# PH : PHYSICS

*Duration* : Three Hours *Maximum Marks* :150

**Read the following instructions carefully**

1. This question paper contains **24** printed pages including pages for rough work. Please check all pages and report discrepancy, if any.
2. Write your registration number, your name and name of the examination centre at the specified locations on the right half of the ORS.
3. Using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your paper code.
4. All the questions in this question paper are of objective type.
5. Questions must be answered on **Objective Response Sheet (ORS)** by darkening the appropriate bubble (marked A, B, C, D) using HB pencil against the question number on the left hand side of the ORS. **Each question has only one correct answer**. In case you wish to change an answer, erase the old answer completely. More than one answer bubbled against a question will be treated as a wrong answer.
6. Questions 1 through 20 are 1-mark questions and questions 21 through 85 are 2-mark questions.
7. Questions 71 through 73 is one set of common data questions, questions 74 and 75 is another pair of common data questions. The question pairs (76, 77), (78, 79), (80, 81), (82, 83) and (84, 85) are questions with linked answers. The answer to the second question of the above pairs will depend on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is un-attempted, then the answer to the second question in the pair will not be evaluated.
8. Un-attempted questions will carry zero marks.
9. **NEGATIVE MARKING**: For Q.1 to Q.20, ***0.25*** mark will be deducted for each wrong answer. For Q.21 to Q.75, ***0.5*** mark will be deducted for each wrong answer. For the pairs of questions with linked answers, there will be negative marks only for wrong answer to the first question, i.e. for Q.76, Q.78, Q.80, Q.82 and Q.84, ***0.5*** mark will be deducted for each wrong answer. There is no negative marking for Q.77, Q.79, Q.81, Q.83 and Q.85.
10. Calculator **without data connectivity** is allowed in the examination hall.
11. Charts, graph sheets and tables are NOT allowed in the examination hall.
12. Rough work can be done on the question paper itself. Additional blank pages are given at the end of the question paper for rough work.

**Q. 1 – Q. 20 carry one mark each.**

Q.1 For arbitrary matrices $E, F, G$ and $H$, if $EF - FE = 0$ then $Trace(EFGH)$ is equal to

(A) $Trace(HGFE)$
(B) $Trace(E).Trace(F).Trace(G).Trace(H)$
(C) $Trace(GFEH)$
(D) $Trace(EGHF)$

Q.2 An unitary matrix $\begin{pmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{pmatrix}$ is given, where $a, b, c, d, \alpha$ and $\beta$ are real. The inverse of the matrix is

(A) $\begin{pmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{pmatrix}$
(B) $\begin{pmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{pmatrix}$
(C) $\begin{pmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{pmatrix}$
(D) $\begin{pmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{pmatrix}$

Q.3 The curl of a vector field $\vec{F}$ is $2\hat{x}$. Identify the appropriate vector field $\vec{F}$ from the choices given below.

(A) $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$
(B) $\vec{F} = 3z\hat{y} + 5y\hat{z}$
(C) $\vec{F} = 3x\hat{y} + 5y\hat{z}$
(D) $\vec{F} = 2\hat{x} + 5y\hat{z}$

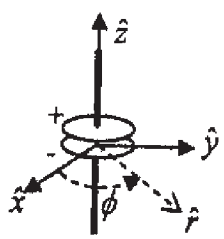
Q.4 A rigid body is rotating about its center of mass, fixed at the origin, with an angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$. If the torque acting on it is $\vec{\tau}$ and its angular momentum is $\vec{L}$, the rate of change of its kinetic energy is

(A) ½ $\vec{\tau}.\vec{\omega}$
(B) ½ $\vec{L}.\vec{\omega}$
(C) ½ $(\vec{\tau}.\vec{\omega} + \vec{L}.\vec{\alpha})$
(D) ½ $\vec{L}.\vec{\alpha}$

Q.5 A cylinder of mass $M$ and radius $R$ is rolling down without slipping on an inclined plane of angle of inclination $\alpha$. The number of generalized coordinates required to describe the motion of this system is

(A) 1
(B) 2
(C) 4
(D) 6

Q.6 A parallel plate capacitor is being discharged. What is the direction of the energy flow in terms of the Poynting vector in the space between the plates?



Q.7 Unpolarized light falls from air to a planar air-glass interface (refractive index of glass is 1.5) the reflected light is observed to be plane polarized. The polarization vector and the angle of incidence $\theta_i$ are

(A) perpendicular to the plane of incidence and $\theta_i = 42^\circ$.

(B) parallel to the plane of incidence and $\theta_i = 56^\circ$.

(C) perpendicular to the plane of incidence and $\theta_i = 56^\circ$.

(D) parallel to the plane of incidence and $\theta_i = 42^\circ$.

Q.8 A finite wave train, of an unspecified nature, propagates along the positive $x$ axis with a constant speed $v$ and without any change of shape. The differential equation among the four listed below, whose solution it must be, is

(A) $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,t) = 0$

(B) $\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t) = 0$

(C) $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - i\hbar\frac{\partial}{\partial t}\right)\psi(x,t) = 0$

(D) $\left(\nabla^2 + \frac{\partial}{\partial t}\right)\psi(\vec{r},t) = 0$

Q.9 Let $|\psi_0\rangle$ denote the ground state of the hydrogen atom. Choose the correct statement from those given below:

(A) $[L_x, L_y]\,|\psi_0\rangle = 0$

(B) $L^2\,|\psi_0\rangle = 0$

(C) $\vec{L}\cdot\vec{S}\,|\psi_0\rangle \neq 0$

(D) $[S_x, S_y]\,|\psi_0\rangle = 0$

Q.10 Thermodynamic variables of a system can be volume $V$, pressure $P$, temperature $T$, number of particles $N$, internal energy $E$ and chemical potential $\mu$, etc. For a system to be specified by Microcanonical (MC), Canonical (CE) and Grand Canonical (GC) ensembles, the parameters required for the respective ensembles are:

(A) MC: $(N,V,T)$; CE: $(E,V,N)$; GC: $(V,T,\mu)$

(B) MC: $(E,V,N)$; CE: $(N,V,T)$; GC: $(V,T,\mu)$

(C) MC: $(V,T,\mu)$; CE: $(N,V,T)$; GC: $(E,V,N)$

(D) MC: $(E,V,N)$; CE: $(V,T,\mu)$; GC: $(N,V,T)$

Q.11 The pressure versus temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid-to-solid transition region. The change in the specific volume remains constant in this region. The conclusion one can get from the above is

(A) the entropy of solid is zero in this temperature region.

(B) the entropy increases when the system goes from liquid to solid phase in this temperature region.

(C) the entropy decreases when the system transforms from liquid to solid phase in this region of temperature.

(D) the change in entropy is zero in the liquid-to-solid transition region.

Q.12 The radial wave function of the electrons in the state of $n$=1 and $l$=0 in a hydrogen atom is

Q.13 The last two terms of the electronic configuration of manganese (Mn) atom is $3d^5 4s^2$. [illegible] factor of $Mn^{4+}$ ion is

(A) $^4D_{1/2}$ (B) $^4F_{3/2}$ (C) $^3F_{9/2}$ (D) $^3D_{7/2}$

Q.14 The coherence length of laser light is

(A) directly proportional to the length of the active lasing medium.

(B) directly proportional to the width of the spectral line.

(C) inversely proportional to the width of the spectral line.

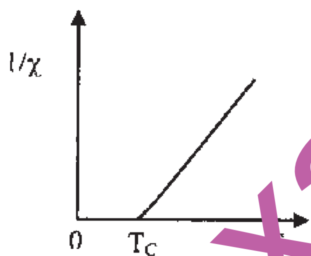
(D) inversely proportional to the length of the active lasing medium.

Q.15 Metallic monovalent sodium crystallizes in body centered cubic structure. If the length of the unit cell is $4 \times 10^{-8}$ cm, the concentration of conduction electrons in metallic sodium is

(A) $6.022 \times 10^{23}$ cm$^{-3}$ (B) $3.125 \times 10^{22}$ cm$^{-3}$ (C) 2.562 [illegible] cm$^{-3}$ (D) $1.250 \times 10^{20}$ cm$^{-3}$

Q.16 The plot of inverse magnetic susceptibility $1/\chi$ versus temperature $T$ of an antiferromagnetic sample corresponds to

(A)

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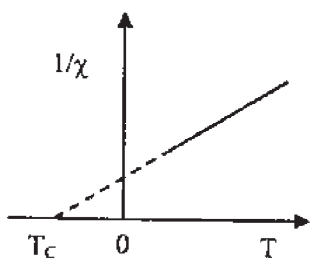


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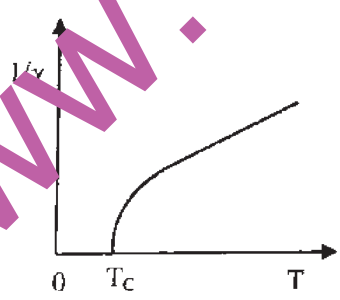


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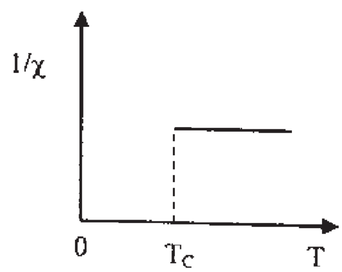



Q.17 According to the quark model, the $K^+$ meson is composed of the following quarks:

(A) u u d (B) u $\bar{c}$ (C) u $\bar{s}$ (D) s $\bar{u}$

Q.18 An $O^{16}$ nucleus is spherical and has a charge radius $R$ and a volume $V = \frac{4}{3}\pi R^3$. According to the

Q 19 A common emitter transistor amplifier circuit is operated under a fixed bias. In this operating point

(A) remains fixed with an increase in temperature.

(B) moves towards cut-off region with an increase in temperature.

(C) moves towards the saturation region with a decrease in temperature.

(D) moves towards the saturation region with an increase in temperature.

Q.20 Under normal operating conditions, the gate terminal of an $n$-channel junction field effect transistor (JFET) and an $n$-channel metal oxide semiconductor field effect transistor (MOSFET) are

(A) both biased with positive potentials.

(B) both biased with negative potentials.

(C) biased with positive and negative potentials, respectively.

(D) biased with negative and positive potentials, respectively.

**Q. 21 to Q.75 carry two marks each.**

Q 21 The eigenvalues of the matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ are

(A) $\frac{1}{2}\left(\sqrt{3} \pm i\right)$ when $\theta = 45^\circ$

(B) $\frac{1}{2}\left(\sqrt{3} \pm i\right)$ when $\theta = 30^\circ$

(C) $\pm 1$ since the matrix is unitary

(D) $\frac{1}{\sqrt{2}}(1 \pm i)$ when $\theta = 30^\circ$

Q.22 If the Fourier transform $F[\delta(x-a)] = \exp(-i2\pi\nu a)$, then $F^{-1}(\cos 2\pi a\nu)$ will correspond to

(A) $\delta(x-a)\delta(x+a)$

(B) a constant

(C) $\frac{1}{2}[\delta(x-a) + i\delta(x+a)]$

(D) $\frac{1}{2}[\delta(x-a) + \delta(x+a)]$

Q.23 If $I \equiv \oint_C dz\, Ln(z)$, where $C$ is the unit circle taken anticlockwise and $Ln(z)$ is the principal branch of the Logarithm function, which one of the following is correct?

(A) $I = 0$ by residue theorem

(B) $I$ is not defined since $Ln(z)$ has a branch cut.

(C) $I \neq 0$

(D) $\oint_C dz\, Ln(z^2) = 2I$

Q.25 Consider the Bessel equation $(\nu = 0)$, $\frac{d^2y}{dz^2} + \frac{1}{z}\frac{dy}{dz} + y = 0$. Which one of the following statements is correct?

(A) Equation has regular singular points at $z = 0$ and $z = \infty$.

(B) Equation has 2 linearly independent solutions that are entire.

(C) Equation has an entire solution and a second linearly independent solution singular at $z = 0$.

(D) Limit $z \to \infty$, taken along $x$ axis, exists for both the linearly independent solutions.

Q.26 Under a certain rotation of coordinate axes, a rank-1 tensor $v_a$ $(a=1,2,3)$ transforms according to the orthogonal transformation defined by the relations $v_1' = \frac{1}{\sqrt{2}}(v_1 + v_2)$ ; $v_2' = \frac{1}{\sqrt{2}}(-v_1 + v_2)$ ; $v_3' = v_3$. Under the same rotation a rank-2 tensor $T_{a,b}$ would transform such that

(A) $T'_{1,1} = T_{1,1}T_{1,2}$

(B) $T'_{1,1} = T_{1,1}$

(C) $T'_{1,1} = T_{1,1} + 2T_{2,2} - T_{2,1}$

(D) $T'_{1,1} = \frac{1}{2}(T_{1,1} + T_{2,2} + T_{1,2} + T_{2,1})$

Q.27 The Lagrangian of a system is given by $L = \frac{1}{2}\dot{q}^2 + q\dot{q} - \frac{1}{2}q^2$. It describes the motion of

(A) a harmonic oscillator.

(B) a damped harmonic oscillator.

(C) an anharmonic oscillator.

(D) a system with unbounded motion.

Q.28 The moment of inertia tensor of a rigid body is given by $I = \begin{pmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{pmatrix}$. The magnitude of the moment of inertia about an axis $\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ is

(A) 6

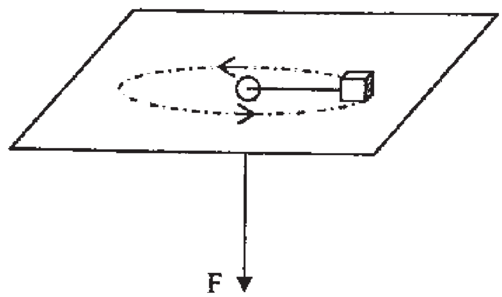
(B) 5

(C) 2

(D) 8/3

Q.29 A hoop of radius R is pivoted at a point on the circumference. The period of small oscillations in the plane of the hoop is

Q.30 A mass $m$ is constrained to move on a horizontal frictionless surface. It is set in circular motion with radius $r_0$ and angular speed $\omega_0$ by an applied force $\vec{F}$ communicated through an inextensible thread that passes through a hole on the surface as shown in the figure. This force is then suddenly doubled. The magnitude of the radial velocity of the mass




(A) increases till the mass falls into the hole.

(B) decreases till the mass falls into the hole.

(C) remains constant.

(D) becomes zero at a radius $r_1$ where $0 < r_1 < r_0$.

Q.31 For a simple harmonic oscillator the Lagrangian is given by $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$. If $A(p,q) = \frac{p + iq}{\sqrt{2}}$ and $H(p,q)$ is the Hamiltonian of the system, the Poisson bracket $\{A(p,q), H(p,q)\}$ is given by

(A) $iA(p,q)$ (B) $iA^*(p,q)$ (C) $-iA^*(p,q)$ (D) $-iA(p,q)$

Q.32 A plane electromagnetic wave is given by $E_0(\hat{x} + e^{i\delta}\hat{y})\exp\{i(kz - \omega t)\}$. At a given location, the number of times $\vec{E}$ vanishes in one second is

(A) An integer near $\frac{\omega}{\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, $n$ is integer

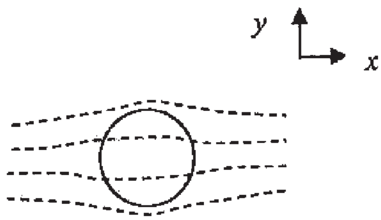
(B) An integer near $\frac{\omega}{\pi}$ and is independent of $\delta$

(C) An integer near $\frac{\omega}{2\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, $n$ is integer

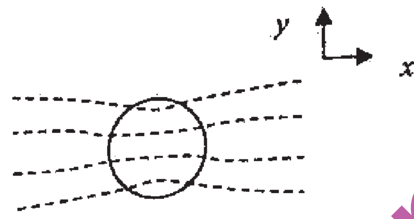
(D) An integer near $\frac{\omega}{2\pi}$ and is independent of $\delta$

Q.33 A dielectric sphere is placed in a uniform electric field directed along the positive y-axis. Which one of the following represents the correct equipotential surfaces?

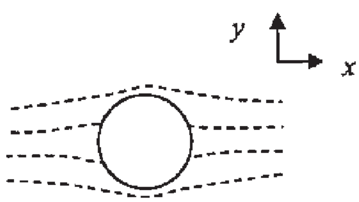
(A)



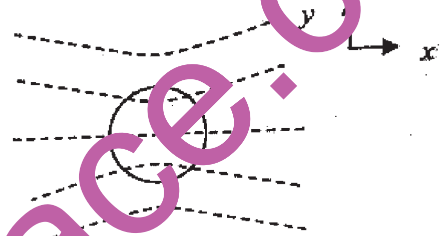

(B)



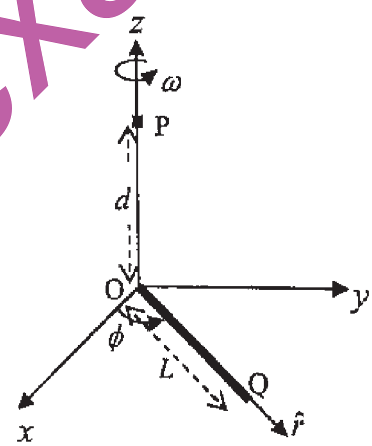

(C)




(D)




Q.34 A rod of length $L$ with uniform charge density $\lambda$ per unit length is in the $xy$-plane and rotating about $z$-axis passing through one of its edge with an angular velocity $\vec{\omega}$ as shown in the figure below. $(\hat{r},\hat{\phi},\hat{z})$ refer to the unit vectors at Q, $\vec{A}$ is the vector potential at a distance $d$ from the origin O along z-axis for $d >> L$ and $\vec{J}$ is the current density due to the motion of the rod. Which one of the following statements is correct?




(A) $\vec{J}$ along $\hat{r}$; $\vec{A}$ along $\hat{z}$; $|\vec{A}| \propto \dfrac{1}{d}$

(B) $\vec{J}$ along $\hat{\phi}$; $\vec{A}$ along $\hat{\phi}$; $|\vec{A}| \propto \dfrac{1}{d^2}$

(C) $\vec{J}$ along $\hat{r}$; $\vec{A}$ along $\hat{z}$; $|\vec{A}| \propto \dfrac{1}{d^2}$

(D) $\vec{J}$ along $\hat{\phi}$; $\vec{A}$ along $\hat{\phi}$; $|\vec{A}| \propto \dfrac{1}{d}$

Q.35 A circular disc of radius

Q.36 At time $t=0$, a charge distribution $\rho(\vec{r},0)$ exists within an ideal homogeneous conductor of permittivity $\varepsilon$ and conductivity $\sigma$. At a later time $\rho(\vec{r},t)$ is given by

(A) $\rho(\vec{r},t)=\rho(\vec{r},0)\exp\left(-\frac{\sigma t}{\varepsilon}\right)$

(B) $\rho(\vec{r},t)=\frac{\rho(\vec{r},0)}{1+(\sigma t/\varepsilon)^2}$

(C) $\rho(\vec{r},t)=\rho(\vec{r},0)\exp\left[-\left(\frac{\sigma t}{\varepsilon}\right)^2\right]$

(D) $\rho(\vec{r},t)=\rho(\vec{r},0)\frac{\varepsilon}{\sigma t}\sin\left(\frac{\sigma t}{\varepsilon}\right)$

Q.37 A nonrelativistic charged particle moves along the positive $x$-axis with a constant positive acceleration $a\hat{x}$. The particle is at the origin at $t=0$. Radiation is observed at $t=0$ at a distant point $(0,d,0)$ on the $y$-axis. Which one of the following statements is correct?

(A) The radiation is unpolarized.

(B) The radiation is plane polarized with polarization parallel to the $x$-axis.

(C) The radiation is plane polarized with polarization parallel to the $xy$-plane along a line inclined to the $x$ axis.

(D) The radiation is elliptically polarized.

Q.38 For a physical system, two observables $O_1$ and $O_2$ are known to be compatible. Choose the correct implication from amongst those given below:

(A) Every eigenstate of $O_1$ must necessarily be an eigenstate of $O_2$.

(B) Every non-degenerate eigenstate of $O_1$ must necessarily be an eigenstate of $O_2$.

(C) When an observation of $O_1$ is carried out on an arbitrary state $|\Psi\rangle$ of the physical system, a subsequent observation of $O_2$ leads to an unambiguous result.

(D) Observation of $O_1$ and $O_2$, carried out on an arbitrary state $|\Psi\rangle$ of the physical system, lead to the identical results irrespective of the order in which the observations are made.

Q.39 An exact measurement of the position of a simple harmonic oscillator (SHO) is made with the result $x=x_0$. [The SHO has energy levels $E_n$ ($n=0,1,2,\ldots$) and associated normalized wave-functions $\psi_n$]. Subsequently, an exact measurement of energy $E$ is made. Using the general notation $Pr(E=E')$ denoting the probability that a result $E'$ is obtained for this measurement, the following statements are written. Which one of the following statements is correct?

(A) $Pr(E=E_0)=0$

(B) $Pr(E=E_n)=1$ for *some* value of $n$.

(C) $Pr(E=E_n)\propto\psi_n(x)$

(D) $Pr(E>E'')>0$ for *any* $E''$.

Q.40 Consider the combined system of proton and electron in the hydrogen atom in its

Q.41 A particle is placed in a one dimensional box of size $L$ along the $x$-axis ($0 < x < L$). Wh following is true?

(A) In the ground state, the probability of finding the particle in the interval ($L/4, 3L/4$) is half.

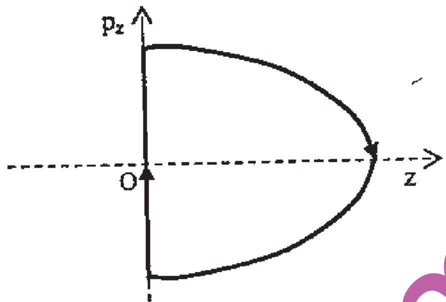
(B) In the first excited state, the probability of finding the particle in the interval ($L/4, 3L/4$) is half. This also holds for states with $n = 4, 6, 8, \ldots$

(C) For an arbitrary state $|\Psi\rangle$, the probability of finding the particle in the left half of the well is half.

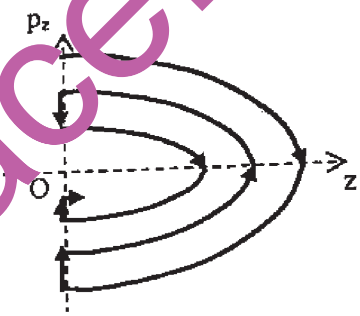
(D) In the ground state, the particle has a definite momentum.

Q.42 An inelastic ball of mass $m$ has been thrown vertically upwards from the ground at $z = 0$. The initial kinetic energy of the ball is $E$. The phase trajectory of the ball after successive bouncing on the ground is

(A)




(B)

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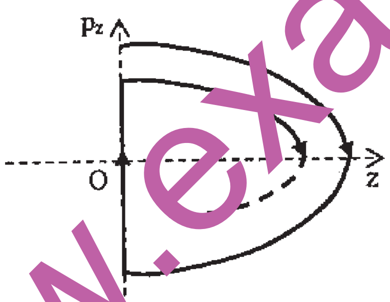


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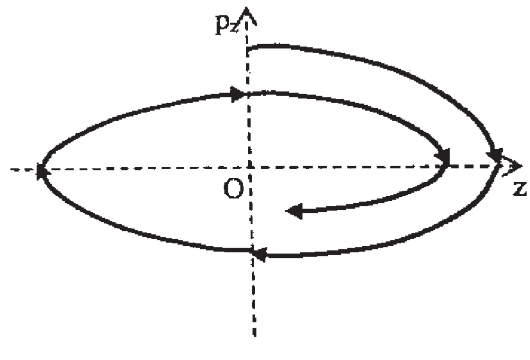



Q.43 A system containing $N$ non-interacting localized particles of spin ½ and magnetic moment $\mu$ each is kept in constant external magnetic field $B$ and in thermal equilibrium at temperature $T$. The magnetization of the system is,

(A) $N\mu \coth\left(\frac{\mu B}{k_B T}\right)$ (B) $N\mu \tanh\left(\frac{\mu B}{k_B T}\right)$

(C) $N\mu \sinh\left(\frac{\mu B}{k_B T}\right)$ (D) $N\mu \cosh\left(\frac{\mu B}{k_B T}\right)$

Q.44 Two identical particles have to be distributed among three energy levels. Let $r_B$, $r_F$ and $r_C$ represent the ratios of probability of finding two particles to that of fin

Q.45 A photon gas is at thermal equilibrium at temperature $T$. The mean number of photons in a state $\varepsilon = \hbar\omega$ is

(A) $\exp\left(\frac{\hbar\omega}{k_BT}\right)+1$ (B) $\exp\left(\frac{\hbar\omega}{k_BT}\right)-1$ (C) $\left(\exp\left(\frac{\hbar\omega}{k_BT}\right)+1\right)^{-1}$ (D) $\left(\exp\left(\frac{\hbar\omega}{k_BT}\right)-1\right)^{-1}$

Q.46 Consider a system of $N$ atoms of an ideal gas of type $A$ at temperature $T$ and volume $V$. It is kept in diffusive contact with another system of $N$ atoms of another ideal gas of type $B$ at the same temperature $T$ and volume $V$. Once the combined system reaches equilibrium,

(A) the total entropy of the final system is the same as the sum of the entropy of the individual system always.

(B) the entropy of mixing is $2Nk_B \ln 2$.

(C) the entropy of the final system is less than that of sum of the initial entropies of the two gases.

(D) the entropy of mixing is non-zero when the atoms $A$ and $B$ are of the same type.

Q.47 Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E = 0, \varepsilon$ and $2\varepsilon$ having degeneracies $g(E) = 1, 2$ and $4$ respectively. The mean energy of the system is

(A)
$$\varepsilon\left(\frac{4\exp\left(-\varepsilon/k_BT\right)+8\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)$$

(B)
$$\varepsilon\left(\frac{2\exp\left(-\varepsilon/k_BT\right)+8\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)$$

(C)
$$\varepsilon\left(\frac{2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}{1+2\exp\left(-\varepsilon/k_BT\right)+4\exp\left(-2\varepsilon/k_BT\right)}\right)^2$$

(D)
$$\varepsilon\left(\frac{\exp\left(-\varepsilon/k_BT\right)+2\exp\left(-2\varepsilon/k_BT\right)}{1+\exp\left(-\varepsilon/k_BT\right)+\exp\left(-2\varepsilon/k_BT\right)}\right)$$

Q.48 Three consecutive absorption lines at 64.275 $cm^{-1}$, 77.130 $cm^{-1}$ and 89.985 $cm^{-1}$ have been observed in a microwave spectrum for a linear rigid diatomic molecule. The moments of inertia $I_A$ and $I_B$ are ($I_A$ is with respect to the bond axis passing through the centre of mass and $I_B$ is with respect to an axis passing through the centre of mass and perpendicular to bond axis)

(A) both equal to $\frac{\hbar^2}{12.855hc}$ gm $cm^2$ (B) zero and $\frac{\hbar^2}{12.855hc}$ gm $cm^2$

(C) both equal to $\frac{\hbar^2}{6.427hc}$ gm $cm^2$ (D) zero and $\frac{\hbar^2}{6.427hc}$ gm $cm^2$

Q.49 A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic

Q.50 Which one of the following statement is **INCORRECT** in vibrational spectroscopy with anharmonicity?

(A) The selection rule for vibrational spectroscopy is $\Delta v = \pm 1, \pm 2, \ldots$

(B) Anharmonicity leads to multiple absorption lines.

(C) The intensities of hot band lines are stronger than the fundamental absorption.

(D) The frequencies of hot band lines are smaller than the fundamental absorption.

Q.51 The molecular spectra of two linear molecules O-C-O and O-C-S are recorded in the microwave region. Which one of the following statement is correct?

(A) Both the molecules would show absorption lines.

(B) Both the molecules would not show absorption lines.

(C) O-C-O would show absorption lines, but not O-C-S.

(D) O-C-S would show absorption lines, but not O-C-O.

Q.52 When the refractive index $\mu$ of the active medium changes by $\Delta\mu$ in a laser resonator of length $L$, the change in the spectral spacing between the longitudinal modes of the laser is ($c$ is the speed of light in free space)

(A) $\dfrac{c}{2(\mu+\Delta\mu)L}$. (B) $\dfrac{c}{2\Delta\mu L}$. (C) $\dfrac{c}{2L}\dfrac{\Delta\mu}{\mu(\mu+\Delta\mu)}$. (D) zero.

Q.53 The primitive translation vectors of the body centered cubic lattice are $\vec{a} = \frac{a}{2}(\hat{x}+\hat{y}-\hat{z})$, $\vec{b} = \frac{a}{2}(-\hat{x}+\hat{y}+\hat{z})$ and $\vec{c} = \frac{a}{2}(\hat{x}-\hat{y}+\hat{z})$. The primitive translation vectors $\vec{A}, \vec{B}$ and $\vec{C}$ of the reciprocal lattice are

(A) $\vec{A} = \frac{2\pi}{a}(\hat{x}-\hat{y})$; $\vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z})$; $\vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

(B) $\vec{A} = \frac{2\pi}{a}(\hat{x}+\hat{y})$; $\vec{B} = \frac{2\pi}{a}(\hat{y}-\hat{z})$; $\vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

(C) $\vec{A} = \frac{2\pi}{a}(\hat{x}+\hat{y})$; $\vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z})$; $\vec{C} = \frac{2\pi}{a}(\hat{x}-\hat{z})$

(D) $\vec{A} = \frac{2\pi}{a}(\hat{x}+\hat{y})$; $\vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z})$; $\vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

Q.54 The structure factor of a single cell of identical atoms of form factor $f$ is given by $S_{hkl} = f\sum_j \exp\left(-i2\pi\left(x_j h + y_j k + z_j l\right)\right)$ where $\left(x_j, y_j, z_j\right)$ is the coordinate of an atom, and $hkl$ are the Miller indices. Which one of the following statement is correct for the diffraction peaks of body centered cubic (BCC) and face centered cubic (FCC) lattices?

(A) BCC : (200); (110); (222)
FCC : (111); (311); (400)

(B) BCC : (210); (110); (222)
FCC : (111); (311); (400)

Q.55 The lattice specific heat $C$ of a crystalline solid can be obtained using the Dulong Petit, Einstein model and Debye model. At low temperature $\hbar\omega >> k_B T$, which one of the following statements is true ( $a$ and $A$ are constants)

(A) Dulong Petit : $C \propto \exp(-a/T)$ ; Einstein : $C =$ . constant ; Debye : $C \propto \left(\frac{T}{A}\right)^3$

(B) Dulong Petit : $C =$ . constant ; Einstein : $C \propto \left(\frac{T}{A}\right)^3$ ; Debye : $C \propto \exp(-a/T)$

(C) Dulong Petit : $C =$ . constant ; Einstein : $C \propto \frac{e^{-a/T}}{T^2}$ ; Debye : $C \propto \left(\frac{T}{A}\right)^3$

(D) Dulong Petit : $C \propto \left(\frac{T}{A}\right)^3$ ; Einstein : $C \propto \frac{e^{-a/T}}{T^2}$ ; Debye : $C =$ . constant

Q.56 A linear diatomic lattice of lattice constant $a$ with masses $M$ and $m$ $(M > m)$ are coupled by a force constant $C$. The dispersion relation is given by

$$\omega_{\pm}^2 = C\left(\frac{M+m}{Mm}\right) \pm \left[C^2\left(\frac{M+m}{Mm}\right)^2 - \frac{4C^2}{Mm}\sin^2\frac{ka}{2}\right]^{1/2}.$$

Which one of the following statements is **INCORRECT**?

(A) The atoms vibrating in transverse mode correspond to the optical branch.

(B) The maximum frequency of the acoustic branch depends on the mass of the lighter atom $m$.

(C) The dispersion of frequency in the optical branch is smaller than that in the acoustic branch.

(D) No normal modes exist in the acoustic branch for any frequency greater than the maximum frequency at $k = \pi/a$.

Q.57 The kinetic energy of a free electron at a corner of the first Brillouin zone of a two dimensional square lattice is larger than that of an electron at the mid-point of a side of the zone by a factor $b$. The value of $b$ is

(A) $b=\sqrt{2}$ (B) $b=2$ (C) $b=4$ (D) $b=8$

Q.58 An intrinsic semiconductor with mass of a hole $m_h$ and mass of an electron $m_e$ is at a finite temperature $T$. If the top of the valence band energy is $E_v$ and the bottom of the conduction band energy is $E_c$, the Fermi energy of the semiconductor is

(A) $E_F = \left(\frac{E_v + E_c}{2}\right) - \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$

(B) $E_F = \left(\frac{k_B T}{2}\right) + \frac{3}{4}(E_v + E_c)\ln\left(\frac{m_h}{m_e}\right)$

(C) $E_F = \left(\frac{E_v + E_c}{2}\right) + \frac{3}{4}k_B T \ln\left(\frac{m_h}{m_e}\right)$

(D) $E_F = \left(\frac{k_B T}{2}\right) - \frac{3}{4}(E_v + E_c)\ln\left(\frac{m_h}{m_e}\right)$

Q.59 Choose the correct statement from the following:

(A) The reaction $K^+K^- \rightarrow p\bar{p}$ can proceed irrespective of the kinetic energies of $K^+$ and $K^-$.

Q.60 The following gives a list of pairs containing (i) a nucleus (ii) one of its properties. Find which is **INAPPROPRIATE**.

(A) (i) $_{10}Ne^{20}$ nucleus; (ii) stable nucleus

(B) (i) A spheroidal nucleus; (ii) an electric quadrupole moment

(C) (i) $_8O^{16}$ nucleus; (ii) nuclear spin J=1/2

(D) (i) $U^{238}$ nucleus; (ii) Binding energy = 1785 MeV (approximately)

Q.61 The four possible configurations of neutrons in the ground state of $_4Be^9$ nucleus, according to the shell model, and the associated nuclear spin are listed below. Choose the correct one.

(A) $(1s_{1/2})^2(1p_{3/2})^3$; J=3/2

(B) $(1s_{1/2})^2(1p_{1/2})^2(1p_{3/2})^1$, J=3/2

(C) $(1s_{1/2})^1(1p_{3/2})^4$; J=1/2

(D) $(1s_{1/2})^2(1p_{3/2})^2(1p_{1/2})^1$, J=1/2

Q.62 The mass difference between the pair of mirror nuclei $_6C^{11}$ and $_5B^{11}$ is given to be $\Delta MeV/c^2$. According to the semi-empirical mass formula, the mass difference between the pair of mirror nuclei $_9F^{17}$ and $_8O^{17}$ will approximately be (rest mass of proton $m_p$ = 938.27 MeV/$c^2$ and rest mass of neutron $m_n$ = 939.57 MeV/$c^2$)

(A) 1.39$\Delta$ MeV/$c^2$

(B) ([illegible]39$\Delta$ [illegible]) MeV/$c^2$

(C) 0.86$\Delta$ MeV/$c^2$

(D) (1.[illegible] + 0.78) MeV/$c^2$

Q.63 A heavy nucleus is found to contain more neutrons than protons. This fact is related to which one of the following statements.

(A) The nuclear force between neutrons is stronger than that between protons.

(B) The nuclear force between protons is of a shorter range than those between neutrons, so that a smaller number of protons are held together by the nuclear force.

(C) Protons are unstable, so their number in a nucleus diminishes.

(D) It costs more energy to add a proton to a (heavy) nucleus than a neutron because of the Coulomb repulsion between protons.

Q.64 A neutral pi meson ($\pi^0$) has a rest-mass of approximately 140 MeV/$c^2$ and a lifetime of $\tau$ sec. A $\pi^0$ produced in the laboratory is found to decay after 1.25 $\tau$ sec into two photons. Which of the following sets represents a *possible* set of energies of the two photons as seen in the laboratory?

(A) 70 MeV and 70 MeV

(B) 35 MeV and 100 MeV

(C) 75 MeV and 100 MeV

(D) 25 MeV and 150 MeV

Q.65 An a.c. voltage of 220 $V_{rms}$ is applied to the primary of a 10:1 step-down transformer. The secondary of the transformer is centre tapped and connected to a full wave rectifier with a load resistance. The d.c. voltage appearing across the load is

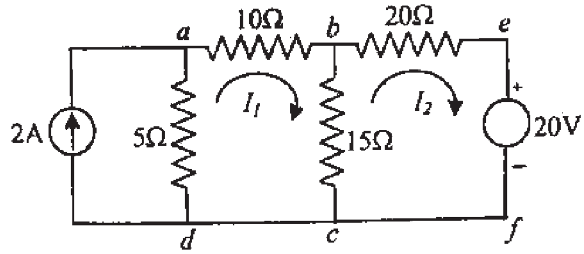
(A) 22

(B) 31

(C) 62

(D) 44

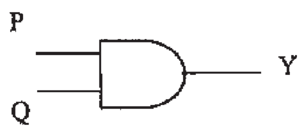
Q.66 Let $I_1$ and $I_2$ represent mesh currents in the loop *abcda* and *befcb* respectively. The correct expression describing Kirchoff's voltage loop law in one of the following loops is,



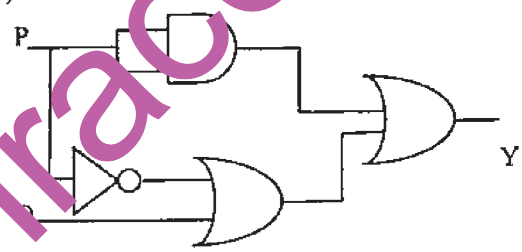

(A) $30I_1 - 15I_2 = 10$
(B) $-15I_1 + 20I_2 = -20$
(C) $30I_1 - 15I_2 = -10$
(D) $-15I_1 + 20I_2 = 20$

Q.67 The simplest logic gate circuit corresponding to the Boolean expression, $Y = P + \overline{P}Q$ is

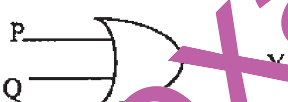
(A)



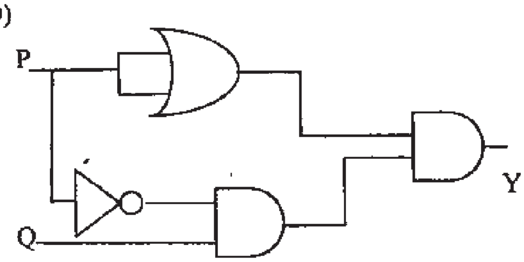

(B)




(C)



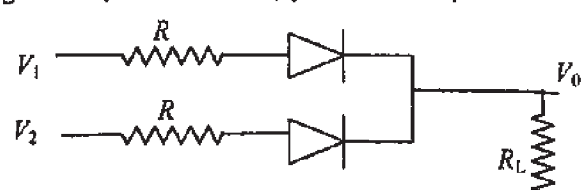

(D)



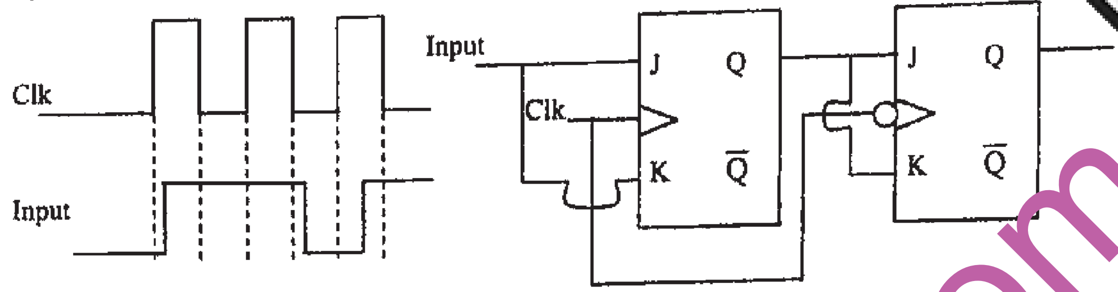

Q.68 An analog voltage V is converted into 2-bit binary number. The minimum number of comparators required and their reference voltages are

(A) $3, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}\right)$
(B) $3, \left(\frac{V}{3}, \frac{2V}{3}, V\right)$
(C) $4, \left(\frac{V}{5}, \frac{2V}{5}, \frac{3V}{5}, \frac{4V}{5}\right)$
(D) $4, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}, V\right)$

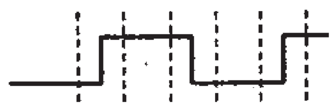
Q.69 The following circuit (where $R_L >> R$) performs the operation of



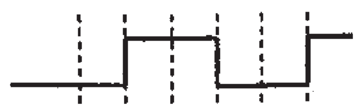
Q.70 In the T type master-slave JK flip flop is shown along with the clock and input waveforms. The output of flip flop was zero initially. Identify the correct output waveform.




(A)



(B)

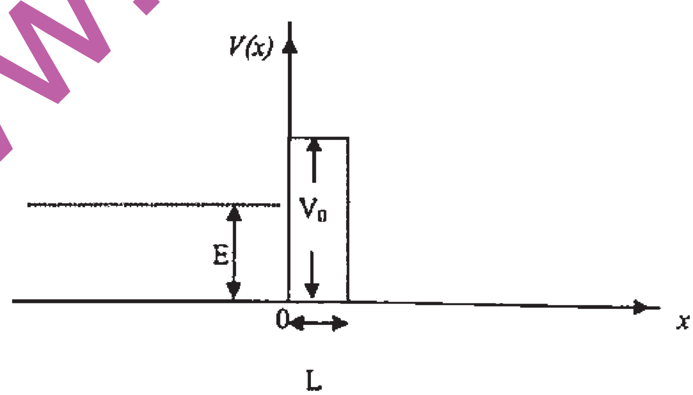


(C)

(D)

## Common Data Questions

**Common Data for Questions 71, 72 and 73:** A beam of identical particles of mass $m$ and energy $E$ is incident from left on a potential barrier of width $L$ (between $0 < x < L$) and height $V_0$ as shown in the figure ($E < V_0$).




For $x > L$, there is tunneling with a transmission coefficient $T > 0$. Let $A_0, A_R$ and $A_T$ denote the amplitudes for the incident, reflected and the transmitted waves, respectively:

Q.72 Let the probability current associated with the incident wave be $S_0$. Let $R$ be the coefficient. Then

(A) the probability current vanishes in the classically forbidden region.

(B) the probability current is $TS_0$ for $x > L$.

(C) for, $x < 0$, the probability current is $S_0(1+R)$

(D) for $x > L$, the probability current is complex.

Q.73 The ratio of the reflected to the incident amplitude $A_R/A_0$ is

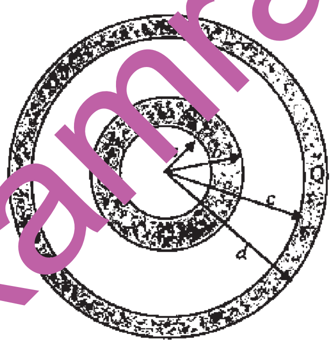
(A) $1 - A_T/A_0$

(B) $\sqrt{(1-T)}$ in magnitude

(C) a real negative number

(D) $\sqrt{\left(1-\left|\frac{A_T}{A_0}\right|^2\right)\frac{E}{V_0 - E}}$

**Common Data for Questions 74 and 75:** Consider two concentric conducting spherical shells with inner and outer radii $a$, $b$ and $c$, $d$ as shown in the figure. Both the shells are given $Q$ amount of positive charges.




Q.74 The electric field in different regions are

(A) $\vec{E} = 0$ for $r < a$ ; $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $a < r < b$

$\vec{E} = 0$ for $b < r < c$ ; $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

(B) $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < a$ ; $\vec{E} = 0$ for $a < r < b$

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b < r < c$ ; $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

(C) $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < a$ ; $\vec{E} = 0$ for $a < r < b$

$\vec{E} = 0$ for $b < r < c$ ; $\vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

Q.75 In order to have equal surface charge densities on the outer surfaces of both the following conditions should be satisfied

(A) $d = 4b$ and $c = 2a$
(B) $d = 2b$ and $c = \sqrt{2}a$
(C) $d = \sqrt{2}b$ and $c > a$
(D) $d > b$ and $c = \sqrt{2}a$

## Linked Answer Questions: Q.76 to Q.85 carry two marks each.

**Statement for Linked Answer Questions 76 and 77:**
Consider the β–decay of a free neutron at rest in the laboratory.

Q.76 Which of the following configurations of the decay products correspond to the largest energy of the anti-neutrino $\bar{\nu}$? (rest mass of electron $m_e$ = 0.51 MeV/$c^2$, rest mass of proton $m_p$ = 938.27 MeV/$c^2$ and rest mass of neutron $m_n$ = 939.57 MeV/$c^2$)

(A) In the laboratory, proton is produced at rest.
(B) In the laboratory, momenta of proton, electron and the anti-neutrino all have the same magnitude.
(C) In the laboratory, proton and electron fly-off with (nearly) equal and opposite momenta.
(D) In the laboratory, electron is produced at rest.

Q.77 Using the result of the above problem, answer the following. Which of the following represents approximately the maximum allowed energy of the anti-neutrino $\bar{\nu}$?

(A) 1.3 MeV (B) 0.8 MeV (C) 0.5 MeV (D) 2.0 MeV

**Statement for Linked Answer Questions 78 and 79:**
Consider a two dimensional electron gas of $N$ electrons of mass $m$ each in a system of size $L \times L$.

Q.78 The density of states between energy $\varepsilon$ and $\varepsilon + d\varepsilon$ is

(A) $\dfrac{4\pi L^2 m}{h^2} d\varepsilon$ (B) $\dfrac{4\pi L^2 m}{h^2} \dfrac{1}{\sqrt{\varepsilon}} d\varepsilon$ (C) $\dfrac{4\pi L^2 m}{h^2} \sqrt{\varepsilon}\, d\varepsilon$ (D) $\dfrac{4\pi L^2 m}{h^2} \varepsilon\, d\varepsilon$

Q.79 The ground state energy $E_0$ of the system in terms of the Fermi energy $E_F$ and the number of electrons $N$ is given by

(A) $\dfrac{1}{3} N E_F$ (B) $\dfrac{1}{2} N E_F$ (C) $\dfrac{2}{3} N E_F$ (D) $\dfrac{3}{5} N E_F$

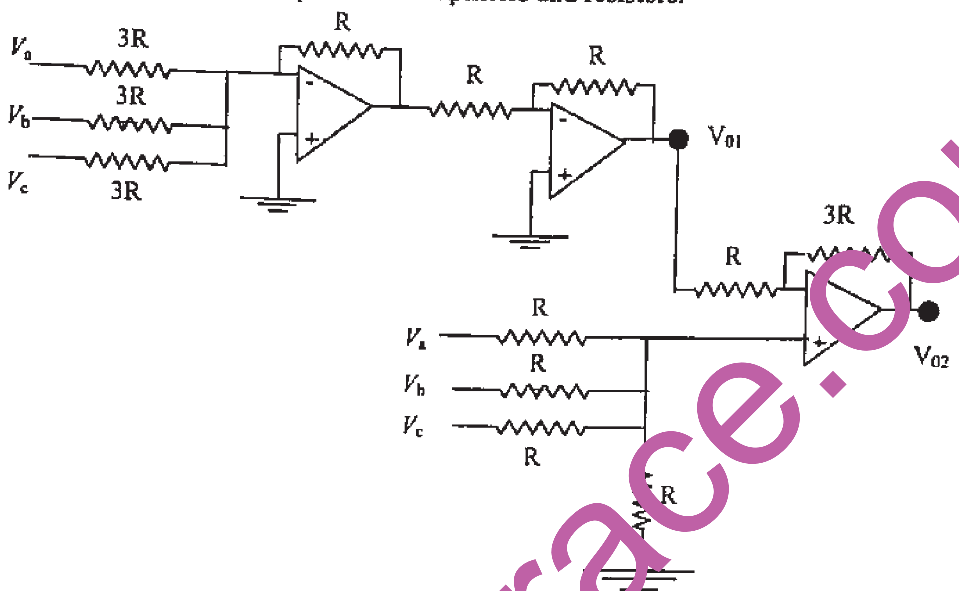
**Statement for Linked Answer Questions 80 and 81:**
The rate of a clock in a spaceship "Suryashakti" is observed from earth to be 3/5 of the rate of the clocks on earth.

Q.80 The speed of the spaceship "Suryashakti" relative to earth is

(A) $\dfrac{4}{5}c$ (B) $\dfrac{3}{5}c$ (C) $\dfrac{9}{10}c$ (D) $\dfrac{2}{5}c$

Q.81 The rate of a clock in a spaceship "Aakashganga" is observed from earth to be 5/13 of the rate of the clocks on earth. If both Aakashganga and Suryashakti

**Statement for Linked Answer Questions 82 and 83:**
The following circuit contains three operational amplifiers and resistors.




Q.82 The output voltage at the end of second operational amplifier $V_{01}$ is

(A) $V_{01}=3(V_a+V_b+V_c)$

(B) $V_{01}=-\frac{1}{3}(V_a+V_b+V_c)$

(C) $V_{01}=\frac{1}{3}(V_a+V_b+V_c)$

(D) $V_{01}=\frac{4}{3}(V_a+V_b+V_c)$

Q.83 The output $V_{02}$ ( at the end of third op amp) of the above circuit is

(A) $V_{02}=2(V_a+V_b+V_c)$

(B) $V_{02}=3(V_a+V_b+V_c)$

(C) $V_{02}=-\frac{1}{2}(V_a+V_b+V_c)$

(D) Zero

**Statement for Linked Answer Questions 84 and 85:**
The set $V$ of all polynomials of a real variable x of degree two or less and with real coefficients, a real linear vector space $V \equiv \{c_0 + c_1 x + c_2 x^2 : c_0, c_1, c_2 \in R\}$.

Q.84 For $f(x) = a_0 + a_1 x + a_2 x^2 \in V$ and $g(x) = b_0 + b_1 x + b_2 x^2 \in V$, which one of the following constitutes an acceptable scalar product?

(A) $(f, g) = a_0^2 b_0 + a_1^2 b_1 + a_2^2 b_2$

(B) $(f, g) = a_0^2 b_0^2 + a_1^2 b_1^2 + a_2^2 b_2^2$

(C) $(f, g) = a_0 b_0 - a_1 b_1 + a_2 b_2$

(D) $(f, g) = a_0 b_0 + \frac{a_1 b_1}{2} + \frac{a_2 b_2}{3}$

Q.85 Using the scalar product obtained in the above question, identify the subspace of $V$ that is orthogonal to $(1 + x)$:

(A) $\{f(x) : b(1 - x) + cx^2; b, c \in R\}$

(B) $\{f(x) = b(1 - 2x) + cx^2; b, c \in R\}$

(C) $\{f(x) : b + cx^2; b, c \in R\}$

(D) $\{f(x) : bx + cx^2; b, c \in R\}$

**END OF THE QUESTION PAPER**