OCR ✩
RECOGNISING ACHIEVEMENT

FREE-STANDING MATHEMATICS QUALIFICATION
ADVANCED LEVEL

Additional Mathematics   **6993**

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
• Printed answer book 6993

Other materials required:
• Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

•   The question paper will be found in the centre of the printed answer book.
•   Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
•   **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
•   Use black ink. Pencil may be used for graphs and diagrams only.
•   Read each question carefully. Make sure you know what you have to do before starting your answer.
•   Answer **all** the questions.
•   Do **not** write in the bar codes.
•   You are permitted to use a scientific or graphical calculator in this paper.
•   Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

•   The number of marks is given in brackets [ ] at the end of each question or part question on the question paper.
•   You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
•   The total number of marks for this paper is **100**.
•   The printed answer book consists of **20** pages. The question paper consists of **8** pages. Any blank pages are indicated.
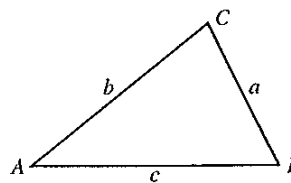
INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

•   Do not send this question paper for marking; it should be retained in the centre or destroyed.

Formulae Sheet: 6993 Additional Mathematics

In any triangle ABC

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Answer all questions on the Printed Answer Book provided.

Section A

1   Determine whether the point (5, 2) lies inside or outside the circle whose equation is $x^2 + y^2 = 30$.
     You must show your working.   [3]

2   The equation of a curve is $y = x^3 - x^2 - 2x - 3$.

     Find the equation of the tangent to this curve at the point (3, 9).   [5]

3   In the triangle PQR, PQ = 8 cm, RQ = 9 cm and RP = 7 cm.

     (i) Find the size of the largest angle.   [4]

     (ii) Calculate the area of the triangle.   [3]

4   Solve the equation $5 \sin 2x = 2 \cos 2x$ in the interval $0° \leqslant x \leqslant 360°$.
     Give your answers correct to 1 decimal place.   [5]

5   The coordinates of the points A, B and C are (−2, 1), (5, 2) and (4, 9) respectively.

     (a) Find the coordinates of the midpoint, M, of the line AC.   [1]

     (b) Show that BM is perpendicular to AC.   [3]

     (c) (i) Use the result of part (b) to state the mathematical name of the triangle ABC.   [1]

          (ii) Prove this by another method.   [2]

6   Solve the inequality $x^2 - 12x + 35 \leqslant 0$.   [4]

7   (a) Determine whether or not each of the following is a factor of the expression $x^3 - 7x + 6$.
          You must show your working.

          (i) $(x - 2)$ [2]

          (ii) $(x + 1)$ [1]

     (b) (i) Factorise the function $f(x) = x^3 - 7x + 6$.   [3]

          (ii) Solve the equation $f(x) = 0$.   [1]

8   (i) On the axes given, indicate the region for which the following inequalities hold. shade the region which is **not** required.

$$5x + 3y \geqslant 30$$
$$3x + y \geqslant 12$$
$$y \geqslant 0$$
$$x \geqslant 0$$

[5]

   (ii) Find the minimum value of $6x + y$ subject to these conditions. [2]

9   The gradient function of a curve is given by $\dfrac{dy}{dx} = 3x^2 - 2x + 4$.

Find the equation of the curve, given that it passes through the point (2, 2). [4]

10   You are given that $\sin \theta = \frac{2}{5}$ with $0° \leqslant \theta \leqslant 90°$.

Using the identity $\sin^2 \theta + \cos^2 \theta = 1$, find an exact value for $\cos \theta$. [3]

Section B

11   Eggs are delivered to a supermarket in boxes of 6.
For each egg, the probability that it is cracked is 0.05 independently of other eggs.

Find the probability that

(i) in one box there are no cracked eggs,   [2]

(ii) in one box there is exactly 1 cracked egg.   [4]

The manager checks the eggs as follows.

- He takes a box at random from the delivery.
- He accepts the whole delivery if this box contains no cracked eggs.
- He rejects the whole delivery if the box contains 2 or more cracked eggs.
- If the box contains 1 cracked egg then he chooses another box at random.
- He accepts the delivery only if this second box contains no cracked eggs.

(iii) Find the probability that the delivery is rejected.   [6]

12   Two cars, A and B, move from rest away from a point O on a straight road starting at the same time.

(a) Car A moves with constant acceleration of $2 \, m \, s^{-2}$.

Express the displacement of car A after time t seconds as a function of t.   [2]

(b) Car B moves with acceleration given by $a = \frac{1}{2}t + 1$.

Express the displacement of car B after time t seconds as a function of t.   [4]

(c)   (i) Find the time at which the cars are the same distance from O.   [2]

(ii) Find the distance they have travelled at that time.   [2]

(d) Draw a sketch graph of the velocity of each car on the axes given.   [2]

13 A pyramid has a square base, ABCD, with vertex E. E is directly above the centre of the shown in Fig. 13.
The lengths of the sides of the base are each $2x$ metres and the height is h metres.
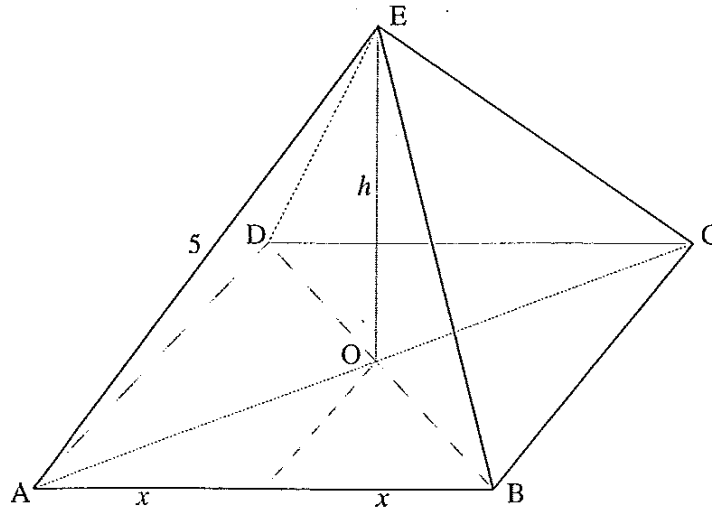The lengths of the sloping edges, AE, BE, CE and DE, are each 5 metres.

Fig. 13

(i) Show that $2x^2 = 25 - h^2$. [2]

(ii) Show that the volume of the pyramid, $V\,\text{m}^3$, is given by $V = \dfrac{50h - 2h^3}{3}$. [2]

(iii) As h varies, find the value of h for which V has a stationary value. [4]

(iv) Prove that this stationary value is a maximum. [2]

(v) Calculate the angle between the edge AE and the base when h takes this value. [2]

[Volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height.]

14 The cross-section of a speed hump is modelled by the region enclosed by the x-axis and t

$$y = \frac{1 - (x-1)^4}{5}.$$
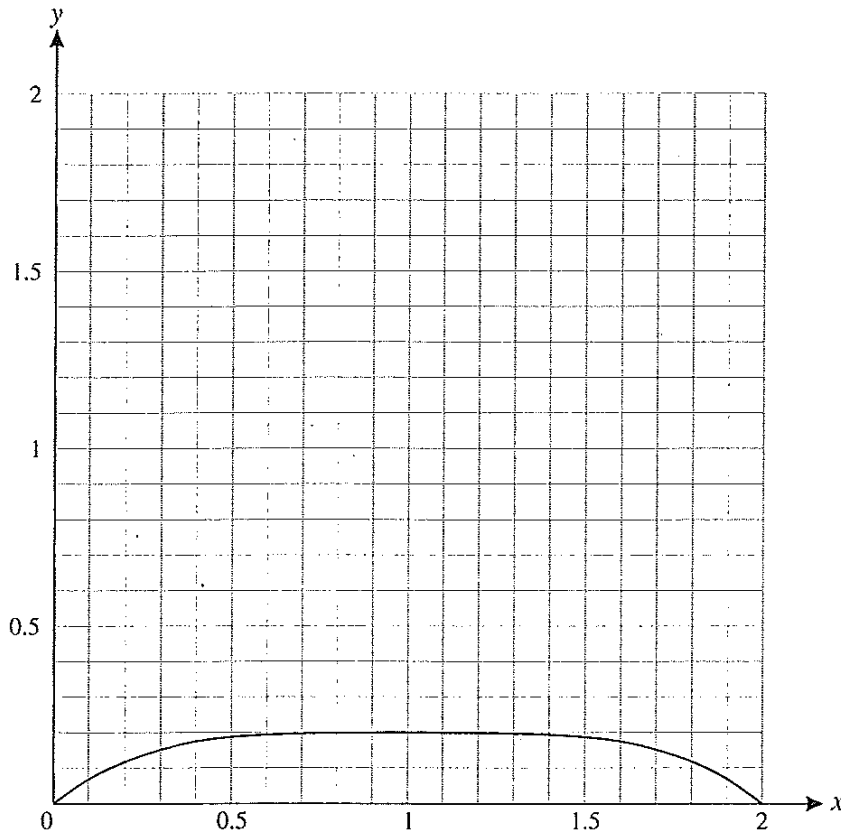
The graph is shown in Fig. 14.
Units are metres.

Fig. 14

(a) (i) Write down the maximum value of $1 - (x-1)^4$. [1]

(ii) Hence write down the maximum height of the speed hump. [1]

(b) Show that $y = \frac{1}{5}(4x - 6x^2 + 4x^3 - x^4)$. [3]

(c) Find the area of the cross-section of the speed hump. [7]

Additional FSMQ

Free Standing Mathematics Qualification

6993: Additional Mathematics

Mark Scheme for June 2011

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1. Subject-specific Marking Instructions

1. **M** (method) marks are not lost for purely numerical errors.
   A (accuracy) marks depend on preceding **M** (method) marks. Therefore **M0 A1** <u>cannot</u> be awarded.
   B (independent) marks are independent of **M** (method) marks and are awarded for a correct final answer or a correct intermediate stage.

2. Subject to 1, two situations may be indicated on the mark scheme conditioning the award of **A** marks or **B** marks:
   i.   Correct answer correctly obtained (no symbol)
   ii.   Follows correctly from a previous answer whether correct or not (**FT** on mark scheme and on the annotations tool).

3. Always mark the greatest number of significant figures seen, even if this is then rounded or truncated in the answer.

4. Where there is clear evidence of a misread, a penalty of 1 mark is generally appropriate. This may be achieved by awarding **M** marks but not an **A** mark, or awarding one mark less than the maximum.

5. For methods not provided for in the mark scheme give as far as possible equivalent marks for equivalent work. If in doubt, consult your team leader.

6. Where a follow through (**FT**) mark is indicated on the mark scheme for a particular part question, you must ensure that you refer back to the answer of the previous part question if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

2. Abbreviations

The following abbreviations are commonly found in Mathematics mark schemes.

- Where you see **oe** in the mark scheme it means **or equivalent.**

- Where you see **cao** in the mark scheme it means **correct answer only.**

- Where you see **soi** in the mark scheme it means **seen or implied.**

- Where you see **www** in the mark scheme it means **without wrong working.**

- Where you see **rot** in the mark scheme it means **rounded or truncated.**

- Where you see **seen** in the mark scheme it means that you should award the mark if that number/expression is seen anywhere in the answer space, even if it is not in the method leading to the final answer.

- Where you see **figs 237**, for example, this means any answer with only these digits. You should ignore leading or trailing zeros and any decimal point e.g. 237000, 2·37, 2·370, 0·00237 would be acceptable but 23070 or 2374 would not.

ection A

Question		Answer	Marks	Part Marks and Guidance
1		For (5, 2) use $x^2 + y^2$ $= 29$	M1 A1	Substitute or use Pythagoras **soi** or √29 As usual only award A marks if the M mark has been awarded. **Alternative method:** Sub of $x = 5$ or $y = 2$ in $x^2 + y^2 = 30$ to find y or x **M1** $y = √5$ or $x = √26$ **A1**
		so inside	A1 **3**	Conclusion (dependent on M1A1 awarded)
2		$\dfrac{dy}{dx} = 3x^2 - 2x - 2$ At $x = 3$ gradient $= 27 - 6 - 2 = 19$ $\Rightarrow y - 9 = $ "their" $19(x - 3)$	M1 A1 A1 M1	Differentiate All three terms correct 19 **isw** (**dep** on first **M1**) Find line using correct point and *their* 19 At least one power decreased by 1. "*their*" 19 means: the value of the derivative
		$\Rightarrow y = 19x - 48$ oe	A1 **5**	Only 3 terms
3	i	eg $\cos P = \dfrac{8^2 + 7^2 - 9^2}{2.8.7}$ oe	M1 A1	Cosine formula correctly used to find any angle Anything that rounds to 73.4°, 48.2° or 58.4°
		$\Rightarrow P = 73.4°$	M1 A1 **4**	For identifying correct angle Anything that rounds to 73.4°

Question		Answer	Marks		Part Marks and Guidance
3	ii	Area $= \dfrac{1}{2} \times 7 \times 8 \times \sin(their\ angle\ \text{P})$	**M1** **A1**		Use of formula Correct substitution from *their* **(i)**
		$= 26.8$	**A1**		Anything that rounds to 26.8
					Accept complete alternative methods
			3		
4		$5\sin 2x = 2\cos 2x$	**M1** **A1**		Use of tan 0.4
		$\Rightarrow \tan 2x = 0.4$			allow $\tan x = 0.4$ for **M1A1**
		$\Rightarrow 2x = 21.8,\ 201.8$	**A1**		**Alternative method** Use of Pythagoras to get
		$\Rightarrow x = 10.9,\ 100.9$	**A1**		$\sin 2x = \dfrac{2}{\sqrt{29}}$ or $\cos 2x = \dfrac{5}{\sqrt{29}}$
		Also $x = 190.9,\ 280.9$	**A1**		**M1A1** *and the last three marks are still available, ignore extra solutions*
					10.9 Any 2ⁿᵈ value 3ʳᵈ and 4ᵗʰ values (*ignore extra solutions*)
			5		
5	a	*M is* $\left(\dfrac{-2+4}{2},\ \dfrac{1+9}{2}\right)$ *which is* $(1,5)$	**B1**		
			1		
5	b	Gradient of AC is $\dfrac{9-1}{4+2} = \dfrac{4}{3}$	**B1**		One gradient
		Gradient of BM is $\dfrac{2-5}{5-1} = -\dfrac{3}{4}$	**B1**		Second gradient
		$\dfrac{4}{3} \times -\dfrac{3}{4} = -1$ oe	**B1**		*Their* $m_1 \times m_2 = -1$ Eg One is the negative reciprocal of the other
			3		
5	c i	Isosceles	**B1**		Allow right-angled isosceles Accept wrong spelling Do not accept right-angled triangle
			1		

4

Question		Answer	Marks	Part Marks and Guidance
c	ii	$AB^2 = 7^2 + 1^2 = 50$ $BC^2 = 7^2 + 1^2 = 50$ $\Rightarrow$ two sides equal in length	**M1** **A1** **2**	Using Pythagoras on AB and BC　Attempt by vectors **AB** and **BC M1** Or fully labelled diagram with correct sides shown　**Alternative** If answer to (**c**)(**i**) was right-angled, then accept proof that it is (requires all three lengths.) **Alternative:** If (**c**)(**i**) was equilateral or scalene then **M1** (only) for attempt at all three sides. NB If nothing is written in (**i**) then no credit in this part.
6		$(x \pm 5)(x \pm 7)$ Boundaries $x = 5, x = 7$ $\Rightarrow 5 \leq x \leq 7$	**M1** **A1** **B2** **4**	Or use of correct formula (allow one error in substitution) or correct shaped graph seen **soi** Accept $x \geq 5$, $x \leq 7$ for **B1, B1**　Condone < or >
a	i	Attempt to find f(2) by substitution of 2 = 0, So Yes	**M1** **A1** **2**	Remainder theorem or attempt to divide (justification is sight of $x^3 - 2x^2$) Or: attempt to factorise, justification is sight of $(x^2 \dots 3)$ Correct working only
a	ii	$f(-1) = -1 + 7 + 6 = 12$ so no.	**B1** **1**	Sight of 12 or correct evidence, conclusion required
b	i	$f(x) = (x - 2)(x^2 + 2x - 3)$ $= (x - 2)(x + 3)(x - 1)$	**M1** **A1** **A1** **3**	Attempt to factorise or use long division (justifications as in (**a**)(**i**)) Sight of correct quadratic **soi** Answer　**Alternative:** Use of Remainder theorem **M1** Sight of 2nd factor **A1** All correct **A1**
b	ii	$x = 1, 2, -3$	**B1** **1**	**FT** their brackets　Must be three roots

5

Question	Answer	Marks	Part Marks and Guidance
3 i		**B1** for one line	*If there is work here that is not crossed out, then mark it and ignore anything on Page 18.*
		B1 for correct shading	Helpful hint:
		B1 for other line	Lines go through (0, 12) and (4,0)
		B1 for correct shading	(0, 10) and (6, 0)
		B1 for correct shading to give $x \geq 0$, $y \geq 0$	Intersection at (1.5, 7.5)
		5	If **B0** for a line allow **B1** for shading if negative gradient and lines intersect
			NB If intercepts are within 1 small square of the correct points then give the marks for the lines
3 ii	$6x + y$ is minimum at (0, 12) *(can be implied by correct answer)* So is 12	**B1** **B1** **2**	
9	$\dfrac{dy}{dx} = 3x^2 - 2x + 4 \Rightarrow (y) = x^3 - x^2 + 4x + c$	**M1** Integrate **A1** Ignore c	At least one term with power increased by 1. (NB do not accept multiplying throughout by x)
	(2,2) satisfies $\Rightarrow 2 = 8 - 4 + 8 + c$ $\Rightarrow c = -10$	**M1** (**dep** on 1st **M1** mark) Substitute	
	$\Rightarrow y = x^3 - x^2 + 4x - 10$	**A1** **cao** **4**	(ie must be $y = \dots$)
10	$\sin\theta = \dfrac{2}{5} \Rightarrow \sin^2\theta = \dfrac{4}{25} \Rightarrow 0.16 + \cos^2\theta = 1$	**M1**	Sight of a triangle with sides 2, 5, $\sqrt{21}$ acceptable for **M1** Then **A2** for $\cos\theta$
	$\cos^2\theta = \dfrac{21}{25}$	**A1** $\cos^2\theta$	
	$\Rightarrow \cos\theta = \dfrac{1}{5}\sqrt{21}$ oe	**A1** eg $\sqrt{0.84}$ or $\sqrt{\dfrac{21}{25}}$ **isw** **3**	NB **M0** if calculator used to find θ in order to find $\cos\theta$

6

ection B

Question		Answer	Marks	Part Marks and Guidance
1	i	$P(0) = (0.95)^6$ $= 0.735(09189....)$	M1 A1 **2**	Correct p plus correct power Not 2sf
1	ii	$P(1) = 6 \times (0.95)^5 \times (0.05)^1$ $= 0.232(134281....)$	M1 B1 B1 A1 **4**	Correct p and q and powers add to 6 Coefficient **soi** Correct powers for correct p and q **soi** Coefficient may be missing
1	iii	P(1st box contains 2 or more eggs) = 1 – (their(**i**) + their (**ii**)) = 1 – (0.7351 + 0.2321) =1 – 0.9672 = 0.0328 P(2nd box has any cracked eggs) = 1 – *their* (**i**) = 0.2649 P(consignment is rejected) = 0.0328 + 0.2649 × their (**ii**) = 0.0328 + 0.0615 = 0.0943	M1 A1 M1 A1 M1 A1 **6**	Accept anything rounding to 0.033 Accept anything rounding to 0.265 In either method, accept answers which lie between 0.094 and 0.095 Alternative P(accepted) **M1** Ans(**ii**) × Ans(**i**) **A1** 0.1706 **soi** (Accept 0.171) **M1(dep)** Add to this Ans(**i**) **A1** 0.9057 (Accept 0.906) **M1** P(consignment is rejected) = 1 – 0.9057 **A1** = 0.09428

7

Question		Answer	Marks	Part Marks and Guidance
2	a	$s = ut + \frac{1}{2}at^2$ with $u = 0$ and $a = 2$ $\Rightarrow s = t^2$	**M1** **A1** **2**	Constant acceleration formulae or integrate twice – ignore c
2	b	$(v) = \frac{t^2}{4} + t$ $s = \frac{t^3}{12} + \frac{t^2}{2}$ Ignore c	**M1** **A1** **M1** **A1** **4**	Integrate Integrate
2	c i	$\frac{t^3}{12} + \frac{t^2}{2} = t^2$ $\Rightarrow \frac{t}{12} + \frac{1}{2} = 1$ $\Rightarrow t = 6$	**M1** **A1** **2**	Equate their functions
2	c ii	$s = 6^2$ or $s = \frac{6^3}{12} + \frac{6^2}{2}$ Displacement = 36 (m)	**M1** **A1** **2**	Substitute *their non-zero* (**c**)(**i**) in *their* (**a**) or (**b**) **soi**
2	d		**B1** **B1** **2**	One clearly straight line through origin with positive gradient Other clearly a curve through the origin of correct shape with first part below the line as per diagram Ignore labels

8

Question	Answer	Marks	Part Marks and Guidance
3 i	$AO^2 = x^2 + x^2 = 2x^2$ or $AC^2 = (2x)^2 + (2x)^2 = 8x^2$ $h^2 + AO^2 = AE^2 \Rightarrow h^2 + 2x^2 = 25$ $\Rightarrow 2x^2 = 25 - h^2$	**M1** **A1** **2**	Correct application of Pythagoras on the base NB Answer is given Algebra must be convincing
3 ii	$V = \frac{1}{3} \times$ base area $\times$ height $= \frac{1}{3} \times 4x^2 h$ $= \dfrac{50h - 2h^3}{3}$	**M1** **A1** **2**	Formula seen including $4x^2$ Care: the answer is given
3 iii	$\dfrac{dV}{dh} = \dfrac{50 - 6h^2}{3}$ $= 0$ when $50 - 6h^2 = 0$ $\Rightarrow h^2 = \dfrac{25}{3}$ $\Rightarrow h = \sqrt{\dfrac{25}{3}} = \dfrac{5}{\sqrt{3}} = \dfrac{5}{3}\sqrt{3} = 2.89$	**M1** **A1** **M1** **A1** **4**	**SC3** $h = 2.89$ with either $\dfrac{dV}{dh}$ or $\dfrac{1}{3}$ missing Differentiation **cao** **dep** Set (numerator) = 0 Numerical value must be 2.89 Any of these answers is acceptable
3 iv	$\dfrac{d^2V}{dh^2} = -4h$ < 0 so maximum	**M1** **A1** **2**	Accept $-12h$ Or alternatives: Complete method to investigate value of derivative Or: complete method to investigate the value of V either side **and** at the turning point
3 v	At this point sinEAO $\dfrac{h}{5} = \dfrac{1}{3}\sqrt{3}$ $\Rightarrow$ Angle EAO = 35.3°	**M1** **A1** **2**	Use of a correct ratio with *their h* (and/or *x*) Accept 35.2 which comes from $h = 2.88$

9

Mark Scheme

Question			Answer	Marks	Part Marks and Guidance
4	**a**	**i**	Max value = 1	**B1** **1**	Not from any use of 0.2 from graph
4	**a**	**ii**	Height = 0.2 (m) or 20 cm	**B1** **1**	
4	**b**		$x^4 - 4x^3 + 6x^2 - 4x + 1$ $\Rightarrow y = \dfrac{1}{5}\left(4x - 6x^2 + 4x^3 - x^4\right)$	**B2** **B1** **3**	-1 each error An error is signs, powers, coefficients, failure to include the 1 at end **Dep on B2** convincing algebra (means sight of an extra correct step **www**)
4	**c**		Area $= \displaystyle\int_0^2 \dfrac{1}{5}\left(4x - 6x^2 + 4x^3 - x^4\right).dx$ $= \dfrac{1}{5}\left[2x^2 - 2x^3 + x^4 - \dfrac{x^5}{5}\right]_0^2$ $= \dfrac{1}{5}\left(8 - 16 + 16 - \dfrac{32}{5}\right) = \dfrac{8}{25}$ $= 0.32$ Area of cross section $= 0.32m^2 =$ $3200cm^2$	**M1** **A3** **M1** **A1** **A1** **7**	**Alternative method:** Integrate original function is OK, but in dealing with limits $x = 0$ must then be seen. Integrate (ignore c) **A2** if one error, **A1** if two errors Omission of $\dfrac{1}{5}$ is one error. (**Dep** on 1st **M1**) Deal with limits correctly (Putting $x = 0$ does not need to be seen) Multiply by $\dfrac{1}{5}x$ or $\dfrac{1}{5x}$, ie integrating $\dfrac{1}{5}$ gives **A0** Units

10

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