Centre Number

Candidate Number

Rewarding Learning

General Certificate of Secondary Education
January 2015

Further Mathematics

Unit 1

Pure Mathematics

[GMF11]

GMF11

FRIDAY 16 JANUARY, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Complete in blue or black ink only. Do not write with a gel pen.
All working should be clearly shown since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all sixteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

Formula Sheet

PURE MATHEMATICS

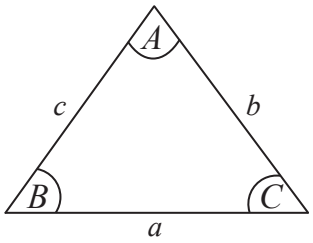
Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$



Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: $\text{Mean} = \frac{\Sigma fx}{\Sigma f}$ $\quad \text{Median} = L_1 + \frac{\left\{\frac{N}{2} - (\Sigma f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
N is total frequency
$(\Sigma f)_1$ is the sum of the frequencies up to but not including the median class
f_{median} is the frequency of the median class
c is the width of the median class

$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

Examiner Only	
Marks	Remark

1 Matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} -2 & 3 \\ 5 & -4 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -7 \\ 4 & -2 \end{bmatrix}$$

Evaluate

(i) $\mathbf{A}^2$

Answer _____ [2]

(ii) $\mathbf{A}^2 - 3\mathbf{B}$

Answer _____ [2]

2 Solve the equation $x^2 = 14x + 2$ by completing the square.

Give your answer in the form $a \pm \sqrt{b}$, where a and b are whole numbers.

Answer _____ [4]

Examiner Only	
Marks	Remark

3 If $y = 2x^3 - \frac{1}{4x^5}$ find $\frac{d^2y}{dx^2}$

Examiner Only	
Marks	Remark

Answer _____ [4]

Examiner Only	
Marks	Remark

4 **(i)** Solve the equation

$$\sin\theta = 0.75$$

for $0° \leqslant \theta \leqslant 180°$

Answer _____ [2]

(ii) **Hence** solve the equation

$$\sin\left(\frac{x + 10°}{2}\right) = 0.75$$

for $0° \leqslant x \leqslant 360°$

Answer _____ [3]

5 Matrices **P** and **R** are defined by

$$\mathbf{P} = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{R} = \begin{bmatrix} 13 & 4 \\ 7 & 6 \end{bmatrix}$$

(i) Find the matrix $\mathbf{P}^{-1}$, the inverse of **P**

Answer _____ [2]

Examiner Only	
Marks	Remark

(ii) Hence find the matrix **Q** such that **PQ = R**

Answer _____ [3]

Examiner Only	
Marks	Remark

6 Simplify the following expressions

(i) $\dfrac{3x+1}{2x+1} \div \dfrac{1}{2x^2-x-1}$

Answer _____ [3]

Examiner Only	
Marks	Remark

(ii) $\dfrac{3x}{x-4} - \dfrac{2x+1}{x^2-16}$

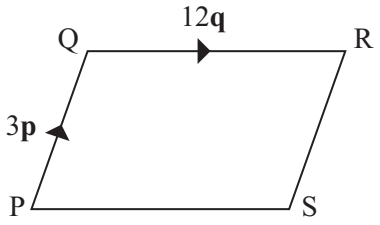
Examiner Only

Marks	Remark

Answer _____ [4]

Examiner Only	
Marks	Remark

7 In the parallelogram PQRS below, $\overrightarrow{PQ}$ represents the vector 3**p** and $\overrightarrow{QR}$ represents the vector 12**q**



(i) Express in terms of **p** and **q**

(a) $\overrightarrow{PR}$

Answer _____ [1]

(b) $\overrightarrow{QS}$

Answer _____ [1]

A point T is on QS produced such that $\overrightarrow{ST} = -5\mathbf{p} + 20\mathbf{q}$

(ii) Find the vector $\overrightarrow{TR}$, giving your answer in its simplest form.

Answer _____ [2]

(iii) Given that $\overrightarrow{QS} = k\ \overrightarrow{QT}$, where k is a constant, find the value of k

Answer _____ [2]

Examiner Only	
Marks	Remark

8 Solve the set of simultaneous equations

$$4x + 2y - z = 16$$
$$5x - 3y - z = 3$$
$$14x + 2y - 3z = 40$$

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Marks	Remark

Answer $x =$ _____ , $y =$ _____ , $z =$ _____ [8]

You may use this page if needed.
(Questions continue overleaf.)

9 **(a)** Given that $\log_4 16 = 2x$, find the value of x

Examiner Only	
Marks	Remark

Answer _____ [2]

(b) If $\log_7 2 = p$ and $\log_7 6 = q$, express in terms of p and q

(i) $\log_7 12$

Answer _____ [1]

(ii) $\log_7 21$

Answer _____ [2]

Examiner Only	
Marks	Remark

10 At Fitness One Gym there are:

8 women in the Circuits class.

6 more men in the Circuits class than men in the Spinning class.

2 less women than men in the Spinning class.

Let x be the number of men in the Spinning class.

Write down in terms of x

(i) the number of women in the Spinning class,

Answer _____ [1]

(ii) the number of men in the Circuits class.

Answer _____ [1]

Examiner Only	
Marks	Remark

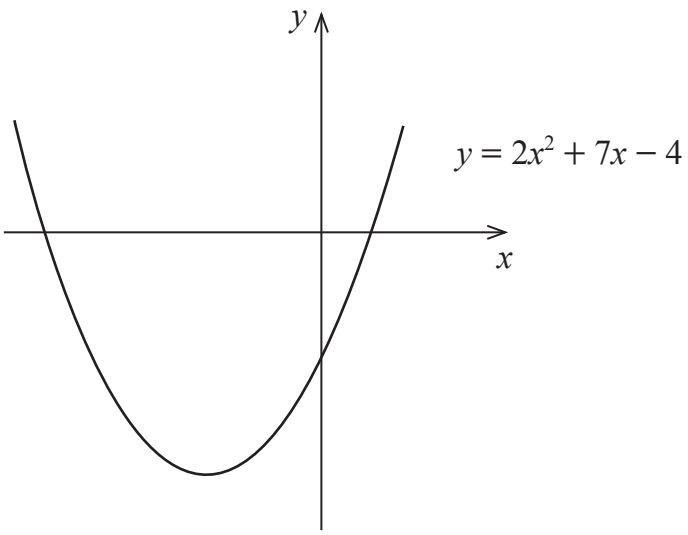
The ratio of men to women in each class is the same.

(iii) Form a quadratic equation in x and hence find the number of men in the Spinning class.

Answer _____ [5]

Examiner Only	
Marks	Remark

11 The sketch below shows the curve with equation $y = 2x^2 + 7x - 4$



(i) Find the x-coordinates of the points where the curve crosses the x-axis.

Answer _____ [2]

Examiner Only	
Marks	Remark

(ii) **Hence** find the area enclosed between this curve, the **negative** x-axis and the y-axis.

Answer _____ [5]

Examiner Only	
Marks	Remark

12 The point P lies on the curve $y = x^2 + 5x - 1$
The gradient of the **normal** to the curve at the point P is $\frac{1}{3}$

(i) State the gradient of the **tangent** to the curve at the point P.

Answer _____ [1]

(ii) **Hence** find the coordinates of the point P.

Answer _____ [3]

Examiner Only	
Marks	Remark

(iii) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$

Answer _____ [2]

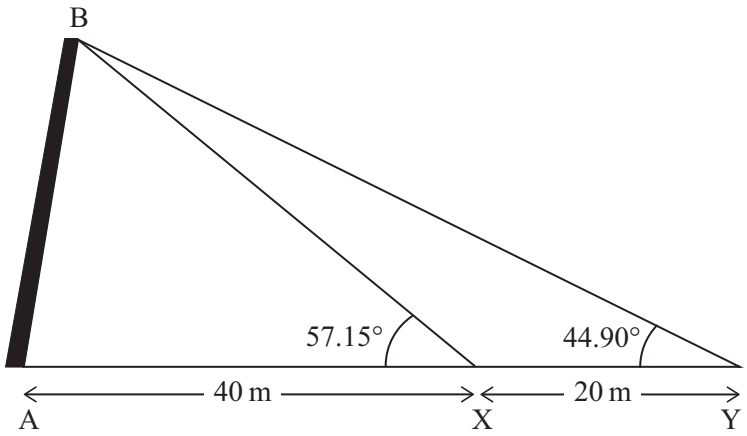
Examiner Only	
Marks	Remark

Examiner Only	
Marks	Remark

13 Two tourists were in the square in front of the Leaning Tower of Pisa, AB, in Italy.

One was at a point X, 40 m from the base of the tower on horizontal ground, and the other was at a point Y, 20 m further out from X. The points A, X and Y were in a straight line and in the same vertical plane as the tower.

The angles of elevation of the top of the tower from X and Y were 57.15° and 44.90° respectively.



Calculate

(i) the size of the angle $X\hat{B}Y$,

Answer _____° [1]

(ii) the distance BX,

Answer _____ m [2]

(iii) the distance AB, from the base to the top of the tower,

Answer _____ m [2]

Examiner Only	
Marks	Remark

(iv) the size of the angle $\mathrm{B\hat{A}X}$.

Answer _____° [2]

(v) **Hence** write down the size of the angle by which the tower is leaning from the vertical.

Answer _____° [1]

Examiner Only	
Marks	Remark

14 Solve the equation

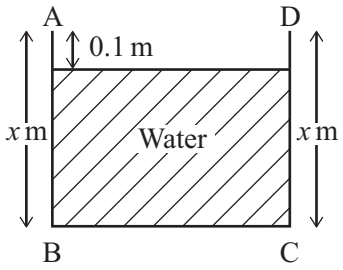
$$2^{3x-2} = 5^{x-1}$$

Answer _____ [5]

Examiner Only	
Marks	Remark

Examiner Only	
Marks	Remark

15 A long sheet of metal, 2 m wide, is bent to form a water channel.
The two vertical sides, AB and DC, are of equal length.
The base BC is horizontal. The channel is open at the top.
A cross section of the channel is shown in the diagram below.



The total length of AB, BC and CD is 2 m.

The water level in the channel is 0.1 m from the top.

Let x m be the height of the sides of the channel.

(i) Write down the length of the base BC in terms of x

Answer _____ [1]

(ii) Show that the cross-sectional area A of the water is given by

$$A = 2.2x - 2x^2 - 0.2$$

[2]

(iii) Find the value of x which will maximise A, proving that it is a maximum area.

Answer _____ [3]

(iv) **Hence** find the dimensions of the channel which will give the maximum cross-sectional area of water.

Answer _____ [1]

Examiner Only	
Marks	Remark

16 A curve is defined by the equation $y = 4x - 2x^2 - x^3$

(i) Find the coordinates of the points where the curve crosses the x-axis, giving values correct to 1 decimal place where necessary.

Answer _____ [3]

Examiner Only	
Marks	Remark

(ii) Find the coordinates of the turning points of the curve.

Examiner Only	
Marks	Remark

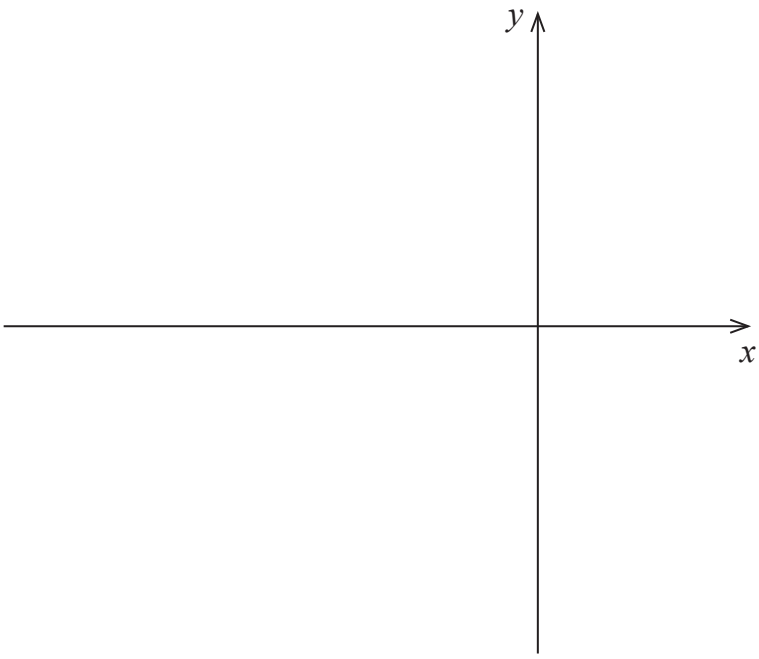
Answer _____ [5]

(iii) Identify each turning point as either a maximum or a minimum point. You must show working to justify your answers.

Examiner Only	
Marks	Remark

Answer _____ [2]

(iv) Sketch the curve on the axes below. Your sketch must show the turning points and where the curve crosses the x-axis.



[2]

Examiner Only	
Marks	Remark

THIS IS THE END OF THE QUESTION PAPER

For Examiner's use only	
Question Number	**Marks**
1	
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