Rewarding Learning

General Certificate of Secondary Education
2006

Additional Mathematics

Paper 1
Pure Mathematics

[G0301]

TUESDAY 16 MAY, AFTERNOON

MARK SCHEME

GCSE ADDITIONAL MATHEMATICS 2006

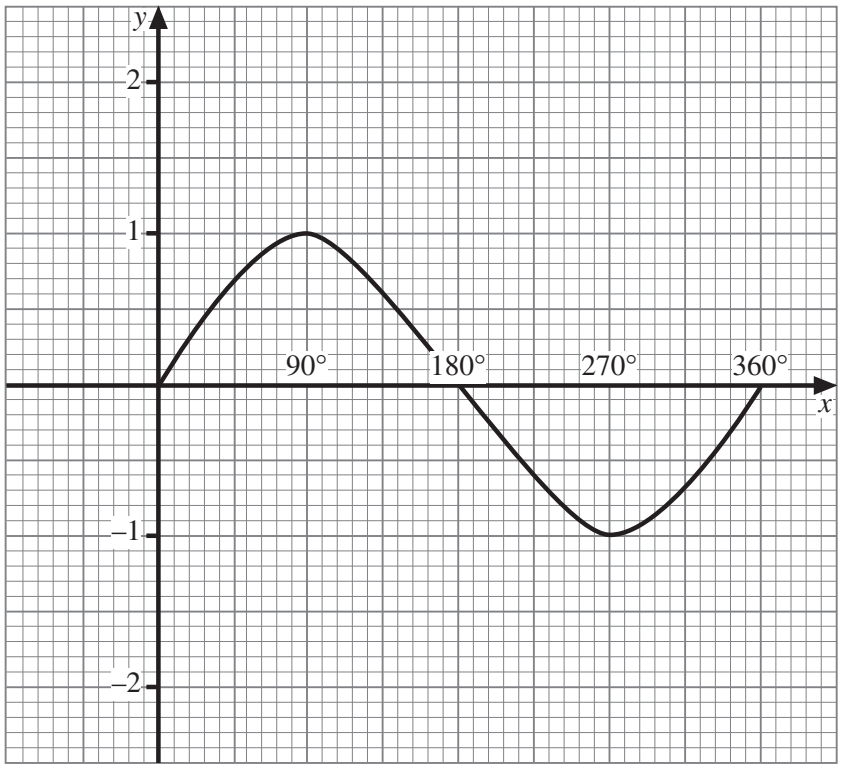
Paper 1

Pure Mathematics

Mark Scheme

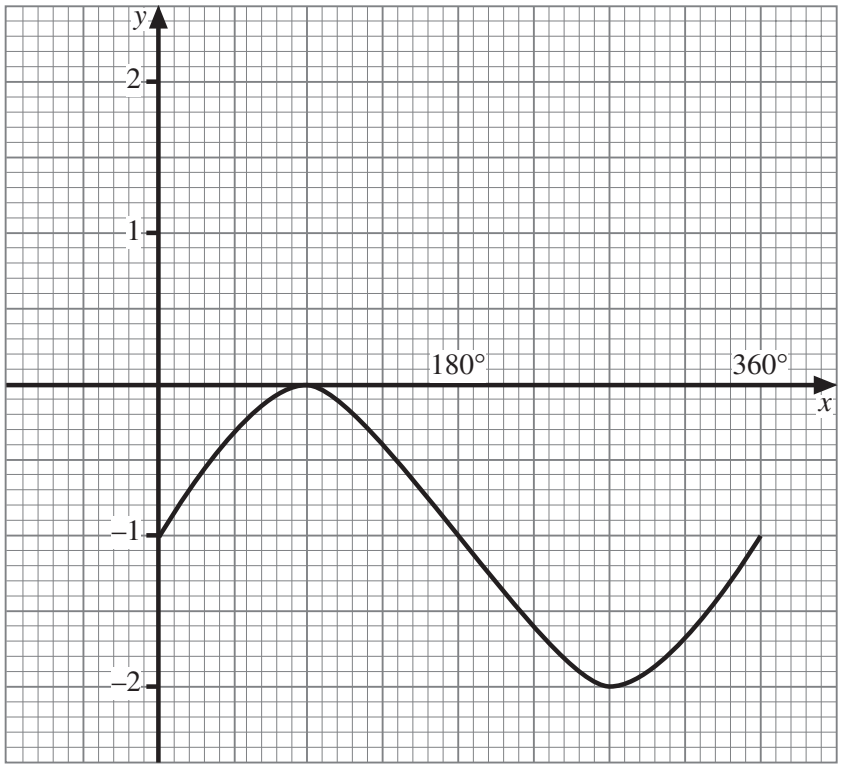
AVAILABLE MARKS

1 **(i)** $y = \sin x$



M1 (shape)
W1 (position)

(ii) $y = \sin x - 1$



M1 (shape)
MW1 (position)

4

			AVAILABLE MARKS
2	$\sin(\frac{1}{2}x + 80°) = 0.6$		
	$\therefore \frac{1}{2}x + 80° = 36.87°$ **or** $143.13°$	M1, W1, W1	
	$\therefore \frac{1}{2}x = -43.13°$ **or** $63.13°$		
	$\therefore x = -86.26°$ **or** $126.26°$	W1, W1	5
3	**(i)** $\det \mathbf{A} = 18 - 16 = 2$	MW1	
	$\therefore \mathbf{A}^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -8 \\ -2 & 3 \end{bmatrix}$	MW1	
	(ii) $\begin{bmatrix} 3 & 8 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 9 \end{bmatrix}$	M1	
	$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \mathbf{A}^{-1}\begin{bmatrix} 13 \\ 9 \end{bmatrix}$		
	$= \frac{1}{2}\begin{bmatrix} 6 & -8 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 13 \\ 9 \end{bmatrix}$	M2	
	$= \frac{1}{2}\begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0.5 \end{bmatrix}$	W1	
	$\therefore x = 3, \ y = 0.5$		6
4	**(a)** $y = 2x^7 - \frac{5}{x^4} - 7$		
	$\frac{dy}{dx} = 14x^6 + 20x^{-5}$		
	$= 14x^6 + \frac{20}{x^5}$	MW1, MW1, MW1	
	(b) $\int\left(\frac{2}{x^7} - 5x^4\right)dx$		
	$= -\frac{2}{6}x^{-6} - \frac{5}{5}x^5 + c$		
	$= -\frac{1}{3x^6} - x^5 + c$	MW1, MW1, MW1	6

			AVAILABLE MARKS
5	**(i)** $\dfrac{3x-1}{x+2}+\dfrac{2x-1}{x+4}$		
	$=\dfrac{(3x-1)(x+4)+(2x-1)(x+2)}{(x+2)(x+4)}$	M2	
	$=\dfrac{(3x^2+11x-4)+(2x^2+3x-2)}{x^2+6x+8}$	W1, W1	
	$=\dfrac{5x^2+14x-6}{x^2+6x+8}$		
	(ii) Equation is equivalent to		
	$\dfrac{5x^2+14x-6}{x^2+6x+8}=3$		
	$\therefore\ 5x^2+14x-6=3(x^2+6x+8)$	M2	
	$\therefore\ 5x^2+14x-6=3x^2+18x+24$		
	$\therefore\ 2x^2-4x-30=0$		
	$\therefore\ x^2-2x-15=0$	W1	
	$\therefore\ (x-5)(x+3)=0$		
	$\therefore\ x=5$ **or** $x=-3$	W1	8
6	**(a)** $4^a=64\ \therefore\ a=3$	MW1	
	(b) $\log_3 18=\log_3(2\times 9)$		
	$=\log_3 2+\log_3 9$	M1	
	$=\log_3 2+\log_3 3^2$		
	$=\log_3 2+2\log_3 3$	M1	
	$=b+2$	W1	
	(c) $5^{(1+\frac{1}{2}x)}=21$		
	$\therefore (1+\tfrac{1}{2}x)\log 5=\log 21$	M1, M1	
	$\therefore 1+\tfrac{1}{2}x=\dfrac{\log 21}{\log 5}$	M1	
	$\therefore x=2\left(\dfrac{\log 21}{\log 5}-1\right)$		
	$=1.783$	W1	8

			AVAILABLE MARKS
7	**(i)** $y = x^2 - 4x + 7$		
	$\frac{dy}{dx} = 2x - 4$	MW1	
	At $x = 1$, $\frac{dy}{dx} = -2$	W1	
	$\therefore$ Equation of tangent is		
	$y - 4 = -2(x - 1)$	M1	
	i.e. $y = -2x + 6$	W1	
	(ii) $\frac{dy}{dx} = 6$ when $2x - 4 = 6$		
	i.e. $x = 5$	MW1	
	When $x = 5$, $y = 12$	W1	
	$\therefore$ Equation of tangent is		
	$y - 12 = 6(x - 5)$		
	i.e. $y = 6x - 18$	MW1	
	(iii) $y = -2x + 6$		
	If $x = 3$ $y = -6 + 6 = 0$		
	$y = 6x - 18$		
	If $x = 3$ $y = 18 - 18 = 0$		
	$\therefore$ tangents meet at (3, 0)	MW1	8
8	**(i)** $\hat{OXA} = 180 - (65 + 15) = 100°$	W1	
	$\frac{OX}{\sin 15} = \frac{OA}{\sin 100}$	M2	
	$\therefore OX = \frac{1200 \times \sin 15}{\sin 100} = 315$ m	W1	
	(ii) $\hat{BOX} = 180 - 65 = 115°$	W1	
	$BX^2 = BO^2 + OX^2 - 2 \times BO \times OX \times \cos 115$	M2	
	$= 500^2 + 315^2 - 2 \times 500 \times 315 \times \cos 115$		
	$\therefore BX = 695$ m	W1	
	(iii) $\frac{BX}{\sin 115} = \frac{OX}{\sin \hat{XBO}}$	M1	
	$\therefore \sin \hat{XBO} = \frac{315 \times \sin 115}{695} = 0.4108$		
	$\therefore \hat{XBO} = 24.3°$ This is the required angle.	W1	10

		AVAILABLE MARKS
9	**(i)** $10x + 15y + 20z = 45$	
	$\therefore 2x + 3y + 4z = 9 \quad (1)$ — MW1	
	(ii) $14x + 12y + 20z = 50$	
	$\therefore 7x + 6y + 10z = 25 \quad (2)$ — MW1	
	(iii) $15x + 12y + 36z = 60$	
	$\therefore 5x + 4y + 12z = 20 \quad (3)$ — MW1	
	(iv) $(2) \times 2 - (1) \times 5 \rightarrow 4x - 3y = 5 \quad (4)$	
	$(1) \times 3 - (3) \rightarrow x + 5y = 7 \quad (5)$ — M2, W2	
	$(5) \times 4 - (4) \rightarrow 23y = 23$	
	$\therefore y = 1$ — M2	
	$\therefore x = 7 - 5y = 2$ $\therefore z = \frac{9 - 2x - 3y}{4} = 0.5$ — M2	
	$\therefore$ high – 2 Mb medium – 1 Mb — W1 low – 0.5 Mb	
	(v) Storage space left = 256 – 45 – 50 – 60 = 101 Mb — MW1	
	2 Mb gives 6 secs video	
	$\therefore$ 1 Mb gives 3 secs	
	$\therefore$ amount of video = 101×3 secs	
	= 303 secs	
	= 5 mins 3 secs	
	i.e. just over 5 mins — MW1	14

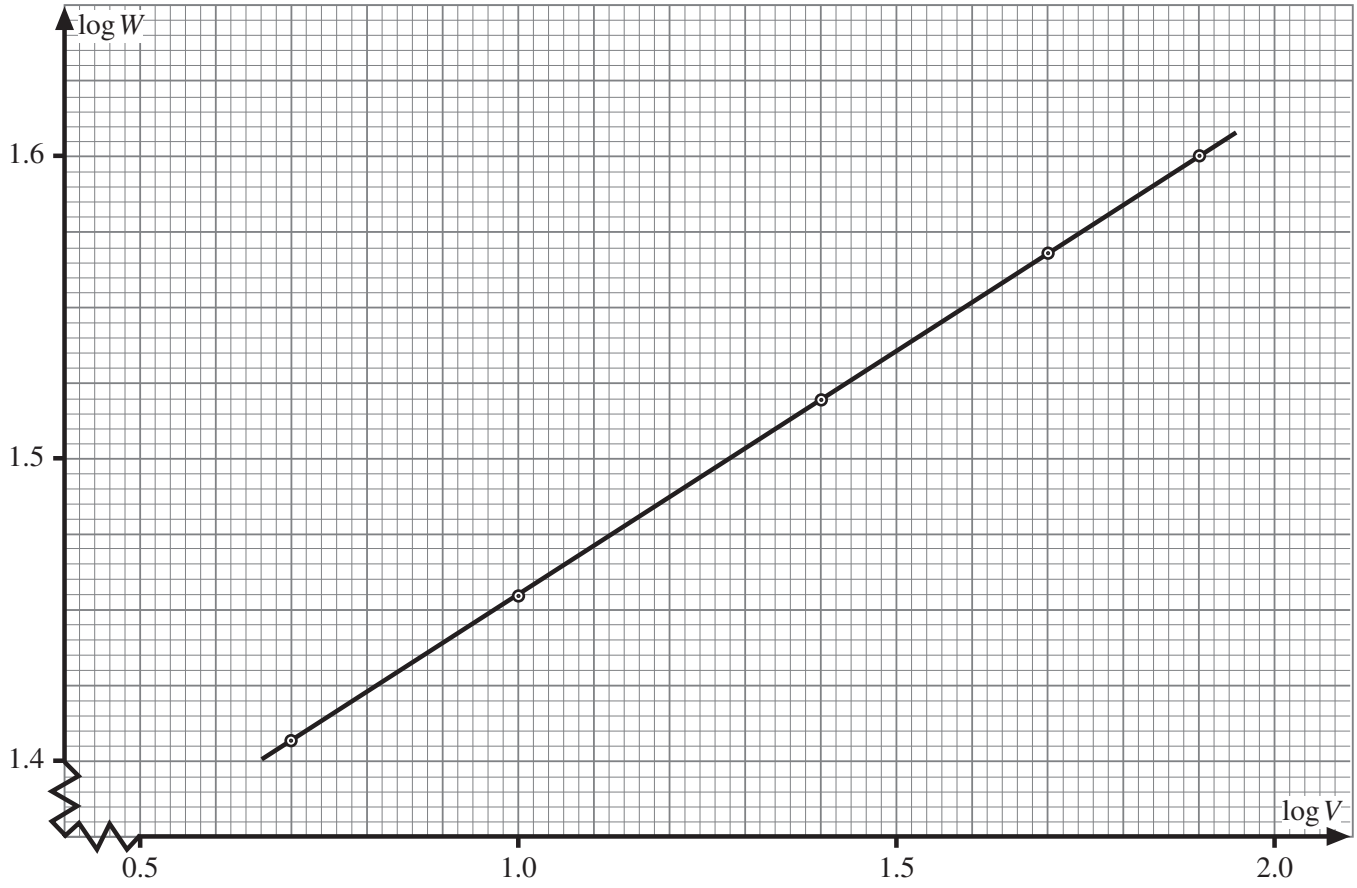
AVAILABLE MARKS

10 (i) $W = kV^n$

$\therefore \log W = n \log V + \log k$ M1

$\log V$	$\log W$
0.699	1.407
1.000	1.455
1.398	1.519
1.699	1.567
1.903	1.600

W2



W1 (labels)
W1 (points)
W1 (line)

AVAILABLE MARKS

Straight line graph so relationship is verified.

			AVAILABLE MARKS
(ii)	$\log W = n \log V + \log k$		
	$\therefore\ 1.407 = 0.699n + \log k$		
	$1.600 = 1.903n + \log k$		
	$\therefore\ n = \dfrac{1.600 - 1.407}{1.903 - 0.699} = 0.16$	M1, W1	
	$\log k = 1.600 - 1.903 \times 0.16 = 1.296$		
	$\therefore\ k = 19.75$	M1, W1	
	so $W = 19.75V^{0.16}$		
(iii)	$W = 19.75 \times 100^{0.16} = 41.3$	M1, W1	
	Assume that the formula holds for higher values of V than given in the table.	M1	
(iv)	$35 = 19.75V^{0.16}$		
	$\therefore V = \left(\dfrac{35}{19.75}\right)^{\frac{1}{0.16}}$	M1	
	$= 35.7$ km/h	W1	15

AVAILABLE MARKS

11 **(i)** $y = 0$ when $x = 0, -1$ or 2

So points are $(0, 0)$, $(-1, 0)$ and $(2, 0)$ MW1

(ii) $y = x(x + 1)(x - 2) = x(x^2 - x - 2) = x^3 - x^2 - 2x$ W1

So

$$\frac{dy}{dx} = 3x^2 - 2x - 2 = 0$$ MW1, M1

when

$$x = \frac{2 \pm \sqrt{4 + 24}}{6}$$

$$= 1.22 \text{ **or** } -0.55$$ W1, W1

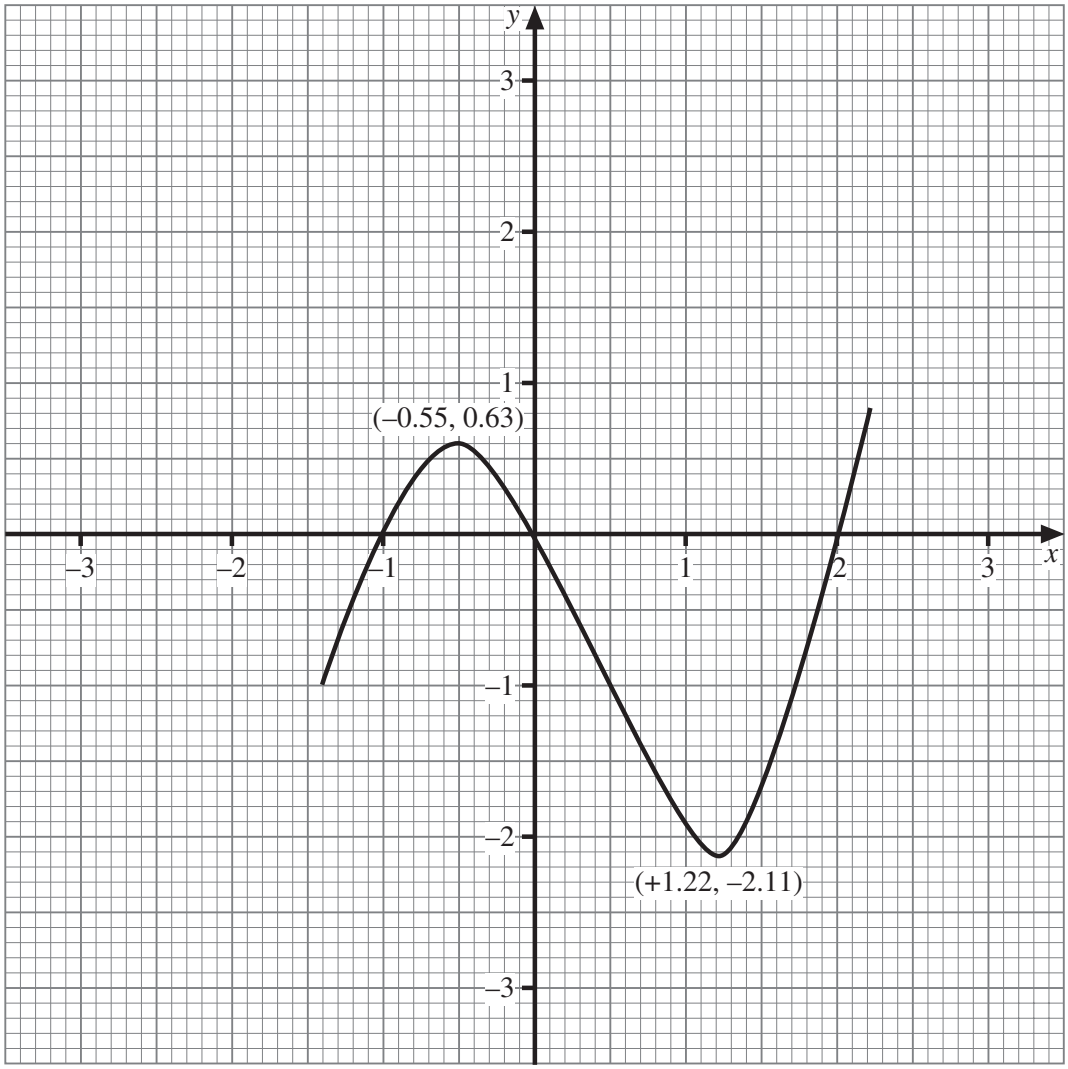
$$\frac{d^2y}{dx^2} = 6x - 2$$ MW1

When $x = 1.22$, $y = -2.11$, $\frac{d^2y}{dx^2} = 5.32 > 0$ M1

When $x = -0.55$, $y = 0.63$, $\frac{d^2y}{dx^2} = -5.3 < 0$

So minimum at $(1.22, -2.11)$

maximum at $(-0.55, 0.63)$ W1

		AVAILABLE MARKS
(iii)		
	W1 (shape) W1 (max) W1 (min)	
$\text{Area} = \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx$	M1	
$= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2 \right]_{-1}^{0}$	MW2	
$= \left[0 - \left(\frac{1}{4} + \frac{1}{3} - 1 \right) \right]$		
$= \frac{5}{12} = 0.417$	W1	16
	Total	**100**