## ONE MARKS QUESTIONS (1-20)

1. The value of the contour integral, $\left|\int_{C} \vec{r} \times d \vec{\theta}\right|$, for a circle C of radius r with center at the origin r
(a.) $2 \pi r$
(b.) $r^{2} / 2$
(c.) $\pi r^{2}$
(d.) $r$
2. An electrostatic field $\vec{E}$ exists in a given region R. Choose the WRONG statement.
(a.) Circulation of $\vec{E}$ is zero
(b.) $\vec{E}$ can alway se expressed as the gradient of a scalar field
(c.) The potential difference between any two arbitrary points in the region R is zero
(d.) The work done in a closed path lying entirely in $R$ is zero
3. The Lagrangian of a flee particle in spherical polar co-ordinates is given by $L=\frac{1}{2} m\left(\dot{r}^{2}+r^{2} \dot{\theta}^{2}+r^{2} \dot{\phi}^{2} \sin ^{2} \theta\right)$. The quantity that is conserved is
(a.) $\frac{\partial L}{\partial \dot{r}}$
(b.) $\frac{\partial L}{\partial \dot{\theta}}$
(c.) $\frac{\partial L}{\partial \dot{\phi}}$
(d.) $\frac{\partial L}{\partial \dot{\phi}}+\dot{r} \dot{\theta}$
4. A conducting loop L of surface area S is moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}(\vec{r}, t)=\vec{B}_{0} t^{2}, B_{0}$
(a.) $-\int_{S} \frac{\partial \vec{B}}{\partial t} d \vec{S}$
(b.) $\oint_{L}(\vec{v} \times \vec{B}) \cdot d \vec{L}$
(d.) $-\oint_{S} \frac{\partial \vec{B}}{\partial t} \cdot d \vec{S}+\oint_{L}(\vec{v} \times \vec{B}) \cdot d \vec{L}$
5. The eigenvalues of the matrix $\mathrm{A}=\left(\begin{array}{ll}0 & i \\ i & 0\end{array}\right)$ are
(a.) real and distinct
(b.) complex and distinct
(c.) complex and coinciding
(d.) real and coinciding
6. $\quad \sigma_{i}(i=1,2,3)$ represent the Pauli spin matrices. Which one of the following is NOT true?
(a.) $\sigma_{\mathrm{i}} \sigma_{\mathrm{j}}+\sigma_{\mathrm{j}} \sigma_{\mathrm{i}}=2 \delta_{\mathrm{ij}}$
(b.) $\operatorname{Tr}\left(\sigma_{\mathbf{i}}\right)=0$
(c.) The givenvalues of $\sigma_{\mathrm{i}}$ are $\pm 1$
(d.) $\operatorname{det}\left(\sigma_{\mathrm{i}}\right)=1$
7. Which one of the functions given below represents the bound state eigenfunction of the operator $-\frac{d^{2}}{d x^{2}}$ in the region, $0 \leq \mathrm{x}<\infty$, with the givenvalue -4 ?
(a.) $A_{0} e^{2 x}$
(b.) $A_{0} \cosh 2 x$
(c.) $A_{o} e^{-2 x}$
(d.) $A_{0} \sinh 2 x$
8. Pick the WRONG statement.
(a.) The nuclear force is independent of electric charge
(b.) The Yukawa potential is proportional to $\mathrm{r}^{-1} \exp \left[\frac{m c}{h} r\right]$, where r is the separation between two nucleons
(c.) The range of nuclear force is of the order of $10^{-15} \mathrm{~m}-10^{-14} \mathrm{~m}$
(d.) The nucleons interact among each other by the exchange of mesons
9. If p and q are the position and momentum variables, which one of the following is NOT a canonical transformation?
(a.) $\mathrm{Q} \alpha \mathrm{q}$ and $\mathrm{P}=\frac{1}{\alpha} \mathrm{p}$, for $\alpha \neq 0$
(b.) $Q=\alpha q+\beta p$ and $P=\beta q+\alpha p$ for $\alpha, \beta$ real and $\alpha^{2}-\beta^{2}=1$
(c.) $\mathrm{Q}=\mathrm{p}$ and $\mathrm{P}=\mathrm{q}$
(d.) $Q=p$ and $P=-q$
(a.) 10
(b.) 0.1
(c.) 30 dB
(d.) 10 dB
10. In an insulating solid which one of the following physical phenomena is a consequen exclusion principle?
(a.) Ionic conductivity
(b.)Ferromagnetism
(c.) Paramagnetism
(d.)Ferroelectricity
11. Which one of the following curves gives the solution of the differential equation $k_{1} \frac{d x}{d t}+k_{2} x=k_{3}$, , where $\mathrm{k}_{1}, \mathrm{k}_{2}$ and $\mathrm{k}_{3}$ are positive constants with initial conditions $\mathrm{x}=0$ at $\mathrm{t}=0$ ?
(a.)

(b.)

(c.)

(d.)

12. Identify which one is a first order phase transition?
(a.) A liquid to gas transition at its critical temperature.
(b.) A liquid to gas transition close to its triple point.
(c.) A paramagnetic to ferromagnetic transition in the absence of a magnetic field.
(d.) A metal to superconductor transition in the absence of a magnetic field.
13. Group I lists some physical phenomena while Group II gives some physical parameters. Match the phenomena with the corresponding parameter.

## Group I

F. Doppler Broadening
Q. Natural Broadening
R. Rotational spectrum
S. Total internal reflection

Group II

1. Moment of inertia
2. Refractive index
3. Lifetime of the en ergy level
4. Pressure
(a.) P-4, Q-3, R-1, S-2
(b.) P-3, Q-2, R-1, S-4
(c.) P-2, Q-3, R-4, S-1
(d.) P-1, Q-4, R-2, S-3
5. The separation between the first Stokes and corresponding anti-Stokes lines of the rotational Raman spectrum in terms of the rotational constant, $B$ is
(a.) 2 B
(b.) 4 B
(c.) 6 B
(d.) 12 B
6. A superconducting ring is cooled in the presence of a magnetic field below its critical temperature $\left(\mathrm{T}_{\mathrm{C}}\right)$. The total magnetic flux that passes through the ring is
(b.) $n \frac{h}{2 e}$
(c.) $\frac{n h}{4 \pi e}$
(d.) $\frac{n e^{2}}{h c}$
7. In a cubic crystal, atoms of mass $\mathrm{M}_{1}$ lie on one set of planes and atoms of mass $\mathrm{M}_{2}$ lie on p interleaved between those of the first set. If C is the force constant between nearest neighbour plan the frequency of lattice vibrations for the optical phonon branch with wavevector $\mathrm{k}=0$ is
(a.) $\sqrt{2 C\left(\frac{1}{M_{1}}+\frac{1}{M_{2}}\right)}$
(b.) $\sqrt{C\left(\frac{1}{2 M_{1}}+\frac{1}{M_{2}}\right)}$
(c.) $\sqrt{C\left(\frac{1}{M_{1}}+\frac{1}{2 M_{2}}\right)}$
(d.) 0
8. In the quark model which one of the following represents a proton?
(a.) udd
(b.) uud
(c.) $\mathrm{u} \bar{b}$
(d.) $\bar{c}$
9. 



The circuit shown above
(a.) is a common-emitter amplifier
(b.)uses a pnp transistor
(c.) is an oscillator
(d.) has a voltage gain less than one
(a.) $\mathrm{Nm}_{\mathrm{n}}+\mathrm{Z} \mathrm{m}_{\mathrm{p}}$
(b.) $\mathrm{Nm}_{\mathrm{p}}+\mathrm{Zm}_{\mathrm{n}}$
(c.) $\mathrm{Nm}_{\mathrm{n}}+\mathrm{Zm}_{\mathrm{p}}+\frac{B E}{c^{2}}$
(d.) $\mathrm{Nm}_{\mathrm{p}}+\mathrm{Zm}_{\mathrm{n}}+\frac{B E}{c^{2}}$

## TWO MARKS QUESTIONS (21-60)

21. The magnetic field (in $\mathrm{A} \mathrm{m}^{-1}$ ) inside along solid cylindrical conductor of radius $\mathrm{a}=0.1 \mathrm{~m}$ is, $\vec{H}=\frac{10^{4}}{r}\left[\frac{1}{\alpha^{2}} \sin (\alpha r)-\frac{r}{\alpha} \cos (\alpha r)\right] \hat{\phi}$, where $\alpha=\frac{\pi}{2 a}$. What is the total current (n A) in the conductor?
(a.) $\frac{\pi}{2 a}$
(b.) $\frac{800}{\pi}$
(c.) $\frac{400}{\pi}$
(d.) $\frac{300}{\pi}$
22. Which one of the following current densities, $\vec{J}$, can generate the magnetic vector potential $\vec{A}=\left(y^{2} \hat{i}+x^{2} \hat{j}\right)$ ?
(a.) $\frac{2}{\mu_{0}}(x \hat{i}+y \hat{j})$
(b.) $-\frac{2}{\mu_{0}}(\hat{i}+\hat{j})$
(c.) $\frac{2}{\mu_{0}}(\hat{i}-\hat{j})$
(d.) $\frac{2}{\mu_{0}}(x \hat{i}-y \hat{j})$
23. The value of the integral $\int_{C} \frac{e^{z}}{z^{2}-3 z+2} d z$, where the contour C is the circle $|z|=3 / 2$ is
(d.) $-\pi i e$
24. a non-conducting medium characterized by $\varepsilon=\varepsilon_{0}, \mu=\mu_{0}$ and conductivity in $\left(\mathrm{Vm}^{-1}\right)$ is given by $\vec{E}=20 \sin \left[10^{8} t-k z\right] \hat{j}$. The magnetic field, $\vec{H}\left(\operatorname{in~} A m^{-1}\right)$,
(a.) $20 k \cos \left[10^{8} t-k z\right] \hat{i}$
(b.) $\frac{20 k}{10^{8} \mu_{0}} \sin \left[10^{8} t-k z\right] \hat{j}$
(c.) $-\frac{20 k}{10^{8} \mu_{0}} \sin \left[10^{8} t-k z\right] \hat{i}$
(d.) $-20 k \cos \left[10^{8} t-k z\right] \hat{i}$
25. A cylindrical rod of length $L$ and radius $r$, made of an inhomogeneous dielectric, is placed with its axis along the $z$ direction with one end at the origin as shown below.


If the rod carries a polarization, $\vec{P}=\left(5 z^{2}+7\right) \hat{k}$, the volume bound charge inside the diel ectric is
(a.) Zero
(b.) $10 \pi r^{2} L$
(c.) $-5 \pi r^{2} L$
(d.) $-5 \pi r^{2} L^{2}$
26. Let $T_{i j}=\sum_{k} \varepsilon_{i j k} a_{k}$ and $\beta_{k}=\sum_{i, j} \varepsilon_{i j k} T_{i j}$, where $\varepsilon_{i j \mathrm{k}}$ is the Levi-Cavita density, defined to be zero if two of the indices coincide and +1 and -1 depending on whether ijk is even or odd permutation of $1,2,3$. Then $\beta_{3}$ is equal to
(a.) $2 \mathrm{a}_{3}$
(b.) $-2 a_{3}$
(c.) $a_{3}$
(d.) $-a_{3}$
27. The dependence of the magnetic susceptibility ( $\chi$ ) of a material with temperature (I) can be represented by $\chi \propto \frac{1}{T-\theta}$, Where $\theta$ is the Curie-Weiss temperature. The plot of magnetic susceptibility versus temperature is sketched in the figure, as curves $P, Q$ and $R$ with curve $Q$ having ( $\theta=0$. Which one of the following statements is correct?

(a.) Curve R represents a paramagnet and Q a ferromagnet
(b.) Curve Q represents a ferromagnet and P an antiferromagnet
(c.) Curve R represents an antiferromagnet arid Q a paramagnet
(d.) Curve R represents an antiferromagnet and Q a ferromagnet
28. The dielectric constant of a material at optical frequencies is mainly clue to
(a.) ionic polarizability
(b.)electronic p olarizability
(c.) dip olar polarizability
(d.)ionic and dip olar polarizability
29. An electron of wave vector $\vec{k}_{e}$, velocity $\vec{v}_{e}$ and effective mass $\mathrm{m}_{\mathrm{e}}$ is removed from a filled energy band. The resulting hole has wave vector $\vec{k}_{h}$, velocity $\vec{v}_{h}$, and effective mass $m_{h}$. Which one of the following statements is correct?
(a.) $\vec{k}_{h}=\vec{k}_{e} ; \vec{v}_{h}=-\vec{v}_{e} ; m_{h}=-m_{e}$
(b.) $\vec{k}_{h}=\vec{k}_{e} ; \vec{v}_{h}=\vec{v}_{e} ; m_{h}=m_{e}$
(c.) $\vec{k}_{h}=\vec{k}_{e} ; \vec{v}_{h}=-\vec{v}_{e} ; m_{h}=-m_{e}$
(d.) $\vec{k}_{h}=-\vec{k}_{e} ; \vec{v}_{h}=\vec{v}_{e} ; m_{h}=-m_{e}$
30. In a diatomic molecule, the internuclear separation of the ground and first excited electronic state are the same as shown in the figure. If the molecule is initially in the lowest vibrational state of the ground state, then the absorption spectrum will appear as


(b.)

(c.)

(d.)

31. Five energy levels of a sy stem including the ground state are shown below. Their lifetimes and the allowed electric dipole transitions are also marked.


Which one of the following transitions is the most suitable the a continuous wave (CW) laser?
(a.) $1 \rightarrow 0$
(b.) $2 \rightarrow 0$
(c.) $4 \rightarrow 2$
(d.) $4 \rightarrow 3$
32. Assuming the mean life time of a muon (in its rest frame) to be $2 \times 10^{-6} \mathrm{~s}$, its life time in the laboratory frame, when it is moving with a velocity 095 c is
(a.) $6.4 \times 10^{-6} \mathrm{~s}$
(b.) $0.62 \times 10^{-6} \mathrm{~s}$
(c.) $2.16 \times 10^{-6} \mathrm{~s}$
(d) $\cap 10$ v $1 n^{-6}$ c
(a.) 10 lines
(b.) 4 v
(c.) 6 lines
(d.) 14 lines
34. The probability that an energy level C at a temperature T is unoecunied by a fermion potential $\mu$ is given by
(a.) $\frac{1}{e^{(\varepsilon-\mu) / k_{B} T}+1}$
(b.) $\frac{1}{e^{(\varepsilon-\mu) / k_{s} T}-1}$
(c.) $\frac{1}{e^{(\mu-\varepsilon) / k_{B} T}+1}$
(d.) $\frac{1}{e^{(\mu-\varepsilon) / k_{B} T}-1}$
35. Consider the following expression for the mass of a nucleus with Z protons and A nucleons:
$M(A, Z)=\frac{1}{c^{2}}\left(f(A)+y Z+z Z^{2}\right)$. Here $f(A)$ is a function of A ,
$y=-4 a_{A}$,
$z=a_{c} A^{-1 / 3}+4 a_{A} A^{-1}$,
$\mathrm{a}_{\mathrm{A}}$ and $\mathrm{a}_{\mathrm{c}}$ are constants of suitable dimensions. For a fixed A , the expression of Z for the most stable nucleus is
(a.) $Z=\frac{A / 2}{1+\left(\frac{a_{c}}{a_{A}}\right) A^{2 / 3}}$
(b.) $Z=\frac{A / 2}{1+\left(\frac{a_{c}}{4 a_{A}}\right) A^{2 / 3}}$
(c.) $Z=\frac{A}{1+\left(\frac{a_{c}}{4 a_{A}}\right) A^{2 / 3}}$
(d.) $Z=1+A^{2 / 3}$
36. The de Broglie wavelength of particles of mass $m$ with average momentum $p$ at a temperature T in three dimensions is given by
(a.) $\lambda=\frac{h}{\sqrt[m]{m}}$
(b.) $\lambda=\frac{h}{\sqrt{3 m k_{B} T}}$
(c.) $\lambda=\frac{h}{\sqrt{2 k_{B} T}}$
(d.) $\lambda=\frac{h}{\sqrt{3 m}}$


Assuming an ideal voltage source, Thevenin's resistance and Thevenin's voltage respectively for the above circuit are
(a.) $15 \Omega$ and 7.5 V
(b.) $20 \Omega$ and 5 V
(c.) $10 \Omega$ and 10 V
(d.) $30 \Omega$ and 15 V
38. Let $|\mathrm{n}\rangle$ and $|\mathrm{p}\rangle$ denote the isospin states with $\mathrm{I}=1 / 2=\mathrm{I}_{3}=1 / 2$ and $\mathrm{I}=1 / 2$, and $\mathrm{I}_{3}=-1 / 2$ of a nucleon respectively. Which one of the following two-nucelon states has $\mathrm{I}=0, \mathrm{I}_{3}=0$ ?
(a.) $\frac{1}{\sqrt{2}}(|n n\rangle-|p p\rangle)$
(b.) $\frac{1}{\sqrt{2}}(|n \mathrm{n}\rangle+|\mathrm{pp}\rangle)$
(c.) $\frac{1}{\sqrt{2}}(|n p\rangle-|p n\rangle)$
(d.) $\frac{1}{\sqrt{2}}(|n p\rangle-|p n\rangle)$
39. An amplifier of gain 1000 is made into a feedback amplifier by feeding $9.9 \%$ of its output voltage in series with the input opposing. If $f_{L}=20 \mathrm{~Hz}$ and $f_{H}=200 \mathrm{kHz}$ for the amplifier without feedback, then due to the feedback
(a.) the gain decreases by 10 times
$\qquad$


Pick the correct statement based on the above circuit.
(a.) The maximum Zener current, $\mathrm{I}_{\mathrm{Z}(\max )}$, when $\mathrm{R}_{\mathrm{L}}=10 \mathrm{k} \Omega$ is 15 mA
(b.) The minimum Zener current, $\mathrm{I}_{\mathrm{Z}(\min )}$, when $\mathrm{R}_{\mathrm{L}} 10 \mathrm{k} \Omega$ is 5 mA
(c.) With $\mathrm{V}_{\mathrm{in}},=20 \mathrm{~V}, \mathrm{I}_{\mathrm{L}}=\mathrm{Iz}$, when $\mathrm{R}_{\mathrm{L}}=2 \mathrm{k} \Omega$
(d.) The power dissipated across the Zener when $R_{L}=10 \mathrm{k} \Omega$ and $V_{\text {in }}=20 \mathrm{~V}$ is 100 mW
41. The disintegration energy is defined to be the difference in the rest energy between the initial and final states. Consider the following process:
${ }_{94}^{240} \mathrm{Pu} \rightarrow{ }_{92}^{236} \mathrm{U}+{ }_{2}^{4} \mathrm{He}$.
The emitted $\alpha$ particle has a kinetic en ergy 5.17 MeV . The value of the disintegration energy is
(a.) 5.26 MeV
(b.) 5.17 MeV
(c.) 5.08 MeV
(d.) 2.59 MeV
42. A classical particle is moving in an external potential field $V(x, y, z)$ which is invariant under the following infinitesimal transformations
$x \rightarrow x^{\prime}=x+\partial x$,
$y \rightarrow y^{\prime}=y+\partial y$,
$\binom{x}{y} \rightarrow\binom{x^{\prime}}{y^{\prime}}=R z\binom{x}{y}$,
Where $R_{Z}$ is the matrix corresponding to rotation about the $z$ axis. The conserved quantities are (the sy mbols have their usual meaning)
(a.) $\mathrm{p}_{\mathrm{x}}, \mathrm{p}_{\mathrm{z}}, \mathrm{L}_{\mathrm{z}}$
(b.) $p_{x}, p_{y}, L_{z}, E$
(c.) $p_{y}, L_{z}, E$
(d.) $p_{y}, p_{z}, L_{x}, E$
43. The spin function of a free particle, in the $b$ asis in which $S_{Z}$ is is diagonal, $\binom{1}{1}$ with eigenvalues $++\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively. In the given basis, the norman of $\mathrm{S}_{\mathbf{y}}$, with eigenvalue $-\frac{\hbar}{2}$
(a.) $\frac{1}{\sqrt{2}}\binom{1}{i}$
(b.) $\frac{1}{\sqrt{2}}\binom{0}{i}$
(c.) $\frac{1}{\sqrt{2}}\binom{i}{0}$
(d.) $\frac{1}{\sqrt{2}}\binom{i}{1}$
44. $\quad \hat{A}$ and $\hat{B}$ represent two physical characteristics of a quantum system. If $\hat{A}$ is Hermitian, then for the product $\hat{A} \hat{B}$ to be Hermitian, it is sufficient that
(a.) $\hat{B}$ is Hermitian
(b.) $\hat{B}$ is anti-Hermitian
(c.) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ commute
(d.) $\hat{B}$ is Hermitian and $\hat{A}$ and $\hat{B}$ anti-commute
45. Consider the set of vectors in three-dimensional real vector space
$\Re^{3}, S=\{(1,1,1),(1,-1,1),(1,1,-1)\}$.
Which one of the following statements is true?
(a.) S is not a linearly independent set.
(b.) S is a basis for $\mathbf{R}^{3}$.
(c.) The vectors in S are orthogonal.
(d.) An orthogonal set of vectors cannot be generated from S .
46. For a Fermi gas of N particles in three dimensions at $\mathrm{T}=0 \mathrm{~K}$, the Fermi energy, $\mathrm{E}_{\mathrm{F}}$ is proportional to
(a.) $\mathrm{N}^{2 / 3}$
(b.) $\mathrm{N}^{3 / 2}$
(c.) $\mathrm{N}^{3}$
(d.) $\mathrm{N}^{2}$
(a.) $\pm\left(\frac{k}{m}\right)^{1 / 2}$
(b.) $\pm\left(\frac{k}{m}\right)^{1 / 4}$
(c.) $\pm\left(\frac{k}{2 m}\right)^{1 / 4}$
(d.) $\pm\left(\frac{k}{2 m}\right)^{1 / 2}$
48. A particle is in the normalized state $|\psi\rangle$ which is is a superposition of the energy eigenstates $\mid \mathrm{E}_{\mathrm{o}}=$ $10 \mathrm{eV}\rangle$ and $\left|\mathrm{E}_{1}=30 \mathrm{eV}\right\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is given by
(a.) $\frac{1}{2}\left|E_{o}=10 \mathrm{eV}\right\rangle+\frac{\sqrt{3}}{4}\left|E_{1}=30 \mathrm{eV}\right\rangle$
(b.) $\frac{1}{\sqrt{3}}\left|E_{o}=10 \mathrm{eV}\right\rangle+\sqrt{\frac{2}{3}}\left|E_{1}=30 \mathrm{eV}\right\rangle$
(c.) $\frac{1}{2}\left|E_{o}=10 \mathrm{eV}\right\rangle-\frac{\sqrt{3}}{4}\left|E_{1}=30 \mathrm{eV}\right\rangle$
(d.) $\frac{1}{\sqrt{2}}\left|E_{o}=10 \mathrm{eV}\right\rangle-\frac{1}{\sqrt{2}}\left|E_{1}=30 \mathrm{eV}\right\rangle$
49. The Lagrangian of a particle of mass $m$ moving in one dimension is $L=\exp (\alpha \mathrm{t})\left[\frac{m \dot{x}^{2}}{2}-\frac{k x^{2}}{2}\right]$, where $\alpha$ and k are positive constants. The equation of motion of the particle is
(a.) $\ddot{x}+\alpha \dot{x}=0$
(b.) $\ddot{x}+\frac{k}{m} x=0$
(c.) $\ddot{x}-\alpha \dot{x}+\frac{k}{m} x=0$
(d.) $\ddot{x}+\alpha \dot{x}+\frac{k}{m} x=0$
50. Two monochromatic waves having frequencies $\omega$ and $\omega+\Delta \omega(\Delta \omega \ll \omega)$ and corresponding wavelengths $\lambda$ and $\lambda-\Delta \lambda(\Delta \lambda \ll \lambda)$ of same polarization, traveling along $x$-axis are superimposed on each other. The phase velocity and group velocity of the resultant wave are respectively given by
(a.) $\frac{\omega \lambda}{2 \pi}, \frac{\Delta \omega \lambda^{2}}{2 \pi \lambda \lambda}$
.. $\Delta \omega \lambda^{2}$
(c.) $\frac{\omega \Delta \lambda}{2 \pi}, \frac{\Delta \omega \Delta \lambda^{2}}{2 \pi}$
(d.) $\omega \Delta \lambda, \omega \Delta \lambda$

## Common Data Questions

## Common Data for Questions 51 and 52:

Consider a two level quantum sy stem with energies $\varepsilon_{1}=0$ and $\varepsilon_{2}=\varepsilon$.
51. The Helmholtz free energy of the sy stem is given by
(a.) $-k_{B} T \ln \left(1+e^{-\varepsilon / k_{B} T}\right)$
(b.) $k_{B} T \ln \left(1+e^{-\varepsilon / k_{B} T}\right)$
(c.) $\frac{3}{2} k_{B} T$
(d.) $\varepsilon-k_{B} T$
52. The specific heat of the sy stem is given by
(a.) $\frac{\varepsilon}{k_{B} T} \frac{e^{-\varepsilon / k_{B} T}}{\left(1+e^{-\varepsilon / k_{B} T}\right)^{2}}$
(b.) $\frac{\varepsilon^{2}}{k_{B} T^{2}} \frac{e^{-\varepsilon / k_{B} T}}{\left(1+e^{-\varepsilon / k_{B} T}\right)}$
(c.) $\frac{\varepsilon^{2} e^{-\varepsilon / k_{B} T}}{\left(1+e^{-\varepsilon / k_{B} T}\right)^{2}}$
(d.) $\frac{\varepsilon^{2}}{k_{B} T^{2}} \frac{e^{-\varepsilon / k_{B} T}}{\left(1+e^{-\varepsilon / k_{B} T}\right)^{2}}$

## Common Data for Questions 53 and 54:

A free particle of mass moves along the $x$ direction. At $t=0$, the normalized wave function of the particle is given by $\psi(x, 0)=\frac{1}{(2 \pi \alpha)^{1 / 4}} \exp \left[-\frac{x^{2}}{4 \alpha^{2}}+i x\right]$, where $\alpha$ is real constant.
(b.) $\hbar \sqrt{\alpha}$
(c.) $\alpha$
(d.) $\hbar / \sqrt{\alpha}$
54. The expectation value of the particle energy is
(a.) $\frac{\hbar}{2 m} \frac{1}{2 \alpha^{3 / 2}}$
(b.) $\frac{\hbar}{2 m} \alpha^{2}$
(c.) $\frac{\hbar^{2}}{2 m} \frac{4 \alpha^{2}+1}{4 \alpha^{3 / 2}}$
(d.) $\frac{\hbar^{2}}{8 m \alpha^{3 / 2}}$

## Common Data for Questions 55 and 56:

Consider the Zeeman splitting of a single electron system for the $3 \mathrm{~d} \rightarrow 3$ pelectric dipole transition.
55. The Zeeman spectrum is
(a.) randomly polarized
(b.) only $\pi$ polarized
(c.) only $\sigma$ polarized
(d.) both $\pi$ and $\sigma$ polarized
56. The fine structure line having the longest wavelength will split into
(a.) 17 components
(b.) 10 components
(c.) 8 components
(d.) 4 components

## Linked Answer Questions

Statement for Linked Answer Questions 57 and 58:
The primitive translation vectors of the face centered cubic (fcc) lattice are

$$
\hat{a}_{1}=\frac{a}{2}(\hat{j}+\hat{k}) ; \hat{a}_{2}=\frac{a}{2}(\hat{i}+\hat{k}) ; \hat{a}_{3}=\frac{a}{2}(\hat{i}+\hat{j})
$$

$\hat{b}_{1}=\left(\frac{2 \pi}{a}\right)(-\hat{i}+\hat{j}+\hat{k}) ; \hat{b}_{2}=\left(\frac{2 \pi}{a}\right)(\hat{i}-\hat{j}+\hat{k}): \hat{b}_{3}=\left(\frac{2 \pi}{a}\right)(\hat{i}+\hat{j}-\hat{k})$
(b.)
$\hat{b}_{1}=\left(\frac{\pi}{a}\right)(-\hat{i}+\hat{j}+\hat{k}) ; \hat{b}_{2}=\left(\frac{\pi}{a}\right)(\hat{i}-\hat{j}+\hat{k}) ; \hat{b}_{3}=\left(\frac{\pi}{a}\right)(\hat{i}+\hat{j}-\hat{k})$
(c.)
$\hat{b}_{1}=\left(\frac{\pi}{2 a}\right)(-\hat{i}+\hat{j}+\hat{k}) ; \hat{b}_{2}=\left(\frac{\pi}{2 a}\right)(\hat{i}-\hat{j}+\hat{k}) ; \hat{b}_{3}=\left(\frac{\pi}{2 a}\right)(\hat{i}+\hat{j}-\hat{k})$
(d.)
$\hat{b}_{1}=\left(\frac{3 \pi}{a}\right)(-\hat{i}+\hat{j}+\hat{k}) ; \hat{b}_{2}=\left(\frac{3 \pi}{a}\right)(\hat{i}-\hat{j}+\hat{k}) ; \dot{b}_{3}=\left(\frac{3 \pi}{a}\right)(\hat{i}+\hat{j}-\hat{k})$
58. The volume of the primitive cell of the fcc reciprocal lattice is
(a.) $4\left(\frac{2 \pi}{a}\right)^{3}$
(b.) $4\left(\frac{\pi}{a}\right)^{3}$
(c.) $4\left(\frac{\pi}{2 a}\right)^{3}$
(d.) $4\left(\frac{3 \pi}{a}\right)^{3}$

## Statement for Linked Answer Questions 59 and 60:

The Kamaugh map of a logic circuit is shown below:

59. The minimized logic expression for the above map is
(a.) $Y=\bar{P} \bar{R}+\bar{Q}$
(b.) $Y=\bar{Q} \cdot P R$
60. The corresp onding logic implementation using gates is given as:
(a.)

(b.)

(c.)

(d.)


