Rewarding Learning

General Certificate of Secondary Education
2011

Additional Mathematics

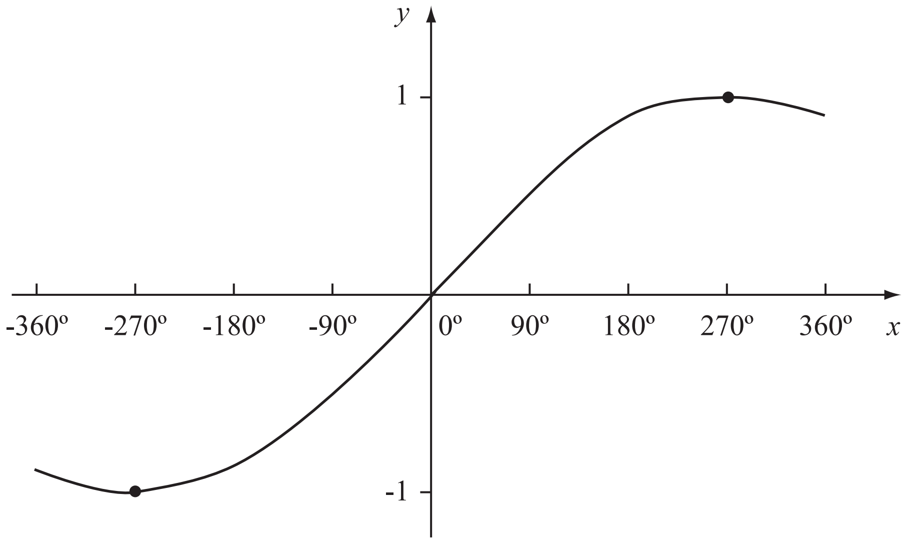
Paper 1
Pure Mathematics

[G0301]

TUESDAY 17 MAY, MORNING

MARK SCHEME

1



M1 sin between $\pm$ 1
M1 max/min
M1 values at $\pm$ 360º
W1 sketch — 4

2 **(i)** $\cos\theta = 0.1$

$\therefore \theta = 84.26^\circ$ or 275.74° — MW1, MW1

(84º or 276º to nearest degree)

(ii) $\cos\left(\frac{4}{5}x + 30^\circ\right) = 0.1$

From (i)

$\frac{4}{5}x + 30^\circ = 84.26^\circ$ or 275.74° — M1

$\therefore x = 67.83^\circ$ or 307.17° — W1, W1

(68º or 307º to nearest degree) — 5

AVAILABLE MARKS

		AVAILABLE MARKS
3 **(i)** $\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 1 & -6 \end{bmatrix}$		
$\therefore\ \mathbf{A}^{-1} = -\frac{1}{15}\begin{bmatrix} -6 & -3 \\ -1 & 2 \end{bmatrix}$	$2 \times$ MW1	
$= \frac{1}{15}\begin{bmatrix} 6 & 3 \\ 1 & -2 \end{bmatrix}$		
(ii) $2x + 3y = 7$, $x - 6y = 6$		
$\therefore\ \begin{bmatrix} 2 & 3 \\ 1 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$	M1	
$\therefore\ \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -6 \end{bmatrix}^{-1}\begin{bmatrix} 7 \\ 6 \end{bmatrix}$	M2	
$= \frac{1}{15}\begin{bmatrix} 6 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 6 \end{bmatrix}$		
$= \frac{1}{15}\begin{bmatrix} 60 \\ -5 \end{bmatrix} = \begin{bmatrix} 4 \\ -\frac{1}{3} \end{bmatrix}$		
$\therefore x = 4,\ y = -\frac{1}{3}$	W1	6
4 **(a)** $y = ax^3 - \frac{2}{x}$		
$\frac{dy}{dx} = 3ax^2 + 2x^{-2}$	$2 \times$ MW1	
$= 3ax^2 + \frac{2}{x^2}$		
(b) $\int (2x^5 + \frac{3}{4x^4} - 6)\,dx$		
$= \frac{1}{3}x^6 - \frac{1}{4}x^{-3} - 6x + c$	$4 \times$ MW1	
$= \frac{1}{3}x^6 - \frac{1}{4x^3} - 6x + c$		6

	AVAILABLE MARKS

5 **(i)** $y = \frac{12}{x^2}$

$\therefore \frac{dy}{dx} = -\frac{24}{x^3}$ MW1

When $x = 2, y = 3, m = \frac{dy}{dx} = -3$ MW1

$y = mx + c$

$\therefore 3 = (-3)(2) + c$ M1

$\therefore c = 9$

So equation of tangent is

$y = -3x + 9$ W1

(ii) When $y = 0, x = 3$ $\therefore$ A(3, 0) MW1

When $x = 0, y = 9$ $\therefore$ B(0, 9) MW1

(iii) area AOB = $\frac{1}{2} \times 3 \times 9 = 13.5$ MW1 7

6 **(i)** $\frac{4-3x}{2x-1} + \frac{1-3x}{1-x}$

$= \frac{(4-3x)(1-x)+(2x-1)(1-3x)}{(2x-1)(1-x)}$ M2

$= \frac{(3x^2-7x+4)+(-6x^2+5x-1)}{(-2x^2+3x-1)}$

$= \frac{-3x^2-2x+3}{-2x^2+3x-1}$ W1

$= \frac{3x^2+2x-3}{2x^2-3x+1}$ M1 for changing signs

(ii) $\frac{4-3x}{2x-1} + \frac{1-3x}{1-x} = 3$

$\therefore \frac{3x^2+2x-3}{2x^2-3x+1} = 3$

$\therefore 3x^2 + 2x - 3 = 3(2x^2 - 3x + 1)$ M2

$\therefore 3x^2 + 2x - 3 = 6x^2 - 9x + 3$

$\therefore 3x^2 - 11x + 6 = 0$ W1

$\therefore (3x - 2)(x - 3) = 0$

$\therefore x = \frac{2}{3}$ or $x = 3$ W1 8

	AVAILABLE MARKS

7 **(a)** $8^{(2-\frac{x}{3})} - 5 = 0$

$\therefore 8^{(2-\frac{x}{3})} = 5$

$\therefore \log 8^{(2-\frac{x}{3})} = \log 5$ M1

$\therefore (2 - \frac{x}{3}) \log 8 = \log 5$ M1

$\therefore 2 - \frac{x}{3} = \frac{\log 5}{\log 8}$ M1

$\therefore x = 3(2 - \frac{\log 5}{\log 8})$

$\therefore x = 3.678$ to 3 d.p. (or 3.68 to 3 s.f.) W1

(b) $\log_5 25 + \log_2 8$

$= 2 + 3$ MW1, MW1

$= 5$

(c) $\log_a 4 = \frac{1}{3}$

$\therefore a^{\frac{1}{3}} = 4$ M1

$\therefore a = 4^3$

$\therefore a = 64$ W1 8

AVAILABLE MARKS

8 **(i)** $\cos X\hat{Y}A = \dfrac{AY^2 + XY^2 - AX^2}{2 \times AY \times XY}$

$= \dfrac{3115^2 + 310^2 - 3045^2}{2 \times 3115 \times 310}$ M2

$\therefore X\hat{Y}A = 74.16^\circ$ W1

(ii) $S\hat{A}C = X\hat{Y}A = 74.16^\circ$ W1

(iii) $A\hat{S}C = 180^\circ - S\hat{A}C - A\hat{C}S$

$= 180^\circ - 74.16^\circ - 48.50^\circ = 57.34^\circ$ MW1

(iv) $\dfrac{AS}{\sin A\hat{C}S} = \dfrac{AC}{\sin A\hat{S}C}$

$\therefore AS = \dfrac{1000 \sin 48.50}{\sin 57.34} = 890 \text{ m}$ M1, W1

(v) $\dfrac{\sin X\hat{A}Y}{XY} = \dfrac{\sin X\hat{Y}A}{AX}$

$\therefore \sin X\hat{A}Y = \dfrac{310 \sin 74.16}{3045}$ M1

$\therefore X\hat{A}Y = 5.62^\circ$ W1

As this is greater than 5º he can't see the full length of the ship in his binoculars. M1

(vi) $\dfrac{BS}{XY} = \dfrac{AS}{AY}$

$\therefore BS = \dfrac{890 \times 310}{3115} = 89 \text{ m}$ M1, W1 12

9 **(i)** $\log A = n\log P + \log k$ M1

AVAILABLE MARKS

$\log P$	$\log A$
3.085	3.422
2.983	3.356
2.948	3.334
2.729	3.194
2.679	3.161

M1 for taking logs
W1 for all answers correct to 3 decimal places

Straight line graph (see graph page) W1 for labels
W1 for all points plotted accurately
W1 for straight line drawn through these points

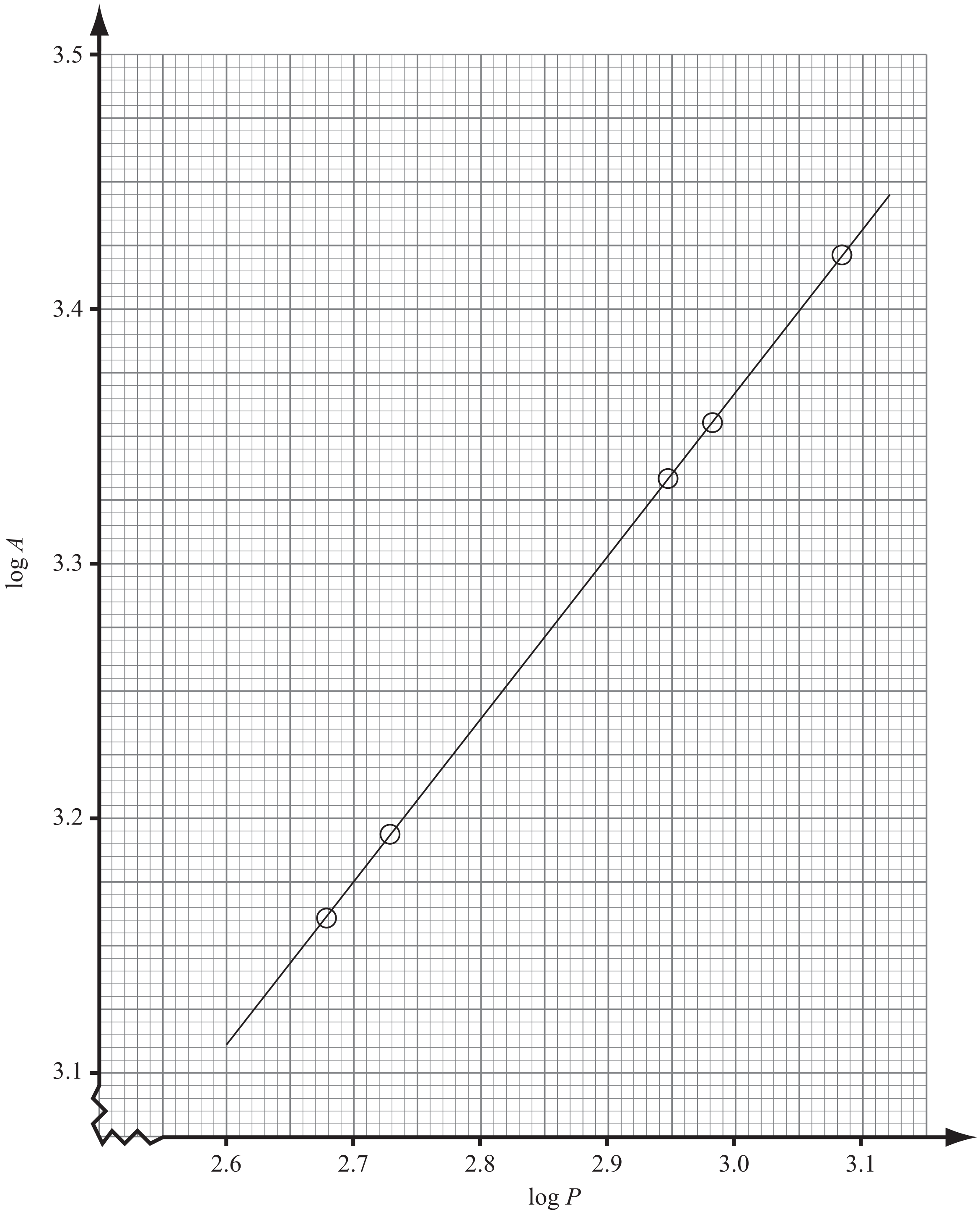
(ii) $n = \dfrac{3.422 - 3.161}{3.085 - 2.679} = 0.64$ M1, W1

$A = kP^{0.64}$
$2640 = k(1217^{0.64})$
$k = 27.99$ M1, W1

(iii) $A = 27.99\ (508^{0.64}) = 1509$ MW1

(iv) $27.99P^{0.64} = 2190$
$P^{0.64} = 78.24\ldots$ MW1
$P = \sqrt[0.64]{78.24\ldots}$ or $0.64\log P = \log 78.24\ldots$
$\log P = 2.9585\ldots$
$P = 908.87$ $P = 908.87$ MW1

(v) Attendance is not directly proportional to the profit MW1 14


3.5
3.4
3.3
3.2
3.1
log A
2.6
2.7
2.8
2.9
3.0
3.1
log P

		Marks	AVAILABLE MARKS
10 **(i)** $84x + 60y + 68z = 824$		M1	
$\therefore 21x + 15y + 17z = 206$	(1)	W1	
(ii) $7x + 11y + 6z = 100$	(2)	W1	
$3x + 9y + z = 58$	(3)	W1	
(iii) $3 \times (2) - (1) \rightarrow 18y + z = 94$	(4)	M1, W1	
$7 \times (3) - (1) \rightarrow 48y - 10z = 200$			
$\therefore 24y - 5z = 100$	(5)	M1, W1	
$5 \times (4) + (5) \rightarrow 114y = 570$		M1	
$\therefore y = 5$		W1	
From (4) $z = 94 - 18y$			
$\therefore z = 4$			
From (3) $x = \frac{1}{3}(58 - 9y - z)$		M1	
$\therefore x = 3$			
So amounts are: Cornflakes − 300 g			
Porridge − 500 g			
Crispies − 400 g		W1	
(iv) $12p + 9q = 120$	(6)		
$10p + 7.5q = 100$	(7)	MW1	
$10 \times (6) - 12 \times (7) \rightarrow 0p + 0q = 0$			
and so there is no unique solution		M1	14

11 **(i)** $-2x(3x + 1)(2x - 3) = 0$ M1

$x = 0$ or $-1/3$ or $3/2$

$(0,0)$ $(-1/3,0)$ $(3/2,0)$ W1

(ii) $y = -2x(3x + 1)(2x - 3)$

$y = -2x(6x^2 - 7x - 3)$

$= -12x^3 + 14x^2 + 6x$ MW1

$\frac{dy}{dx} = -36x^2 + 28x + 6$ MW1

$-36x^2 + 28x + 6 = 0$ M1

$2(-18x^2 + 14x + 3) = 0$

$-18x^2 + 14x + 3 = 0$

$x = \frac{-14 \pm \sqrt{196 + 216}}{-36} = \frac{-14 \pm \sqrt{412}}{-36}$ M1

$x = \frac{-14 \pm 20.298}{-36} = \frac{-14 + 20.298}{-36}$ or $\frac{-14 - 20.298}{-36}$

$x = -0.17$ or 0.95 W1

$y = -0.56$ or 8.05

$(-0.17, -0.56)$ and $(0.95, 8.05)$ M1, W1

(iii) $\frac{d^2y}{dx^2} = -72x + 28$

$x = -0.17$

$\frac{d^2y}{dx^2} = 40.24$ Minimum TP $(-0.17, -0.56)$

$x = 0.95$

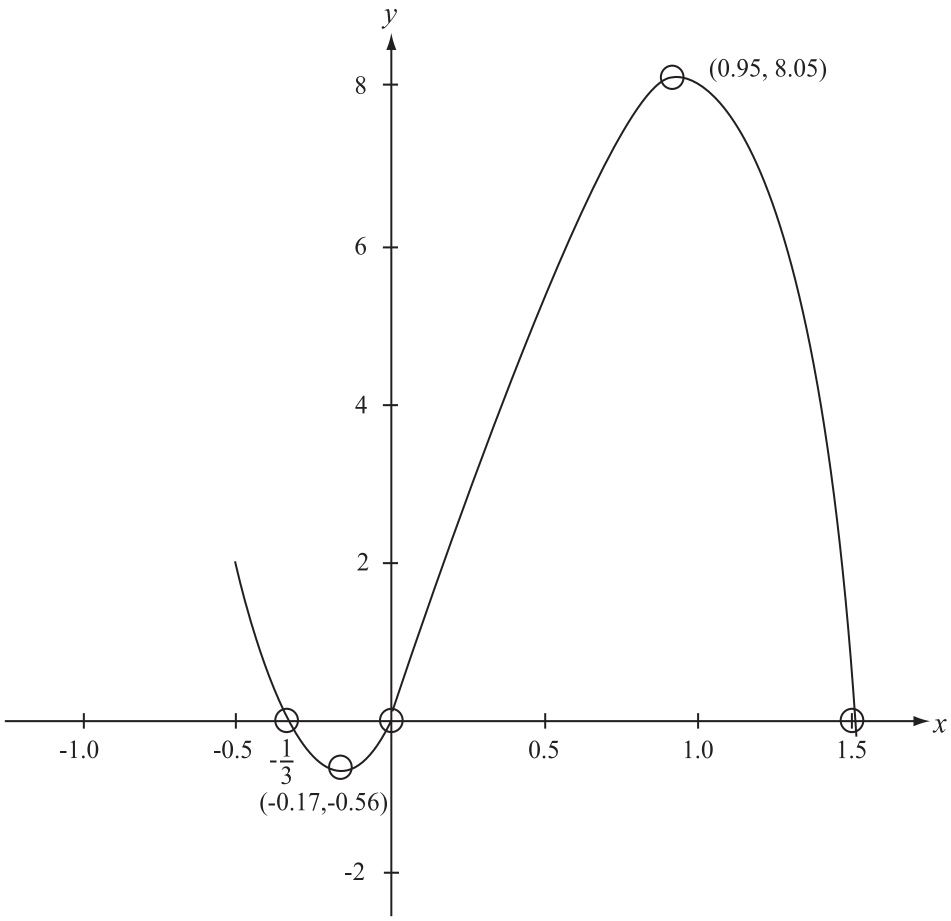
$\frac{d^2y}{dx^2} = -40.4$ Maximum TP $(0.95, 8.05)$

M1 (full method) W1

AVAILABLE MARKS

	AVAILABLE MARKS

(iv)



W1 max and min
W1 intercepts on x-axis

(v)

$$\text{Area} = \int_0^{1.5} (-12x^3 + 14x^2 + 6x)\,\mathrm{d}x$$ M1

$$= [-3x^4 + \frac{14}{3}x^3 + 3x^2]_0^{1.5}$$ MW1

$$= -3(1.5)^4 + \frac{14}{3}(1.5)^3 + 3(1.5)^2$$

$$= 7.3125$$ MW1 16

Total **100**