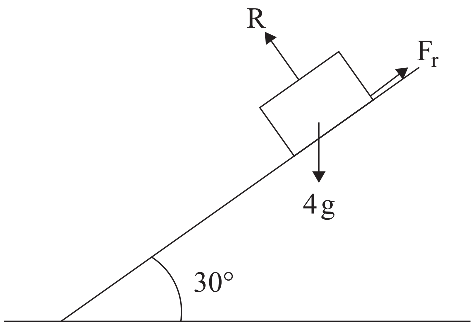
F_r = 4 g sin 25° MW1

F_r = 16.905 N

F_r = μ R

$\mu = \frac{16.905}{36.252} = 0.466 \rightarrow 0.47$ W1

(iii)



Resolve perpendicular to slope

R = 4 g cos 30° MW1

R = 34.641 N

F_r = μR = 0.466 × 34.641

= 16.143 N MW1

Resolve parallel to slope and use $F = ma$

$4\text{ g} \sin 30° - F_r = 4a$ M1, MW1

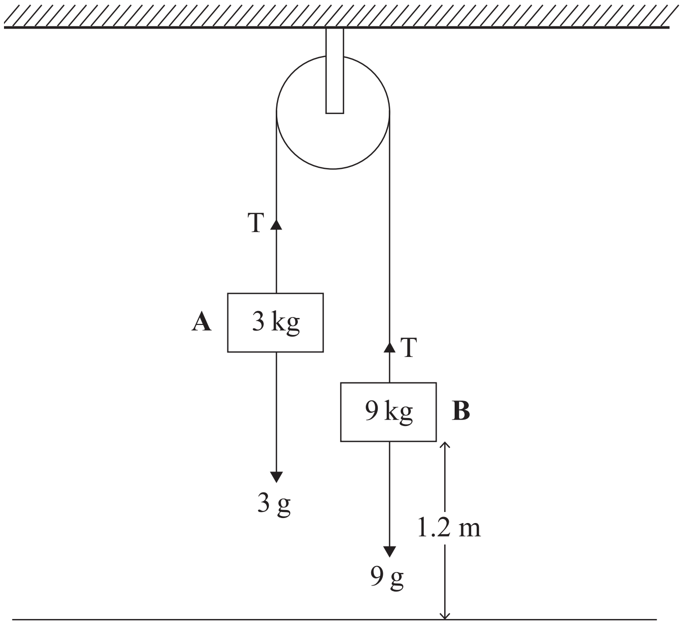
$4a = 20 - 16.143$

$a = 0.96 \text{ m/s}^2$ W1

AVAILABLE MARKS: 10

AVAILABLE MARKS

7 **(i)**



MW1 (forces at A)
MW1 (forces at B)

(ii) For B: $9\,g - T = 9a$ — MW1

For A: $T - 3\,g = 3a$ — MW1

$\therefore\ 6\,g = 12a$

$\therefore\ 12a = 60$

$\therefore\ a = 5\,\text{m/s}^2$ — W1

(iii) $T - 3\,g = 3a$

$T = 3\,g + 3a$

$= 30 + 15$ — M1

$= 45\,\text{N}$ — W1

(iv) $s = 1.2,\ u = 0,\ a = 5$

$v^2 = u^2 + 2as$

$= 0 + 2 \times 5 \times 1.2$ — MW1

$= 12$

$v = \sqrt{12} = 3.46$ m/s — W1 — 9

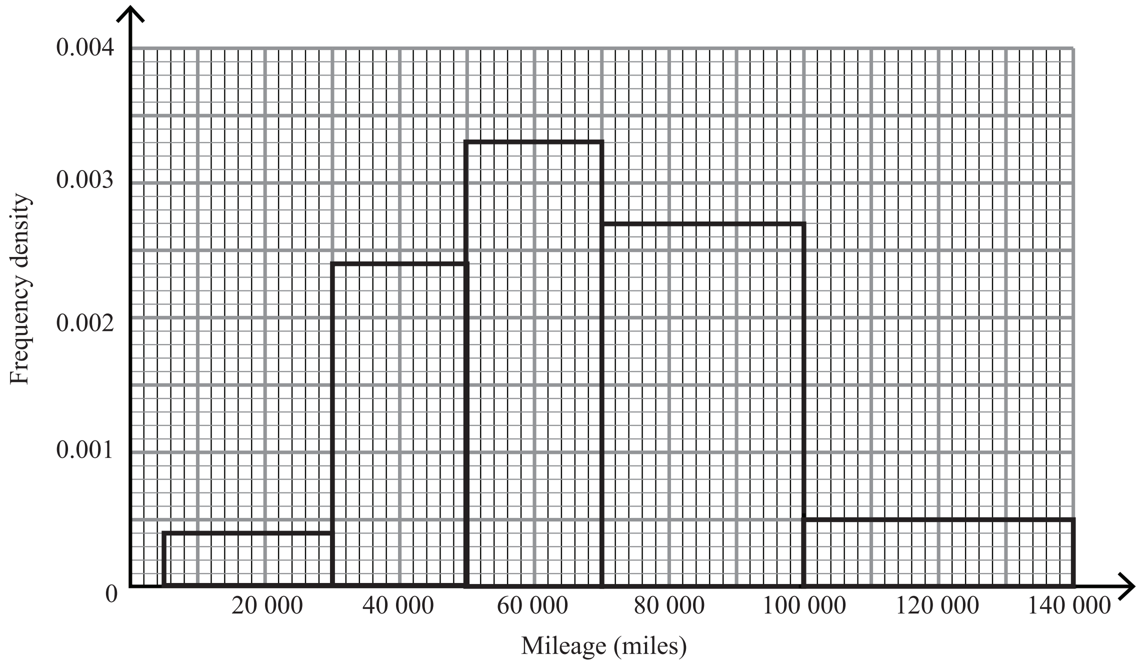
8 **(i)** mean $= \dfrac{(63 \times 47) + (48 \times 21) + (72 \times 49)}{47 + 21 + 49}$ — M1

$= 64.08$ (2 d.p.) — W1

(ii) French – It has the smallest standard deviation. — M1 — 3

9 Frequency densities are:
0.0004 0.0024 0.0033 0.0027 0.0005 M1, W1

AVAILABLE MARKS



W1 (labels)
W1 (widths)
W1 (heights) 5

10 **(i)** Modal class is $210 \leqslant t < 240$ M1

(ii) Median class is $240 \leqslant t < 270$ M1

Number of runners = 2299

$$\text{Median} = 240 + \frac{\frac{2299}{2} - 823}{435} \times 30$$ MW2

= 262.52 minutes W1

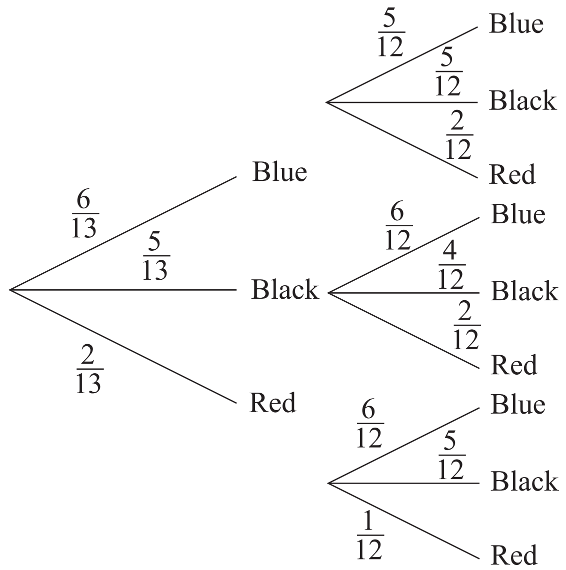
(iii) $\sum fx =$ (5 × 135) + (37 × 165) + (226 × 195) + (555 × 225) + (435 × 255) + (527 × 285) + (338 × 315) + (176 × 345)

= 604 035 MW1

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{604\,035}{2299}$$ M1

= 262.74 minutes W1 8

11



(i) $\frac{5}{12}$ MW1

(ii) (a) P (same) $= \left(\frac{6}{13}\times\frac{5}{12}\right)+\left(\frac{5}{13}\times\frac{4}{12}\right)+\left(\frac{2}{13}\times\frac{1}{12}\right)$ M1

$= \frac{30}{156}+\frac{20}{156}+\frac{2}{156}$

$= \frac{52}{156} = \frac{1}{3}$ W1

(b) P (different) = 1 – P (same) $= 1-\frac{1}{3}=\frac{2}{3}$ M1, W1

(iii) P (both red | same) $= \dfrac{\frac{2}{13}\times\frac{1}{12}}{\frac{1}{3}}$ M1, M1

$= \frac{1}{26}$ W1

AVAILABLE MARKS
8

AVAILABLE MARKS

12 **(i)** The possible two digit numbers are:
10–16, 30–36, 50–56, 70–76 and 90–96 — M1
So number of possibilities is
7 + 7 + 7 + 7 + 7 = 35 — W1

(ii) Numbers >55 are 56, 70–76 and 90–96
So 1 + 7 + 7 = 15 — MW1

Hence P (>55) = $\frac{15}{35} = \frac{3}{7}$ — MW1

Alternative solution

(i) Sample space

Units

Tens	0	1	2	3	4	5	6
1	10	11	12	13	14	15	16
3	30	31	32	33	34	35	36
5	50	51	52	53	54	55	56
7	70	71	72	73	74	75	76
9	90	91	92	93	94	95	96

M1

Number of possibilities = 5 × 7 = 35 — W1

(ii) Numbers >55 are shown

Number = 15 — MW1

So P (>55) = $\frac{15}{35} = \frac{3}{7}$ — MW1 — 4

13 **(i)**

Ranks (heights)	4	8	1	6	9	2	3	7	5	MW1
Ranks (shoe sizes)	2	7	2	6	8	2	5	9	4	MW1

Alternatively:

Ranks (heights)	6	2	9	4	1	8	7	3	5	MW1
Ranks (shoe sizes)	8	3	8	4	2	8	5	1	6	MW1

(ii)

d^2	4	1	1	0	1	0	4	4	1

M1,W1

$\sum d^2 = 16$

$r = 1 - \frac{6 \times 16}{9 \times 80}$ M1

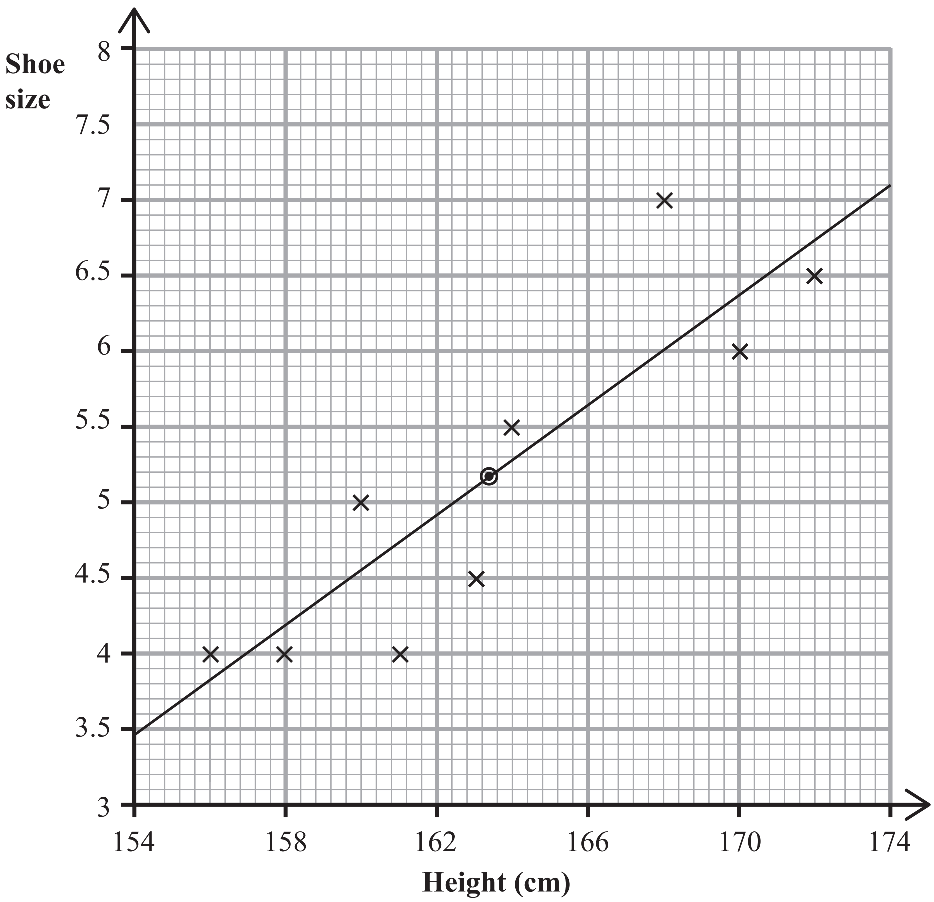
$= 0.87$ W1

(iii) (Strong) positive correlation M1

(iv) Mean height $= \frac{1472}{9} = 163.56$ cm

Mean shoe size $= \frac{46.5}{9} = 5.17$ W1

(v)



W1, W1

AVAILABLE MARKS

(vi) $m = \frac{6.00 - 4.54}{168 - 160} = 0.18$ M1

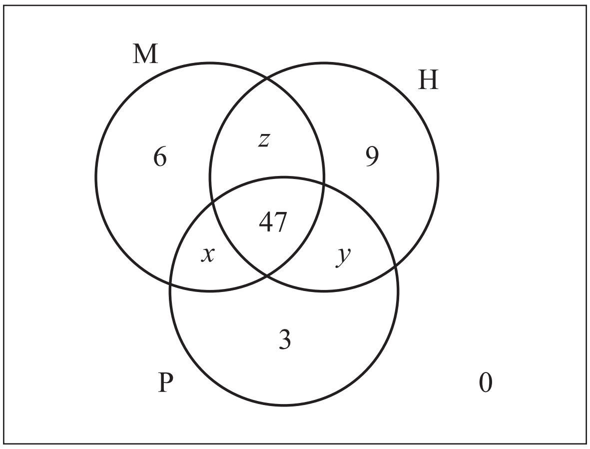
Passes through means

So 5.17 = 0.18 (163.56) + c M1

$\therefore$ c = –24.27

$\therefore y = 0.18\,x - 24.27$ W1 13

14 (i)



MW2

23 did not choose ham, so

$6 + x + 3 = 23$

$x = 14$ MW1

So P (mushrooms and peppers) $= \frac{14 + 47}{100}$ M1

$= \frac{61}{100}$ W1

(ii) P (2 toppings) $= \frac{x + y + z}{100}$ M2

$= \frac{100 - (6 + 3 + 9 + 47)}{100}$ MW1

$= \frac{35}{100} = \frac{7}{20}$ W1 9

Total **100**