Rewarding Learning

Number

Candidate Num

General Certificate of Secondary Education
2014

Further Mathematics

Unit 2

Mechanics and Statistics

GMF21

[GMF21]

FRIDAY 13 JUNE, MORNING

TIME

2 hours.
Candidates should spend approximately one hour on each section.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Complete in blue or black ink only. Do not write with a gel pen.
All working should be clearly shown since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all thirteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

Formula Sheet

PURE MATHEMATICS

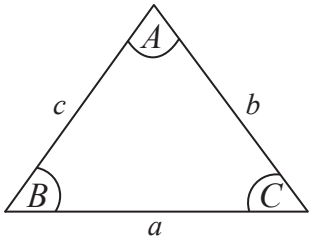
Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\,cosA$

Area of triangle $= \frac{1}{2}ab \sin C$



Differentiation: If $y = ax^n$ then $\dfrac{dy}{dx} = nax^{n-1}$

Integration: $\int ax^n \, dx = \dfrac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: Mean $= \dfrac{\Sigma fx}{\Sigma f}$ $\qquad$ Median $= L_1 + \dfrac{\left\{\frac{N}{2} - (\Sigma f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
N is total frequency
$(\Sigma f)_1$ is the sum of the frequencies up to but not including the median class
f_{median} is the frequency of the median class
c is the width of the median class

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

Section A

Mechanics

You should spend approximately **one hour** on this section.

Take $\mathbf{g = 10\,m/s^2}$ when required.

Examiner Only	
Marks	Remark

1 (Throughout this question **i** and **j** denote unit vectors parallel to a set of standard *x*-*y* axes.)

Vectors **a** and **b** are defined by

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} \quad \text{and} \quad \mathbf{b} = 4\mathbf{i} - 2\mathbf{j}$$

Find

(i) the vector $3\mathbf{a} - \mathbf{b}$ in terms of **i** and **j**,

Answer _____ [2]

(ii) the magnitude of the vector $3\mathbf{a} - \mathbf{b}$,

Answer _____ [1]

(iii) the acute angle the vector $3\mathbf{a} - \mathbf{b}$ makes with the positive x-axis.

Examiner Only	
Marks	Remark

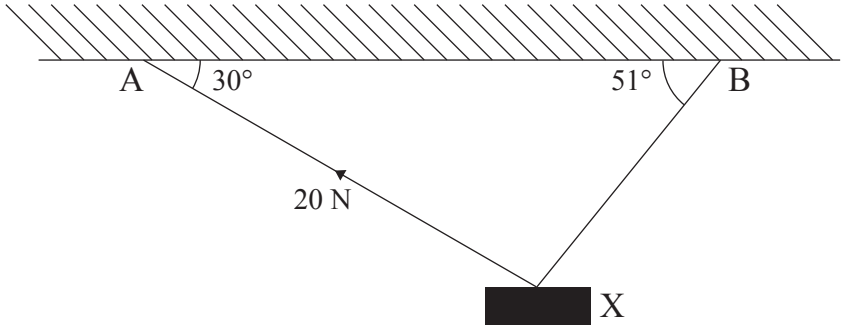
Answer _____ ° [1]

Examiner Only	
Marks	Remark

2 A block X, of mass *m* kg, is held in equilibrium by two light inextensible strings XA and XB, attached to a horizontal ceiling. The strings are inclined to the ceiling at angles of 30° and 51°, as shown in the diagram below.

The tension in the string XA is 20 N.

(i) Mark on the diagram the other forces acting on the block.



[1]

(ii) Calculate the tension in the string XB.

Answer _____ N [2]

(iii) Calculate the value of m.

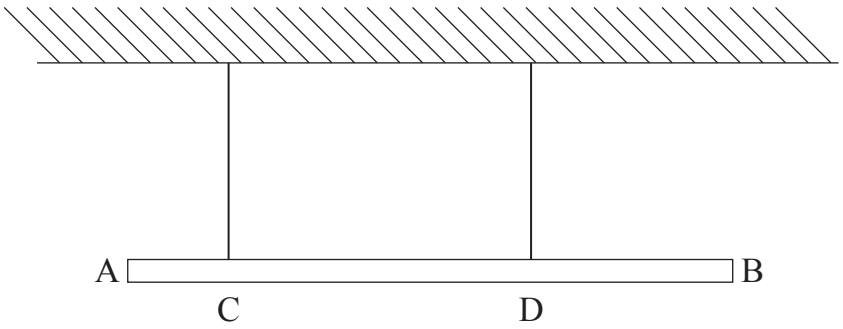
Examiner Only	
Marks	Remark

Answer _____ [3]

Examiner Only	
Marks	Remark

3 A uniform plank AB, of length 6 m and mass 12 kg, is held horizontally by two vertical strings attached to a ceiling. The strings are connected to the plank at the points C and D, as shown in the diagram below, where AC = 1 m and AD = 4 m.

(i) On the diagram mark all the forces acting on the plank.



[1]

(ii) Calculate the tension in each of the two strings.

Answer _____ N and _____ N [3]

A mass M kg is attached to the end B so that the plank is on the point of turning about D.

(iii) Write down the tension in the string at C.

Answer _____ N [1]

(iv) Calculate the value of M.

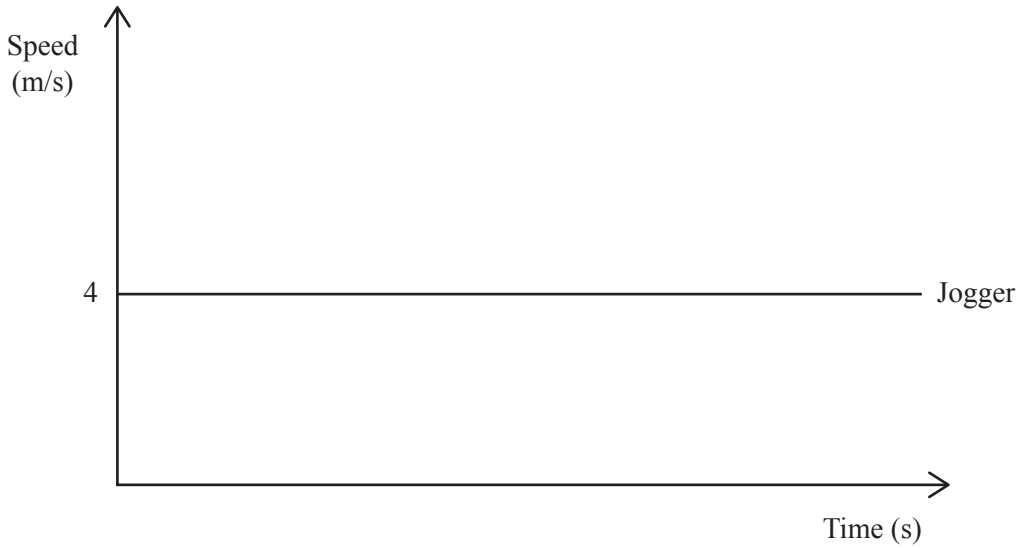
Answer _____ [3]

Examiner Only	
Marks	Remark

4 A man is jogging at a constant speed of 4 m/s when he passes a stationary cyclist.
Ten seconds after he passes the cyclist, she gives chase.

The cyclist accelerates uniformly for 6 seconds until she reaches a speed of 9 m/s.
She maintains this speed until she reaches the jogger.

(i) On the axes below, sketch the speed-time graph for the cyclist.



[2]

Examiner Only	
Marks	Remark

Hence, or otherwise, calculate

(ii) the time taken for the cyclist to reach the jogger,

Answer _____ s [5]

(iii) the distance travelled by the cyclist when she reaches the jogger.

Answer _____ m [2]

Examiner Only	
Marks	Remark

Examiner Only	
Marks	Remark

5 A missile is fired vertically upwards from the ground. It starts from rest and rises with a constant acceleration of $5\,m/s^2$. The missile's fuel burns out after 10 seconds.

(i) Calculate the height of the missile after 10 seconds.

Answer _____ m [2]

(ii) Calculate the speed of the missile after 10 seconds.

Answer _____ m/s [2]

After the fuel runs out, the missile continues to rise vertically, and can be modelled as a particle travelling freely under gravity.

(iii) Calculate the maximum height above the **ground** reached by the missile.

Answer _____ m [3]

(iv) Calculate the time taken from the moment the missile leaves the ground until it reaches the ground again.

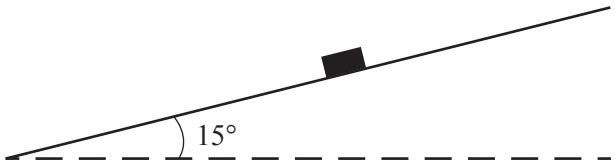
Examiner Only

Marks	Remark

Answer _____ s [4]

Examiner Only	
Marks	Remark

6 A block of mass 5 kg is on a rough plane which is inclined at an angle of 15° to the horizontal, as shown in the diagram below.



The block is projected up the plane with a speed of 8 m/s, and travels up the plane for 2 seconds before coming to rest momentarily.

(i) Show that the deceleration of the block is 4 m/s^2.

[1]

(ii) Calculate the magnitude of the normal reaction between the block and the plane.

Answer _____ N [1]

(iii) Calculate the magnitude of the frictional resistance.

Answer _____ N [3]

(iv) Calculate the coefficient of friction.

Answer _____ [2]

The block now slides down the plane.

(v) Calculate the acceleration of the block down the plane.

Answer _____ m/s^2 [5]

Examiner Only	
Marks	Remark

Section B

Statistics

You should spend approximately **one hour** on this section.

Examiner Only	
Marks	Remark

7 While waiting for his flight home, Dominic recorded the time intervals, in minutes and seconds, between planes taking off from one runway at Heathrow Airport. His results are given in the table below.

Minutes	Seconds
0	58
2	17
0	50
1	04
1	21
0	58
0	52
1	56
1	11
2	13

(i) Calculate the mean of these times. Give your answer in minutes and seconds.

Answer _____ [2]

(ii) Calculate the standard deviation of these times.

Examiner Only	
Marks	Remark

Answer _____ s [2]

8 When training for a ski jump competition Gerry recorded the length of each of his jumps, correct to the nearest metre. His results are summarised in the table below.

Distance (m)	31–50	51–70	71–90	91–100	101–110	111–120
Number of jumps	2	11	16	23	8	1

(i) Calculate an estimate for the median distance jumped.

Answer _____ m [4]

(ii) Why is this an **estimate** for the median distance jumped?

Answer _____

_____

_____ [1]

Examiner Only

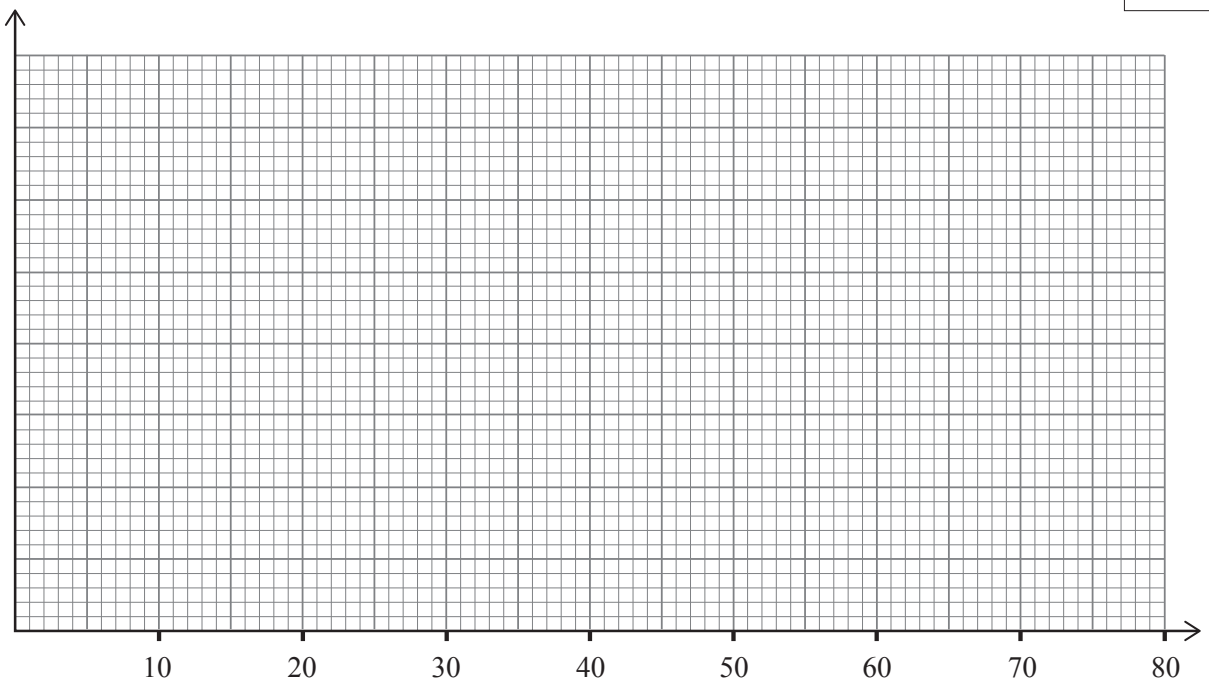
Marks	Remark

9 On Monday a librarian recorded the ages of people visiting the library. The results are summarised in the table below.

Examiner Only	
Marks	Remark

Age (years)	4–11	12–17	18–37	38–57	58–77
Number of visitors	44	36	63	56	48

Using the axes below, draw a histogram to represent this information.
Label each axis clearly.



[5]

10 Molly is playing a game with a regular six-sided die.

She throws the die once, and if the score is not a 6 then this score is recorded.

If she throws a 6 on her first throw, then she is given a second throw and the total of both throws is recorded.

Find the probability of Molly getting

(i) a total score of 6,

Answer _____ [1]

(ii) a total score of 8,

Answer _____ [2]

(iii) a score greater than 8,

Answer _____ [3]

Examiner Only	
Marks	Remark

(iv) a score of 12, given that she scored greater than 8

Answer _____ [3]

Examiner Only	
Marks	Remark

11 A bag contains 4 red sweets and some yellow sweets.

One sweet is selected at random.

By letting x equal the number of yellow sweets in the bag,

(i) write down the probability, in terms of x, that the sweet selected is red.

Answer _____ [1]

A second sweet is selected at random from the remaining sweets in the bag.

(ii) Write down the probability, in terms of x, that **both** sweets selected are red.

Answer _____ [2]

Examiner Only	
Marks	Remark

The probability that both sweets selected are red is $\frac{1}{6}$

(iii) By forming an equation in x, determine the number of yellow sweets that were in the bag.

Answer _____ [2]

Examiner Only	
Marks	Remark

12 Throughout the month of May, Aoife recorded the number of days on which she revised for her three science subjects, Chemistry, Physics and Biology.

On 4 days she revised all 3 science subjects.
On 8 days she revised both Biology and Physics.
On 11 days she revised both Chemistry and Biology.
On 9 days she revised both Chemistry and Physics.
On 19 days she revised Chemistry.
On 15 days she revised Physics.
On 16 days she revised Biology.

(i) Illustrate this information on a Venn Diagram.

[3]

(ii) On how many days did she **not** revise any of her science subjects?

Answer _____ [1]

Examiner Only	
Marks	Remark

(iii) What is the probability that on a day chosen at random Aoife revised **exactly** two science subjects?

Answer _____ [2]

On a day chosen at random, Aoife did not revise Physics.

(iv) What is the probability that she revised Chemistry on that day?

Answer _____ [3]

Examiner Only	
Marks	Remark

13 The marks achieved by 10 students for each of their two Learning for Life and Work coursework pieces are recorded in the table below.

First piece	36	26	34	32	22	47	33	46	38	35
Second piece	28	24	28	25	16	33	22	36	22	20

(i) Find the rank orders for the marks in each of these coursework pieces.

[2]

(ii) Calculate Spearman's coefficient of rank correlation.

Answer _____ [4]

Examiner Only	
Marks	Remark

(iii) What significance, if any, do you attach to the value you obtained in **(ii)**?

Answer _____ [1]

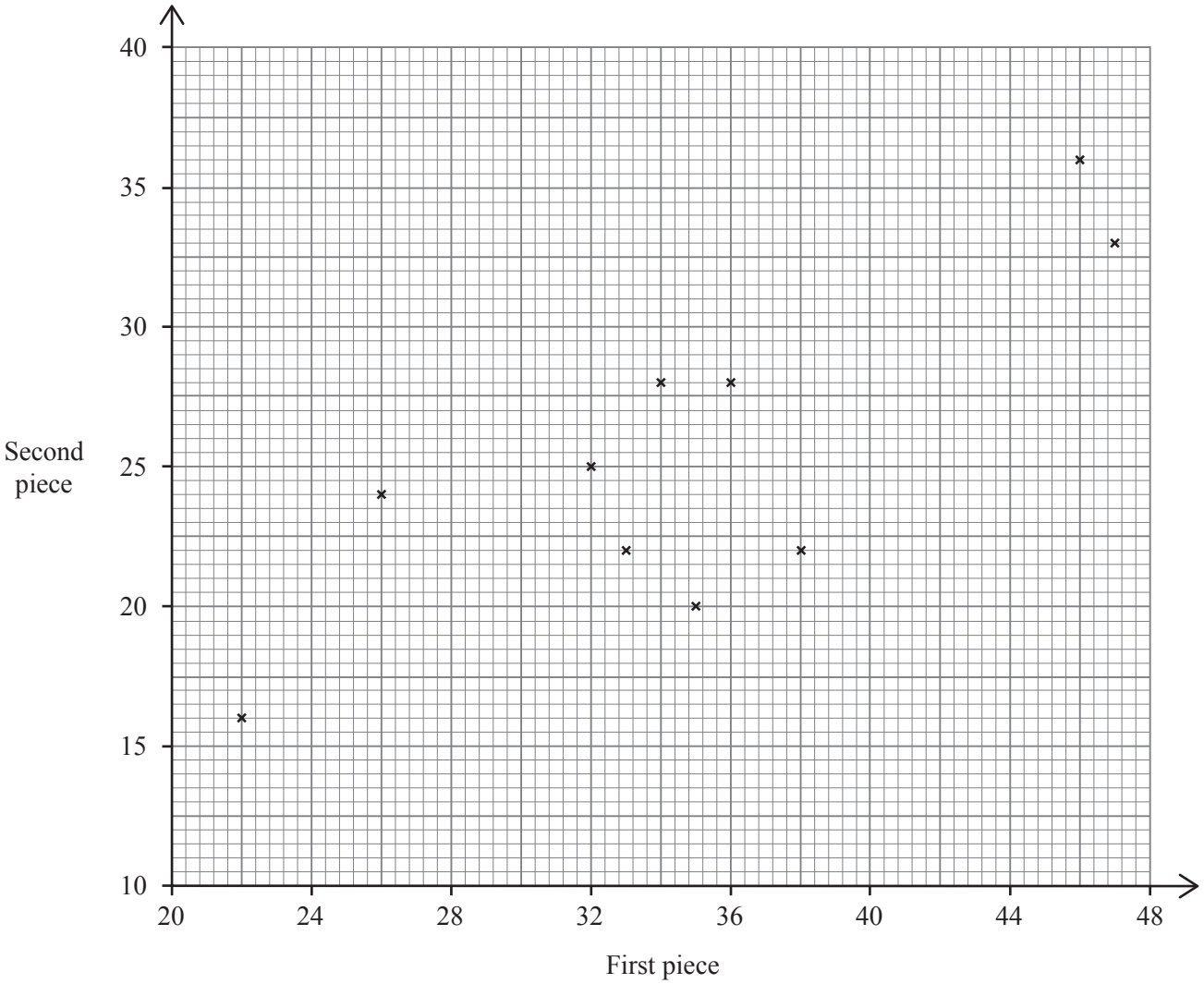
(iv) Calculate the mean score in each coursework piece.

Answer First piece _____

Second piece _____ [1]

Examiner Only	
Marks	Remark

The data from the table are plotted on the graph below.



(v) Draw your line of best fit on the graph. [2]

Examiner Only	
Marks	Remark

(vi) Determine the equation of the line of best fit which you have drawn.

Examiner Only	
Marks	Remark

Answer _____ [3]

THIS IS THE END OF THE QUESTION PAPER

For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
Total Marks	

Examiner Number