

GCS

**Additional
Mathematics**

Summer 2007

Mark Schemes

Issued: October 2007

**NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE)
AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)
MARK SCHEMES (2007)**

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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Rewarding Learning

**General Certificate of Secondary Education
2007**

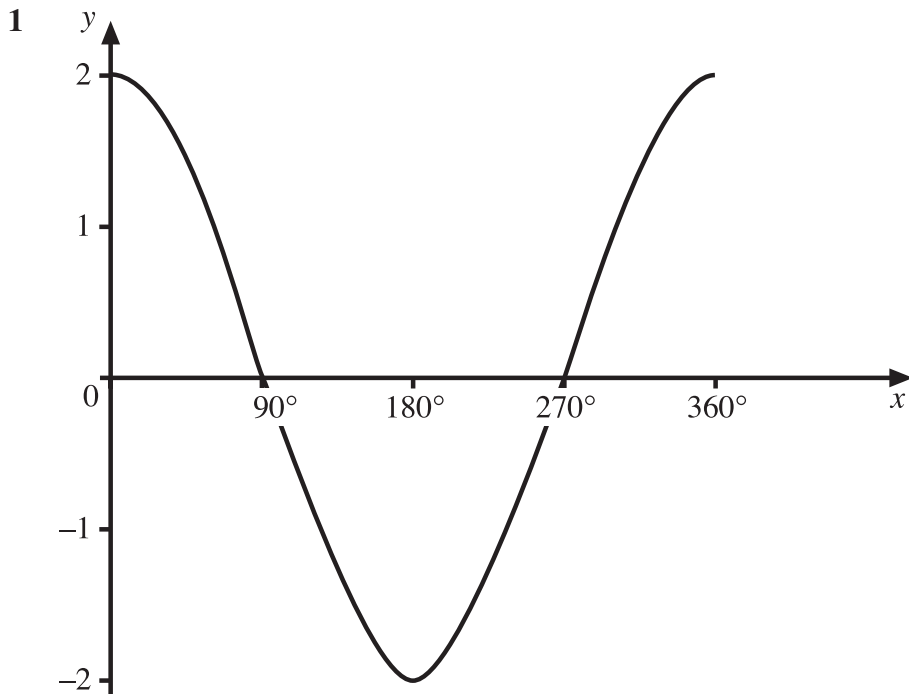
Additional Mathematics

Paper 1
Pure Mathematics

[G0301]

TUESDAY 15 MAY, AFTERNOON

**MARK
SCHEME**



M1 (cos curve)

M1 (± 2)

W1 (cutting axis)

M1 (drawing)

AVAILABLE MARKS

4

2 (i) $\cos \theta = 0.1$
 $\therefore \theta = 84.26^\circ$ or -84.26° to 2 dp

2 \times MW1

(ii) $\cos(2x - 10^\circ) = 0.1$

From (i)

$2x - 10 = 84.26$ or -84.26

M1

$\therefore 2x = 94.26$ or -74.26

$\therefore x = 47.13$ or -37.13 to 2 dp

W1

4

3 (i) $\det \mathbf{A} = -13$

$$\begin{aligned} \therefore \mathbf{A}^{-1} &= -\frac{1}{13} \begin{bmatrix} 1 & -2 \\ -5 & -3 \end{bmatrix} \\ &= \frac{1}{13} \begin{bmatrix} -1 & 2 \\ 5 & 3 \end{bmatrix} \end{aligned}$$

MW1, MW1

(ii) $\begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ -5 \end{bmatrix}$

M1

$$\begin{aligned} \therefore \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 16 \\ -5 \end{bmatrix} \\ &= \frac{1}{13} \begin{bmatrix} -1 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 16 \\ -5 \end{bmatrix} \\ &= \frac{1}{13} \begin{bmatrix} -26 \\ 65 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix} \end{aligned}$$

M2

$\therefore x = -2, y = 5$

W1

6

4 (a) $y = 3x^2 + \frac{2}{x}$

$\therefore \frac{dy}{dx} = 6x - \frac{2}{x^2}$

2 × MW1

(b) $\int \left(x^5 - \frac{1}{x^2} + 2 \right) dx$

$$= \frac{x^6}{6} - \left(-\frac{1}{x} \right) + 2x + c$$

$$= \frac{x^6}{6} + \frac{1}{x} + 2x + c$$

4 × MW1

6

- 5 (i) Cuts x -axis when $y = 0$,
i.e. $4 - \frac{8}{x} = 0$
 $\therefore x = 2$ MW1
 \therefore point P is (2, 0)
- (ii) $\frac{dy}{dx} = \frac{8}{x^2} = 2$ at $x = 2$ MW1
 \therefore Equation of tangent is
 $y - 0 = 2(x - 2)$
i.e. $y = 2x - 4$ MW1
- (iii) $\frac{dy}{dx} = \frac{1}{2}$
 $\therefore \frac{8}{x^2} = \frac{1}{2}$ M1
 $\therefore x = 4$ or $x = -4$; but $x > 0$
 \therefore point Q is (4, 2) W1
- (iv) Equation of tangent is
 $y - 2 = \frac{1}{2}(x - 4)$ MW1
i.e. $y - 2 = \frac{1}{2}x - 2$
i.e. $y = \frac{1}{2}x$ W1
This passes through (0, 0)
- 6 (i) $\frac{1-2x}{x-5} - \frac{x+1}{2x+5}$
 $= \frac{(1-2x)(2x+5) - (x+1)(x-5)}{(x-5)(2x+5)}$ M2
 $= \frac{(-4x^2 - 8x + 5) - (x^2 - 4x - 5)}{2x^2 - 5x - 25}$ W1, W1
 $= \frac{-5x^2 - 4x + 10}{2x^2 - 5x - 25}$
- (ii) $\frac{-5x^2 - 4x + 10}{2x^2 - 5x - 25} = -2$
 $\therefore -5x^2 - 4x + 10 = -2(2x^2 - 5x - 25)$ M2
 $\therefore -5x^2 - 4x + 10 = -4x^2 + 10x + 50$
 $\therefore -x^2 - 14x - 40 = 0$
 $\therefore x^2 + 14x + 40 = 0$ W1
 $\therefore (x + 10)(x + 4) = 0$
 $\therefore x = -10$ or $x = -4$ W1

7

8

- 7 (a) $4^{2x+3} = 3$
 $\therefore (2x+3)\log 4 = \log 3$ M2
 $\therefore 2x+3 = \frac{\log 3}{\log 4}$ M1
 $\therefore x = \frac{\frac{\log 3}{\log 4} - 3}{2}$
 $= -1.104$ to 3 dp W1
- (b) $\log_2 a = 3$
 $\therefore a = 2^3 = 8$ MW1
- (c) $\log 36 = \log 6^2 = 2 \log 6 = 2p$ MW1
 $\log 9 = \log \frac{36}{4}$
 $= \log 36 - \log 4$ M1
 $= 2p - q$ W1

8

- 8 (i) $\hat{XOP} = 75 - 47.5 = 27.5^\circ$ MW1
- (ii) $PX^2 = OX^2 + OP^2 - 2 \times OX \times OP \cos \hat{XOP}$ M2
 $= 6.25^2 + 3.5^2 - 2 \times 6.25 \times 3.5 \times \cos 27.5$
 $\therefore PX = 3.54 \text{ km}$ W1
- (iii) $\frac{\sin \hat{OXP}}{OP} = \frac{\sin \hat{XOP}}{PX}$ M2
 $\therefore \sin \hat{OXP} = \frac{3.5 \sin 27.5}{3.54}$
 $\therefore \hat{OXP} = 27.16^\circ$ W1
- (iv) $\hat{XZP} = 180 - 47.5 = 132.5^\circ$ MW1
- (v) $\hat{XPZ} = 180 - (132.5 + 27.16) = 20.34^\circ$ MW1
- (vi) $\frac{ZX}{\sin \hat{XPZ}} = \frac{PX}{\sin \hat{XZP}}$ M2
 $\therefore ZX = \frac{3.54 \sin 20.34}{\sin 132.5}$
 $\therefore ZX = 1.67 \text{ km}$ W1

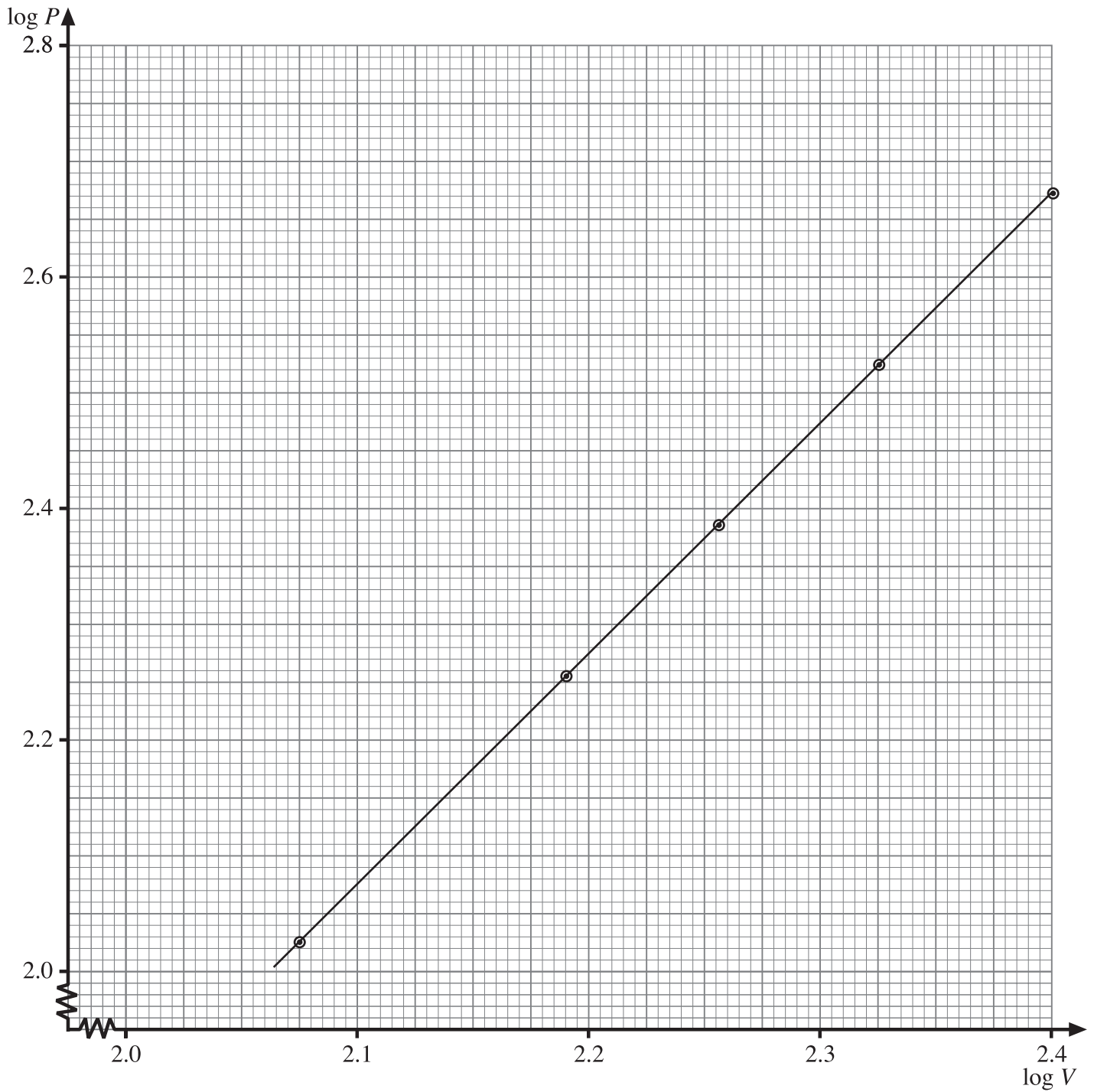
- 9 (i) $P = kV^n$
 $\therefore \log P = n \log V + \log k$ M1

$\log V$	$\log P$
2.076	2.029
2.189	2.256
2.252	2.381
2.324	2.525
2.397	2.671

W2

AVAILABLE
MARKS

12



Straight line graph so relationship is verified.

W1 – labels
 W1 – points
 W1 – straight line

AVAILABLE MARKS

$$(ii) \quad n = \text{gradient} = \frac{2.671 - 2.029}{2.397 - 2.076} = 2$$

M1, W1

$$P = kV^n \therefore k = \frac{P}{V^n} = \frac{469.2}{249.5^2} = 0.00754$$

M1, W1

$$\text{so } P = 0.00754V^2$$

$$(iii) \quad 400 = 0.00754V^2$$

$$\therefore V = \left(\frac{400}{0.00754} \right)^{\frac{1}{2}}$$

M1

$$= 230.3 \text{ km/h}$$

W1

$$(iv) \quad P = 0.00754 \times 62.8^2$$

$$= 29.7 \text{ kg/m}^2$$

MW1

Assume the formula holds beyond the range of given values.

M1

14

- 10 (i)** $20x + 8y + 12z = 860$ MW1
 $\therefore 5x + 2y + 3z = 215$ (1)
- (ii)** $9x + 9y + 6z = 570$ MW1
 $\therefore 3x + 3y + 2z = 190$ (2)
- (iii)** $10(1.2x) + 10(y + 6) + 10(z - 5) = 780$ $3 \times$ MW1
 $\therefore 12x + 10y + 10z = 780 - 60 + 50 = 770$
 $\therefore 6x + 5y + 5z = 385$ (3)
- (iv)** $2 \times (1) - 3 \times (2) \rightarrow x - 5y = -140$ (4)
 $5 \times (2) - 2 \times (3) \rightarrow 3x + 5y = 180$ (5) M2, W2
 $(4) + (5) \rightarrow 4x = 40$ M2
 $\therefore x = 10$
 $\therefore y = \frac{180 - 3x}{5} = 30$
 $\therefore z = \frac{215 - 5x - 2y}{3} = 35$ M1, W1
- (v)** Jack would have spent
 $29 \times 12 + 17 \times 36 + 18 \times 30 = 1500$ M1
 i.e. £15.00 W1

15

11 (i) For other points when $y = 0$

$$3x^2 + 2x - 5 = 0$$

M1

$$(3x + 5)(x - 1) = 0$$

$$\therefore x = -\frac{5}{3} \text{ or } x = 1$$

So points are $(-1.67, 0)$ and $(1, 0)$ (to 2 dp)

W1

(ii) $y = 3x^3 + 2x^2 - 5x$

$$\frac{dy}{dx} = 9x^2 + 4x - 5 = 0$$

MW1, M1

$$\therefore (9x - 5)(x + 1) = 0$$

M1

$$\therefore x = \frac{5}{9} \text{ or } x = -1$$

W1

$$\frac{d^2y}{dx^2} = 18x + 4$$

MW1

$$\text{at } x = \frac{5}{9}, \frac{d^2y}{dx^2} = 14 > 0 \therefore \text{min}$$

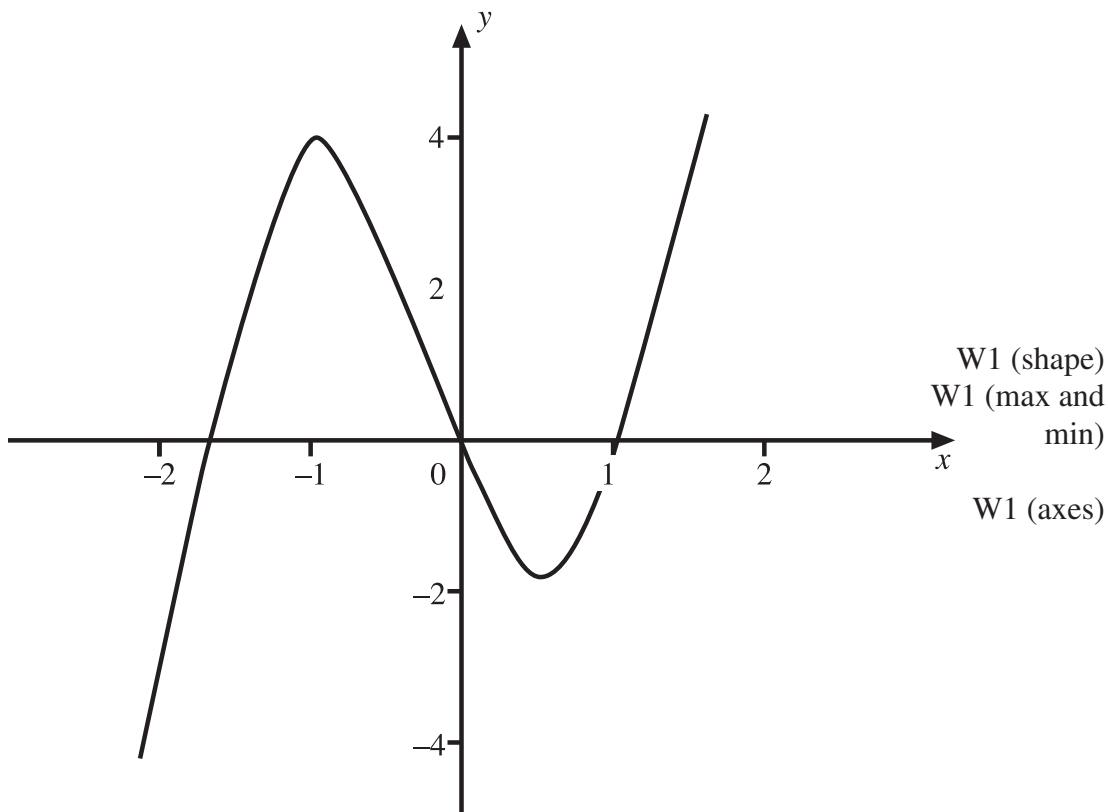
$$\text{at } x = -1, \frac{d^2y}{dx^2} = -14 < 0 \therefore \text{max}$$

M1

So minimum at $(0.56, -1.65)$ (to 2 dp)

M1, W1

maximum at $(-1, 4)$



$$\begin{aligned}
 \text{(iv) Area} &= -\int_0^1 (3x^3 + 2x^2 - 5x) dx \\
 &= -\left[\frac{3}{4}x^4 + \frac{2}{3}x^3 - \frac{5x^2}{2} \right]_0^1 \\
 &= -\left[\frac{3}{4} + \frac{2}{3} - \frac{5}{2} \right] = \frac{13}{12} \\
 &= 1.08 \text{ to 2 dp}
 \end{aligned}$$

M1

MW1

W1

Total

**AVAILABLE
MARKS**

16

100



Rewarding Learning

**General Certificate of Secondary Education
2007**

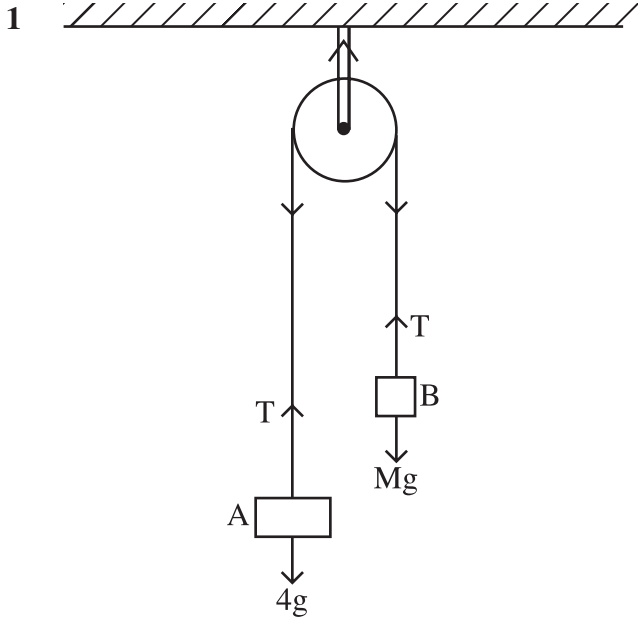
Additional Mathematics

Paper 2
Mechanics and Statistics

[G0302]

MONDAY 21 MAY, AFTERNOON

**MARK
SCHEME**



(i) At A $4g - T = 4(2.5)$

$$T = 40 - 10$$

$$T = \underline{30\text{N}}$$

MW1

W1

(ii) At B $T - Mg = M(2.5)$

$$30 - Mg = 2.5M$$

$$10M + 2.5M = 30$$

$$12.5M = 30$$

$$M = 2.4\text{ kg}$$

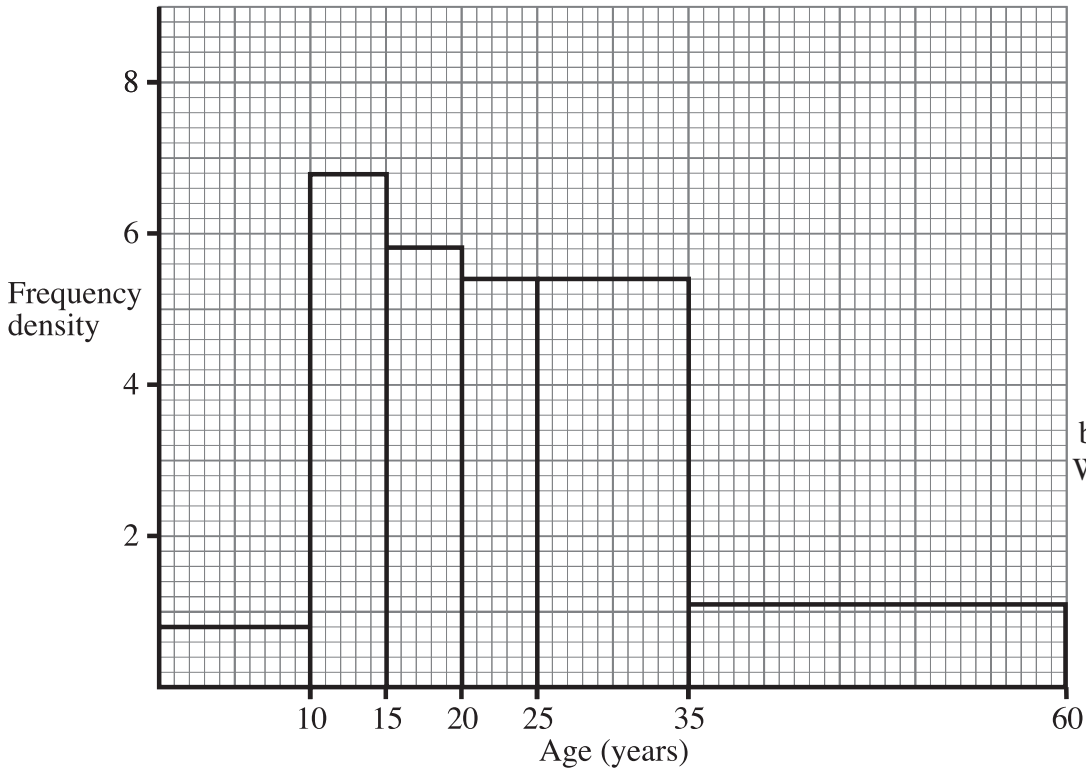
MW1

W1

4

2	class width	10	5	5	5	10	25
	frequency density	0.8	6.8	5.8	5.4	5.4	1.12

M1, W1



5

3 (i) $p(4\mathbf{i} + \mathbf{j}) + q(2\mathbf{i} - 3\mathbf{j}) = 5\mathbf{i} - 4\mathbf{j}$

$$4p\mathbf{i} + p\mathbf{j} + 2q\mathbf{i} - 3q\mathbf{j} = 5\mathbf{i} - 4\mathbf{j}$$

$$(4p + 2q)\mathbf{i} + (p - 3q)\mathbf{j} = 5\mathbf{i} - 4\mathbf{j}$$

MW1

$$\Rightarrow 4p + 2q = 5$$

$$\Rightarrow p - 3q = -4 \quad \times -4$$

MW1

$$-4p + 12q = 16$$

$$\underline{4p + 2q = 5}$$

$$14q = 21$$

$$\Rightarrow q = 1.5$$

$$p - 4.5 = -4$$

W1

$$\Rightarrow p = 0.5$$

$$\left. \begin{array}{l} p = 0.5 \\ q = 1.5 \end{array} \right\}$$

(ii) $3\mathbf{a} - 3\mathbf{b} + 2\mathbf{c} = 3(4\mathbf{i} + \mathbf{j}) - 3(2\mathbf{i} - 3\mathbf{j}) + 2(5\mathbf{i} - 4\mathbf{j})$

$$12\mathbf{i} + 3\mathbf{j} - 6\mathbf{i} + 9\mathbf{j} + 10\mathbf{i} - 8\mathbf{j}$$

$$\Rightarrow 16\mathbf{i} + 4\mathbf{j}$$

MW1

$$|16\mathbf{i} + 4\mathbf{j}| = \sqrt{16^2 + 4^2} = \sqrt{272} = 16.49$$

$$\Rightarrow |16\mathbf{i} + 4\mathbf{j}| = \underline{16.5 \text{ to 1 decimal place}}$$

MW1

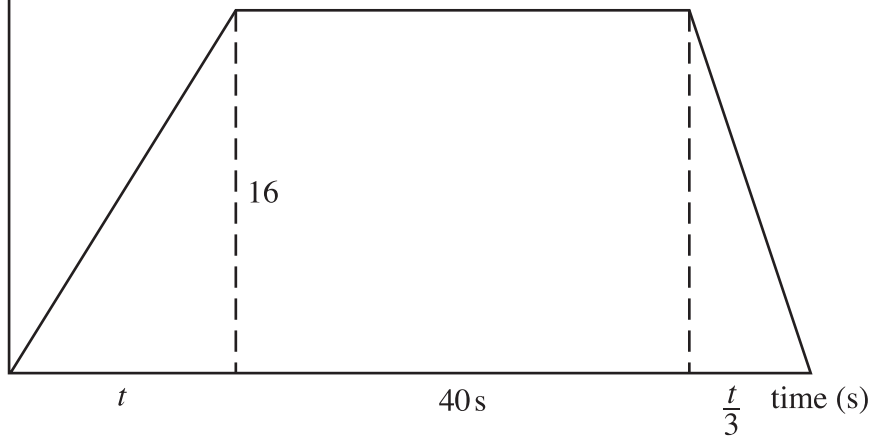
(iii) $\tan \theta = \frac{4}{16}$

$$\Rightarrow \theta = \underline{14.0^\circ}$$

MW1

6

4 (i) Speed
(m/s)



W1

W1

(ii) Total time = $t + 40 + \frac{t}{3} = \frac{4}{3}t + 40$

MW1

$$\Rightarrow \frac{4}{3}t + 40 = 56$$

$$\Rightarrow \frac{4}{3}t = 16$$

$$\Rightarrow t = \frac{3 \times 16}{4} = \underline{12\text{ s}}$$

W1

(iii) Total distance travelled = $\frac{1}{2}(56 + 40) \cdot 16$
 $= 96 \times 8 = \underline{768\text{ m}}$

MW1, MW1

W1

or

using $v = u + at$

$$16 = 0 + 12a \Rightarrow a = \frac{16}{12} = \frac{4}{3}\text{ m/s}^2$$

MW1

Distance while accelerating $s_1 = 0 + \frac{1}{2} \times \frac{4}{3} \times 144 = 96\text{ m}$

Distance while at constant speed $s_2 = 40 \times 16 = 640\text{ m}$

MW1

Distance while retarding $s_3 = (16 \times 4) + \frac{1}{2}(-4)(16) = 64 - 32$
 $= 32\text{ m}$

Total distance travelled = $96 + 640 + 32$
 $= \underline{768\text{ m}}$

W1

7

5	(i)	rank music	rank photos	$ d $	d^2	
		5.5	5	0.5	0.25	
		7	3	4	16	
		3	4	1	1	
		4	7	3	9	M1, W1
		2	6	4	16	
		5.5	1.5	4	16	
		1	1.5	0.5	<u>0.25</u>	
					58.5	

(ii) $r = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$ M2 $\sum d^2$

$r = 1 - \frac{6 \times 58.5}{7 \times 48} = -0.04$ M1, W1

(iii) No significance M1 7

6 (i) mean = $\frac{75}{5} = 15$ MW1

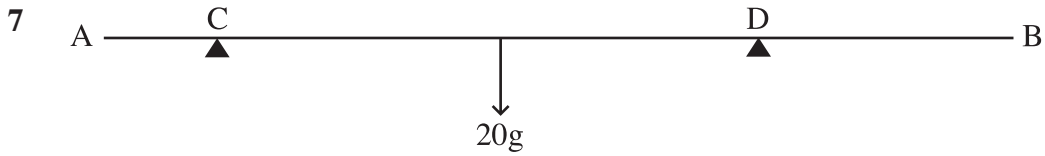
s.d. = $\sqrt{\left(\frac{1305}{5} - 225\right)}$ M1

= 6 W1

(ii) $a = 2$ M1

$\frac{75(2) + 5b}{5} = 40$

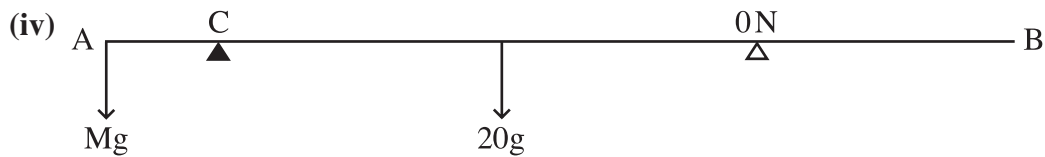
$b = 10$ M2, W1 7



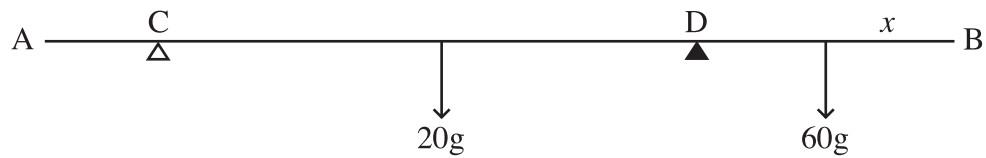
(i) At \hat{D} $C \times 5 = 20g \times 2$ MW1
 $5C = 40g$
 $C \uparrow = 8g N = \underline{80 N}$ W1

(ii) $C \uparrow + D \uparrow = 20g$
 $80 + D \uparrow = 200$
 $\Rightarrow D \uparrow = 120 N$ W1

(iii) The reaction at D is zero W1

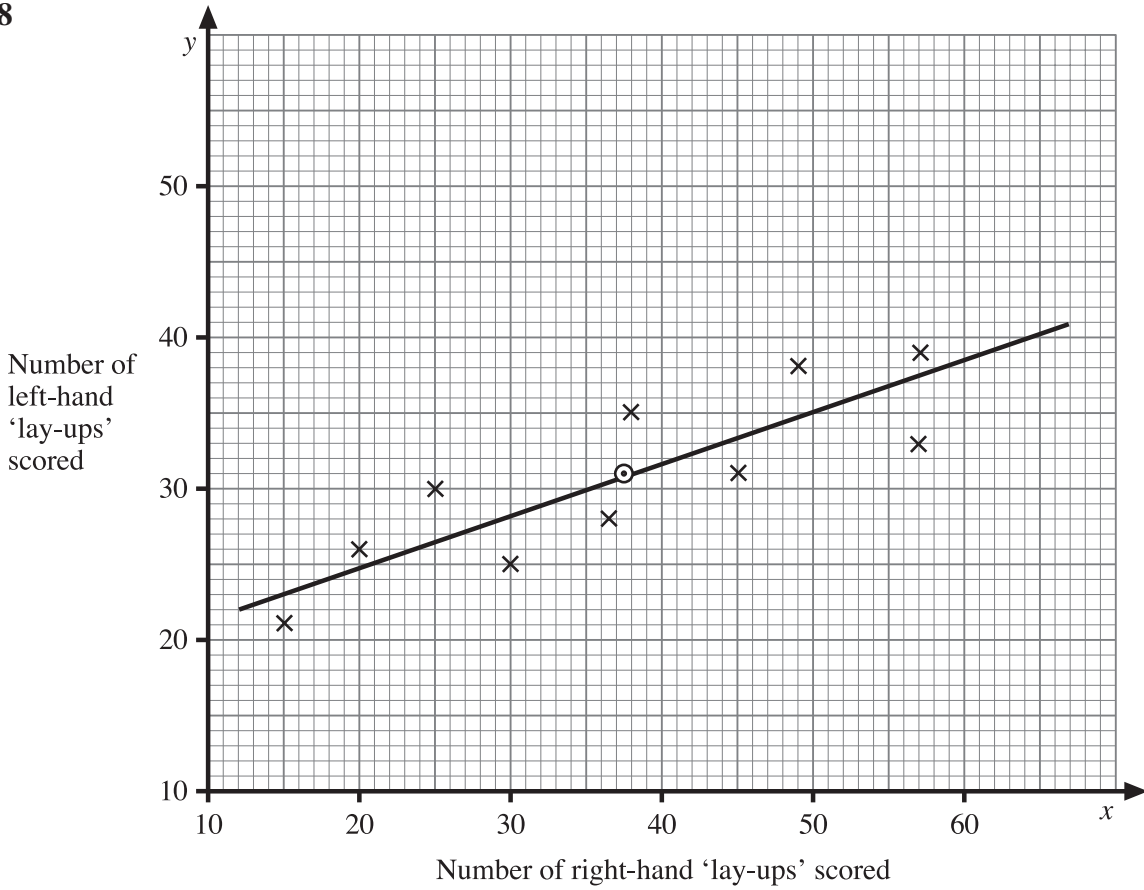


At \hat{C} $Mg \times 1 = 20g \times 3$ MW1
 $\Rightarrow M = 60 \text{ kg}$ W1



(v) At \hat{D} $20g \times 2 = 60g(2 - x)$ MW1, MW1
 $40 = 120 - 60x$
 $\Rightarrow 60x = 80$
 $x = \frac{8}{6} = \frac{4}{3} = \underline{1\frac{1}{3} \text{ m}}$ W1

8



- (a) (i) Mean right = 37.3 }
 Mean left = 30.6 }

MW1

- (ii) On graph

MW1 (mean)

MW1 (slope)

- (iii) $m = 0.33$

M1, W1

$c = 18.35$

M1, W1

$y = 0.33x + 18.35$

- (iv) $0.33 \times 50 + 18.35 \approx 35$

MW1

- (b) 30.5

MW1

9

AVAILABLE MARKS

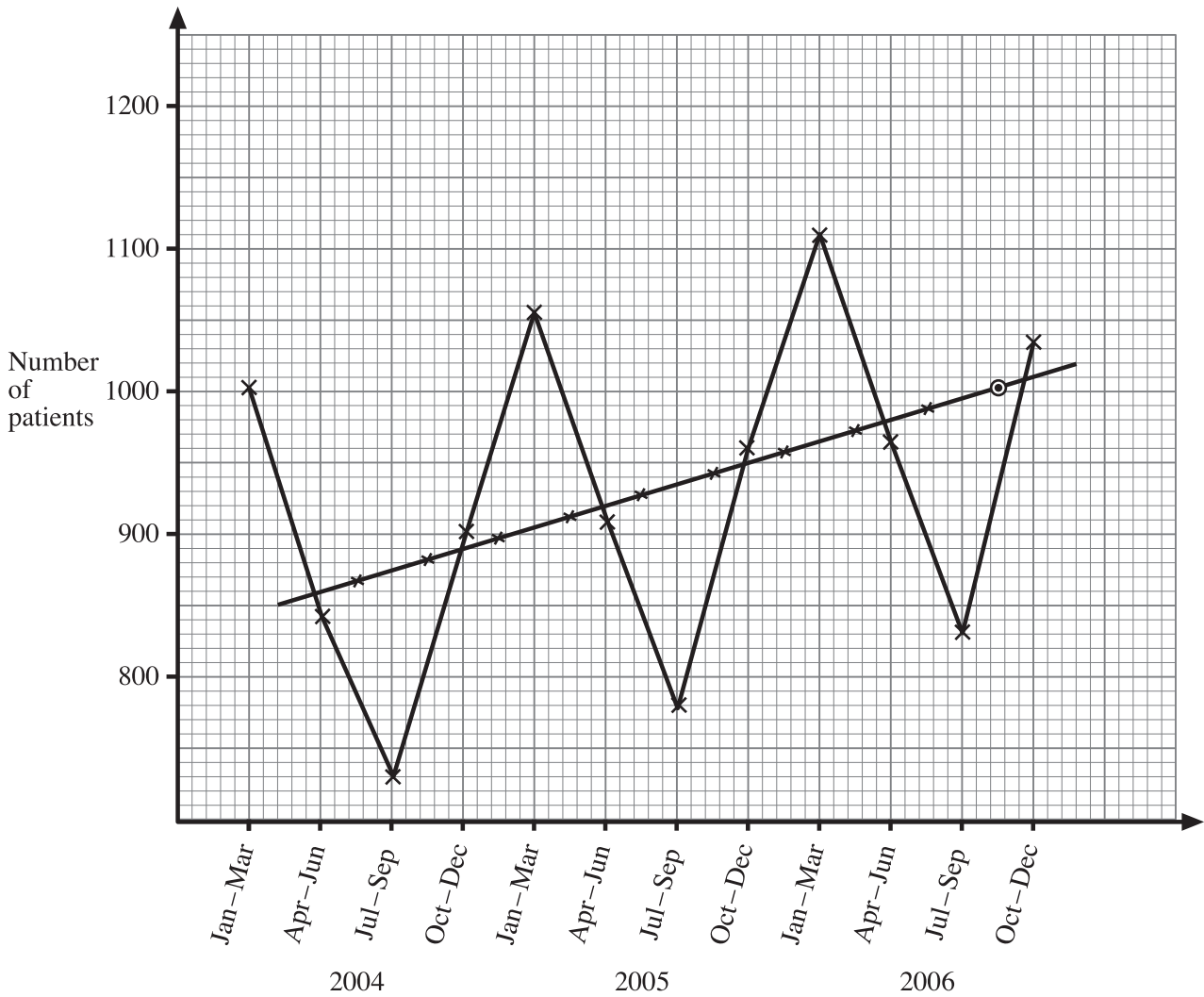
9 (i) 4 pt moving averages

- 868
- 881
- 897
- 911
- 928
- 942
- 957
- 970
- 987

M2, W1

(ii)

M2 set, W1 line



(iii) $\frac{x + 1036 + 836 + 965}{4} = 1000$

$x = 1163$

M1, W1 reading

M1, W1

AVAILABLE MARKS

10

10 At maximum height $v = 0$

$$\Rightarrow v^2 = u^2 + 2as$$

$$\Rightarrow 0 = 8^2 - 2(10) \cdot s$$

MW1

$$\Rightarrow s = \frac{64}{20} = \underline{3.2\text{ m}}$$

W1

So maximum height above pavement = $10 + 3.2 = \underline{13.2\text{ m}}$

MW1

(ii) $v^2 = u^2 + 2as$

$$\Rightarrow v^2 = 0 + 2 \times 10(13.2) = 264$$

MW1

$$\Rightarrow v = \sqrt{264} = \underline{16.2\text{ m/s}} \text{ (to 1 decimal place)}$$

W1

(iii) time to reach greatest height = t_1

$$v = u + at$$

$$\Rightarrow 0 = 8 - 10t_1$$

$$\Rightarrow t_1 = 0.8 \text{ secs}$$

MW1

time from greatest height to pavement = t_2

$$s = ut + \frac{1}{2}at^2$$

$$13.2 = 0 + \frac{1}{2} \times 10 \times t_2^2$$

MW1

$$\Rightarrow t_2^2 = 2.64$$

$$\Rightarrow t_2 = \sqrt{2.64} = 1.6\text{ s}$$

W1

$$\text{Total time} = 0.8 + 1.6 = \underline{2.4\text{ s}}$$

MW1

(iv) Speed of rebound = 8.1 m/s

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0 = 8.1^2 + 2(-10)s$$

MW1

$$\Rightarrow s = \frac{8.1^2}{20} = 3.28 = \underline{3.3\text{ m}} \text{ to 1 decimal place}$$

W1

11

11 (i)

			red			
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
blue	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	3,1	3,2	3,3	3,4	3,5	3,6
	3,1	3,2	3,3	3,4	3,5	3,6

$$P(\text{both prime}) = \frac{15}{36} = \frac{5}{12}$$

M1, W1

(ii) $P(\text{total prime}) = \frac{16}{36} = \frac{4}{9}$

M1, W1

(iii) $P(1,1/\text{prime}) = \frac{1}{16}$

M2, W1

(iv) $P(b > r) = \frac{8}{36} = \frac{2}{9}$

M1, W1

(v) $P(\text{no double}) = \left(\frac{30}{36}\right)^6 = \left(\frac{5}{6}\right)^6 = \frac{15625}{46656} \approx 0.33$

MW1, MW1

(vi) $P(\text{at least 1 double}) = 1 - \left(\frac{5}{6}\right)^6$
 $= \frac{31031}{46656} \approx 0.67$

MW1

12

alternative solution on next page

or

11 (i)

	Blue	Red				
	2	2, 3, 5	$2/6 \times 3/6 = 6/36$	}	$15/36 = 5/12$	M1 W1
	3	2, 3, 5	$3/6 \times 3/6 = 9/36$			

(ii)

	Blue	Red				
	1	1, 2, 4, 6	$1/6 \times 4/6 = 4/36$	}	$16/36 = 4/9$	M1 W1
	2	1, 3, 5	$2/6 \times 3/6 = 6/36$			
	3	2, 4	$3/6 \times 2/6 = 6/36$			

(iii) From (ii), $P(\text{both 1/total is prime}) = 1/(4 + 6 + 6) = 1/16$ M2
W1

(iv)

	Blue	Red				
	2	1	$2/6 \times 1/6 = 2/36$	}	$8/36 = 2/9$	M1 W1
	3	1, 2	$3/6 \times 2/6 = 6/36$			

(v)

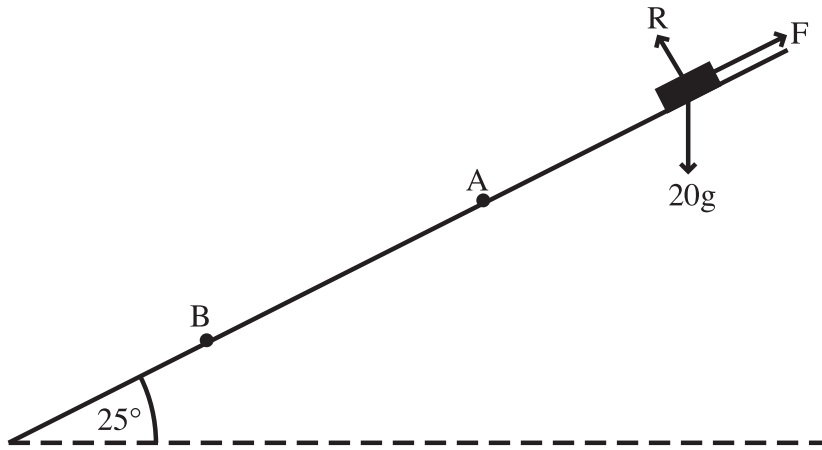
	Blue	Red				
	1	1	$1/6 \times 1/6 = 1/36$	}	$6/36 = 1/6$	
	2	2	$2/6 \times 1/6 = 2/36$			
	3	3	$3/6 \times 1/6 = 3/36$			

$P(\text{double}) = 1/6 \Rightarrow P(\text{no double}) = 5/6$ M1

$P(\text{no double in 6 throws}) = (5/6)^6 \approx 0.33$ W1

(vi) From (v), $P(\text{at least 1 double in 6 throws}) \approx 1 - 0.33 \approx 0.67$ MW1

12 (i)



2 × W1

(ii) between A and B using $v^2 = u^2 + 2as$

$$4^2 = 2^2 + 2a(4)$$

MW1

$$16 = 4 + 8a$$

$$\Rightarrow 8a = 12$$

$$\Rightarrow a = \underline{1.5 \text{ m/s}^2}$$

W1

(iii) $20g \sin 25 - F = 20(1.5)$

MW1 (direction of F)

$$\Rightarrow F = 20g \sin 25 - 30$$

MW1 ($20g \sin 25$)

$$\Rightarrow F = 84.523 - 30$$

MW1 (equation)

$$\Rightarrow F = 54.5 \text{ N (to 1 decimal place)}$$

W1 (answer)

(iv) $R = 20g \cos 25 = 181.26 = 181.3 \text{ N}$

MW1

$$F = \mu R$$

$$\Rightarrow 54.5 = \mu \times 181.3$$

MW1

$$\Rightarrow \mu = \frac{54.5}{181.3} = 0.3 \text{ (to 1 decimal place)}$$

W1

(v) using $v = u + at$

$$\Rightarrow v = 2 + (1.5) \times 2 = 2 + 3 = \underline{5 \text{ m/s}}$$

MW1

(vi) $S = ut + \frac{1}{2}at^2$

$$S = 2(2) + \frac{1}{2}(1.5) \times 2^2 = 4 + 3 = \underline{7 \text{ m}}$$

MW1

13

Total

100

