Version 1.0

General Certificate of Secondary Education

Linked Pair Pilot

Methods in Mathematics 9365
Applications of Mathematics 9370

Delegate Materials

Launch Meetings – spring 2010

Delegate Materials

Principal Examiners have prepared these delegate materials as specimens. These materials have not, therefore, been through the normal process of standardising that would take place for live papers.

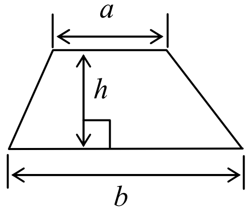
Further copies of these Delegate Materials are available from:

The GCSE Mathematics Department, AQA Devas Street, Manchester, M15 6EX
Telephone: 0161 953 3852   Fax: 161 957 3871   e mail: sprior@aqa.org.uk

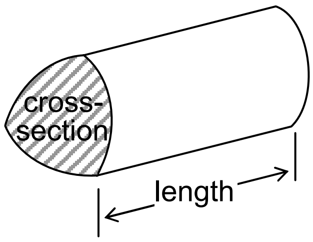
Set and published by the Assessment and Qualifications Alliance.

Formulae Sheet: Foundation Tier (A2 and M2)

Area of trapezium = $\frac{1}{2}(a + b)h$

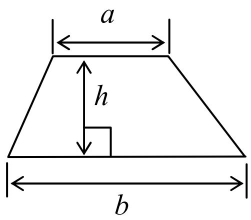


Volume of prism = area of cross-section × length

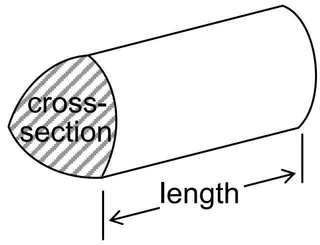


Formulae Sheet: Higher Tier (A2 and M2)

Area of trapezium = $\frac{1}{2}(a+b)h$

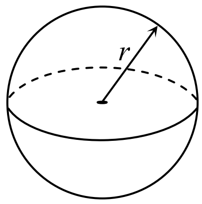


Volume of prism = area of cross-section × length



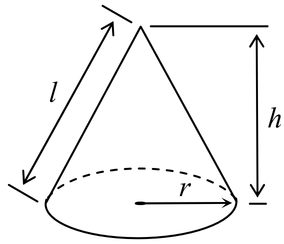
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$

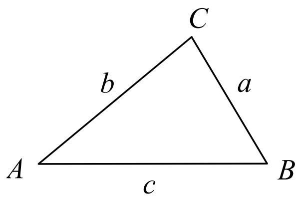


In any triangle ABC

Area of triangle = $\frac{1}{2}ab\sin C$

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$



The Quadratic Equation

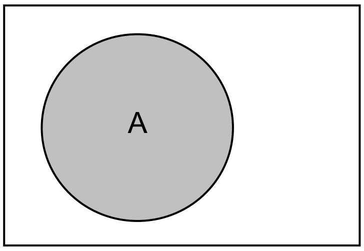
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

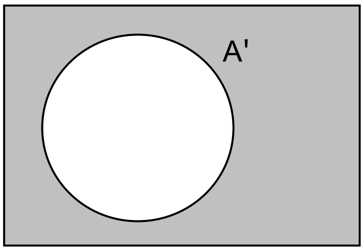
Formulae Sheet: Higher Tier (M1)

Set notation

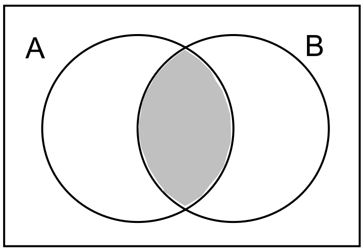
A



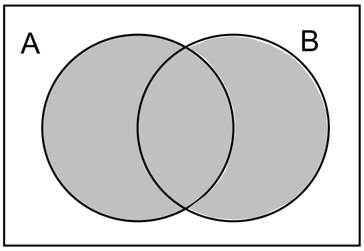
A'



$A \cap B$



$A \cup B$



Problem solving exemplar

An ice cream van sells 5 different flavours of ice cream.
A group of children line up to buy ice cream.
Each child has a double scoop with **two** flavours.
Each child chooses a different combination of flavours.
Every possible combination is chosen.

Ice Cream

Choose from
Vanilla
Strawberry
Chocolate
Lemon
Mint

Double scoop only 99 p

How many children are there?

(2 marks)

Functional mathematics example

1 The timetable shows the train times between Huddersfield and Penistone.

Train Timetable: Huddersfield to Penistone						
Huddersfield:	*depart*	08:20	10:20	12:20	14:20	16:20
Denby Dale:	*depart*	08:45	10:45	12:45	14:45	16:45
Penistone:	*arrive*	08:55	10:55	12:55	14:55	16:55

(a) How long does the journey take between Huddersfield and Penistone?

(1 mark)

(b) Vidal and Sonia live in Huddersfield.
It takes them 25 minutes to travel from home to Huddersfield station.
They want to go to Penistone for a concert that starts at 3:15 pm.

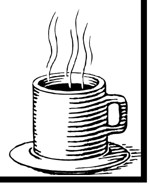
What is the latest time they can leave home to get to the concert on time?

(3 marks)

(c) The cost of food and drink from the Trolley service on the train is shown.

MENU

Tea	*£1.50*
Coffee	*£1.50*
Cold drink	*95p*
Crisps	*50p*
Chocolate bar	*85p*



They want two teas and two chocolate bars.
They only have £5.

Can they afford them?

You **must** show your working.

(3 marks)

(d) During one journey the trolley attendant keeps this tally of what he sells.

	Tally	Totals
Tea	𝍸 𝍷𝍷𝍷	
Coffee	𝍸 𝍸 𝍷𝍷𝍷𝍷	
Cold drink	𝍸 𝍸 𝍸 𝍸 𝍷	
Crisps	𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷	
Chocolate bar	𝍷𝍷𝍷	

Work out the amount of money he takes during the journey.

Show clearly how you work out your answer.

(4 marks)

ASSESSMENT and QUALIFICATIONS ALLIANCE

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Methods in Mathematics 9365

Exemplar Material

Simultaneous equations

One linear, one non-linear, where non-linear could have squared terms in both unknowns.

Example

Solve the simultaneous equations

$$y = x + 1$$
$$x^2 + y^2 = 25$$

(*6 marks*)

This could be a QWC question as the steps to solve are well defined and need to be followed through in order.

Step 1: Substitute linear into non-linear

$$x^2 + (x + 1)^2 = 25$$

Step 2: Expand bracket and rearrange into a quadratic

$$x^2 + x^2 + 2x + 1 = 25$$
$$2x^2 + 2x - 24 = 0$$
$$x^2 + x - 12 = 0$$

Step 3: Solve the quadratic to get the values of x

$$(x + 4)(x - 3) = 0$$
$$x = -4 \text{ or } 3$$

Step 4: Find the equivalent values of y for the values of x to get the pairs of solutions.

When $x = -4$, $y = -3$

When $x = 3$, $y = 4$

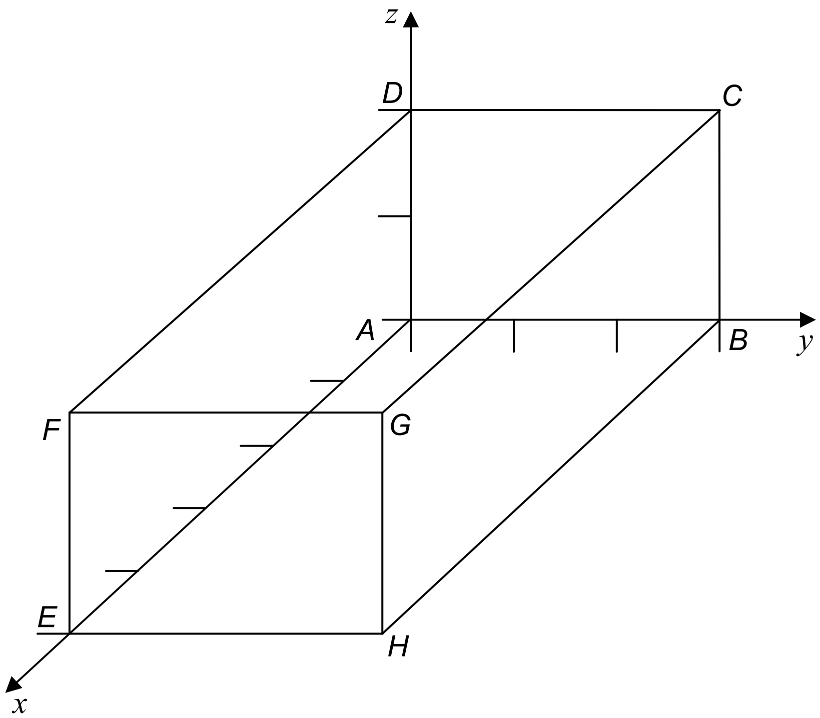
3D coordinate systems

One of the coordinates will always be zero.

Example

ABCDEFG is a cuboid with sides of 2 cm, 3 cm and 5 cm.

The vertex *A* is placed at the origin, with the sides aligned along the axes of a 3-dimensional centimetre coordinate system so that point *F* is at (5, 0, 2).



Not drawn accurately

(a) Write down the coordinates of the point *G*.

(1 mark)

(b) Write down the coordinates of the mid-point of the side *ABCD*.

(1 mark)

(c) Work out the length of the diagonal *AG*

(3 marks)

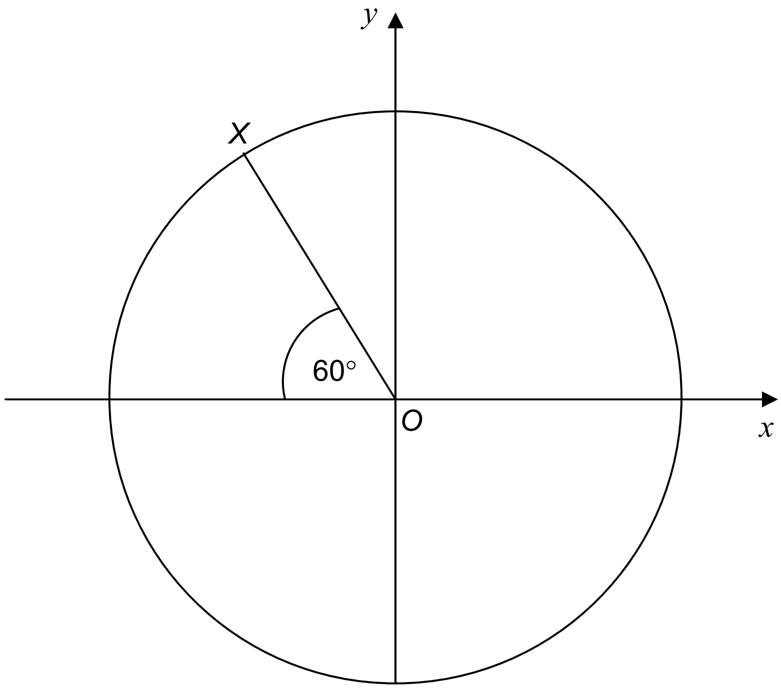
Cartesian equation

Understand and use the Cartesian equation of a circle centred at origin and link to trigonometrical functions.

Example

The circle $x^2 + y^2 = 16$ is shown.

The point *X* is such that the angle between *OX* and the x-axis is 60°.



Not drawn accurately

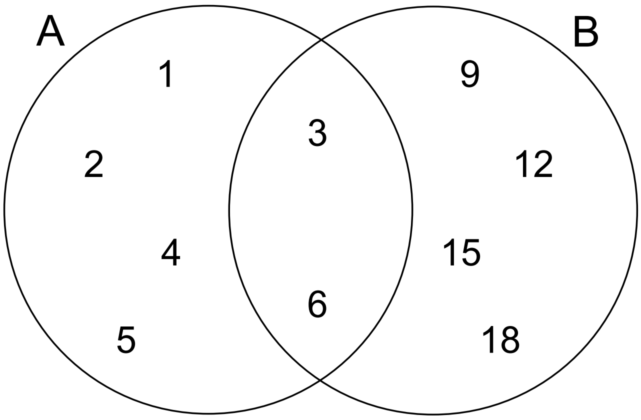
Work out the correct coordinates of the point *X*.

(3 marks)

Understand and use set notation to describe events and compound events.
Use Venn diagrams to represent the number of possibilities and hence find probabilities.

Understand and use Venn diagrams to solve problems.

The Venn diagram shows two sets, A and B.



A number is selected at random from set A.

What is the probability that it is an odd number?

(1 mark)

What is the probability that it is a factor of 12?

(1 mark)

A number is selected at random from A $\cup$ B.

What is the probability that it is a multiple of 3?

(2 marks)

Form quadratic expressions to describe the *n*th term of a sequence.

Example

A sequence is 3, 8, 15, 24, 35, 48, ….

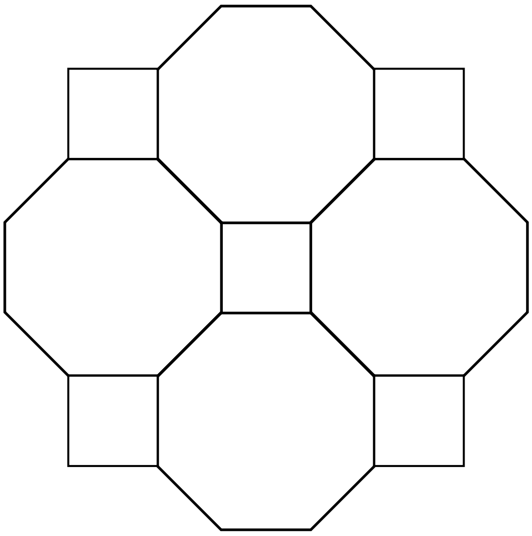
Find an expression for the *n*th term.

(3 marks)

Tiling patterns and tessellations

Example

A tessellation is made from regular octagons and squares.



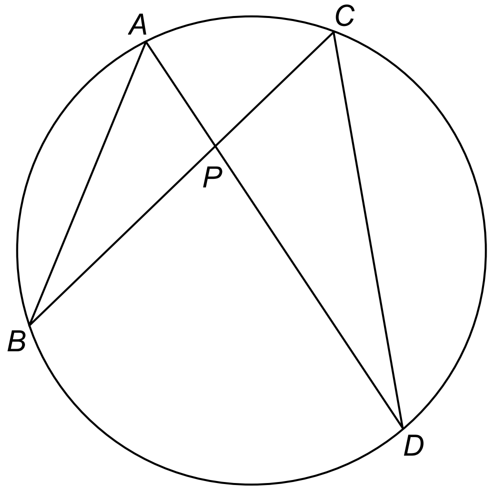
By reference to the angles of the octagon and square, explain why these two shapes will tessellate.

(3 marks)

Intersecting chords theorem

AD and *BC* are two chords of a circle that intersect at *P*.

$AP \times PD = BP \times PC$



Proof

Angle *BAP* = angle *DCP* (Angles in same segment)

Angle *ABP* = Angle *CDP* (Angles in same segment)

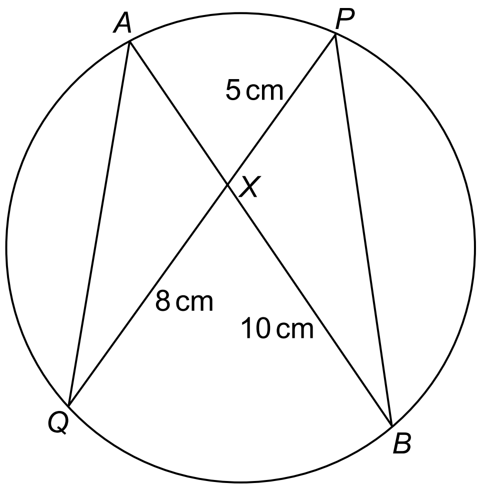
Angle *BPA* = Angle *DPC* (Vertically opposite angles)

Hence triangle *PAB* is similar to angle *PCD* (Equiangular)

Hence $\dfrac{AP}{BP} = \dfrac{PC}{PD}$

Hence $AP \times PD = BP \times PC$

Example question



Not drawn accurately

AB and *QP* are two chords in a circle that intersect at *X*.

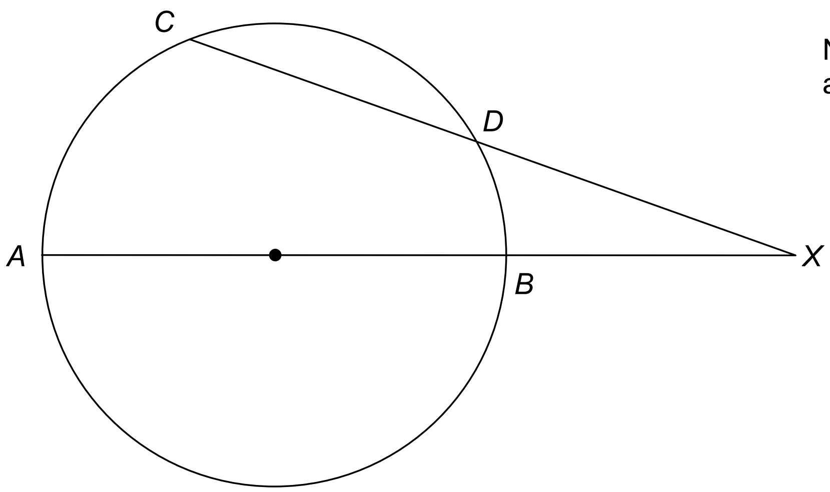
$XP = 5$ cm, $QX = 8$ cm and $XB = 10$ cm.

Work out the length *AX*.

(2 marks)

Example question

AB is the diameter and *CD* a chord of a circle.
AB and *CD* are extended to meet at *X*.
AX is 20 cm, *CD* is 7 cm and *DX* is 8 cm.



Not drawn accurately

Calculate *AB*, the diameter of the circle.

(3 marks)

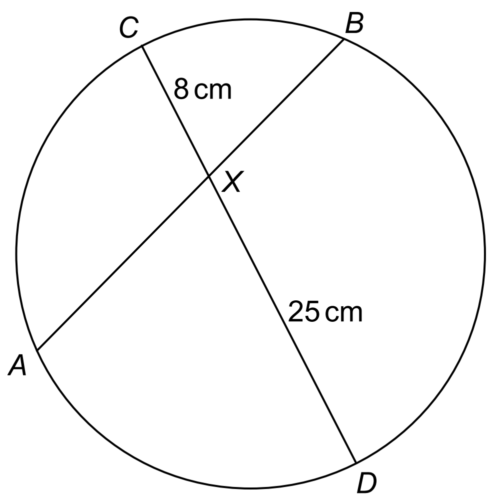
Example question

AB and *CD* are two chords of a circle that intersect at *X*.

CX = 8 cm, *DX* = 25 cm.

AX = 2 × *BX*

Work out the length of *AX*.



Not drawn accurately

Let *BX* = x

$2x \times x = 8 \times 25$

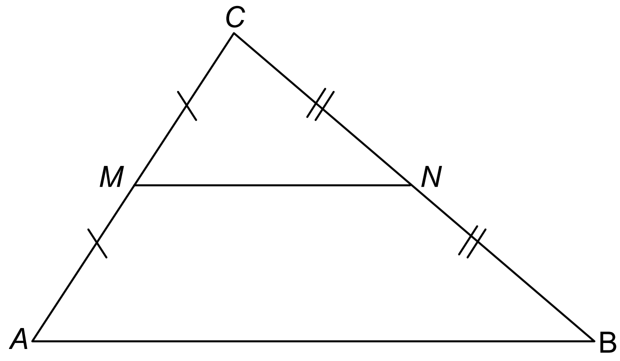
$x = 10$

AX = 20 cm

(3 marks)

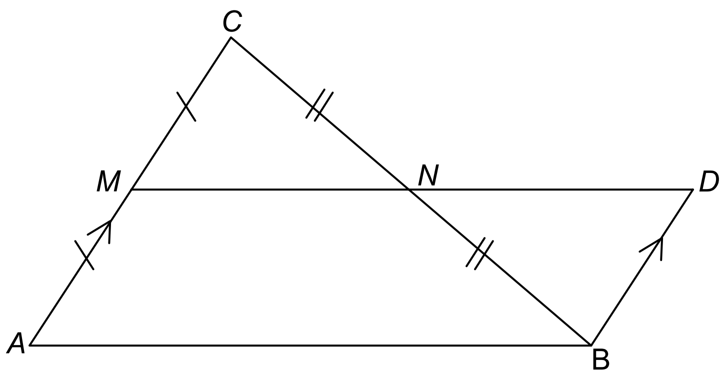
Midpoint theorem

(a) The line joining the midpoints of any two sides of a triangle is parallel to the third side of the triangle and equal to half its length.



Proof

Draw a line from B parallel to AC. Extend MN to a point D that is on the line from B.



Angle CNM = Angle BND (Vertically opposite angles)

Angle MCN = Angle NBD (Alternate angles)

$CN = NB$ (Given)

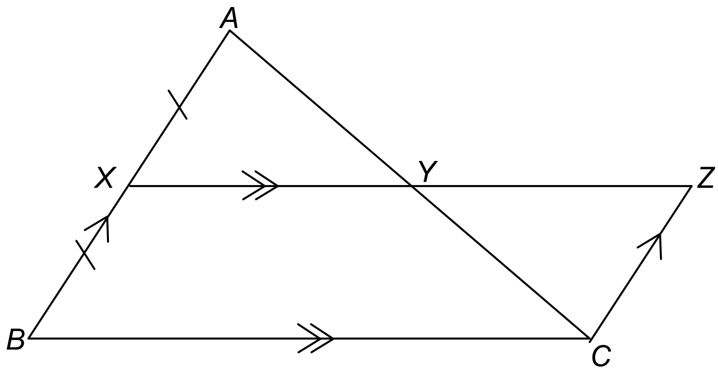
Hence triangles MNC and NDB are congruent. (ASA)

Therefore $MN = ND$ and $CM = BD = AM$

Hence $MDBA$ is a parallelogram so MN is parallel to AB

$MD = AB$, hence $MN = \frac{1}{2} AB$

(b) The line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side.



Proof

Produce *XY* so that *CZ* is parallel to *BX.*

BCZX is a parallelogram (2 sides parallel)

BX = *CZ* = *AX*

Angle *YCZ* = Angle *YAX* (Alternate angles)

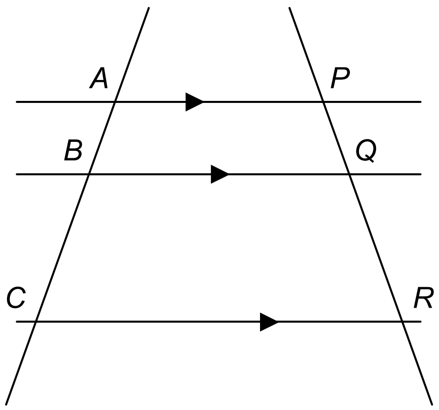
Angle *CYZ* = Angle *AYX* (Vertically opposite angles)

Hence triangles *CYZ* and *AYX* are congruent. (AAS)

Therefore *CY* = *AY*

Intercept theorem

If three or more parallel straight lines make intercepts on one transversal, they will make intercepts on any other transversal so that the ratios of lengths on the transversals are equal.



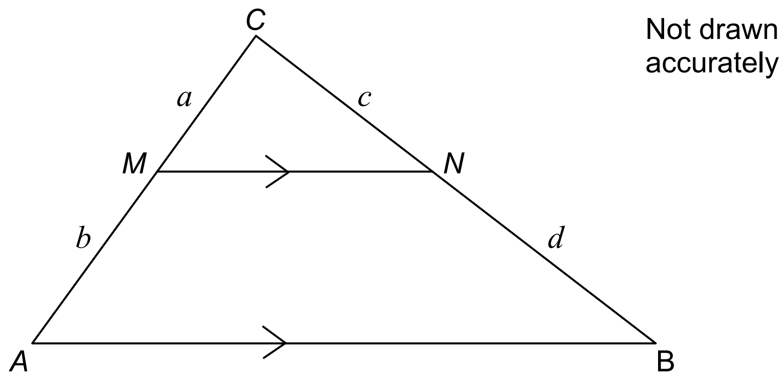
Proof

Draw a transversal parallel to *PR* so that it intersects *AB*.

This gives lots of similar triangles which can be used to show that *AB* : *BC* = PQ : QR

Corollary - Ratio theorem

A line *MN* drawn parallel to the side *AB* of the triangle *ABC* divides the sides *AC* and *BC* such that *AM* : *MC* = *BN* : *NC*



Proof

CMN and *CAB* are similar triangles so $\frac{AC}{MC} = \frac{BC}{NC}$

Hence $\frac{a+b}{a} = \frac{c+d}{c}$

$$1 + \frac{b}{a} = 1 + \frac{d}{c}$$

$$\frac{b}{a} = \frac{d}{c}$$

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Exemplar Material

Calculations relating to enterprise, saving a borrowing, appreciation and depreciation.

Mr Jones buys a new car for £ 18 245 in June 2004.
He sells it for £ 8500 in June 2009.

Here is a formula to work out the annual depreciation.

$$\text{Annual depreciation} = \frac{\text{Original price (£)} - \text{Final price (£)}}{\text{Number of years}}$$

(a) Use the formula to work out the annual depreciation of the car.
Give your answer to the nearest £ 10.

(3 marks)

(b) Estimate the value of the car in June 2010.

(2 marks)

Personal and domestic finance, RPI, CPI exchange rates etc

Terri goes on holiday to Germany.
She buys some euros for £500.
On her holiday, Terri spends 484 Euros.
When she arrives back in Britain she sells the remaining euros for pounds.

When she buys euros	£ 1 = 1.14 Euros
When she sells euros	£ 1 = 1.18 Euros

How much money, in pounds, does she get back after her holiday?

(4 marks)

Here is a formula for working out the Annual Equivalent Rate of interest (AER).

$$AER = 100\left(\left(1+\frac{r}{100n}\right)^{n}-1\right)$$

r is the rate of interest used.
n is the number of times each year that it is worked out.

The Cardiff Building Society work out interest every six months using an interest rate of 5%.

The Dover Building Society work out interest daily (n = 365) using an interest rate of 4.6%.

Compare the AER of these Building Societies, assuming an investment for a year.

(4 marks)

Spreadsheets in context of finance and business

This spreadsheet gives details of the weights of chocolate and packaging in two Easter Eggs.

	A	B	C	D	E
1	**Easter Egg**	**Weight of Chocolate (g)**	**Weight of packaging (g)**	**Total weight of Easter Egg (g)**	**% of chocolate in Easter Egg by weight**
2	Chokky	340	170	510	66.7
3	Dairy Crisp	575	240		

(a) Tom writes formulae to complete the spreadsheet.

This is the formula he writes for column D row 2 $= B2 + C2$

What formula does he write for column D row 3?

(1 mark)

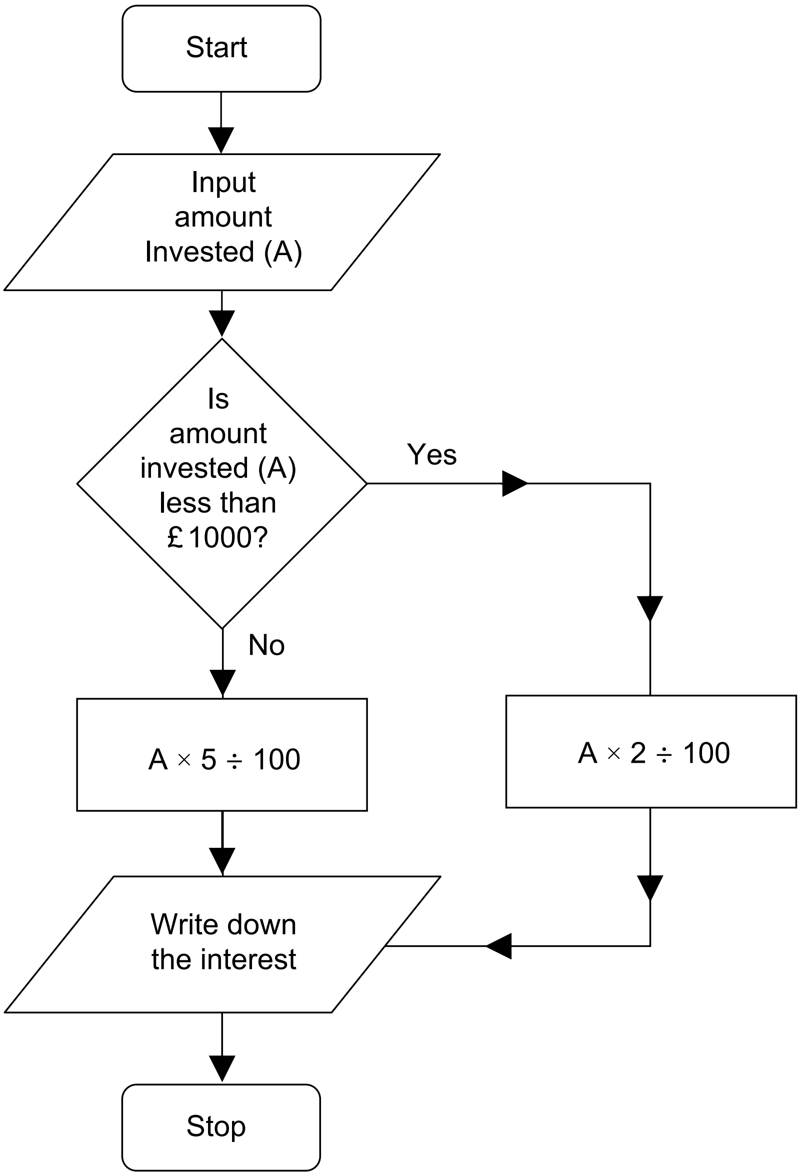
(b) His formula for column E row 2 is $= B2 \div D2 \times 100$

Use this information to complete the spreadsheet.

(2 marks)

Flow charts in context of finance and business

Here is a flowchart for working out the interest paid on a one year's savings bond.



Work out the interest when £1500 is invested.

(2 marks)

Linear programming

A school asks a bus company to transport some students on a trip.

L is the number of large buses used.

S is the number of small buses used.

The company has 4 large buses and 5 small buses available.

Two inequalities that represent the number of buses that the company can use to transport the students are $L \le 4$ and $S \le 5$

These inequalities are represented on the grid opposite.

(a) The company has a maximum of 7 drivers.
The large bus can transport 25 students.
The small bus can transport 15 students.
The company has been asked to transport a total of 90 students.

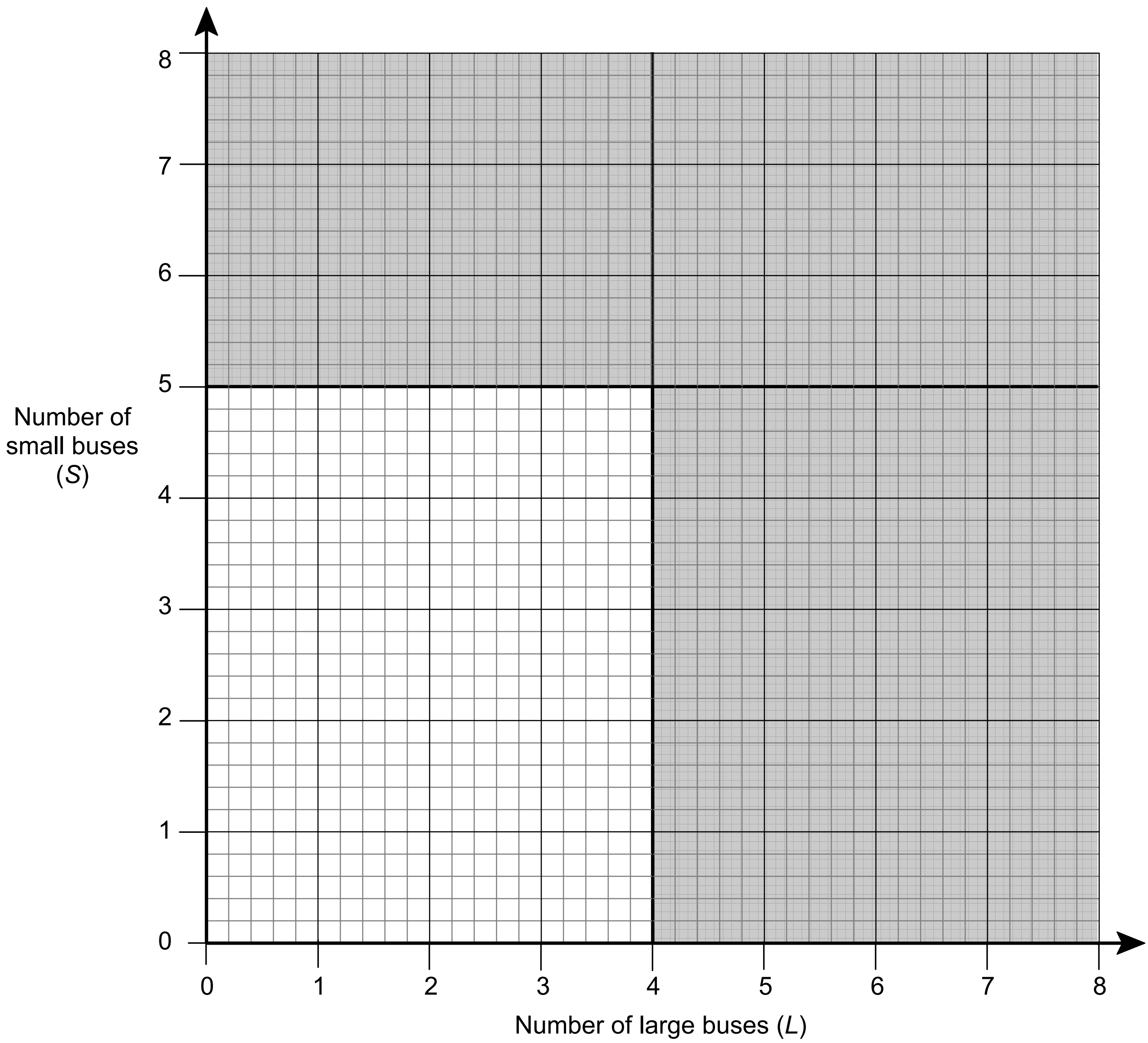
Write down **two** inequalities that fit these conditions and represent them on the graph.

(5 marks)

(b) The company charge £250 for each large bus and £100 for each small bus.

What is the cheapest way the bus company can transport the 90 students?

(2 marks)


Number of small buses (S)
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
Number of large buses (L)

Moving averages

The table shows Steve's electricity bills from March 2008 to June 2009.

The entry for December 2008 is missing.

Date	March 2008	June 2008	Sept 2008	Dec 2008	March 2009	June 2009
Amount (£)	34.70	23.80	19.40	....	37.60	27.00

(a) The value of the first four-point moving average is £28.50.

Calculate Steve's electricity bill for December 2008.

(2 marks)

(b) Will the second four-point moving average be greater or less than £28.50?

Give a reason for your answer

(1 mark)

Discuss and start to estimate risk

Example question

Beryl lives in Manchester.

Once a week she needs to fly to Belfast for her job.

She is going to fly to Belfast every Friday for 20 weeks.

If her flight is late she is paid £50 less for the day.

Four airlines agree to give her a bulk deal if she books all 20 flights with them.

The table shows the charges and the probability that each airline's morning flights are late.

Airline	**Cost per flight**	**Probability of being late**
Fly Q	£50	0.5
Jeteasy	£52.50	0.4
Jet4	£60	0.2
Baby Jet	£40	0.9

Which airline should Beryl choose to fly with?
Show all working to justify your answer.

(3 marks)

Mark Scheme

Works out the cost and penalty for any one airline eg, Fly Q $20 \times 50 + 10 \times 50 =$ £1500	M1
Calculates at least 3 correctly eg, Fly Q £1500, Jeteasy £1450, Jet4 £1400, Baby Jet £1700	M1
Arrives at conclusion with all working shown	A1

Recognise graphs that represent direct and inverse proportion

Example question

x, y and z are all positive numbers.

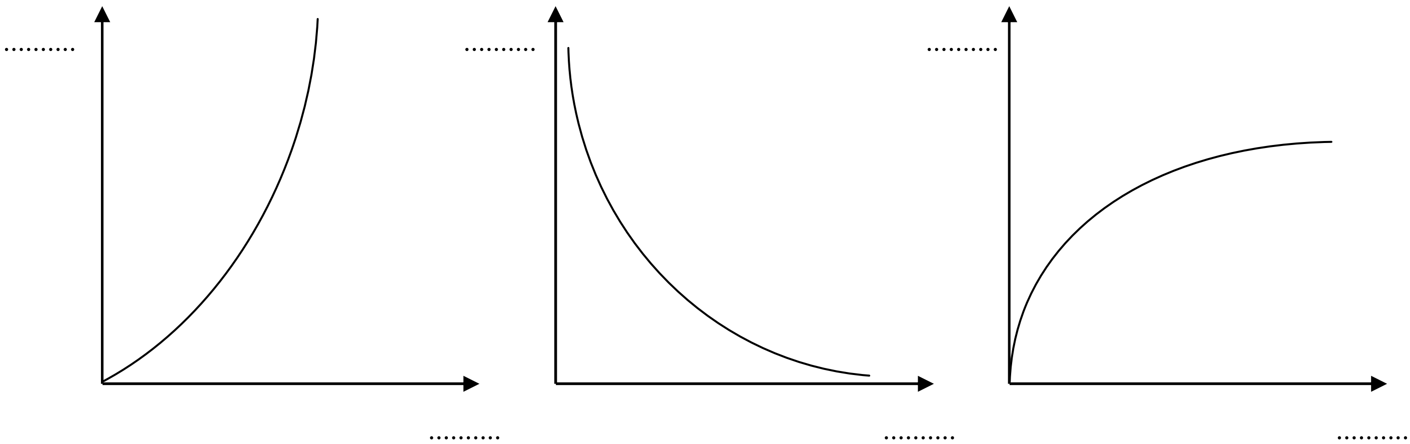
y is inversely proportional to x.

z is directly proportional to y^3.

z is directly proportional to the square root of x.

The three graphs represent these relationships.

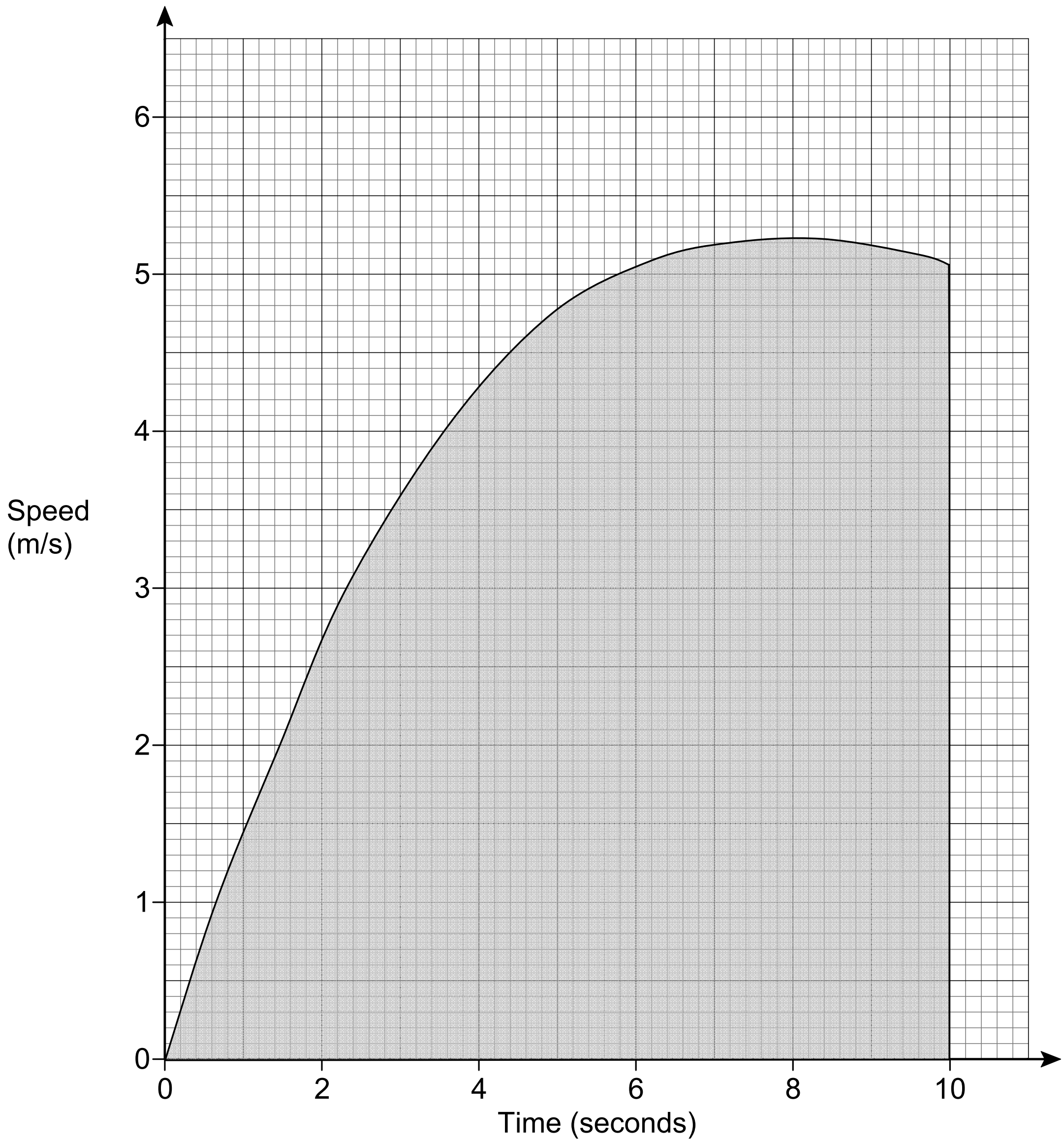
Label the axes of each graph with x, y or z as appropriate.



(3 marks)

Gradient at a point on a curve.
Areas under curves and travel graphs and interpret the result

The graph shows the speed of a runner during the first 10 seconds of a race.



(a) (i) Use an appropriate estimation method to calculate the shaded area on the graph.

(4 marks)

(a) (ii) What does this area represent?

(1 mark)

(b) (i) Estimate the gradient of the graph 5 seconds after the start of the race.

(3 marks)

(b) (ii) What does this gradient represent?

(1 mark)