OCR
RECOGNISING ACHIEVEMENT

FREE-STANDING MATHEMATICS QUALIFICATION
ADVANCED LEVEL

Additional Mathematics 6993

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 16 page Answer Booklet
- Graph paper

Other Materials Required:
None

Tuesday 15 June 2010
Morning

Duration: 2 hours

6993

INSTRUCTIONS TO CANDIDATES

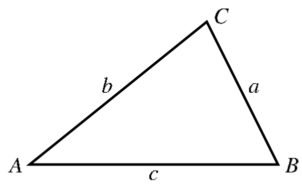
- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- You are not allowed a formulae booklet in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- This document consists of **8** pages. Any blank pages are indicated.

Formulae Sheet: 6993 Additional Mathematics

In any triangle *ABC*



Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Section A

1 Solve the inequality $3 - x < 4(x - 1)$.

2 Expand $(1 - x)^{12}$ in ascending powers of x up to the term in x^3, and simplify your answer. **[3]**

3 The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = x^3 - 5x^2 + 2x + 8$.

(i) Find the remainder when $\mathrm{f}(x)$ is divided by $(x + 1)$. **[2]**

(ii) Solve the equation $\mathrm{f}(x) = 0$. **[3]**

4 In a game 4 fair dice are thrown.

Calculate the probability that

(i) no six is thrown, **[2]**

(ii) at least 2 sixes are thrown. **[4]**

5 The curve $y = x^3 - 3x^2 - 9x + 7$ has two turning points, one of which is where $x = 3$.

(i) Find the coordinates of the other turning point and determine whether it is a maximum or minimum point. **[5]**

(ii) Sketch the curve. **[1]**

6 An aeroplane touches down at a point A on a runway, travelling at $90\,\mathrm{m\,s^{-1}}$. It then decelerates uniformly until it reaches a speed of $6\,\mathrm{m\,s^{-1}}$ at a point B on the runway, 2016 m from A.

(i) Find the deceleration. **[3]**

(ii) Find the time taken to travel from A to B. **[2]**

7 It is required to solve the equation $\sin\theta\cos\theta = \frac{1}{4}$.

(i) Show that $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\sin\theta\cos\theta}$. **[**

(ii) Hence show that the equation $\sin\theta\cos\theta = \frac{1}{4}$ is equivalent to $\tan\theta + \dfrac{1}{\tan\theta} = 4$. **[2]**

(iii) By expressing this equation as a quadratic equation in t, where $t = \tan\theta$, find the two values of θ, in the range $0^\circ \leqslant \theta \leqslant 180^\circ$, that satisfy the equation. **[4]**

8 A train moves between two stations, taking 5 minutes for the journey.
The velocity of the train may be modelled by the equation $v = 60(t^4 - 10t^3 + 25t^2)$ where v is measured in metres per minute and t is measured in minutes.

Calculate the distance between the two stations. **[5]**

9 The diameter of a circle is PQ, where P and Q are the points (1, 3) and (15, 1) respectively.

(i) Find the centre of the circle. **[2]**

(ii) Show that the radius of the circle is $5\sqrt{2}$. **[2]**

(iii) Hence find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. **[2]**

10 John and Paul are carrying out an experiment.

The table shows their results for x and y.

x	0	2	3	4
y	4	0	0.25	0

Paul proposes that the relationship should be modelled by $y = k(x-2)(x-4)$. This is shown in Fig. 10.

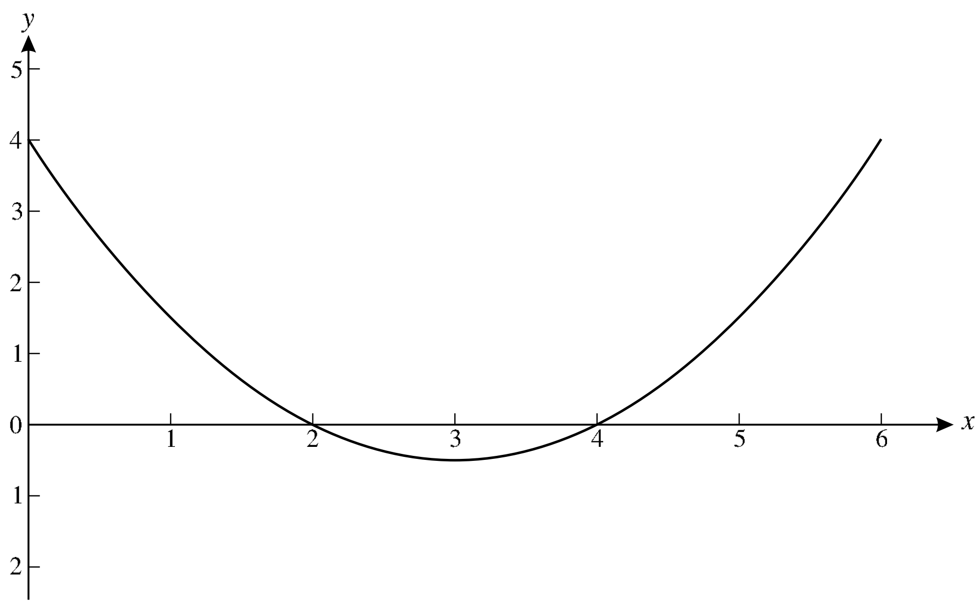


Fig. 10

(i) Find the value of k for which the points (0, 4), (2, 0) and (4, 0) satisfy this equation. **[2]**

John proposes a different model, using $y = c(x-2)^2(x-4)$.

(ii) Find the value of c for which the points (0, 4), (2, 0) and (4, 0) satisfy this equation. **[2]**

(iii) Which is the better model for John and Paul's results? Give a reason for your answer. **[2]**

Section B

11 Michael is at a point A and the base of a church tower is at a point F, as shown in Fig. 11.
He measures the bearing of the tower to be 060°.
Michael walks 100 metres due North to the point B from where he measures the bearing of F to be 110°.
The triangle ABF is in the horizontal plane.

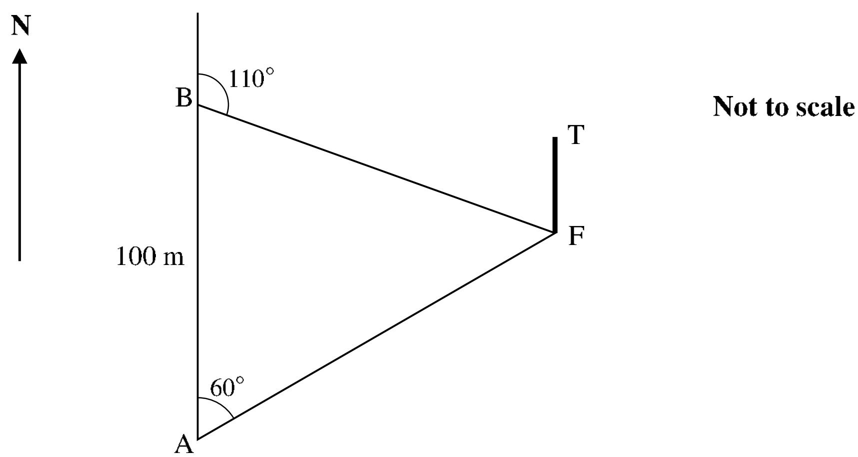


Fig. 11

(i) Show that AF = 122.7 m, correct to 4 significant figures, and find BF. **[5]**

Michael finds that the angle of elevation of the top of the tower, T, from A is 10°.

(ii) Find the height of the tower. **[2]**

C is the point on AB that is nearest to F.

(iii) Find CF and the angle of elevation from C to the top of the tower, correct to 1 decimal place. **[5]**

12 Fig. 12 shows the shape AOB that is to be made from card.

B is the point (5, 0) and OB is part of the curve with equation $y = 0.3x^2 - 1.5x$.

The line AB is the normal to the curve at B.

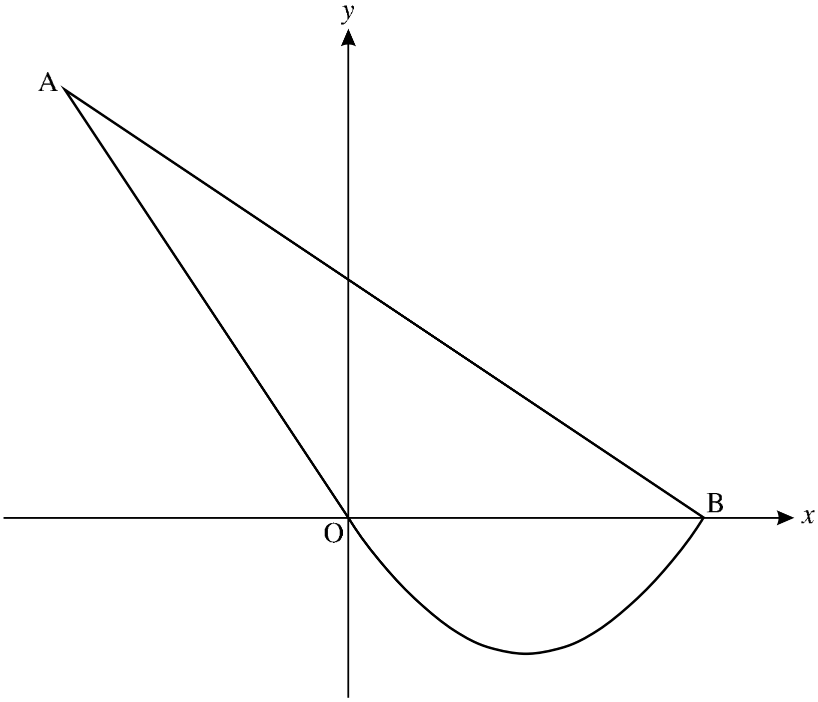


Fig. 12

(i) Find the equation of the line AB. **[4]**

The equation of the line AO is $2y + 3x = 0$.

(ii) Find the coordinates of the point A. **[3]**

(iii) Find the area of the shape AOB. **[5]**

[Questions 13 and 14 are printed overleaf.]

13 Ali and Beth make components in a factory. Ali works faster than Beth and makes 3 more co per hour. As a result he takes 2 hours less time than Beth to make 72 components.

Let t hours be the time that Ali takes to make 72 components.

(i) Write expressions for the numbers of components made per hour by Ali and by Beth. **[3]**

(ii) Hence derive the equation $3t(t+2) = 144$. **[5]**

(iii) Solve this equation to find the times that Ali and Beth take to make 72 components. **[4]**

14 A firm has to transport 1500 packages to a site. It has a number of large vans which will transport 200 packages each and a number of small vans which will transport 100 packages each.

Let x be the number of large vans and let y be the number of small vans used.

(i) Write down an inequality based on the number of packages transported. **[2]**

The firm needs to use at least as many small vans as large vans.

(ii) Write a second inequality. **[1]**

(iii) Plot these two inequalities on a graph, using 1 cm to represent one van on each axis. Indicate the region for which these inequalities hold. Shade the area that is **not** required. **[3]**

A large van costs £80 to complete the trip and a small van costs £60 to complete the trip.

(iv) Write down the objective function and hence find from your graph the number of each type of van that will minimise the cost, and work out that cost. **[4]**

(v) What choice of vans should be made to minimise the cost if the restriction about the large and small vans is removed? Work out the cost in this case. **[2]**

ADVANCED FSMQ

MARK SCHEME

Maximum mark: 100

Syllabus/component:

6993 Additional Mathematics

Paper date: June 15, 2010

Mark Scheme, 2010

OCR - ADDITIONAL MATHEMATICS 6993
Marking instructions.

The total mark for the paper is **100.**

Marks for method are indicated by an **M**. A method that is dependent on previous work is **DM.**

Marks for accuracy are of two kinds:

(i) **A** mark indicates correct work only and
(ii) **F** mark indicates that a "follow through" is allowed.

If an **M** mark is not gained then nor do any of the accuracy marks associated with it.

Marks not associated with a method are denoted **B**, which should be treated as "correct only", and **E** which may be wrong because of a previous error.

Marks are not divisible except as indicated. e.g. A 2,1 means that 2 are awarded for a correct answer and 1 for an answer that is only partially correct, as agreed at the meeting of Examiners.

When the method of solution is not one that has been discussed and does not fit the existing scheme then an alternative scheme should be devised which maintains the same number of M, A, F, B and E marks. You should also bring this to the attention of the Principal Examiner.

The rubric says that the norm is for answers to be given to 3 s.f. except where indicated. Where this rubric is ignored then 1 mark should be deducted once in the paper, at the point where it is first met. This should be indicated -1, TMSF or -1TFSF. Details will be discussed at the meeting of examiners.

Misreading of a question (including the candidate's own working) should normally be penalised by the loss of the relevant accuracy mark or two marks (whichever is less); but if the question is made substantially easier then further penalties may be imposed.

Sub-marks should be shown near to the relevant work. If these are individual marks then the appropriate letter should be given. Sub-marks are given in the question paper and the mark scheme. For substantially correct solutions a number of sub-marks may be combined, even up to the total mark for the question for a totally correct question. The sum of the sub-marks are then added and ringed at the end of the question. (This means that a totally correct question has the total mark written twice - once as a "sum of sub-marks" and unringed and once ringed as the total for the question.) The total mark for the paper should be given on the front page, top right and ringed.

Work that is crossed out and not replaced should be marked. If work has been crossed out and replaced then the replacement work should be marked even if it is incorrect and the crossed out work correct.

Any notation that is understandable may be used to support your marking. In particular:
isw – ignore subsequent working
www – without wrong working
soi – seen or implied

An independent person should be used to check the summation of marks. You should add the [illegible] marks on the paper to check the addition and the independent checker should add the unringed marks. There is a fee paid for this checking - if you are unable to find anyone to do this work the Board and the Principal Examiner must be informed.

Please mark in red.

Section A

1		$3 - x < 4(x-1)$ $\Rightarrow 3 - x < 4x - 4$ $\Rightarrow 7 < 5x$ $\Rightarrow x > \frac{7}{5}$	B1 B1 B1 **3**	Sight of $4x - 4$ Sight of ax and b where either $a = 5$ or $b = 7$ oe Final answer WWW
2		$= 1 - \binom{12}{1}x + \binom{12}{2}x^2 - \binom{12}{3}x^3$ $= 1 - 12x + 66x^2 - 220x^3$ *Ignore terms of higher power*	B1 B1 B1 **3**	Signs and powers 2 out of 3 coefficients worked out All coefficients and 1
3	**(i)**	Remainder is f(−1) $= -1 - 5 - 2 + 8 = 0$ *For long division $x^3 + x^2$ in working and x^2 in quotient must be seen for M1* *Or by inspection $(x + 1)(x^2 + \ldots)$ for M1*	M1 A1 **2**	Or long division 0 must be seen or implied
	(ii)	$x^3 - 5x^2 + 2x + 8 = 0$ $\Rightarrow (x+1)(x^2 - 6x + 8) = 0$ $\Rightarrow (x+1)(x-2)(x-4) = 0$ $\Rightarrow x = -1, 2, 4$ *Allow ans with no working*	M1 DM1 A1 **3**	Factorise cubic to give $(x + 1)(ax^2 + bx + c)$ Solve their quadratic
		Alt: Trial to find one root: $x = 2, 4$ M1, A1 $\Rightarrow x = -1, 2, 4$ A1		

4	**(i)**	$\left(\frac{5}{6}\right)^4 = \frac{625}{1296} = 0.4823$	M1 A1 **2**	Either form or 0.482 isw
	(ii)	$1-\left(\frac{5}{6}\right)^4 - 4\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)$ $= 1 - \frac{625}{1296} - \frac{500}{1296} = 1 - 0.4823 - 0.3858$ $= \frac{171}{1296} = \frac{19}{144} = 0.1319$	M1 B1 B1 A1 **4**	1 − 2 terms 4 soi Powers Ans in either form or 0.132
		Alt: Add three terms — M1 $6\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2 + 4\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^4$ — B1 both coeffs; B1 powers $= 0.11574 + 0.01543 + 0.00077$ $= 0.1319$ — A1 ans		

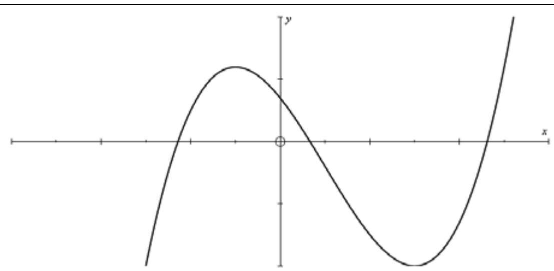
<table>
<tr><td>5</td><td>(i)</td><td>$\frac{dy}{dx} = 3x^2 - 6x - 9$
$= 0$ when $3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow (x-3)(x+1) = 0 \Rightarrow x = 3, -1$
When $x = -1, y = 12$
$\frac{d^2y}{dx^2} = 6x - 6 < 0$ when $x = -1$ so maximum

Allow SC1 for (–1, 12) with no working</td><td>M1
A1

A1
M1
A1

5</td><td>Diffn **and** set = 0
Derived fn

Stationary point
To find nature of turning points</td></tr>
<tr><td></td><td></td><td>Alternative ways to demonstrate maximum at $x = -1$
Value of y — M1
<table><tr><td>– 1 –</td><td>– 1</td><td>–1 +</td></tr><tr><td>$y < 12$</td><td>$y = 12$</td><td>$y < 12$</td></tr></table>A1

Gradient of tangent — M1
<table><tr><td>– 1 –</td><td>– 1</td><td>–1 +</td></tr><tr><td>$\frac{dy}{dx} > 0$</td><td>$\frac{dy}{dx} = 0$</td><td>$\frac{dy}{dx} < 0$</td></tr><tr><td>/</td><td>—</td><td>\</td></tr></table>A1</td><td></td><td>Allow at most one integer either side (typically, $x = -2, 0$ if turning point is correct)</td></tr>
<tr><td></td><td>(ii)</td><td></td><td>B1

1</td><td>General shape:
turning points in correct quadrants
Intercept on y axis in [0,12]
Does not turn back on itself.</td></tr>
<tr><td>6</td><td>(i)</td><td>$u = 90, v = 6, s = 2016$
$\Rightarrow 6^2 = 90^2 + 2a \times 2016$
$\Rightarrow a = -\frac{90^2 - 6^2}{4032} = -\frac{8064}{4032} = -2 \text{ m s}^{-2}$</td><td>M1
A1
A1

3</td><td>Using correct formula
Correct substitution</td></tr>
<tr><td></td><td>(ii)</td><td>$u = 90, v = 6, a = -2$
$\Rightarrow 6 = 90 - 2t$
$\Rightarrow t = \frac{84}{2} = 42$ secs
The two parts can be the other way round</td><td>M1

A1

2</td><td>Using correct formula</td></tr>
</table>

7	(i)	$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ $=\frac{1}{\sin\theta\cos\theta}$	B1 **1**	
		Alt: $\sin^2\theta+\cos^2\theta=1$ $\Rightarrow \sin\theta+\frac{\cos^2\theta}{\sin\theta}=\frac{1}{\sin\theta}$ $\Rightarrow \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{1}{\sin\theta\cos\theta}$		
	(ii)	$\sin\theta\cos\theta=\frac{1}{4}\Rightarrow\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=4$ $\Rightarrow \tan\theta+\frac{1}{\tan\theta}=4$	M1 A1 **2**	Using **(i)** and tan
	(iii)	$\tan\theta+\frac{1}{\tan\theta}=4\Rightarrow\tan^2\theta+1=4\tan\theta$ $\Rightarrow t^2-4t+1=0$ $t=\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}\ \left(=3.732\text{ and }0.268\right)$ $\Rightarrow\theta=15^0\text{ and }75^0$ *Sp Case B1 for 15 and B1 for 75 with no supporting working*	M1 M1 A1 A1 **4**	Clear fractions to give 3 term quadratic Sub numbers into correct quadratic 3sf or more Rounds to these
8		$v=60\left(t^4-10t^3+25t^2\right)$ $\Rightarrow s=\int_0^5\left(60t^4-600t^3+1500t^2\right)\mathrm{d}t$ $=\left[12t^5-150t^4+500t^3\right]_0^5$ $=6250$ m If 60 is left out then 4/5 only.	M1 A2,1 DM1 A1 **5**	Integrate Terms −1 each error Sub t = 5 Cao

9	**(i)**	Centre is $\left(\frac{1+15}{2},\frac{3+1}{2}\right)=(8,2)$ Nb Working with vectors to give diameter = [14,2] and so radius = [7,1] giving centre (15 – 7, 3 –1) is correct.	B1 B1 **2**	For 8 WWW For 2 WWW
	(ii)	$\lvert PC\rvert=\sqrt{(8-1)^2+(2-3)^2}=\sqrt{50}=5\sqrt{2}$	M1 A1 **2**	For $\sqrt{50}$
		Alt: Length of diameter = $\sqrt{(15-1)^2+(3-1)^2}=\sqrt{14^2+2^2}$ $=\sqrt{200}=10\sqrt{2}$ $\Rightarrow$ Radius = $5\sqrt{2}$		
	(iii)	$(x-8)^2+(y-2)^2=50$ $\Rightarrow x^2+y^2-16x-4y+64+4-50=0$ $\Rightarrow x^2+y^2-16x-4y+18=0$	M1 A1 **2**	Correct use of formula including 50 and using their midpoint.
10	**(i)**	Sub (0,4) Gives $k=\frac{1}{2}$	M1 A1 **2**	
	(ii)	Sub (0, 4) Gives $c=-\frac{1}{4}$	M1 A1 **2**	
	(iii)	When $x=3$ $y=-\frac{1}{4}(3-2)^2(3-4)=0.25$ for cubic Or when $x=3$, $y>0$ for cubic John's model is better	B1 DB1 **2**	

Section B

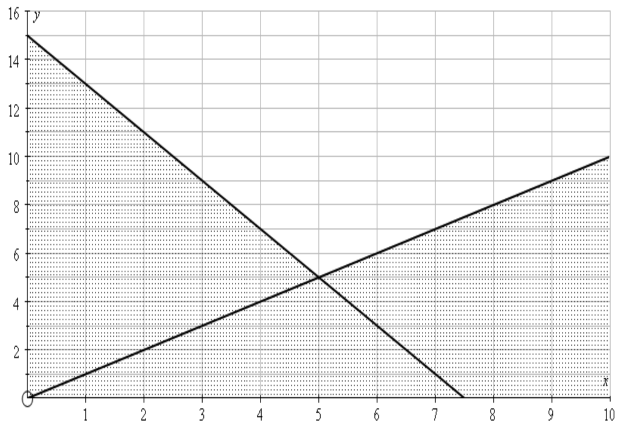
Allow 4 sf in this question

11	**(i)**	$\frac{AF}{\sin 70} = \frac{BF}{\sin 60} = \frac{100}{\sin 50}$ $\Rightarrow AF = \frac{100}{\sin 50} \times \sin 70 \; (= 122.7 \text{ m})$ $\Rightarrow BF = \frac{100}{\sin 50} \times \sin 60 = 113.1 \text{ m}$ oe	M1 A1 A1 M1 A1 **5**	Sin rule applied Sight of 50 and 70 Correct sine rule to find BF
		Alt: Cosine rule for BF: M1 $BF^2 = 100^2 + 122.7^2 - 2\times100\times122.7\times\cos 60$ $= 12785$ BF = 113.1 A1		
	(ii)	$FT = AF \times \tan 10$ $= 122.7 \tan 10 = 21.6 \text{ m}$ *Anything that rounds to 21.6*	M1 A1 **2**	
	(iii)	$CF = 122.7\sin 60$ =106.3 m Or: = *their BF* ×sin70 $\Rightarrow \tan\theta = \frac{TheirFT}{TheirCF}$ $\Rightarrow \theta = 11.5^0$	M1 A1 M1 F1 A1 **5**	Accept 106.2 or 106 Using tan correctly Substituting correctly Accept 11 or 12
		Alt: to find CF. Area of triangle $= \frac{1}{2} \times AF.AB \sin 60 = 5313$ M1 $\Rightarrow \frac{1}{2} \times CF \times 100 = 5313 \Rightarrow CF = 106.3$ A1		

12	**(i)**	$y = 0.3x^2 - 1.5x$ $\frac{dy}{dx} = 0.6x - 1.5$ When $x = 5$ $g_t = 1.5$ $\Rightarrow g_n = -\frac{2}{3}$ AB: $y = -\frac{2}{3}(x-5)$ $\Rightarrow 2x + 3y = 10$	B1 M1 A1 A1 **4**	Derivative Find g_t and use of $m_1 \times m_2 = -1$ For g_n Line in any simplified form
	(ii)	Solve simultaneously: $3y + 2x = 10$ $2y + 3x = 0$ $6y + 4x = 20$ $6y + 9x = 0$ $5x = -20$ $\Rightarrow x = -4, y = 6$ SC1: answer with no working	M1 F1 A1 **3**	Method to eliminate one variable x and y.
	(iii)	Area of triangle = $\frac{1}{2} \times 5 \times$ *their* $y = 15$ Area under curve = $\int_0^5 (0.3x^2 - 1.5x)\,dx$ $= \left[0.1x^3 - 0.75x^2\right]_0^5$ $= -6.25$ $\Rightarrow$ Area of card = $15 + 6.25 = 21.25$ *Other methods, follow scheme* *ieE1 Area of triangle* *M1 area as integral* *A1 Integrand* *A1 value for area* *A1 Final answer*	E1 M1 A1 A1 A1 **5**	Might appear anywhere in this part Ignore limits here Condone lack of –ve sign

13	**(i)**	Ali: $\frac{72}{t}$ Beth: $\frac{72}{t+2}$	M1 A1 A1 **3**	Accept Beth: $\frac{72}{t} - 3$
	(ii)	$\frac{72}{t} - \frac{72}{t+2} = 3$ $\Rightarrow 72(t+2) - 72t = 3t(t+2)$ $\Rightarrow 72t + 144 - 72t = 3t(t+2)$ $\Rightarrow 3t(t+2) = 144$	M1 A1 M1 A1 A1 **5**	Subtraction of their terms = 3 Multiply out and simplify
		Alternative (based on alternative answer to (i) $\frac{72}{\frac{72}{t} - 3} = t + 2$ M1 A1 $\Rightarrow 72t = (72 - 3t)(t+2)$ $\Rightarrow 72t = 72t - 3t^2 + 144 - 6t$ M1 A1 $\Rightarrow 3t^2 + 6t = 144 \Rightarrow 3t(t+2) = 144$ A1		
	(iii)	$3t(t+2) = 144$ $\Rightarrow 3t^2 + 6t - 144 = 0$ $\Rightarrow t^2 + 2t - 48 = 0$ $\Rightarrow (t+8)(t-6) = 0$ $\Rightarrow t = 6$ $\Rightarrow$ Ali takes 6 hours and Beth takes 8 hours. SC1 for answer with no working	 M1 A1 A1 A1 **4**	 For quadratic in simplified form. (See below) www

What is "simplified form"?
Either a quadratic with all three terms on left = 0 ready for the use of the formula
OR:
Divide through by 3 giving $t^2 + 2t = 48$ ready for solving by the completion of the square.

14	**(i)**	$200x + 100y \geq 1500$ oe	M1 A1 **2**	Deriving a linear inequality
	(ii)	$y \geq x$	B1 **1**	
	(iii)		B1 B1 E1 **3**	One line Other line Shading for both, ft their inequalities No Scales: B0, B0, E1 Condone scales not as instructed.
	(iv)	$C = 80x + 60y$ Correct point is (5, 5) Cost = £700 *In absence of OF, 80 ×5 + 60× 5 must be seen*	B1 B1 M1 A1 **4**	 Sub in OF
	(v)	Now minimum cost is at (7, 1) Giving £620 Nb (8, 0) gives £640	B1 B1 **2**	

RECOGNISING ACHIEVEMENT

FSMQ

Additional Mathematics

FSMQ **6993**

Report on the Unit

June 2010

6993/R/10

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

OCR will not enter into any discussion or correspondence in connection with this report.

Any enquiries about publications should be addressed to:

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Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

CONTENTS

Foundations of Advanced Mathematics FSMQ (6989)

REPORT ON THE UNIT

Additional Mathematics – 6993

General Comments

We can report yet another rise in the number of candidates this year, but it is distressing to note that there are still many candidates who are entered inappropriately. We have said before that this specification is intended as an enrichment specification for bright students. Typically, we would expect them to have gained, or be expected to gain, a good grade at Higher Tier of GCSE. Our impression was that this was not likely for many of the candidates. We feel that this examination is not a good experience for candidates who are only going to achieve 20% or so.

As in previous years, the rubric states that final answers should be given to three significant figures where appropriate. Where this was not done, a mark was deducted once in the paper.

There was evidence that some candidates did not finish the paper. We would like to stress that our feeling is not that the paper was too long, but that too much time was wasted taking a long way round a question or tackling it in the wrong way. This can take up a lot of time and in these cases it is not surprising that candidates did not finish.

Examples of where candidates applied long-winded methods were seen in the following questions.

2 Candidates could not derive the binomial coefficients by calculation, finding it necessary to write out Pascal's Triangle for all 12 lines. Some candidates even attempted to multiply $(1 - x)$ by itself 12 times.

3 (i) Failure to use the remainder theorem, depending instead on long division.

3 (ii) Those who knew the remainder theorem attempted to factorise by trying "random" values of x, including such values as $x = 3$ which is not a factor of the constant term, 8. Others, who had to engage in long division, often ignored all the work they had done in part (i) and started afresh.

4 (ii) It is arguable whether 1 – two terms is quicker than adding three terms, except that one of the terms in the subtraction had been found already in part (i). So the difference in the two methods is finding one term or three

5 (i) Most candidates ignored the help that the question gave in that one of the two roots of the quadratic were given. Most candidates solved the quadratic, often by the formula, to find $x = -1$.
In addition to testing for the nature of the turning point at $x = -1$, candidates did the same also for $x = 3$.

5 (ii) The question asked candidates to sketch the curve. Many candidates chose to plot the curve. This will, of course, earn the one mark available but the process of plotting the curve on graph paper will have taken them very much longer than drawing a simple sketch in the answer booklet.

7 (i) Candidates should have been aware that, for one mark, something very straightforward was being asked. Some covered a whole page with attempts to add the fractions.

10 (i) It was surprising how many candidates substituted (0, 4) to obtain a correct value of k and then went on to substitute also $(2, 0)$ and $(4, 0)$. Usually this led, incorrectly, to $k = 0$.

10 (ii) Before substitution, some candidates multiplied out the three brackets. As with part (i) time was then wasted substituting the points on the x-axis which led to $0 = 0 \times c$ giving $c = 0$.

11 (i) Using the cosine rule to find BF took longer than a second application of the sine rule.

11 (ii) Candidates are faced with a right-angled triangle with one side and one angle. It was necessary therefore to use the tan ratio to find the height of the tower. Yet many used the sine rule again.

Q11 (iii) To find the new angle of elevation in a right-angled triangle the tan ratio was once again required. Many found the third side by Pythagoras then used the sin or cosine ratios or even used the cosine rule. Some managed an even more time consuming method. Given the data of the triangle they found the area using $\text{Area} = \frac{1}{2}ab\text{Sin}C$ and then using $\text{Area} = \frac{100 \times \text{CF}}{2}$ to find CF. (Perhaps because their last trial run was using the 2009 paper?)

12 (iii) The total area required candidates to find the area between the x-axis and the curve and then add to that the area of a triangle. Candidates did not see this "extra" bit as a triangle, however, and proceeded to integrate the equations of the lines, usually incorrectly.

It should be clear that candidates, who embark on alternative and much longer methods as described above will rapidly run out of time to finish the paper properly and additionally provide more opportunities to make errors.

Comments on Individual Questions

Section A

1) (Inequality)

Nearly always well done but with frequent errors in algebraic and numerical manipulation. It was disappointing at this level to see

$5x > 7 \Rightarrow x > \frac{5}{7}$ or $-5x < -7 \Rightarrow x < \frac{7}{5}$.

$[x > 1.4]$

2) (Binomial expansion)

As outlined above, many candidates were unable to use the expansion in the form $(1-x)^{12} = 1 - \binom{12}{1}x + \binom{12}{2}x^2 - \binom{12}{3}x^3$ with an understanding of the numeric value of $\binom{12}{n}$. Many candidates who got the powers and coefficients correct did not take account of the signs.

$[1 - 12x + 66x^2 - 220x^3]$

3) **(Remainder Theorem)**
Most of the difficulties lay in the application of long-winded methods as described above. Some candidates failed to obtain full marks due to not answering the question, stopping with the factorised form of the function.
[(i) Remainder = 0, (ii) $x = -1, 2, 4$]

4) **(Binomial probability)**
Some candidates misread the question as throwing 6 dice rather than the stated 4. The requirement "at least" for some did not trigger the idea of taking the unwanted terms from 1.
[(i) $\frac{625}{1296} = 0.4823$, (ii) $\frac{19}{144} = 0.1319$]

5) **(Turning points of a cubic)**
(i) A large number of candidates did not set their derived function to 0, although many of these did imply it by given the "solution" to be $x = -1, 3$. It was expected that candidates should determine the nature of the turning point by one of the three standard methods, but many missed it. Others did not find the y value corresponding to $x = -1$.
[$(-1, 12)$ is a maximum]

(ii) A sketch does not mean a plot on graph paper and only 1 mark was available.

6) **(Variable acceleration)**
It was pleasing to see so many candidates substitute the correct values for u and v and not be concerned about negative values. It was permissible to do the two parts the other way round as the time can be found without knowing the deceleration.
[2 m s^{-2} , 42 secs]

7) **(Trigonometry)**
This question was poorly done, with few candidates making the connection between the question and the trigonometrical identity $\sin^2\theta + \cos^2\theta = 1$.

(i) The context confused even the better candidates, resulting in a lack of appreciation that the identity required the two fractions on the left to be added together.
Some of the good responses came from those who started with the identity and divided throughout by $\sin\theta \cos\theta$.

(ii) There was a general failure also here in not making the connection between the fractions and $\tan\theta$.

(iii) Errors here involved some very poor algebraic manipulation to get from what was given to a quadratic equation in t. Some candidates who got the whole question correct so far, then failed to find values for θ, leaving the two values for t found as their answer.
[15º, 75º.]

8) **(Variable acceleration)**
Most candidates understood that integration was required. Common errors were dividing throughout by 60, integrating the 60 outside the bracket as 60t and some even managed the integral of 60 to be $\frac{60^2}{2}$.
It was also distressing to see so many unrealistic answers. Spectacular answers included 120 m,
600 km and even in excess of 1 000 000 m.
[6250m]

9) (Circle)

(i) The most common error was to state the vector for half the diameter vector as the coordinates for the centre.
(ii) "Show that" was not properly understood by many. Writing
$\sqrt{(8-1)^2+(2-3)^2}=5\sqrt{2}$ is insufficient work for full credit, as this could be simply writing down the answer.
(iii) The form of the circle required was often used well by many to find the correct equation. Others were less sure of the right way to proceed.
[(i) (8, 2), (iii) $x^2 + y^2 - 16x - 4y + 18 = 0$]

10) (Modelling data with curves)

In both parts (i) and (ii) most candidates were able to substitute the point (0, 4) into the given equation to find the constant, though it was disappointing to see many candidates making simple errors such as $4 = -16c \Rightarrow c = -4$.
The main problem was that, having found the value of k and c in each part, candidates then substituted also the other two points as well. Obtaining $0 = 0 \times k$ then caused confusion as most who did this then incorrectly deduced that $k = 0$, contradicting the earlier value found.
(iii) The idea of either function modelling the connection between the data was a concept that was not easy to grasp. Those that tried decided that the criterion for "better" had to be "simpler" and made a deduction accordingly. Not many involved the extra point that was given; those that did were able to deduce the correct answer. For full credit, however, a numerical demonstration was necessary.
[(i) $c = \frac{1}{2}$ (ii) $k = -\frac{1}{4}$, (iii) John's model as it fitted all the points.]

Section B

11) (3D trigonometry)

This question was a source of good marks even for the weaker candidates. However, many became confused by the fact that they could not easily visualise the situation in 3 dimensions. In such cases, drawing diagrams in the dimension required for that particular part is useful and not done enough by candidates.
(i) Many did not read the question fully and missed out the requirement to find BF.
(ii) The candidates who could visualise the fact that the tower was vertical when the rest of the diagram was horizontal did well. There were some, however, who engaged in very long-winded methods (see above).
(iii) A common error was to assume that C was at the midpoint of AB.
[(i) BF = 113.1 m, (ii) FT = 21.6 m, (iii) CF = 106.3 m, θ = 11.5º.]

12) (Tangents, normals and areas)

(i) Most candidates were aware of the tangent/normal rule, and most knew they had to differentiate.
(ii) Many students correctly tried to use simultaneous equations but often with the curve equation.
(iii) Attempts to work out the area were often very confused. The basic fact that the area of a triangle is $\frac{1}{2}\text{base} \times \text{height}$ even when the height is not a side of the triangle was forgotten by some. What resulted was often a plethora of irrelevant subtraction of integrals with resulting overlaps of area. Very few completed this part correctly.
[(i) $2x + 3y = 10$, (ii) (–4, 6), (iii) 21.25]

13) (Problem solving using quadratic equations)

This question was done very badly, which is surprising as many questions of a similar nature have been set in the past.

Many were unable to give valid expressions in part (i) and those who did rarely knew how to proceed in part (ii). Only part (iii) was tackled with any conviction but having obtained a solution to the quadratic equation, some failed to answer the question.

[(i) $\frac{72}{t}, \frac{72}{t+2}$ (iii) Ali takes 6 hours, Beth takes 8 hours.]

14) (Linear programming)

Most candidates scored at least 2 out of the first 3 marks in parts (i) and (ii), sometimes getting the first inequality sign the wrong way round.

(iii) The diagrams illustrating the constraints and feasible region were usually well done.

(iv) A large proportion of candidates failed to write down the objective function, though it was clear that they were able to determine the minimum cost. This was a problem for those who got the inequality sign in (i) the wrong way round as theoretically for their region the answer should have been 0. Few candidates seemed to be upset by this.

(v) This part also caused problems who obtained an incorrect feasible region as, once again, the answer should have been 0.

[$200x + 100y \geq 1500$, (ii) $y \geq x$, (iv) $C = 80x + 60y$, (5, 5), £700, (v) (7, 1) gives £620.]

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