Read the following instructions carefully.

1. Write your name and registration number in the space provided at the bottom of this page.
2. Take out the Optical Response Sheet (ORS) from this Question Booklet without breaking the seal.
3. Do not open the seal of the Question Booklet until you are asked to do so by the invigilator.
4. Write your registration number, your name and name of the examination centre at the specified locations on the right half of the ORS. Also, using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your test paper code ( PH ).
5. This Question Booklet contains 20 pages including blank pages for rough work. After opening the seal at the specified time, please check all pages and report discrepancy, if any.
6. There are a total of 65 questions carrying 100 marks. All these questions are of objective type. Questions must be answered on the left hand side of the ORS by darkening the appropriate bubble (marked A, B, C, D) using HB pencil against the question number. For each question darken the bubble of the correct answer. In case you wish to change an answer, erase the old answer completely. More than one answer bubbled against a question will be treated as an incorrect response.
7. Questions Q. 1- Q. 25 carry 1-mark each, and questions Q. 26 - Q. 55 carry 2-marks each.
8. Questions Q .48 - Q. 51 (2 pairs) are common data questions and question pairs (Q.52, Q.53) and (Q.54, Q.55) are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
9. Questions Q. 56 - Q. 65 belong to General Aptitude (GA). Questions Q. 56 - Q. 60 carry 1-mark each, and questions Q .61 - Q. 65 carry 2-marks each. The GA questions begin on a fresh page starting from page 13 .
10. Unattempted questions will result in zero mark and wrong answers will result in NEGATIVE marks. For Q. 1 - Q. 25 and Q. $56-\mathrm{Q} .60,1 / 3$ mark will be deducted for each wrong answer. For Q. $26-\mathrm{Q} .51$ and Q. 61 - Q. 65 , $2 / 3$ mark will be deducted for each wrong answer. The question pairs (Q.52, Q.53), and (Q.54, Q.55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair, i.e. for Q .52 and $\mathrm{Q} .54,2 / 3$ mark will be deducted for each wrong answer. There is no negative marking for Q. 53 and Q. 55 .
11. Calculator is allowed whereas charts, graph sheets or tables are NOT allowed in the examination hall.
12. Rough work can be done on the question paper itself. Additionally, blank pages are provided at the end of the question paper for rough work.

| Name |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Registration Number | PH |  |  |  |  |  |  |  |

## Some Useful Constants

Speed of light in free space
$c=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$
Boltzmann constant
$k_{B}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$
Planck's constant
$\hbar=h / 2 \pi=1.05 \times 10^{-34} \mathrm{Js}$
Electron charge
$e=1.6 \times 10^{-19} \mathrm{C}$
Permittivity of free space
$\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}$
Permeability of free space
NOTE : In numerical problems, the option closest to the correct answer will be given credit.

## Q. 1 - Q. 25 carry one mark each.

Q. $1 \quad$ Two matrices $A$ and $B$ are said to be similar if $B=P^{-1} A P$ for some invertible matrix $P$. Which of the following statements is NOT TRUE?
(A) $\operatorname{Det} A=\operatorname{Det} B$
(B) Trace of $A=$ Trace of $B$
(C) $A$ and $B$ have the same eigenvectors
(D) $A$ and $B$ have the same eigenvalues
Q. 2 If a force $\vec{F}$ is derivable from a potential function $V(r)$, where $r$ is the distance from the origin of the coordinate system, it follows that
(A) $\vec{\nabla} \times \vec{F}=0$
(B) $\vec{\nabla} \cdot \vec{F}=0$
(C) $\vec{\nabla} V=0$
(D) $\nabla^{2} V=0$
Q. 3 The quantum mechanical operator for the momentum of a particle moving in one dimension is given by
(A) $i \hbar \frac{d}{d x}$
(B) $-i \hbar \frac{d}{d x}$
(C) $i \hbar \frac{\partial}{\partial t}$
(D) $-\frac{\hbar^{2}}{2 m} \frac{d^{2}}{d x^{2}}$
Q. 4 A Carnot cycle operates on a working substance between two reservoirs at temperatures $T_{1}$ and $T_{2}$, with $T_{1}>T_{2}$. During each cycle, an amount of heat $Q_{1}$ is extracted from the reservoir at $T_{1}$ and an amount $Q_{2}$ is delivered to the reservoir at $T_{2}$. Which of the following statements is INCORRECT?
(A) work done in one cycle is $Q_{1}-Q_{2}$
(B) $\frac{Q_{1}}{T_{1}}=\frac{Q_{2}}{T_{2}}$
(C) entropy of the hotter reservoir decreases
(D) entropy of the universe (consisting of the working substance and the two reservoirs) increases
Q. 5 In a first order phase transition, at the transition temperature, specific heat of the system
(A) diverges and its entropy remains the same
(B) diverges and its entropy has finite discontinuity
(C) remains unchanged and its entropy has finite discontinuity
(D) has finite discontinuity and its entropy diverges
Q. 6 The semi-empirical mass formula for the binding energy of nucleus contains a surface corre term. This term depends on the mass number $A$ of the nucleus as
(A) $A^{-1 / 3}$
(B) $A^{1 / 3}$
(C) $A^{2 / 3}$
(D) $A$
Q. 7 The population inversion in a two level laser material CANNOT be achieved by optical pumping because
(A) the rate of upward transitions is equal to the rate of downward transitions
(B) the upward transitions are forbidden but downward transitions are allowed
(C) the upward transitions are allowed but downward transitions are forbidden
(D) the spontaneous decay rate of the higher level is very low
Q. 8 The temperature ( $T$ ) dependence of magnetic susceptibility $(\chi)$ of a ferromagnetic substance with a Curie temperature $\left(T_{c}\right)$ is given by
(A) $\frac{C}{T-T_{c}}$, for $T<T_{c}$
(B) $\frac{C}{T-T_{c}}$, for $T>T_{c}$
(C) $\frac{C}{T+T_{c}}$, for $T>T_{c}$
(D) $\frac{C}{T+T_{c}}$, for all temperatures
where $C$ is constant.
Q. 9 The order of magnitude of the energy gap of a typical superconductor is
(A) 1 MeV
(B) 1 KeV
(C) 1 eV
(D) 1 meV
Q. 10 Which of the following statements is CORRECT for a common emitter amplifier circuit?
(A) The output is taken from the emitter
(B) There is $180^{\circ}$ phase shift between input and output voltages
(C) There is no phase shift between input and output voltages
(D) Both $p-n$ junctions are forward biased
Q. 11 A $3 \times 3$ matrix has elements such that its trace is 11 and its determinant is 36 . The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is
(A) 18
(B) 12
(C) 9
(D) 6
Q. 12 A heavy symmetrical top is rotating about its own axis of symmetry (the $z$-axis). If $I_{1}, I_{2}$ and $I_{3}$ are the principal moments of inertia along $x, y$ and $z$ axes respectively, then
(A) $I_{2}=I_{3} ; I_{1} \neq I_{2}$
(B) $I_{1}=I_{3} ; I_{1} \neq I_{2}$
(C) $I_{1}=I_{2} ; I_{1} \neq I_{3}$
(D) $I_{1} \neq I_{2} \neq I_{3}$
Q. 13 An electron with energy $E$ is incident from left on a potential barrier, given by

$$
\begin{aligned}
V(x) & =0 \text { for } x<0 \\
& =V_{0} \text { for } x>0
\end{aligned}
$$

as shown in the figure.


For $E<V_{0}$, the space part of the wavefunction for $x>0$ is of the form
(A) $e^{\alpha x}$
(B) $e^{-\alpha x}$
(C) $e^{i \alpha x}$
(D) $e^{-i \alpha x}$ where $\alpha$ is a real positive quantity.
Q. 14 If $L_{x}, L_{y}$ and $L_{z}$ are respectively the $x, y$ and $z$ components of angular momentum operator $L$, the commutator $\left[L_{x} L_{y}, L_{z}\right]$ is equal to
(A) $i \hbar\left(L_{x}^{2}+L_{y}^{2}\right)$
(B) $2 i \hbar L_{z}$
(C) $i \hbar\left(L_{x}^{2}-L_{y}^{2}\right)$
(D) 0
Q. 15 The normalized ground state wavefunction of a hydrogen atom is given by $\psi(r)=\frac{1}{\sqrt{4 \pi}} \frac{2}{a^{3 / 2}} e^{-r / a}$, where $a$ is the Bohr radius and $r$ is the distance of the electron from the nucleus, located at the origin. The expectation value $\left\langle\frac{1}{r^{2}}\right\rangle$ is
(A) $\frac{8 \pi}{a^{2}}$
(B) $\frac{4 \pi}{a^{2}}$
(C) $\frac{4}{a^{2}}$
(D) $\frac{2}{a^{2}}$
Q. 16 Two charges $q$ and $2 q$ are placed along the $x$-axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge $q$ is

(A) $\frac{1}{4 \pi \varepsilon_{0}} \frac{7 q^{2}}{2}$
(B) $\frac{1}{4 \pi \varepsilon_{0}} 2 q^{2}$
(C) $\frac{1}{4 \pi \varepsilon_{0}} q^{2}$
(D) $\frac{1}{4 \pi \varepsilon_{0}} \frac{q^{2}}{2}$ plane. The direction of the magnetic field is
(A) along $\hat{i}$ for $z>0$ and along $-\hat{i}$ for $z<0$
(B) along $\hat{k}$ for $z>0$ and along $-\hat{k}$ for $z<0$
(C) along $-\hat{i}$ for $z>0$ and along $\hat{i}$ for $z<0$
(D) along $-\hat{k}$ for $z>0$ and along $\hat{k}$ for $z<0$
Q. 18 A magnetic dipole of dipole moment $\vec{m}$ is placed in a non-uniform magnetic field $\vec{B}$. If the position vector of the dipole is $\vec{r}$, the torque acting on the dipole about the origin is
(A) $\vec{r} \times(\vec{m} \times \vec{B})$
(B) $\vec{r} \times \vec{\nabla}(\vec{m} \cdot \vec{B})$
(C) $\vec{m} \times \vec{B}$
(D) $\vec{m} \times \vec{B}+\vec{r} \times \vec{\nabla}(\vec{m} \cdot \vec{B})$
Q. 19 Which of the following expressions for a vector potential $\vec{A}$ DOES NOT represent a uniform magnetic field of magnitude $B_{0}$ along the $z$-direction?
(A) $\vec{A}=\left(0, B_{0} x, 0\right)$
(B) $\vec{A}=\left(-B_{0} y, 0,0\right)$
(C) $\vec{A}=\left(\frac{B_{0} x}{2}, \frac{B_{0} y}{2}, 0\right)$
(D) $\vec{A}=\left(-\frac{B_{0} y}{2}, \frac{B_{0} x}{2}, 0\right)$
Q. 20 A neutron passing through a detector is detected because of
(A) the ionization it produces
(B) the scintillation light it produces
(C) the electron - hole pairs it produces
(D) the secondary particles produced in a nuclear reaction in the detector medium
Q. 21 An atom with one outer electron having orbital angular momentum $l$ is placed in a weak magnetic field. The number of energy levels into which the higher total angular momentum state splits, is
(A) $2 l+2$
(B) $2 l+1$
(C) $2 l$
(D) $2 l-1$
Q. 22 For a multi-electron atom, $l, \mathrm{~L}$ and S specify the one-electron orbital angular momentum, total orbital angular momentum and total spin angular momentum, respectively. The selection rules for electric dipole transition between the two electronic energy levels, specified by $l, \mathrm{~L}$ and S are
(A) $\Delta \mathrm{L}=0, \pm 1 ; \Delta \mathrm{S}=0 ; \Delta l=0, \pm 1$
(B) $\Delta \mathrm{L}=0, \pm 1 ; \Delta \mathrm{S}=0 ; \Delta l= \pm 1$
(C) $\Delta \mathrm{L}=0, \pm 1 ; \Delta \mathrm{S}= \pm 1 ; \Delta l=0, \pm 1$
(D) $\Delta \mathrm{L}=0, \pm 1 ; \Delta \mathrm{S}= \pm 1 ; \Delta l= \pm 1$
Q. 23 For a three-dimensional crystal having $N$ primitive unit cells with a basis of $p$ atoms, the number of optical branches is
(A) 3
(B) $3 p$
(C) $3 p-3$
(D) $3 N-3 p$
Q. 24 For an intrinsic semiconductor, $m_{e}^{*}$ and $m_{h}^{*}$ are respectively the effective masses of electr holes near the corresponding band edges. At a finite temperature, the position of the Fermi leve
(A) depends on $m_{e}^{*}$ but not on $m_{h}^{*}$
(B) depends on $m_{h}^{*}$ but not on $m_{e}^{*}$
(C) depends on both $m_{e}^{*}$ and $m_{h}^{*}$
(D) depends neither on $m_{e}^{*}$ nor on $m_{h}^{*}$
Q. 25 In the following circuit, the voltage across and the current through the $2 \mathrm{k} \Omega$ resistance are

(A) $20 \mathrm{~V}, 10 \mathrm{~mA}$
(B) $20 \mathrm{~V}, 5 \mathrm{~mA}$
(C) $10 \mathrm{~V}, 10 \mathrm{~mA}$
(D) $10 \mathrm{~V}, 5 \mathrm{~mA}$

## Q. 26 to Q. 55 carry two marks each.

Q. 26 The unit vector normal to the surface $x^{2}+y^{2}-z=1$ at the point $P(1,1,1)$ is
(A) $\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$
(B) $\frac{2 \hat{i}+\hat{j}-\hat{k}}{\sqrt{6}}$
(C) $\frac{\hat{i}+2 \hat{j}-\hat{k}}{\sqrt{6}}$
(D) $\frac{2 \hat{i}+2 \hat{j}-\hat{k}}{3}$
Q. 27 Consider a cylinder of height $h$ and radius $a$, closed at both ends, centered at the origin. Let $\vec{r}=\hat{i} x+\hat{j} y+\hat{k} z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_{S} \vec{r} \cdot \hat{n} d s$ over the closed surface of the cylinder is

(A) $2 \pi a^{2}(a+h)$
(B) $3 \pi a^{2} h$
(C) $2 \pi a^{2} h$
(D) zero
Q. 28 The solutions to the differential equation

$$
\frac{d y}{d x}=-\frac{x}{y+1}
$$

are a family of
(A) circles with different radii
(B) circles with different centres
(C) straight lines with different slopes
(D) straight lines with different intercepts on the $y$-axis
Q. 29 A particle is moving under the action of a generalized potential

$$
V(q, \dot{q})=\frac{(1+\dot{q})}{q^{2}}
$$

The magnitude of the generalized force is
(A) $\frac{2(1+\dot{q})}{q^{3}}$
(B) $\frac{2(1-\dot{q})}{q^{3}}$
(C) $\frac{2}{q^{3}}$
(D) $\frac{\dot{q}}{q^{3}}$
Q. 30 Two bodies of mass $m$ and $2 m$ are connected by a spring of spring constant $k$. The frequency of the normal mode is
(A) $\sqrt{3 k / 2 m}$
(B) $\sqrt{k / m}$
(C) $\sqrt{2 k / 3 m}$
(D) $\sqrt{k / 2 m}$
Q. 31 Let $(p, q)$ and $(P, Q)$ be two pairs of canonical variables. The transformation

$$
\begin{aligned}
& Q=q^{\alpha} \cos (\beta p) \\
& P=q^{\alpha} \sin (\beta p)
\end{aligned}
$$

is canonical for
(A) $\alpha=2, \beta=1 / 2$
(B) $\alpha=2, \beta=2$
(C) $\alpha=1, \beta=1$
(D) $\alpha=1 / 2, \beta=2$
Q. 32 Two particles, each of rest mass $m$ collide head-on and stick together. Before collision, the speed of each mass was 0.6 times the speed of light in free space. The mass of the final entity is
(A) $5 m / 4$
(B) $2 m$
(C) $5 m / 2$
(D) $25 \mathrm{~m} / 8$
Q. 33 The normalized eigenstates of a particle in a one-dimensional potential well

$$
V(x)=\left\{\begin{array}{lr}
0 & \text { if } 0 \leq x \leq a \\
\infty & \text { otherwise }
\end{array}\right.
$$

are given by

$$
\psi_{n}(x)=\sqrt{\frac{2}{a}} \sin \left(\frac{n \pi x}{a}\right), \text { where } n=1,2,3, \cdots
$$

The particle is subjected to a perturbation

$$
\begin{aligned}
V^{\prime}(x) & =V_{0} \cos \left(\frac{\pi x}{a}\right) & & \text { for } 0 \leq x \leq \frac{a}{2} \\
& =0 & & \text { otherwise }
\end{aligned}
$$

The shift in the ground state energy due to the perturbation, in the first order perturbation theory, is
(A) $\frac{2 V_{0}}{3 \pi}$
(B) $\frac{V_{0}}{3 \pi}$
(C) $-\frac{V_{0}}{3 \pi}$
(D) $-\frac{2 V_{0}}{3 \pi}$
Q. 34 If the isothermal compressibility of a solid is $K_{T}=10^{-10}(\mathrm{~Pa})^{-1}$, the pressure required to in its density by $1 \%$ is approximately
(A) $10^{4} \mathrm{~Pa}$
(B) $10^{6} \mathrm{~Pa}$
(C) $10^{8} \mathrm{~Pa}$
(D) $10^{10} \mathrm{~Pa}$
Q. 35 A system of $N$ non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is
(A) $2 k_{B} \ln N$
(B) $3 k_{B} \ln N$
(C) $N k_{B} \ln 2$
(D) $N k_{B} \ln 3$
Q. 36 A system has two energy levels with energies $\varepsilon$ and $2 \varepsilon$. The lower level is 4 -fold degenerate while the upper level is doubly degenerate. If there are $N$ non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature $T$, the fraction of particles in the
upper level is
(A) $\frac{1}{1+e^{-\varepsilon / k_{B} T}}$
(B) $\frac{1}{1+2 e^{\varepsilon / k_{B} T}}$
(C) $\frac{1}{2 e^{\varepsilon / k_{B} T}+4 e^{2 \varepsilon / k_{B} T}}$
(D) $\frac{1}{2 e^{\varepsilon / k_{B} T}-4 e^{2 \varepsilon / k_{B} T}}$
Q. 37 A spherical conductor of radius $a$ is placed in a uniform electric field $\vec{E}=E_{0} \hat{k}$. The potential at a point $P(r, \theta)$ for $r>a$, is given by

$$
\varphi(r, \theta)=\text { constant }-E_{0} r \cos \theta+\frac{E_{0} a^{3}}{r^{2}} \cos \theta
$$

where $r$ is the distance of P from the centre O of the sphere and $\theta$ is the angle OP makes with the z-axis.


The charge density on the sphere at $\theta=30^{\circ}$ is
(A) $3 \sqrt{3} \varepsilon_{0} E_{0} / 2$
(B) $3 \varepsilon_{0} E_{0} / 2$
(C) $\sqrt{3} \varepsilon_{0} E_{0} / 2$
(D) $\varepsilon_{0} E_{0} / 2$
Q. 38 According to the single particle nuclear shell model, the spin-parity of the ground state of ${ }_{8}^{17} \mathrm{O}$ is
(A) $\frac{1}{2}$
(B) $\frac{3}{2}^{-}$
(C) $\frac{3}{2}^{+}$
(D) $\frac{5}{2}^{+}$
Q. 39 In the $\beta$-decay of neutron $n \rightarrow p+e^{-}+\bar{v}_{e}$, the anti-neutrino $\bar{v}_{e}$ escapes detection. Its existo inferred from the measurement of
(A) energy distribution of electrons
(B) angular distribution of electrons
(C) helicity distribution of electrons
(D) forward-backward asymmetry of electrons
Q. 40 The isospin and the strangeness of $\Omega^{-}$baryon are
(A) $1,-3$
(B) $0,-3$
(C) 1,3
(D) 0,3
Q. 41 The lifetime of an atomic state is 1 nanosecond. The natural line width of the spectral line in the emission spectrum of this state is of the order of
(A) $10^{-10} \mathrm{eV}$
(B) $10^{-9} \mathrm{eV}$
(C) $10^{-6} \mathrm{eV}$
(D) $10^{-4} \mathrm{eV}$
Q. 42 The degeneracy of an excited state of nitrogen atom having electronic configuration $1 s^{2} 2 s^{2} 2 \mathrm{p}^{2} 3 \mathrm{~d}^{1}$ is
(A) 6
(B) 10
(C) 15
(D) 150
Q. 43 The far infrared rotational absorption spectrum of a diatomic molecule shows equidistant lines with spacing $20 \mathrm{~cm}^{-1}$. The position of the first Stokes line in the rotational Raman spectrum of this molecule is
(A) $20 \mathrm{~cm}^{-1}$
(B) $40 \mathrm{~cm}^{-1}$
(C) $60 \mathrm{~cm}^{-1}$
(D) $120 \mathrm{~cm}^{-1}$
Q. 44 A metal with body centered cubic (bcc) structure shows the first (i.e. smallest angle) diffraction peak at a Bragg angle of $\theta=30^{\circ}$. The wavelength of X-ray used is $2.1 \AA$. The volume of the PRIMITIVE unit cell of the metal is
(A) $26.2(\AA)^{3}$
(B) $13.1(\AA)^{3}$
(C) $9.3(\AA)^{3}$
(D) $4.6(\AA)^{3}$
Q. 45 In the following circuit, $\operatorname{Tr} 1$ and $\operatorname{Tr} 2$ are identical transistors having $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$. The current passing through the transistor Tr 2 is

(A) 57 mA
(B) 50 mA
(C) 48 mA
(D) 43 mA
$\frac{2011}{\text { Q. } 46}$ The following Boolean expression
$Y=A \bullet \bar{B} \bullet \bar{C} \bullet \bar{D}+\bar{A} \bullet B \bullet \bar{C} \bullet D+\bar{A} \bullet \bar{B} \bullet \bar{C} \bullet D+\bar{A} \bullet \bar{B} \bullet C \bullet D+\bar{A} \bullet B \bullet C \bullet D+A \bullet \bar{B}$ can be simplified to
(A) $\bar{A} \bullet \bar{B} \bullet C+A \bullet \bar{D}$
(B) $\bar{A} \bullet B \bullet \bar{C}+A \bullet \bar{D}$
(C) $A \bullet \bar{B} \bullet \bar{C}+\bar{A} \bullet D$
(D) $A \bullet \bar{B} \bullet C+\bar{A} \bullet D$
Q. 47 Consider the following circuit.


Which of the following correctly represents the output $\mathrm{V}_{\text {out }}$ corresponding to the input $\mathrm{V}_{\text {in }}$ ?

(C)

(B)

(D)


## Common Data Questions

## Common Data for Questions 48 and 49:

Consider a function $f(z)=\frac{z \sin z}{(z-\pi)^{2}}$ of a complex variable $z$.
Q. 48 Which of the following statements is TRUE for the function $f(z)$ ?
(A) $f(z)$ is analytic everywhere in the complex plane
(B) $f(z)$ has a zero at $z=\pi$
(C) $f(z)$ has a pole of order 2 at $z=\pi$
(D) $f(z)$ has a simple pole at $z=\pi$
Q. 49 Consider a counterclockwise circular contour $|z|=1$ about the origin. The integral $\oint f(z) d z$ over this contour is
(A) $-i \pi$
(B) zero
(C) $i \pi$
(D) $2 i \pi$

## Common Data for Questions 50 and 51:

The tight binding energy dispersion ( $E-k$ ) relation for electrons in a one-dimensional array of atoms having lattice constant $a$ and total length $L$ is

$$
E=E_{0}-\beta-2 \gamma \cos (k a),
$$

where $E_{0}, \beta$ and $\gamma$ are constants and $k$ is the wave-vector.
Q. 50 The density of states of electrons (including spin degeneracy) in the band is given by
(A) $\frac{L}{\pi \gamma a \sin (k a)}$
(B) $\frac{L}{2 \pi \gamma a \sin (k a)}$
(C) $\frac{L}{2 \pi \gamma a \cos (k a)}$
(D) $\frac{L}{\pi \gamma a \cos (k a)}$
Q. 51 The effective mass of electrons in the band is given by
(A) $\frac{\hbar^{2}}{\gamma a^{2} \cos (k a)}$
(B) $\frac{\hbar^{2}}{2 \gamma a^{2} \cos (k a)}$
(C) $\frac{\hbar^{2}}{\gamma a^{2} \sin (k a)}$
(D) $\frac{\hbar^{2}}{2 \gamma a^{2} \sin (k a)}$

Linked Answer Questions

## Statement for Linked Answer Questions 52 and 53:

In a one-dimensional harmonic oscillator, $\varphi_{0}, \varphi_{1}$ and $\varphi_{2}$ are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another.
$\psi_{1}$ and $\psi_{2}$ are two states defined by

$$
\begin{aligned}
& \psi_{1}=\varphi_{0}-2 \varphi_{1}+3 \varphi_{2} \\
& \psi_{2}=\varphi_{0}-\varphi_{1}+\alpha \varphi_{2}
\end{aligned}
$$

where $\alpha$ is a constant.
Q. 52 The value of $\alpha$ for which $\psi_{2}$ is orthogonal to $\psi_{1}$ is
(A) 2
(B) 1
(C) -1
(D) -2
Q. 53 For the value of $\alpha$ determined in Q. 52, the expectation value of energy of the oscillator in the state $\psi_{2}$ is
(A) $\hbar \omega$
(B) $3 \hbar \omega / 2$
(C) $3 \hbar \omega$
(D) $9 \hbar \omega / 2$

## Statement for Linked Answer Questions 54 and 55:

A plane electromagnetic wave has the magnetic field given by

$$
\vec{B}(x, y, z, t)=B_{0} \sin \left[(x+y) \frac{k}{\sqrt{2}}+\omega t\right] \hat{k}
$$

where $k$ is the wave number and $\hat{i}, \hat{j}$ and $\hat{k}$ are the Cartesian unit vectors in $x, y$ and $z$ directions, respectively.
Q. 54 The electric field $\vec{E}(x, y, z, t)$ corresponding to the above wave is given by
(A) $c B_{0} \sin \left[(x+y) \frac{k}{\sqrt{2}}+\omega t\right] \frac{(\hat{i}-\hat{j})}{\sqrt{2}}$
(B) $c B_{0} \sin \left[(x+y) \frac{k}{\sqrt{2}}+\omega t\right] \frac{(\hat{i}+\hat{j})}{\sqrt{2}}$
(C) $c B_{0} \sin \left[(x+y) \frac{k}{\sqrt{2}}+\omega t\right] \hat{i}$
(D) $c B_{0} \sin \left[(x+y) \frac{k}{\sqrt{2}}+\omega t\right] \hat{j}$
Q. 55 The average Poynting vector is given by
(A) $\frac{c B_{0}^{2}}{2 \mu_{0}} \frac{(\hat{i}-\hat{j})}{\sqrt{2}}$
(B) $-\frac{c B_{0}^{2}}{2 \mu_{0}} \frac{(\hat{i}-\hat{j})}{\sqrt{2}}$
(C) $\frac{c B_{0}^{2}}{2 \mu_{0}} \frac{(\hat{i}+\hat{j})}{\sqrt{2}}$
(D) $-\frac{c B_{0}^{2}}{2 \mu_{0}} \frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

## General Aptitude (GA) Questions

Q. 56 - Q. 60 carry one mark each.
Q. 56 Choose the most appropriate word from the options given below to complete the following sentence.
If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or $\qquad$ .
(A) hyperbolic
(B) restrained
(C) argumentative
(D) indifferent
Q. 57 Choose the most appropriate word(s) from the options given below to complete the following sentence.
I contemplated $\qquad$ Singapore for my vacation but decided against it.
(A) to visit
(B) having to visit
(C) visiting
(D) for a visit
Q. 58 If $\log (P)=(1 / 2) \log (Q)=(1 / 3) \log (R)$, then which of the following options is TRUE?
(A) $\mathrm{P}^{2}=\mathrm{Q}^{3} \mathrm{R}^{2}$
(B) $Q^{2}=P R$
(C) $\mathrm{Q}^{2}=\mathrm{R}^{3} \mathrm{P}$
(D) $\mathrm{R}=\mathrm{P}^{2} \mathrm{Q}^{2}$
Q. 59 Which of the following options is the closest in the meaning to the word below: Inexplicable
(A) Incomprehensible
(B) Indelible
(C) Inextricable
(D) Infallible
Q. 60 Choose the word from the options given below that is most nearly opposite in meaning to the given word:
Amalgamate
(A) merge
(B) split
(C) collect
(D) separate

## Q. 61 to Q. 65 carry two marks each.

Q. 61 A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4 th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day?
(A) 4
(B) 5
(C) 6
(D) 7
Q. 62 The variable cost $(\mathrm{V})$ of manufacturing a product varies according to the equation $\mathrm{V}=4 \mathrm{q}$, where q is the quantity produced. The fixed cost ( F ) of production of same product reduces with q according to the equation $\mathrm{F}=100 / \mathrm{q}$. How many units should be produced to minimize the total $\operatorname{cost}(\mathrm{V}+\mathrm{F})$ ?
Q. 63 P, Q, R and S are four types of dangerous microbes recently found in a human habitat. Th each circle with its diameter printed in brackets represents the growth of a single microbe sur human immunity system within 24 hours of entering the body. The danger to human beings van proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below:


A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?
(A) P
(B) Q
(C) R
(D) S
Q. 64 Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.

Based on the above passage which topic would not be included in a unit on bereavement?
(A) how to write a letter of condolence
(B) what emotional stages are passed through in the healing process
(C) what the leading causes of death are
(D) how to give support to a grieving friend
Q. 65 A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container?
(A) 7.58 litres
(B) 7.84 litres
(C) 7 litres
(D) 7.29 litres

GATE 2011 - Answer Key - Paper : PH

| Paper | Question no. | Key |
| :---: | :---: | :---: |
| PH | 1 | C |
| PH | 2 | A |
| PH | 3 | B |
| PH | 4 | D |
| PH | 5 | B |
| PH | 6 | C |
| PH | 7 | A |
| PH | 8 | B |
| PH | 9 | D |
| PH | 10 | B |
| PH | 11 | D |
| PH | 12 | C |
| PH | 13 | B |
| PH | 14 | C |
| PH | 15 | D |
| PH | 16 | A |
| PH | 17 | A |
| PH | 18 | D |
| PH | 19 | C |
| PH | 20 | D |
| PH | 21 | A |
| PH | 22 | B |
| PH | 23 | C |
| PH | 24 | C |
| PH | 25 | D |
| PH | 26 | D |
| PH | 27 | - |
| PH | 28 | A |
| PH | 29 | C |
| PH | 30 | A |
| PH | 31. | D |
| PH | 32 | C |
| PH | 33, | A |
| PH | 34 | C |
| PH | 35 | D |
| PH | 36 | B |
| PH | 37 | A |
| PH | 38 | D |
| PH | 39 | A |
| PH | 40 | B |


| Paper | Question no. | Key |
| :---: | :---: | :---: |
| PH | 41 | C |
| PH | 42 | D |
| PH | 43 | C |
| PH | 44 | B |
| PH | 45 | D |
| PH | 46 | C |
| PH | 47 | A |
| PH | 48 | D |
| PH | 49 | B |
| PH | 50 | A |
| PH | 51 | B |
| PH | 52 | C |
| PH | 53 | B |
| PH | 54 | A |
| PH | 55 | D |
| PH | 56 | B |
| PH | 57 | C |
| PH | 58 | B |
| PH | 59 | A |
| PH | 60 | D |
| PH | 61 | C |
| PH | 62 | A |
| PH | 63 | D |
| PH | 64 | C |
| PH | 65 | D |

