Rewarding Learning

General Certificate of Secondary Education
2012

Additional Mathematics

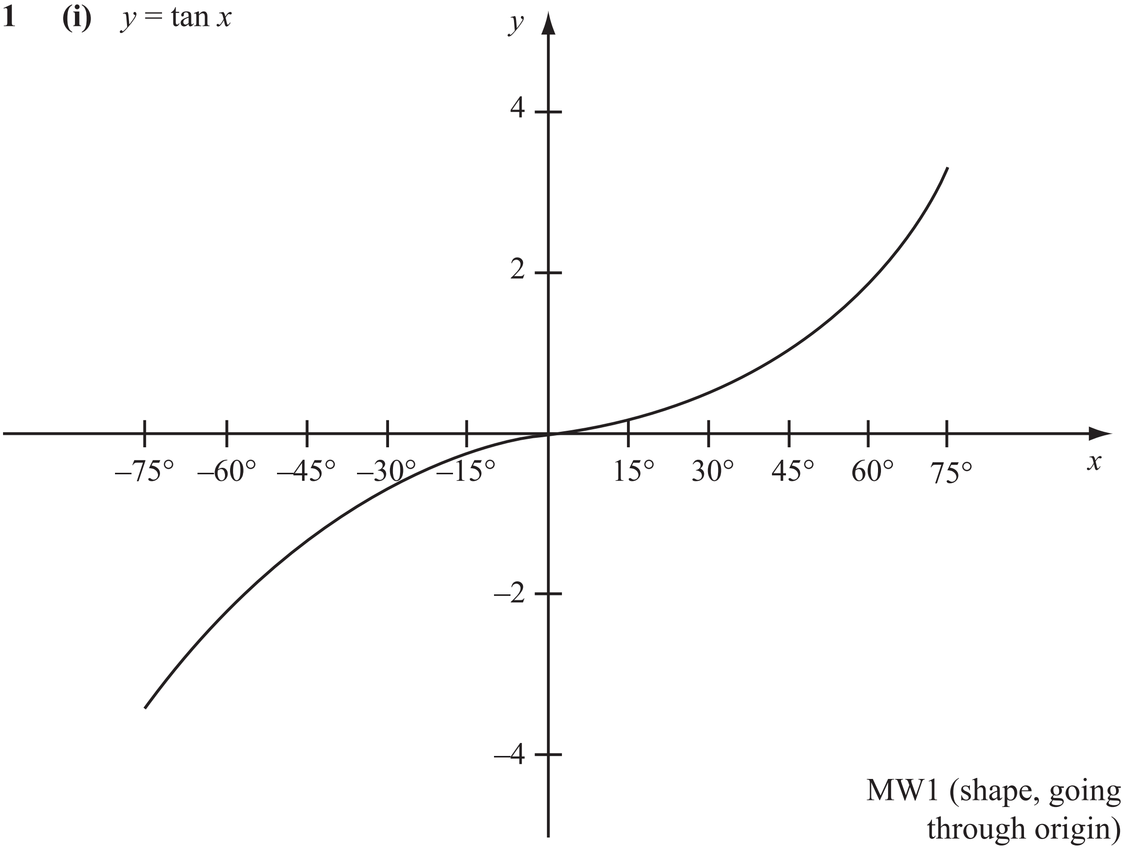
Paper 1
Pure Mathematics

[G0301]

MONDAY 28 MAY, MORNING

MARK SCHEME

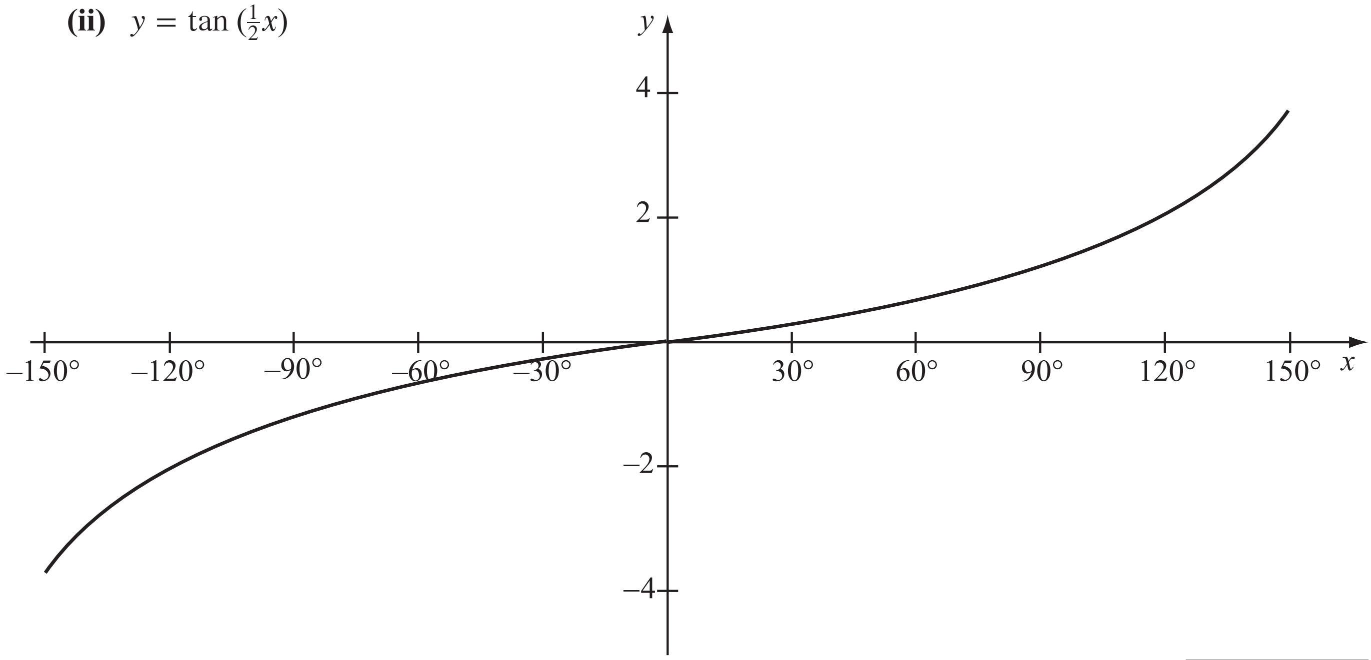
1 (i) $y = \tan x$



MW1 (shape, going through origin)

MW1 (accuracy)

(ii) $y = \tan(\frac{1}{2}x)$



MW1 (shape, going through origin)

MW1 (accuracy)

AVAILABLE MARKS

4

			AVAILABLE MARKS
2 **(i)**	$\sin\theta = -0.4$		
	$\therefore\ \theta = 203.58°$ or $336.42°$	MW1, MW1	
	(204° **or** 336° to nearest degree)		
(ii)	$\sin(2x + 60°) = -0.4$		
	From **(i)**		
	$2x + 60° = 203.58°$ **or** $336.42°$	M1	
	$\therefore\ x = 71.79°$ **or** $138.21°$	W1, W1	
	(72° **or** 138° to nearest degree)		5
3 **(i)**	$A = \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}$		
	$\therefore A^{-1} = -\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$	MW1, MW1	
(ii)	$5x - 4y = -10$		
	$-3x + 2y = 4$		
	$\therefore \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 4 \end{bmatrix}$	M1	
	$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}^{-1}\begin{bmatrix} -10 \\ 4 \end{bmatrix}$		
	$= -\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} -10 \\ 4 \end{bmatrix}$	M2	
	$= -\frac{1}{2}\begin{bmatrix} -4 \\ -10 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$		
	$\therefore x = 2,\ y = 5$	W1	6
4 **(a)**	$y = \frac{3}{2}x^6 - \frac{1}{2x^3} + 2$		
	$= \frac{3}{2}x^6 - \frac{1}{2}x^{-3} + 2$		
	$\frac{dy}{dx} = \frac{18}{2}x^5 - \frac{1}{2}(-3)x^{-4} + 0$	3 × MW1	
	$= 9x^5 + \frac{3}{2x^4}$		
(b)	$\int\left(7x^6 - \frac{3}{x^3}\right)dx$		
	$= \int\left(7x^6 - 3x^{-3}\right)dx$		
	$= \frac{7x^7}{7} - \frac{3x^{-2}}{(-2)} + c$	3 × MW1	
	$= x^7 + \frac{3}{2x^2} + c$		6

			AVAILABLE MARKS
5	**(i)** $y = 6x^3 - 2x^4$		
	$\therefore \dfrac{dy}{dx} = 18x^2 - 8x^3$	MW1	
	At $x = 1, \dfrac{dy}{dx} = 18 - 8 = 10$	MW1	
	$\therefore y = 10x + c$ At $x = 1, y = 4,$ $\therefore 4 = 10 + c$ $\therefore c = -6$		
	$\therefore$ Eqn of tangent is $y = 10x - 6$	MW1	
	(ii) Lines meet when		
	$26x + 18 = 10x - 6$	M1	
	$\therefore 16x = -24$		
	$\therefore x = -\dfrac{3}{2}$	W1	
	When $x = -\dfrac{3}{2}, y = 10x - 6 = -21$	W1	
	So lines meet at P $\left(-\dfrac{3}{2}, -21\right)$		
	(iii) When $x = -\dfrac{3}{2}$ and $y = -21$		
	$y - 14x = -21 + 21 = 0$	MW1	
	So P lies on the line		
	$y - 14x = 0$		7

AVAILABLE MARKS

6 **(i)** $\frac{5x+1}{2x+3} - \frac{3x-7}{1-x}$

$= \frac{(5x+1)(1-x) - (3x-7)(2x+3)}{(2x+3)(1-x)}$ M2

$= \frac{(-5x^2+4x+1) - (6x^2-5x-21)}{-2x^2-x+3}$ W1

$\left.\begin{aligned} &= \frac{-11x^2+9x+22}{-2x^2-x+3} \\ &= \frac{11x^2-9x-22}{2x^2+x-3} \end{aligned}\right\}$ M1

(ii) $\frac{5x+1}{2x+3} - \frac{3x-7}{1-x} = 4$

$\therefore \frac{11x^2-9x-22}{2x^2+x-3} = 4$

$\therefore 11x^2 - 9x - 22 = 4(2x^2+x-3)$

$\therefore 11x^2 - 9x - 22 = 8x^2 + 4x - 12$ M2

$\therefore 3x^2 - 13x - 10 = 0$

$\therefore (3x+2)(x-5) = 0$ W1

$\therefore x = -\frac{2}{3}$ **or** $x = 5$ W1 8

7 **(a)** $8^{(4x-3)} = 50$

$\therefore \log 8^{(4x-3)} = \log 50$ M1

$\therefore (4x-3)\log 8 = \log 50$ M1

$\therefore 4x - 3 = \frac{\log 50}{\log 8}$ M1

$\therefore x = \frac{1}{4}\left(\frac{\log 50}{\log 8} + 3\right)$

$= 1.22$ (to 3 sig. figs.) (1.2 or greater accuracy) W1

(b) **(i)** $\log_{10}(10x^3)$

$= \log_{10} 10 + \log_{10} x^3$ MW1, MW1

$= 1 + 3\log_{10} x$

(ii) $\frac{1}{2}\log_{10} y = 1 + 3\log_{10} x$

$\therefore \log_{10} y^{\frac{1}{2}} = \log_{10}(10x^3)$

$\therefore y^{\frac{1}{2}} = 10x^3$ MW1

$\therefore y = 100x^6$ W1 8

	AVAILABLE MARKS

8 **(i)** $SX^2 = SR^2 + XR^2 - 2 \times SR \times XR \times \cos S\hat{R}X$

$= 9.5^2 + 4.5^2 - 2 \times 9.5 \times 4.5 \times \cos 60°$ M2

$\therefore$ SX = 8.23 km W1

(ii) $\dfrac{\sin X\hat{S}R}{XR} = \dfrac{\sin S\hat{R}X}{SX}$

$\therefore \sin X\hat{S}R = \dfrac{4.5 \sin 60°}{8.23}$ M1

$\therefore X\hat{S}R = 28.26°$ W1

(iii) $R\hat{X}Y = S\hat{R}X + X\hat{S}R = 60° + 28.26° = 88.26°$ W1

(iv) $YR^2 = XR^2 + XY^2 - 2 \times XR \times XY \times \cos R\hat{X}Y$ M1

$= 4.5^2 + 3.15^2 - 2 \times 4.5 \times 3.15 \times \cos 88.26°$

$\therefore$ YR = 5.41 km W1

(v) Time for train to reach X $= \dfrac{SX}{40} = \dfrac{8.23}{40} = 0.206\text{h}\ (12.35\text{m})$ MW1

Time for Frank to reach X $= \dfrac{XR}{20} = \dfrac{4.5}{20} = 0.225\text{h}\ (13.5\text{m})$ MW1

So Frank doesn't arrive in time.

Time for train to reach Y $= \dfrac{SY}{40} = \dfrac{8.23 + 3.15}{40} = 0.285\text{h}\ (17.1\text{m})$ MW1

Time for Jesse to reach Y $= \dfrac{YR}{20} = \dfrac{5.41}{20} = 0.271\text{h}\ (16.23\text{m})$ MW1

So Jesse does arrive in time to warn the driver. 12

9 **(i)** $\log C = q \log A + \log p$ M1

log A	log C
0.699	2.315
0.954	2.145
1.146	2.007
1.362	1.852
1.531	1.730

M1 (taking logs)
W1 (values correct to 3 d.p.)

Straight line graph (see next page) W1 (labels)
W1 (accurate points)
W1 (straight line of best fit)

(ii) $q = \dfrac{2.315 - 1.730}{0.699 - 1.531} = -0.70$ M1, W1

$C = pA^q$

$206.54 = p5^{-0.7}$ M1

$\therefore p = 640$ (to 2 s.f.) W1

i.e. $C = 640A^{-0.7}$

(iii) When $A = 12$, $C = 640 \times 12^{-0.7} = £112.40$ MW1

AVAILABLE MARKS

(iv) Value when Carly sells $= £112.40 \times \frac{1}{2} = £56.20$

When $C = 56.20$,

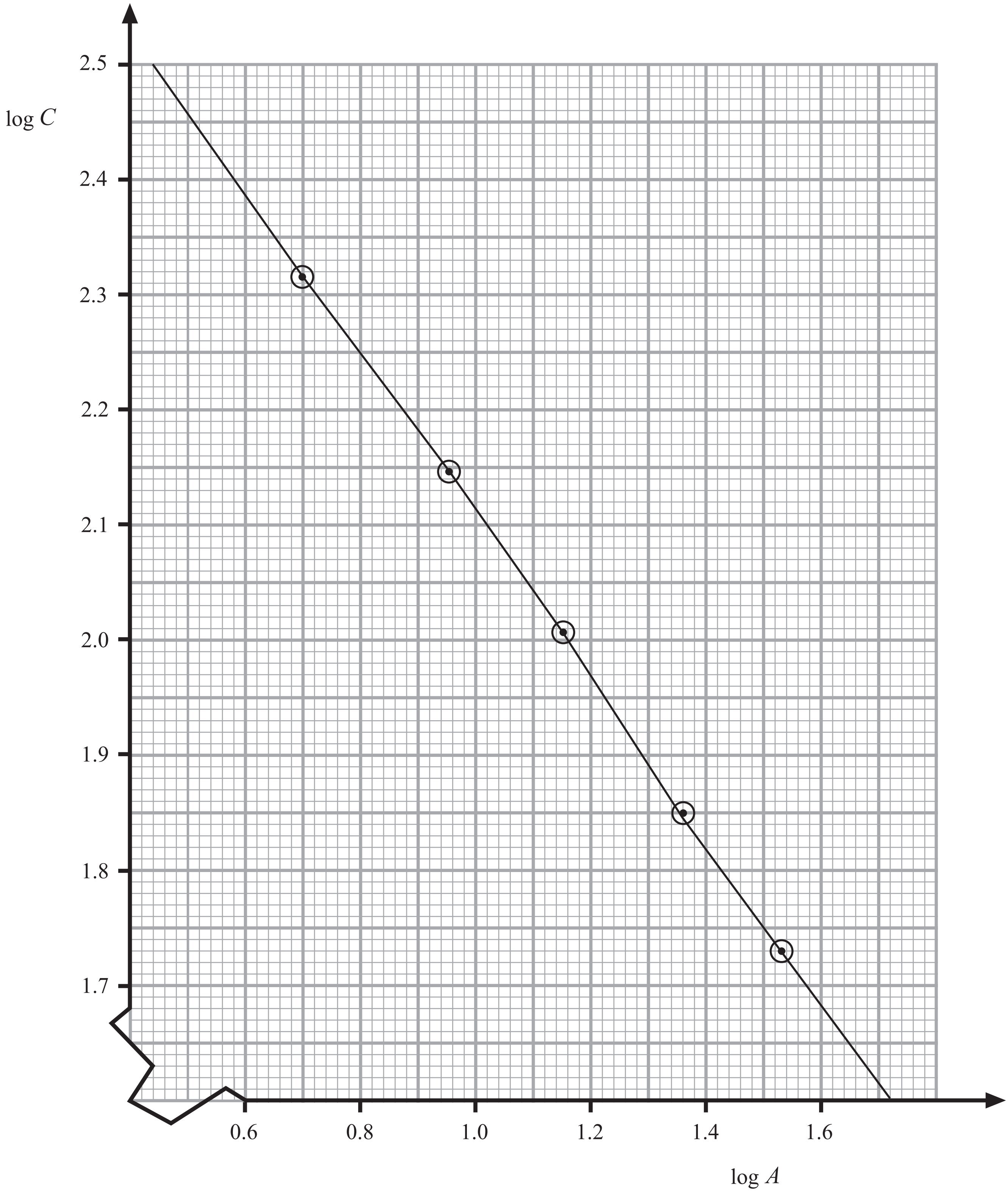
$$56.20 = 640A^{-0.7}$$ M1

$$\therefore A = \left(\frac{56.20}{640}\right)^{-\frac{1}{0.7}}$$

i.e. $A = 32.3$ W1

So Carly should keep her laptop for

$32.3 - 12 = 20.3$ months MW1 14



	AVAILABLE MARKS

10 **(i)** $\frac{5000x}{100} + \frac{3000y}{100} + \frac{2000z}{100} = 500$

$\therefore 50x + 30y + 20z = 500$

$\therefore 5x + 3y + 2z = 50$ MW1

(ii) $\frac{2000x}{100} + \frac{2000y}{100} + \frac{7000z}{100} = 860$

$\therefore 2x + 2y + 7z = 86$ MW1

(iii) $\frac{8000x}{100} + \frac{2000y}{100} + \frac{10\,000z}{100} = 1340$

$\therefore 8x + 2y + 10z = 134$

$\therefore 4x + y + 5z = 67$ MW1

(iv) $5x + 3y + 2z = 50$ ①

$2x + 2y + 7z = 86$ ②

$4x + y + 5z = 67$ ③

③ × 3 – ① → $7x + 13z = 151$ ④ M1, W1

③ × 2 – ② → $6x + 3z = 48$ ⑤ M1, W1

④ × 6 – ⑤ × 7 → $57z = 570$

$\therefore z = 10$ M1

From ⑤ $6x = 48 - 3z$

$\therefore x = 3$ M1

From ③ $y = 67 - 4x - 5z$

$\therefore y = 5$ W1

So low, medium and high risk accounts have interest rates of 3%, 5% and 10% respectively.

(v) $a \times \frac{5}{100} + b \times \frac{10}{100} = 800$ (or $0.05\,a + 0.1\,b = 800$) gets M1, W1

$\therefore a + 2b = 16\,000$ M1, W1

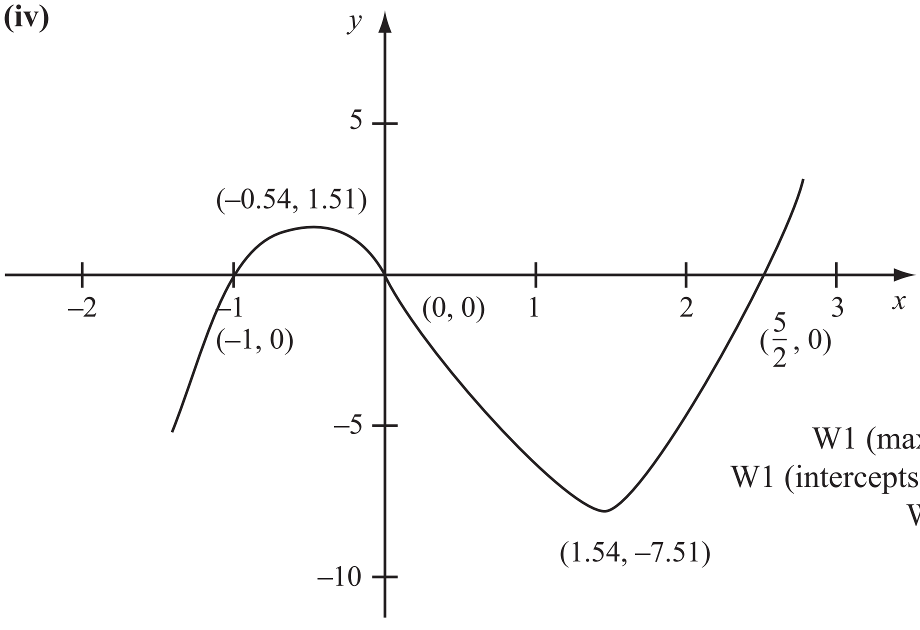
Also $a + b = 10\,000$ (total amount) MW1

(vi) Subtracting gives $b = 6000$, and so $a = 4000$

i.e. £4000 in medium risk account and

£6000 in high risk account MW1 14

			AVAILABLE MARKS
11	**(i)** $y = 2x^3 - 3x^2 - 5x = 0$	M1	
	$\therefore x(2x^2 - 3x - 5) = 0$		
	$\therefore x(2x - 5)(x + 1) = 0$	MW1	
	$\therefore x = 0$ **or** $\frac{5}{2}$ **or** -1	W1	
	So crosses x-axis at (0, 0), (–1, 0) and $\left(\frac{5}{2}, 0\right)$		
	(ii) $\frac{dy}{dx} = 6x^2 - 6x - 5$	MW1	
	At turning points, $6x^2 - 6x - 5 = 0$	M1	
	$\therefore x = \frac{6 \pm \sqrt{36 - 4(6)(-5)}}{12}$	M1	
	$\therefore x = 1.54$ **or** -0.54	W1	
	When $x = 1.54$, $y = -7.51$		
	When $x = -0.54$, $y = 1.51$	(y values) W1	
	So turning points at (1.54, –7.51) and (–0.54, 1.51)		
	(iii) $\frac{d^2y}{dx^2} = 12x - 6$		
	When $x = 1.54$, $\frac{d^2y}{dx^2} = 12.48 > 0$		
	$\therefore$ minimum at (1.54, –7.51)		
	When $x = -0.54$, $\frac{d^2y}{dx^2} = -12.48 < 0$	M1, W1	
	$\therefore$ maximum at (–0.54, 1.51)		
	(iv)		



W1 (max and min)
W1 (intercepts on x-axis)
W1 (shape)

			AVAILABLE MARKS
(v) Area	$= \int_{-1}^{0} (2x^3 - 3x^2 - 5x)\,dx$	M1	
	$= \left[\frac{1}{2}x^4 - x^3 - \frac{5}{2}x^2\right]_{-1}^{0}$	MW1	
	$= [0] - \left[\frac{1}{2} + 1 - \frac{5}{2}\right]$		
	$= 1$	W1	16
		Total	**100**

GCSE ADDITIONAL MATHS SUMMER 2012
PAPER 1
OVERLAY QUESTION 9