Rewarding Learning

General Certificate of Secondary Education
2014

Additional Mathematics

Paper 1
Pure Mathematics

[G0301]

MONDAY 9 JUNE, MORNING

MARK SCHEME

General Marking Instructions

Introduction

Mark schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of students in schools and colleges.

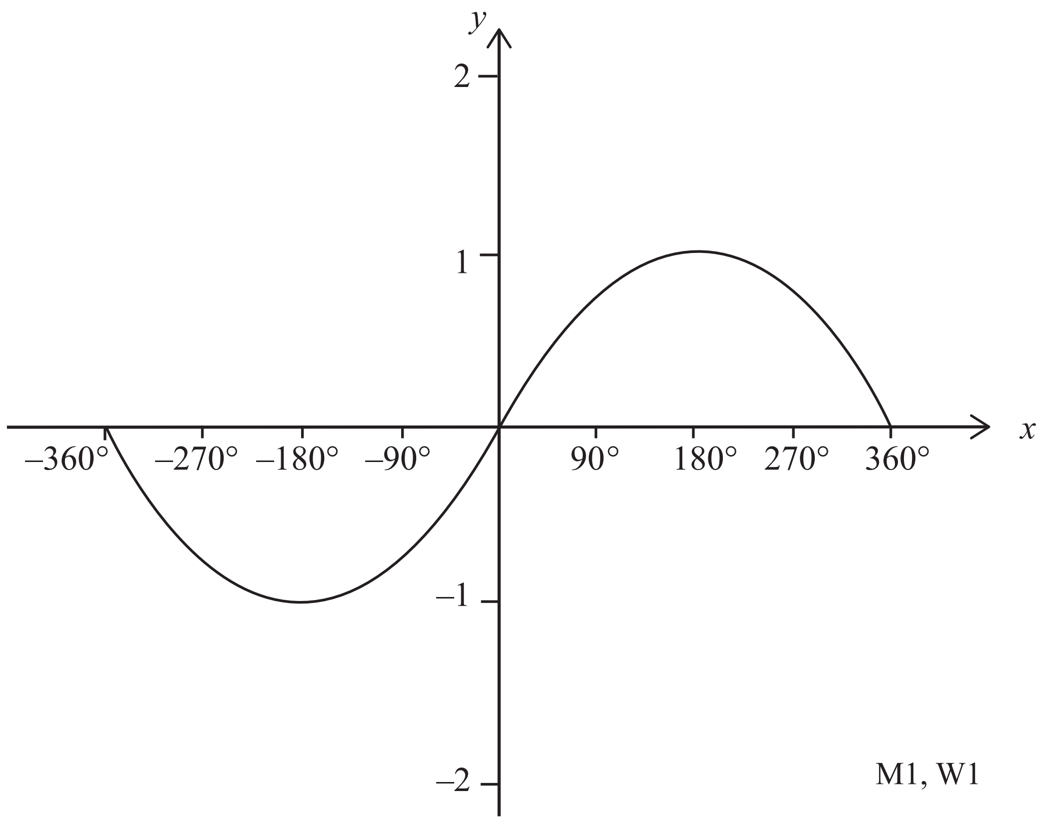
The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes, therefore, are regarded as part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

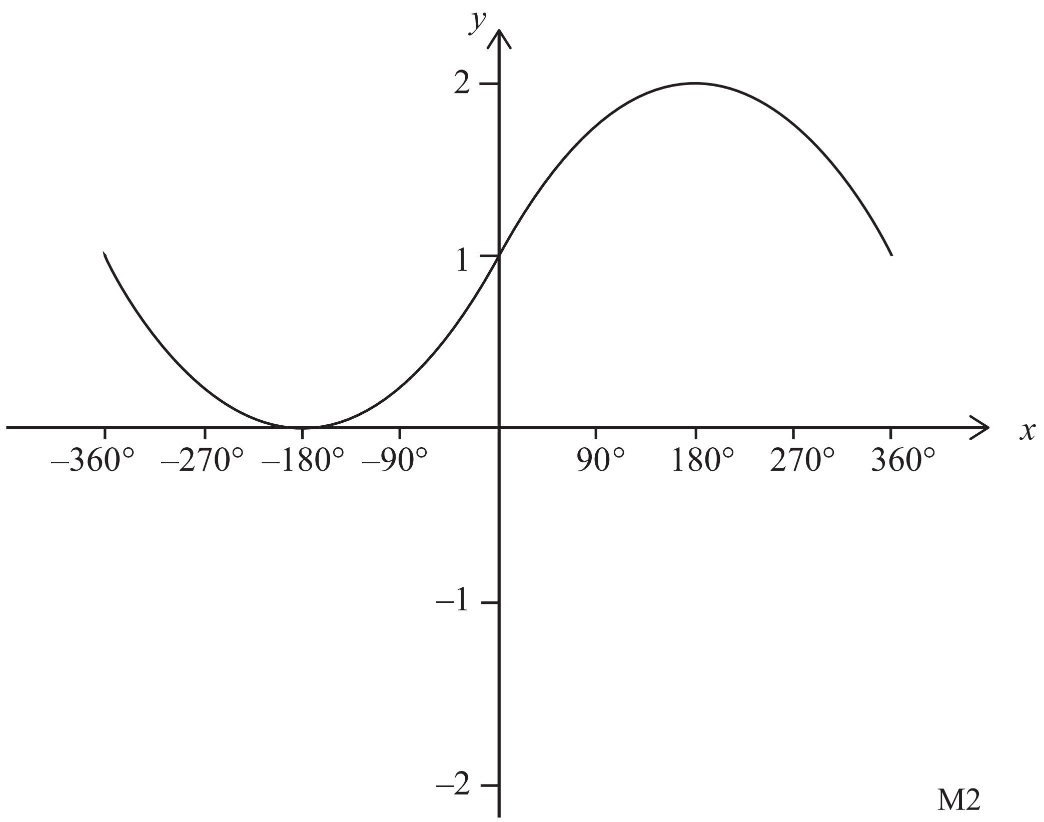
It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

1 (i)



M1, W1

(ii)



M2 4

AVAILABLE MARKS

				AVAILABLE MARKS
2	**(i)**	$\tan\theta = 0.6$		
		$\therefore\ \theta = 30.96°$ **or** $-149.04°$ (31° **or** –149° to nearest degree)	MW1, MW1	
	(ii)	$\tan(\frac{1}{2}x - 60°) = 0.6$		
		From **(i)** $\frac{1}{2}x - 60° = 30.96°$ **or** $-149.04°$	M1	
		$\therefore\ x = 181.92°$ **or** $-178.08°$	W1, W1	
		(182° **or** –178° to nearest degree)		5
3	**(i)**	$\mathbf{A} = \begin{bmatrix} 4 & -3 \\ 2 & -9 \end{bmatrix}$		
		$\therefore\ \mathbf{A}^{-1} = -\frac{1}{30}\begin{bmatrix} -9 & 3 \\ -2 & 4 \end{bmatrix} = \frac{1}{30}\begin{bmatrix} 9 & -3 \\ 2 & -4 \end{bmatrix}$	MW1, MW1	
	(ii)	$\begin{bmatrix} 4 & -3 \\ 2 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$	M1	
		$\therefore\ \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 2 & -9 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ 8 \end{bmatrix}$		
		$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{30}\begin{bmatrix} 9 & -3 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 8 \end{bmatrix}$	M2	
		$= \frac{1}{30}\begin{bmatrix} 30 \\ -20 \end{bmatrix}$		
		$= \begin{bmatrix} 1 \\ -\frac{2}{3} \end{bmatrix}$	W1	
		$\therefore\ x = 1, \quad y = -\frac{2}{3}$		6

		AVAILABLE MARKS
4 **(a)** $y = 4x^3 - \frac{3}{4}x^2 + \frac{4}{3}x^{-2}$		
$\frac{dy}{dx} = 12x^2 - \frac{6}{4}x - \frac{8}{3}x^{-3}$		
or $12x^2 - \frac{3}{2}x - \frac{8}{3x^3}$	3 × MW1	
(b) $\int(4x - 2x^{-3})\,dx$		
$= 2x^2 + x^{-2} + c$		
or $2x^2 + \frac{1}{x^2} + c$	3 × MW1	6
5 **(i)** $y = x^3 - 3x^2 - 18x$		
$\frac{dy}{dx} = 3x^2 - 6x - 18$	MW1	
At $x = 4$, $\frac{dy}{dx} = 6$	MW1	
For line l, $y = 6x - 2$		
So gradient of $l = 6$		
So tangent is parallel to l as gradients are equal.	MW1	
(ii) $\frac{dy}{dx} = 6$		
So $3x^2 - 6x - 18 = 6$	M1	
$3x^2 - 6x - 24 = 0$		
$x^2 - 2x - 8 = 0$	MW1	
$(x - 4)(x + 2) = 0$		
$\therefore x = 4$ **or** $x = -2$	W1	
When $x = -2$, $y = (-2)^3 - 3(-2)^2 - 18(-2)$ $= 16$		
So coordinates of the other point are (–2, 16)	W1	7

			AVAILABLE MARKS
6	**(i)** $\dfrac{3x+8}{x+3} - \dfrac{6x}{2x+1}$		
	$= \dfrac{(3x+8)(2x+1) - 6x(x+3)}{(x+3)(2x+1)}$		
	$= \dfrac{(6x^2+19x+8) - (6x^2+18x)}{2x^2+7x+3}$	MW1, MW1	
	$= \dfrac{x+8}{2x^2+7x+3}$	MW1, MW1	
	(ii) $\dfrac{3x+8}{x+3} - \dfrac{6x}{2x+1} = -2$		
	$\therefore \dfrac{x+8}{2x^2+7x+3} = -2$		
	$\therefore x + 8 = -2(2x^2 + 7x + 3)$	M2	
	$\therefore x + 8 = -4x^2 - 14x - 6$		
	$\therefore 4x^2 + 15x + 14 = 0$	W1	
	$\therefore (4x + 7)(x + 2) = 0$		
	$\therefore x = -\dfrac{7}{4}$ **or** $x = -2$	MW1	8

AVAILABLE MARKS

7 **(a)** $3^{2-3x} = 4$

$\therefore \log 3^{2-3x} = \log 4$ M1

$\therefore (2-3x)\log 3 = \log 4$ M1

$\therefore 2-3x = \dfrac{\log 4}{\log 3}$ MW1

$\therefore x = \dfrac{1}{3}\left(2 - \dfrac{\log 4}{\log 3}\right)$

$\therefore x = 0.246$ W1

Alternative solution:

$2-3x = \log_3 4$ M3

$= 1.2619$

$x = \dfrac{2 - 1.2619}{3}$

$x = 0.246$ W1

(b) $\log_{2x}36 = 2$

$\therefore (2x)^2 = 36$ M1

$\therefore 4x^2 = 36$

$\therefore x^2 = 9$

$\therefore x = 3$ W1

(c) **(i)** $\log_3 20 = \log_3(4 \times 5)$

$= \log_3 4 + \log_3 5$

$= m + n$ MW1

(ii) $\log_3 60 = \log_3(20 \times 3)$

$= \log_3 20 + \log_3 3$ M1

$= m + n + 1$ W1 9

			AVAILABLE MARKS
8	**(i)** $\hat{CAD} = 180° - 84° - 71° = 25°$	MW1	
	(ii) $CD^2 = CE^2 + DE^2 = 20^2 + 25^2$	M1	
	$CD = 32.016\,m \rightarrow 32.02\,m$	W1	
	(iii) $\dfrac{AD}{\sin A\hat{C}D} = \dfrac{CD}{\sin C\hat{A}D}$		
	$\therefore\ AD = \dfrac{32.016 \sin 84°}{\sin 25°}$	M2	
	$= 75.341\,m \rightarrow 75.34\,m$	W1	
	(iv) $AX^2 = AD^2 + XD^2 - 2\ .\ AD.\ XD \cos\ A\hat{D}X$		
	$= 75.341^2 + 73.25^2 - 2 \times 75.341 \times 73.25 \cos 34°$	M2	
	$\therefore\ AX = 43.493\,m \rightarrow\ 43.49\,m$	W1	
	(v) 43.493 m in 150 s		
	$\therefore$ speed $= \dfrac{43.493}{150} = 0.29$ m/s	M1, W1	11

		AVAILABLE MARKS

9 **(i)** $\log M = q \log H + \log p$ — M1

$\log H$	$\log M$
1.301	2.267
1.477	2.346
1.699	2.446
1.903	2.538
2.093	2.623

M1 (taking logs)

W1

Straight line graph, so relationship holds (see next page) — W1 (labels)
W1 (accurate points)
W1 (straight line)

(ii) $q = \dfrac{2.623 - 2.267}{2.093 - 1.301} = 0.45$ (to 2 s.f.) — M1, W1

$M = p\,H^{0.45}$

$185 = p\ 20^{0.45}$

$p = \dfrac{185}{20^{0.45}} = 48$ (to 2 s.f.) — M1, W1

(iii) $M = 48\,(64)^{0.45}$

$= 312\,\text{g}$ — MW1

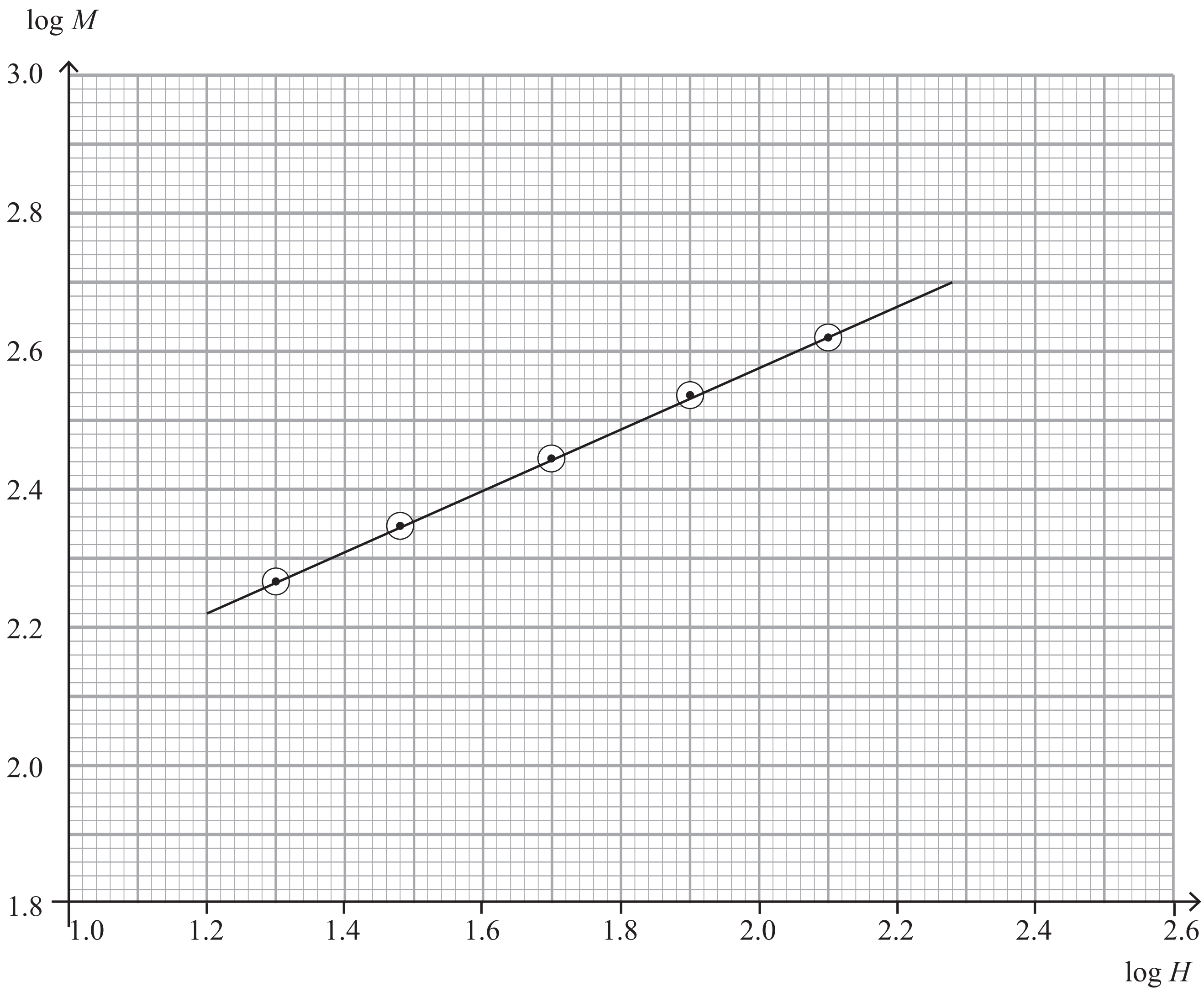
(iv) $500 = 48H^{0.45}$

$H^{0.45} = \dfrac{500}{48}$

$H = \left(\dfrac{500}{48}\right)^{\frac{1}{0.45}}$ — MW1

$= 183\,\text{cm}$ — W1

Assume relationship holds outside given values. — M1 — 14


log M
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
log H

			AVAILABLE MARKS
10	**(i)** $150x + 100y + 50z = 2250$ $\therefore\ 15x + 10y + 5z = 225$ $\therefore\ 3x + 2y + z = 45$ ①	MW1	
	(ii) $195x + 170y + 75z = 3195$ $\therefore\ 39x + 34y + 15z = 639$ ②	MW1	
	(iii) $150x + 75y + 60z = 2130$ $\therefore\ 10x + 5y + 4z = 142$ ③	MW1	
	(iv) ① × 15 – ② → $6x - 4y = 36$ ④	M1, W1	
	① × 4 – ③ → $2x + 3y = 38$ ⑤	M1, W1	
	⑤ × 3 – ④ → $13y = 78$	M1	
	$\therefore\ y = 6$	W1	
	From ⑤ $2x + 18 = 38$ $\therefore\ x = 10$	M1	
	From ① $30 + 12 + z = 45$ $\therefore\ z = 3$		
	So costs are (per bag) coal – £10, logs – £6, peat briquettes – £3	W1	
	(v) Coal is now £7.50 per bag and logs £4.50 per bag.	W1	
	Let n = number of bags of coal bought.		
	Then		
	$7.5n + 4.5 \times 75 + 3 \times 60 = 2130$	M1	
	$\therefore\ n = 215$		
	i.e. 215 bags of coal bought.	W1	14

11 **(i)** $y = x^3 - 5x^2 + 8x = x(x^2 - 5x + 8)$

AVAILABLE MARKS

When $x = 0$, $y = 0$
So curve passes through the origin. MW1

(ii) To cross x-axis again we must have

$$x^2 - 5x + 8 = 0$$ M1

i.e. $x = \dfrac{5 \pm \sqrt{25-32}}{2} = \dfrac{5 \pm \sqrt{-7}}{2}$

So no solution to this equation.
So doesn't cross x-axis at any other points. MW1

(iii) $y = x^3 - 5x^2 + 8x$

$\dfrac{dy}{dx} = 3x^2 - 10x + 8$ M1, W1

For turning points

$3x^2 - 10x + 8 = 0$ M1

$(3x - 4)(x - 2) = 0$

$\therefore\ x = \dfrac{4}{3}$ **or** $x = 2$

When $x = \dfrac{4}{3}$, $y = 4\dfrac{4}{27} = 4.15$

When $x = 2$, $y = 4$

So turning points at $\left(1\dfrac{1}{3}, 4\dfrac{4}{27}\right)$ and $(2, 4)$ W2

(iv) $\dfrac{d^2y}{dx^2} = 6x - 10$

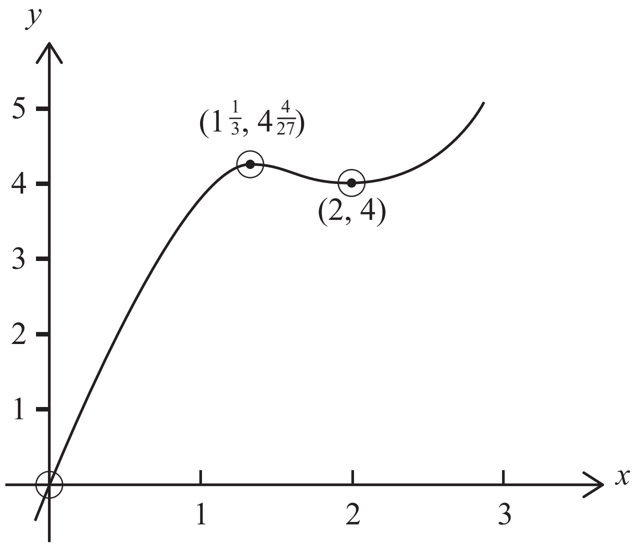
When $x = \dfrac{4}{3}$, $\dfrac{d^2y}{dx^2} = -2 < 0$

$\therefore$ maximum at $\left(1\dfrac{1}{3}, 4\dfrac{4}{27}\right)$

When $x = 2$, $\dfrac{d^2y}{dx^2} = 2 > 0$ M1, W1

$\therefore$ minimum at $(2, 4)$

(v)



M1 (shape)

M1 (positions of points)

(vi) Minimum is at (2, 4), so line is $x = 2$

area $= \int_0^2 (x^3 - 5x^2 + 8x)\,dx$ M1 (integral) MW1 (limits)

$= \left[\dfrac{x^4}{4} - \dfrac{5}{3}x^3 + 4x^2\right]_0^2$ MW1

$= 6\dfrac{2}{3}$ W1

	AVAILABLE MARKS
	16
Total	**100**