2007
PH: Physics

Duration : Three Hours

Maximum Marks :150

Read the following instructions carefully.

1. This question paper contains 85 objective type questions. Q.1 to Q.20 carry **one mark** each and Q.21 to Q.85 carry **two marks each.**

2. Attempt all the questions.

3. Questions must be answered on **Objective Response Sheet (ORS)** by darkening the appropriate bubble (marked A, B, C, D) using HB pencil against the question number on the left hand side of the **ORS**. **Each question has only one correct answer.** In case you wish to change an answer, erase the old answer completely.

4. Wrong answers will carry NEGATIVE marks. In Q.1 to Q.20, **0.25** mark will be deducted for each wrong answer. In Q.21 to Q.76, Q.78, Q.80, Q.82 and in Q.84, **0.5** mark will be deducted for each wrong answer. However, there is no negative marking in Q.77, Q.79, Q.81, Q.83 and in Q.85. More than one answer bubbled against a question will be taken as an incorrect response. Unattempted questions will not carry any marks.

5. Write your registration number, your name and name of the examination centre at the specified locations on the right half of the **ORS**.

6. Using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your paper code.

7. Calculator is allowed in the examination hall.

8. Charts, graph sheets or tables are NOT allowed in the examination hall.

9. Rough work can be done on the question paper itself. Additionally blank pages are given at the end of the question paper for rough work.

10. This question paper contains **20** printed pages including pages for rough work. Please check all pages and report, if there is any discrepancy.

Some useful physical constants, symbols and formulae

Speed of light in free space	c	=	3.0×10^{8} m s^{-1}
Atomic mass unit	amu	=	1.66×10^{-27} kg
Avogadro's number	N_A	=	6.02×10^{23} mole^{-1}
Bohr magneton	μ_B	=	9.27×10^{-24} A m^{2}
Boltzmann constant	k_B	=	1.38×10^{-23} J K^{-1}
Electron charge	e	=	1.60×10^{-19} C
Planck's constant	h	=	6.63×10^{-34} J s
Rest mass of electron	m_e	=	9.11×10^{-31} kg
Reduced Planck's constant ($h/2\pi$)	$\hbar$	=	1.05×10^{-34} J s
Permeability of free space	μ_0	=	1.26×10^{-6} N A^{-2}
Permittivity of free space	ε_0	=	8.85×10^{-12} C^{2} N^{-1} m^{-2}

$\vec{\nabla}$ operator in spherical coordinates:

$$\vec{\nabla}(\ ) = \hat{r}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\ \ ) + \hat{\theta}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\ \ ) + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\ \ )$$

Q. 1 – Q. 20 carry one mark each.

Q.1 The eigenvalues of a matrix are i, $-2i$ and $3i$. The matrix is

(A) unitary.
(B) anti-unitary.
(C) Hermitian.
(D) anti-Hermitian.

Q.2 A space station moving in a circular orbit around the Earth goes into a new bound orbit by firing its engine radially outwards. This orbit is

(A) a larger circle.
(B) a smaller circle.
(C) an ellipse.
(D) a parabola.

Q.3 A power amplifier gives 150W output for an input of 1.5W. The gain, in dB, is

(A) 10 (B) 20 (C) 54 (D) 100

Q.4 Four point charges are placed in a plane at the following positions:
$+Q$ at (1, 0), $-Q$ at (-1, 0), $+Q$ at (0, 1) and $-Q$ at (0, -1).
At large distances the electrostatic potential due to this charge distribution will be dominated by the

(A) monopole moment.
(B) dipole moment.
(C) quadrupole moment.
(D) octopole moment.

Q.5 A charged capacitor (C) is connected in series with an inductor (L). When the displacement current reduces to zero, the energy of the LC circuit is

(A) stored entirely in its magnetic field.
(B) stored entirely in its electric field.
(C) distributed equally among its electric and magnetic fields.
(D) radiated out of the circuit.

Q.6 Match the following

P. Franck-Hertz experiment
Q. Hartree-Fock method
R. Stern-Gerlach experiment
S. Franck-Condon principle

1. electronic excitation of molecules
2. wave function of atoms
3. spin angular momentum of atoms
4. energy levels in atoms

(A)	(B)	(C)	(D)
P-4	P-1	P-3	P-4
Q-2	Q-4	Q-2	Q-1
R-3	R-3	R-4	R-3
S-1	S-2	S-1	S-2

Q.7 The wavefunction of a particle, moving in a one-dimensional time-independent potential $V(x)$, is given by $\psi(x) = e^{-iax+b}$, where a and b are constants. This means that the potential $V(x)$ is of the form

(A) $V(x) \propto x$
(B) $V(x) \propto x^2$
(C) $V(x) = 0$
(D) $V(x) \propto e^{-ax}$

Q.8 The D_1 and D_2 lines of Na ($3^2P_{1/2} \rightarrow 3^2S_{1/2}$, $3^2P_{3/2} \rightarrow 3^2S_{1/2}$) will split on the application of a weak magnetic field into

(A) 4 and 6 lines respectively.
(B) 3 lines each.
(C) 6 and 4 lines respectively.
(D) 6 lines each.

Q.9 In a He-Ne laser, the laser transition takes place in

(A) He only.
(B) Ne only.
(C) Ne first, then in He.
(D) He first, then in Ne.

Q.10 The partition function of a single gas molecule is Z_α. The partition function of N such non-interacting gas molecules is then given by

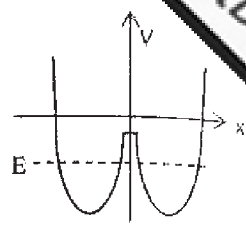
(A) $\frac{(Z_\alpha)^N}{N!}$
(B) $(Z_\alpha)^N$
(C) $N(Z_\alpha)$
(D) $\frac{(Z_\alpha)^N}{N}$

Q.11 A solid superconductor is placed in an external magnetic field and then cooled below its critical temperature. The superconductor

(A) retains its magnetic flux because the surface current supports it.
(B) expels out its magnetic flux because it behaves like a paramagnetic material.
(C) expels out its magnetic flux because it behaves like an anti- ferromagnetic material.
(D) expels out its magnetic flux because the surface current induces a field in the direction opposite to the applied magnetic field.

Q.12 A particle with energy E is in a time-independent double well potential as shown in the figure.

Which of the following statements about the particle is **NOT** correct?



(A) The particle will always be in a bound state.
(B) The probability of finding the particle in one well will be time-dependent.
(C) The particle will be confined to any one of the wells.
(D) The particle can tunnel from one well to the other, and back.

Q.13 It is necessary to apply quantum statistics to a system of particles if

(A) there is substantial overlap between the wavefunctions of the particles.
(B) the mean free path of the particles is comparable to the inter-particle separation.
(C) the particles have identical mass and charge.
(D) the particles are interacting.

Q.14 When liquid oxygen is poured down close to a strong bar magnet, the oxygen stream is

(A) repelled towards the lower field because it is diamagnetic.
(B) attracted towards the higher field because it is diamagnetic.
(C) repelled towards the lower field because it is paramagnetic.
(D) attracted towards the higher field because it is paramagnetic.

Q.15 Fission fragments are generally radioactive as

(A) they have excess of neutrons.
(B) they have excess of protons.
(C) they are products of radioactive nuclides.
(D) their total kinetic energy is of the order of 200 MeV.

Q.16 In a typical *npn* transistor the doping concentrations in emitter, base and collector regions are C_E, C_B and C_c respectively. These satisfy the relation,

(A) $C_E > C_C > C_B$ (B) $C_E > C_B > C_C$ (C) $C_C > C_B > C_E$ (D) $C_E = C_C > C_B$

Q.17 The allowed states for He ($2p^2$) configuration are

(A) ${}^1S_0$, ${}^3S_1$, ${}^1P_1$, ${}^3P_{0,1,2}$, ${}^1D_2$ and ${}^3D_{1,2,3}$ (B) ${}^1S_0$, ${}^3P_{0,1,2}$ and ${}^1D_2$
(C) ${}^1P_1$ and ${}^3P_{0,1,2}$ (D) ${}^1S_0$ and ${}^1P_1$

Q.18 The energy levels of a particle of mass m in a potential of the form

$$V(x) = \infty, \quad x \le 0$$
$$= \tfrac{1}{2}m\omega^2 x^2, \quad x > 0$$

are given, in terms of quantum number $n = 0,1,2,3,\ldots,$ by

(A) $\left(n+\frac{1}{2}\right)\hbar\omega$ (B) $\left(2n+\frac{1}{2}\right)\hbar\omega$ (C) [illegible]

Q.19 The electromagnetic field due to a point charge must be described by Liénard-Weichert potentials when

(A) the point charge is highly accelerated.
(B) the electric and magnetic fields are not perpendicular.
(C) the point charge is moving with velocity close to that of light.
(D) the calculation is done for the radiation zone, i.e. far away from the charge.

Q.20 The strangeness quantum number is conserved in

(A) strong, weak and electromagnetic interactions.
(B) weak and electromagnetic interactions only.
(C) strong and weak interactions only.
(D) strong and electromagnetic interactions only.

Q. 21 to Q. 75 carry two marks each.

Q.21 The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

(A) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
(B) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
(C) 6, 1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
(D) 2, 5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Q.22 A vector field is defined everywhere as $\vec{F} = \frac{y^2}{L}\hat{i} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y, and Z axes, is

(A) $2L^3$ (B) $4L^3$ (C) $8L^3$ (D) $10L^3$

Q.23 If $\vec{r} = x\hat{i} + y\hat{j}$, then

(A) $\vec{\nabla}\cdot\vec{r} = 0$ and $\vec{\nabla}|\vec{r}| = \vec{r}$
(B) $\vec{\nabla}\cdot\vec{r} = 2$ and $\vec{\nabla}|\vec{r}| = \hat{r}$
(C) $\vec{\nabla}\cdot\vec{r} = 2$ and $\vec{\nabla}|\vec{r}| = \frac{\hat{r}}{r}$ ✓
(D) $\vec{\nabla}\cdot\vec{r} = 3$ and $\vec{\nabla}|\vec{r}| = \frac{\hat{r}}{r}$

Q.24 Consider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60^0. The vector $\vec{p}$ in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is

(A) $(1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$
(B) $(1+\sqrt{3})\hat{i}' + 3\hat{j}' + (1-\sqrt{3})\hat{k}'$ ✓
(C) $(1-\sqrt{3})\hat{i}' + (3+\sqrt{3})\hat{j}' + 2\hat{k}'$
(D) $(1-\sqrt{3})\hat{i}' + (3-\sqrt{3})\hat{j}' + 2\hat{k}'$

Q.25 The contour integral $\oint \frac{dz}{z^4 + a^4}$ is to be evaluated on a circle of radius $2a$ [illegible] the origin. It will have contributions only from the points

(A) $\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$

(B) ia and $-ia$

(C) ia, $-ia$, $\frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$

(D) $\frac{1+i}{\sqrt{2}}a$, $-\frac{1+i}{\sqrt{2}}a$, $\frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$

Q.26 Inverse Laplace transform of $\frac{s+1}{s^2-4}$ is

(A) $\cos 2x + \frac{1}{2}\sin 2x$

(B) $\cos x + \frac{1}{2}\sin x$

(C) $\cosh x + \frac{1}{2}\sinh x$

(D) $\cosh 2x + \frac{1}{2}\sinh 2x$

Q.27 The points, where the series solution of the Legendre differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \frac{3}{2}(\frac{3}{2}+1)y = 0$$

will diverge, are located at

(A) 0 and 1

(B) 0 and -1

(C) -1 and 1

(D) $\frac{3}{2}$ and $\frac{5}{2}$

Q.28 Solution of the differential equation $x\frac{dy}{dx} + y = x^4$, with the boundary condition that $y = 1$, at $x = 1$, is

(A) $y = 5x^4$ [illegible]

(B) $y = \frac{x^4}{5} + \frac{4x}{5}$

(C) $y = \frac{4x^4}{5} + \frac{1}{5x}$

(D) $y = \frac{x^4}{5} + \frac{4}{5x}$

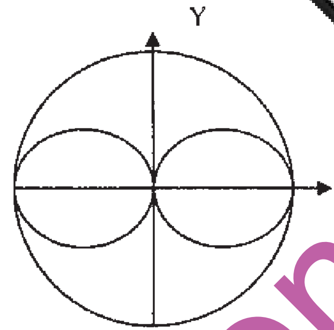
Q.29 Match the following

P. rest mass	1. timelike vector
Q. charge	2. Lorentz invariant
R. four-momentum	3. tensor of rank 2
S. electromagnetic field	4. conserved and Lorentz invariant

(A)	(B)	(C)	(D)
P-2	P-4	P-2	P-4
Q-4	Q-2	Q-4	Q-2
R-3	R-1	R-1	R-3
S-1	S-3	S-3	S-1

Q.30 The moment of inertia of a uniform sphere of radius r about an axis passing through its centre is given by $\frac{2}{5}\left(\frac{4\pi}{3}r^5\rho\right)$.

A rigid sphere of uniform mass density ρ and radius R has two smaller spheres of radius $R/2$ hollowed out of it, as shown in the figure. The moment of inertia of the resulting body about the Y axis is



(A) $\frac{\pi\rho R^5}{4}$ (B) $\frac{5\pi\rho R^5}{12}$ (C) $\frac{7\pi\rho R^5}{12}$ (D) $\pi\rho R$ [illegible]

Q.31 The Lagrangian of a particle of mass m is

$$L = \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] - \frac{V}{2}\left(x^2 + y^2\right) + W \sin \omega t$$ where V, W and ω are constants. The conserved quantities are

(A) energy and z-component of linear momentum only.
(B) energy and z-component of angular momentum only.
(C) z-components of both linear and angular momenta only.
(D) energy and z-components of both linear and angular momenta.

Q.32 Three particles of mass m each situated at $x_1(t)$, $x_2(t)$ and $x_3(t)$ respectively are connected by two springs of spring constant k and un-stretched length ℓ. The system is free to oscillate only in one dimension along the straight line joining all the three particles. The Lagrangian of the system is

(A) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - \ell)^2 + \frac{k}{2}(x_3 - x_2 - \ell)^2$

(B) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_3 - \ell)^2 + \frac{k}{2}(x_3 - x_2 - \ell)^2$

(C) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 + \ell)^2 - \frac{k}{2}(x_3 - x_2 + \ell)^2$

(D) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - \ell)^2 - \frac{k}{2}(x_3 - x_2 - \ell)^2$

Q.33 The Hamiltonian of a particle is $H = \frac{p^2}{2m} + pq$, where q is the generalized and p is the corresponding canonical momentum. The Lagrangian is

(A) $\frac{m}{2}\left(\frac{dq}{dt}+q\right)^2$

(B) $\frac{m}{2}\left(\frac{dq}{dt}-q\right)^2$

(C) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2+q\frac{dq}{dt}-q^2\right]$

(D) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2-q\frac{dq}{dt}+q^2\right]$

Q.34 A toroidal coil has N closely-wound turns. Assume the current through the coil to be I and the toroid is filled with a magnetic material of relative permittivity μ_r. The magnitude of magnetic induction $\vec{B}$ inside the toroid, at a radial distance r from the axis, is given by

(A) $\mu_r\mu_0 NIr$

(B) $\frac{\mu_r\mu_0 NI}{r}$

(C) $\frac{\mu_r\mu_0 NI}{2\pi r}$

(D) $2\pi\mu_r\mu_0 NIr$

Q.35 An electromagnetic wave with $\vec{E}(z,t) = E_0\cos(\omega t - kz)\hat{i}$ is traveling in free space and crosses a disc of radius 2 m placed perpendicular to the z-axis. If $E_0 = 60$ V m^{-1}, the average power, in Watt, crossing the disc along the z-direction is

(A) 30

(B) 60

(C) 120

(D) 270

Q.36 Can the following scalar and vector potentials describe an electromagnetic field?

$\phi(\vec{x},t) = 3xyz - 4t$

$\vec{A}(\vec{x},t) = (2x-\omega t)\hat{i} + (y-2z)\hat{j} + (z-2xe^{i\omega t})\hat{k}$

where ω is a constant.

(A) Yes, in the Coulomb gauge.

(B) Yes, in the Lorentz gauge.

(C) Yes, provided $\omega = 0$.

(D) No.

Q.37 For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V(x) = \frac{1}{2}m\omega^2x^2$, the first excited energy eigenstate is $\psi(x) = xe^{-\alpha x^2}$. The value of α is

(A) $m\omega/4\hbar$

(B) $m\omega/3\hbar$

(C) $m\omega/2\hbar$

(D) $2m\omega/3\hbar$

Q.38 If $[x,p] = i\hbar$, the value of $[x^3,p]$ is

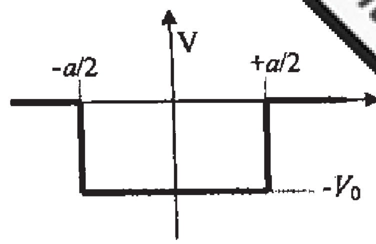
(A) $2i\hbar x^2$

(B) $-2i\hbar x^2$

(C) $3i\hbar x^2$

(D) $-3i\hbar x^2$

Q.39 There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth V_0 of the potential satisfies



(A) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{9\pi^2\hbar^2}{2ma^2}$ (B) $\frac{\pi^2\hbar^2}{ma^2} < V_0 < \frac{2\pi^2\hbar^2}{ma^2}$

(C) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{8\pi^2\hbar^2}{ma^2}$ (D) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{50\pi^2\hbar^2}{ma^2}$

Q.40 An atomic state of hydrogen is represented by the following wavefunction:

$$\psi(r,\theta,\varphi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right) e^{-r/2a_0}\cos\theta .$$

where a_0 is a constant. The quantum numbers of the state are

(A) $\ell = 0, m = 0, n = 1$ (B) $\ell = 1, m = 1, n = 2$

(C) $\ell = 1, m = 0, n = 2$ (D) $\ell = 2, m = 0, n = 3$

Q.41 Three operators X, Y and Z satisfy the commutation relations $[X,Y] = i\hbar Z$, $[Y,Z] = i\hbar X$ and $[Z,X] = i\hbar Y$.
The set of all possible eigenvalues of the operator Z, in units of $\hbar$, is

(A) $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$ (B) $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots\right\}$

(C) $\left\{0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm\frac{5}{2}, \ldots\right\}$ (D) $\left\{-\frac{1}{2}, +\frac{1}{2}\right\}$

Q.42 A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22^0 C by supplying 450 kJ s^{-1}. If the outside temperature is 0^0 C, the heat taken, in kJ s^{-1}, from the outside air is approximately

(A) 487 (B) 470 (C) 467 (D) 417

Q.43 The vapour pressure p (in mm of Hg) of a solid, at temperature T, is expressed by $\ln p = 23 - 3863/T$ and that of its liquid phase by $\ln p = 19 - 3063/T$. The triple point (in Kelvin) of the material is

(A) 185 (B) 190 (C) 195 (D) 200

Q.44 The free energy for a photon gas is given by $F = -\left(\frac{a}{3}\right)VT^4$, where a is a constant.

The entropy S and the pressure P of the photon gas are

(A) $S = \frac{4}{3}aVT^3, \quad P = \frac{a}{3}T^4$
(B) $S = \frac{1}{3}aVT^4, \quad P = \frac{4a}{3}T^3$
(C) $S = \frac{4}{3}aVT^4, \quad P = \frac{a}{3}T^3$
(D) $S = \frac{1}{3}aVT^3, \quad P = \frac{4a}{3}T^4$

Q.45 A system has energy levels E_0, $2E_0$, $3E_0$,...., where the excited states are triply degenerate. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is

(A) 2
(B) 3
(C) 4
(D) 6

Q.46 In accordance with the selection rules for electric dipole transitions, the 4^3P_1 state of helium can decay by photon emission to the states

(A) 2^1S_0, 2^1P_1 and 3^1D_2
(B) 3^1P_1, 2^1D_2 and 3^1S_0
(C) 3^3P_2, 3^3D_3 and 3^3P_0
(D) 2^3S_1, 3^3D_2 and 3^3D_1

Q.47 If an atom is in the $^3D_3$ state, the angle between its orbital and spin angular momentum vectors ($\vec{L}$ and $\vec{S}$) is

(A) $\cos^{-1}\frac{1}{\sqrt{3}}$
(B) $\cos^{-1}\frac{2}{\sqrt{3}}$
(C) $\cos^{-1}\frac{1}{2}$
(D) $\cos^{-1}\frac{\sqrt{3}}{2}$

Q.48 The hyperfine structure of Na ($3^2P_{3/2}$) with nuclear spin $I = 3/2$ has

(A) 1 state.
(B) 2 states.
(C) 3 states.
(D) 4 states.

Q.49 The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecule are expressed as $\varepsilon_J = BJ(J+1)$, where B is the rotational constant and J is a rotational quantum number.

In a system of such diatomic molecules of reduced mass μ, some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass is changed to 1.05μ. In the rotational spectrum of the system, the shift in the spectral line, corresponding to a transition $J = 4 \rightarrow J = 5$, is

(A) 0.475 B
(B) 0.50 B
(C) 0.95 B
(D) 1.0 B

Q.50 The number of fundamental vibrational modes of CO_2 molecule is

(A) four : 2 are Raman active and 2 are infrared active.
(B) four : 1 is Raman active and 3 are infrared active.
(C) three: 1 is Raman active and 2 are infrared active.

Q.51 A piece of paraffin is placed in a uniform magnetic field H_0. The sample contains hydrogen nuclei of mass m_p, which interact only with external magnetic field. An additional oscillating magnetic field is applied to observe resonance absorption. If g_l is the g-factor of the hydrogen nucleus, the frequency, at which resonance absorption takes place, is given by

(A) $\frac{3g_l eH_0}{2\pi m_p}$ (B) $\frac{3g_l eH_0}{4\pi m_p}$ (C) $\frac{g_l eH_0}{2\pi m_p}$ (D) $\frac{g_l eH_0}{4\pi m_p}$

Q.52 The solid phase of an element follows van der Waals bonding with inter-atomic potential $V(r) = -\frac{P}{r^6} + \frac{Q}{r^{12}}$, where P and Q are constants. The bond length can be expressed as

(A) $\left(\frac{2Q}{P}\right)^{-6}$ (B) $\left(\frac{Q}{P}\right)^{-6}$ (C) $\left(\frac{P}{2Q}\right)^{-6}$ (D) $\left(\frac{P}{Q}\right)^{-6}$

Q.53 Consider the atomic packing factor (APF) of the following crystal structures:

P. Simple Cubic
Q. Body Centred Cubic
R. Face Centred Cubic
S. Diamond
T. Hexagonal Close Packed

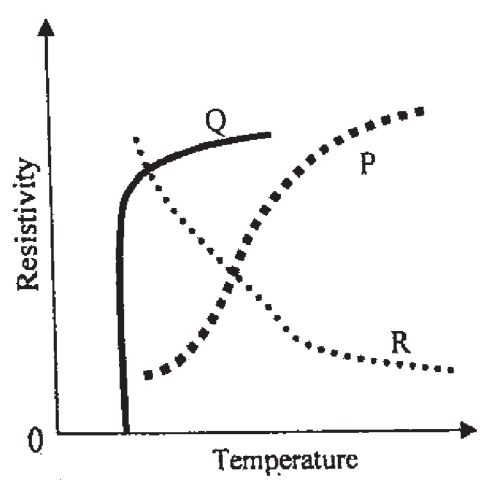
Which two of the above structures have equal APF?

(A) P and Q (B) S and T (C) R and S (D) R and T

Q.54 In a powder diffraction pattern recorded from a face-centred cubic sample using x-rays, the first peak appears at 30^0. The second peak will appear at

(A) 32.8^0 (B) 33.7^0 (C) 34.8^0 (D) 35.3^0

Q.55 Variation of electrical resistivity ρ with temperature T of three solids is sketched (on different scales) in the figure, as curves P, Q and R.



Which one of the following statements describes the variations most appropriately?

Q.56 An extrinsic semiconductor sample of cross-section A and length L is doped in such a way that the doping concentration varies as $N_D(x) = N_0 \exp\left(-\frac{x}{L}\right)$, where N_0 is a constant. Assume that the mobility μ of the majority carriers remains constant. The resistance R of the sample is given by

(A) $R = \frac{L}{A\mu e N_0}[\exp(1.0) - 1]$ (B) $R = \frac{L}{\mu e N_0}[\exp(1.0) - 1]$

(C) $R = \frac{L}{A\mu e N_0}[\exp(-1.0) - 1]$ (D) $R = \frac{L}{A\mu e N_0}$

Q.57 A ferromagnetic mixture of iron and copper having 75% atoms of Fe exhibits a saturation magnetization of 1.3×10^6 A m^{-1}. Assume that the total number of atoms per unit volume is 8×10^{28} m^{-3}. The magnetic moment of an iron atom, in terms of the Bohr Magneton, is

(A) 1.7 (B) 2.3 (C) 2.9 (D) 3.8

$\ln 2 = 0.693$

$T_{1/2} = \frac{0.693}{\lambda}$

$t = \frac{1}{\lambda}\ln\left(\frac{N_0}{N}\right)$

$= \frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_0}{N}\right)$

Q.58 Half life of a radio-isotope is 4×10^8 years. If there are 10^3 radioactive nuclei in a sample today, the number of such nuclei in the sample 4×10^9 years ago were

(A) 128×10^3 (B) 256×10^3 (C) 512×10^3 (D) 1024×10^3

Q.59 In the deuterium + tritium $(d+t)$ fusion more energy is released as compared to deuterium + deuterium $(d+d)$ fusion because

(A) tritium is radioactive.
(B) more nucleons participate in fusion.
(C) the Coulomb barrier is lower for the $d+t$ system than $d+d$ system.
(D) the reaction product ^{4}He is more tightly bound.

Q.60 According to the shell model the ground state spin of the ^{17}O nucleus is

(A) $\frac{3}{2}^+$ (B) $\frac{5}{2}^+$ (C) $\frac{3}{2}^-$ (D) $\frac{5}{2}^-$

Q.61 A relativistic particle travels a length of 3×10^{-3} m in air before decaying. The decay process of the particle is dominated by

(A) strong interactions. (B) electromagnetic interactions.
(C) weak interactions. (D) gravitational interactions.

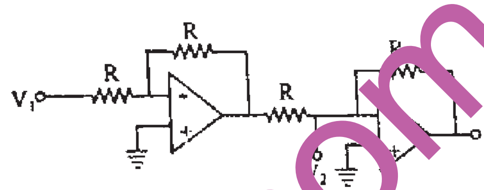
Q.62 The strange baryon Σ^+ has the quark structure

(A) uds (B) uud (C) uus (D) $u\bar{s}$

Q.63 A neutron scatters elastically from a heavy nucleus. The initial and final states of the neutron have the

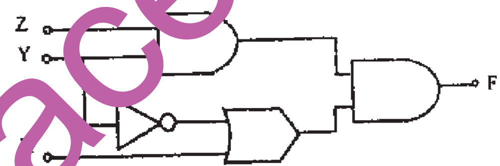
(A) same energy.
(B) same energy and linear momentum.
(C) same energy and angular momentum.
(D) same linear and angular momenta.

Q.64 The circuit shown is based on ideal operational amplifiers. It acts as a



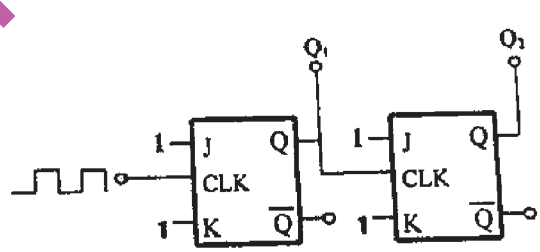
(A) subtractor.
(B) buffer amplifier.
(C) adder.
(D) divider.

Q.65 Identify the function F generated by the logic network shown



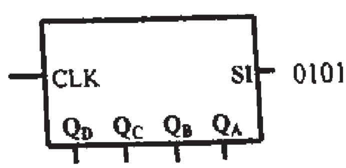
(A) $F = (X + Y) Z$
(B) $F = Z + Y + \bar{Y} X$
(C) $F = ZY (Y + X)$
(D) $F = XYZ$

Q.66 In the circuit shown, the ports Q_1 and Q_2 are in the state $Q_1 = 1, Q_2 = 0$. The circuit is now subjected to two complete clock pulses. The state of these ports now becomes



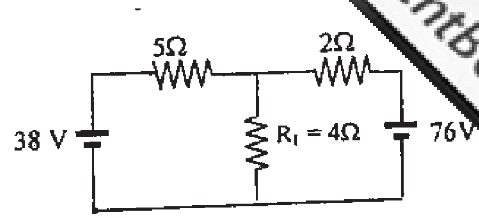
(A) $Q_2 = 1, Q_1 = 0$ (B) $Q_2 = 0, Q_1 = 1$ (C) $Q_2 = 1, Q_1 = 1$ (D) $Q_2 = 0, Q_1 = 0$

Q.67 The registers Q_D, Q_C, Q_B and Q_A shown in the figure are initially in the state 1010 respectively. An input sequence SI = 0101 is applied. After two clock pulses, the state of the shift registers (in the same sequence $Q_D Q_C Q_B Q_A$) is



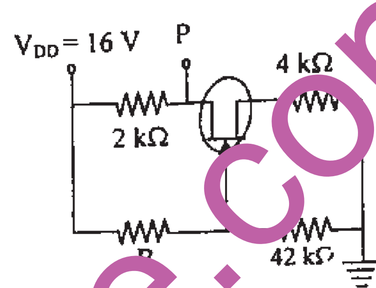
(A) 1001 (B) 0100 (C) 0110 (D) 1010

Q.68 For the circuit shown, the potential difference (in Volts) across R_L is



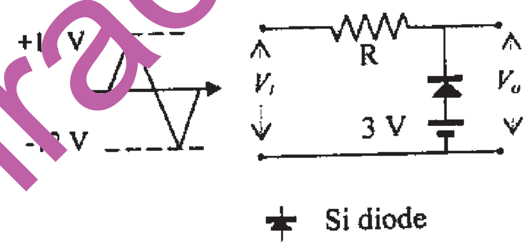
(A) 48 (B) 52 (C) 56 (D) 65

Q.69 In the circuit shown, the voltage at test point P is 12 V and the voltage between gate and source is -2V. The value of R (in kΩ) is

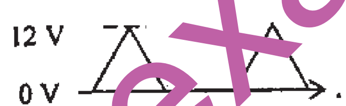


(A) 42 (B) 48 (C) 56 (D) 70

Q.70 When an input voltage V_i, of the form shown, is applied to the circuit given below, the output voltage V_o is of the form

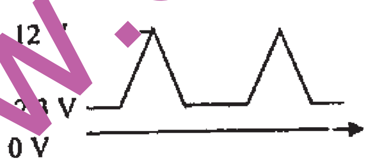


(A)

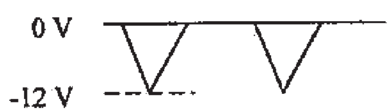


(B)

(C)



(D)



Common Data Questions

Common Data for Questions 71,72,73:
A particle of mass m is confined in the ground state of a one-dimensional box, extending from $x = -2L$ to $x = +2L$. The wavefunction of the particle in this state is $\psi(x) = \psi_0 \cos\frac{\pi x}{4L}$ where ψ_0 is a constant.

Q.71 The normalization factor ψ_0 of this wavefunction is

(A) $\sqrt{\frac{2}{L}}$ (B) $\sqrt{\frac{1}{4L}}$ (C) $\sqrt{\frac{1}{2L}}$ (D) $\sqrt{\frac{1}{L}}$

Q.72 The energy eigenvalue corresponding to this state is

(A) $\frac{\hbar^2\pi^2}{2mL^2}$ (B) $\frac{\hbar^2\pi^2}{4mL^2}$ (C) $\frac{\hbar^2\pi^2}{16mL^2}$ (D) $\frac{\hbar^2\pi^2}{32mL^2}$

Q.73 The expectation value of p^2 (p is the momentum operator) in this state is

(A) 0 (B) $\frac{\hbar^2\pi^2}{32L^2}$ (C) $\frac{\hbar^2\pi^2}{16L^2}$ (D) $\frac{\hbar^2\pi^2}{8L^2}$

Common Data for Questions 74, 75:
The Fresnel relations between the amplitudes of incident and reflected electromagnetic waves at an interface between air and a dielectric of refractive index μ, are

$$E_{\parallel}^{reflected} = \frac{\cos r - \mu\cos i}{\cos r + \mu\cos i} E_{\parallel}^{incident} \quad \text{and} \quad E_{\perp}^{reflected} = \frac{\mu\cos r - \cos i}{\mu\cos r + \cos i} E_{\perp}^{incident}.$$

The subscripts ∥ and ⊥ refer to polarization, parallel and normal to the plane of incidence respectively. Here, i and r are the angles of incidence and refraction respectively.

Q.74 The condition for the reflected ray to be completely polarized is

(A) $\mu\cos i = \cos r$ (B) $\cos i = \mu\cos r$
(C) $\mu\cos i = -\cos r$ (D) $\cos i = -\mu\cos r$

Q.75 For normal incidence at an air-glass interface with $\mu = 1.5$ the fraction of energy reflected is given by

Linked Answer Questions: Q.76 to Q.85 carry two marks e

Statement for Linked Answer Questions 76 & 77:
In the laboratory frame, a particle P of rest mass m_0 is moving in the positive x direction with a speed of $\frac{5c}{19}$. It approaches an identical particle Q, moving in the negative x direction with a speed of $\frac{2c}{5}$.

Q.76 The speed of the particle P in the rest frame of the particle Q is

(A) $\frac{7c}{95}$ (B) $\frac{13c}{85}$ (C) $\frac{3c}{5}$ (D) $\frac{63c}{95}$

Q.77 The energy of the particle P in the rest frame of the particle Q is

(A) $\frac{1}{2}m_0c^2$ (B) $\frac{5}{4}m_0c^2$ (C) $\frac{19}{13}m_0c^2$ (D) $\frac{11}{9}m_0c^2$

Statement for Linked Answer Questions 78 & 79
The atomic density of a solid is $5.85\times10^{28}\,\mathrm{m}^{-3}$. Its electrical resistivity is $1.6\times10^{-8}\,\Omega\,\mathrm{m}$. Assume that electrical conduction is described by the Drude model (classical theory), and that each atom contributes one conduction electron.

Q.78 The drift mobility (in $\mathrm{m}^2\,\mathrm{V}^{-1}\mathrm{s}^{-1}$) of the conduction electrons is

(A) 6.67×10^{-3} (B) 6.67×10^{-5} (C) 7.63×10^{-3} (D) 7.63×10^{-6}

Q.79 The relaxation time (mean free time), in seconds, of the conduction electrons is

(A) 3.98×10^{-15} (B) 3.79×10^{-14} (C) 2.84×10^{-12} (D) 2.64×10^{-11}

Statement for Linked Answer Questions 80 & 81:
A sphere of radius R carries a polarization $\vec{P}=k\vec{r}$, where k is a constant and $\vec{r}$ is measured from the centre of the sphere.

Q.80 The bound surface and volume charge densities are given, respectively, by

(A) $-k|\vec{r}|$ and $3k$ (B)) $k|\vec{r}|$ and $-3k$ (C) $k|\vec{r}|$ and $-4\pi kR$ (D) $-k|\vec{r}|$ and $4\pi kR$

Q.81 The electric field $\vec{E}$ at a point $\vec{r}$ outside the sphere is given by

(A) $\vec{E}=0$ (B) $\vec{E}=\frac{kR(R^2-r^2)}{\varepsilon_0 r^3}\hat{r}$

Statement for Linked Answer Questions 82 & 83:
An ensemble of quantum harmonic oscillators is kept at a finite temperature $T = 1/...$

Q.82 The partition function of a single oscillator with energy levels $(n+\frac{1}{2})\hbar\omega$ is given

(A) $Z = \frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$

(B) $Z = \frac{e^{-\beta\hbar\omega/2}}{1+e^{-\beta\hbar\omega}}$

(C) $Z = \frac{1}{1-e^{-\beta\hbar\omega}}$

(D) $Z = \frac{1}{1+e^{-\beta\hbar\omega}}$

Q.83 The average number of energy quanta of the oscillators is given by

(A) $<n> = \frac{1}{e^{\beta\hbar\omega}-1}$

(B) $<n> = \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}-1}$

(C) $<n> = \frac{1}{e^{\beta\hbar\omega}+1}$

(D) $<n> = \frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

Statement for Linked Answer Questions 84 & 85:
A 16 μA beam of alpha particles, having cross-sectional area 10^{-4} m^2, is incident on a rhodium target of thickness 1 μm. This produces neutrons through the reaction
$\alpha + {}^{100}Rh \rightarrow {}^{101}Pd + 3n.$

Q.84 The number of alpha particles hitting the target per second is

(A) 0.5×10^{14} (B) 1.0×10^{14} (C) 2.0×10^{20} (D) 4.0×10^{20}

Q.85 The neutrons are observed at the rate of 1.806×10^{8} s^{-1}. If the density of rhodium is approximated as 10^4 kg m^{-3} the cross-section for the reaction (in barns) is

(A) 0.1 (B) 0.2 (C) 0.4 (D) 0.8

END OF THE QUESTION PAPER

Space for Rough Work