## FREE-STANDING MATHEMATICS QUALIFICATION ADVANCED LEVEL

ADDITIONAL MATHEMATICS

Candidates answer on the Answer Booklet
OCR Supplied Materials:

- 16 page Answer Booklet
- Graph paper

Other Materials Required:
None

Friday 5 June 2009
Afternoon
Duration: 2 hours


## INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.


## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is $\mathbf{1 0 0}$.
- This document consists of 8 pages. Any blank pages are indicated.


## In any triangle $\boldsymbol{A B C}$

Cosine rule $\quad a^{2}=b^{2}+c^{2}-2 b c \cos A$


## Binomial expansion

When $n$ is a positive integer
$(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n}$
where
$\binom{n}{r}={ }^{n} C_{r}=\frac{n!}{r!(n-r)!}$

## Section A

1 The angle $\theta$ is greater than $90^{\circ}$ and less than $360^{\circ}$ and $\cos \theta=\frac{2}{3}$. Find the exact value of $\tan \theta$.

2 Find the equation of the normal to the curve $y=x^{3}+5 x-7$ at the point $(1,-1)$.

3 A is the point $(1,5)$ and C is the point $(3, p)$.
(i) Find the equation of the line through $A$ which is parallel to the line $2 x+5 y=7$.
(ii) This line also passes through the point C. Find the value of $p$.
$4 \quad \mathrm{AB}$ is a diameter of a circle, where A is $(1,1)$ and B is $(5,3)$.
Find
(i) the exact length of AB ,
(ii) the coordinates of the midpoint of AB ,
(iii) the equation of the circle.

5 Parcels slide down a ramp. Due to resistance the deceleration is $0.25 \mathrm{~m} \mathrm{~s}^{-2}$.
(i) One parcel is given an initial velocity of $2 \mathrm{~ms}^{-1}$. Find the distance travelled before the parcel comes to rest.
(ii) A second parcel is given an initial velocity of $3 \mathrm{~m} \mathrm{~s}^{-1}$ and takes 4 seconds to reach the bottom of the ramp. Find the length of the ramp.

6 The gradient function of a curve is given by $\frac{\mathrm{d} y}{\mathrm{~d} x}=1-4 x+3 x^{2}$.
Find the equation of the curve given that it passes through the point $(2,6)$.
[4]

7 The course of a cross-country race is in the shape of a triangle ABC .
$\mathrm{AB}=8 \mathrm{~km}, \mathrm{BC}=3 \mathrm{~km}$ and angle $\mathrm{ABC}=60^{\circ}$.


Not to scale
(i) Calculate the distance AC and hence the total length of the course.
(ii) The organisers extend the course so that $\mathrm{AC}=9 \mathrm{~km}$.


Calculate the angle BCA.

8 Calculate the $x$-coordinates of the points of intersection of the line $y=2 x+11$ and the curve $y=x^{2}-x+5$. Give your answers correct to 2 decimal places.

9 A car accelerates from rest. At time $t$ seconds, its acceleration is given by $a=4-0.2 t$ nt $t=20$.
(i) Find the velocity after 5 seconds.
(ii) What is happening to the velocity at $t=20$ ?
(iii) Find the distance travelled in the first 20 seconds.

10 (i) Illustrate on one graph the following three inequalities.

$$
\begin{aligned}
& y \geqslant x-1 \\
& x \geqslant 2 \\
& 2 x+y \geqslant 8
\end{aligned}
$$

Draw suitable boundaries and shade areas that are excluded.
(ii) Write down the minimum value of $y$ in this region.

## Section B

11 The shape ABCD below represents a leaf.
The curve ABC has equation $y=-x^{2}+8 x-9$.
The curve ADC has equation $y=x^{2}-6 x+11$.

(i) Find algebraically the coordinates of A and C , the points where the curves intersect.
(ii) Find the area of the leaf.

12 The diagram shows a rectangle $A B E F$ on a plane hillside which slopes at an angle of $30^{\circ}$ to the horizontal. ABCD is a horizontal rectangle. E and F are 100 m vertically above C and D respectively. $\mathrm{AB}=\mathrm{DC}=\mathrm{FE}=500 \mathrm{~m}$.

AE is a straight path.
From $B$ there is a straight path which runs at right angles to $A E$, meeting it at $G$.

(i) Find the distance BE.
(ii) Find the angle that the path AE makes with the horizontal.
(iii) Find the area of the triangle ABE .

Hence find the length BG .

13 In a supermarket chain there are a large number of employees, of whom $40 \%$ are male.
(a) One employee is chosen to undergo training.

What assumption is made if 0.4 is taken to be the probability that this employee is male?
(b) 6 employees are chosen at random to undergo training.
(i) Show that $\mathrm{P}($ all 6 chosen are female $)=0.0467$, correct to 4 decimal places.

Find the probability that
(ii) 3 are male and 3 are female,
(iii) there are more females than males chosen.

14 (a) (i) On the same graph, draw sketches of the curve $y=x^{3}$ and the line $y=3-2 x$.
(ii) Use your sketch to explain why the equation $x^{3}+2 x-3=0$ has only one root.
(b) (i) Show by differentiation that there are no stationary points on the curve $y=x^{3}+3 x-4$.
(ii) Hence explain why the equation $x^{3}+3 x-4=0$ has only one root.
(c) (i) Use the factor theorem to find an integer root of the equation $x^{3}+x-10=0$.
(ii) Write the equation $x^{3}+x-10=0$ in the form $(x-a)\left(x^{2}+p x+q\right)=0$ where $a, p$ and $q$ are values to be determined.
(iii) By considering the quadratic equation $x^{2}+p x+q=0$ found in part (ii), show that the cubic equation $x^{3}+x-10=0$ has only one root.
(d) You are given that $r$ and $s$ are positive numbers. What do the results in parts (a), (b) and (c) suggest about the equation $x^{3}+r x-s=0$ ?

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# Additional Mathematics 

## Mark Schemes for the Units

## June 2009

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## CONTENTS

Foundations of Advanced Mathematics FSMQ (6993)

MARKSCHEME ON THE UNIT
Unit Page
Additional Mathematics - 6993 ..... 1
Grade Thresholds ..... 9

## Additional Mathematics - 6993

## Section A

| 1 |  | Pythagoras for third value: $\begin{aligned} & c=\sqrt{5} \\ & \Rightarrow \tan \theta=-\frac{\sqrt{5}}{2} \end{aligned}$ <br> Alt: $\begin{aligned} & \sin ^{2} \theta=1-\cos ^{2} \theta=1-\frac{4}{9}=\frac{5}{9} \\ & \Rightarrow \sin \theta=\frac{1}{3} \sqrt{5} \\ & \Rightarrow \tan \theta=\frac{\sin \theta}{\cos \theta}=\frac{\sqrt{5} / 3}{2 / 3}=-\frac{\sqrt{5}}{2} \end{aligned}$ | M1 <br> A1 <br> A1 <br> 3 <br> M1 <br> A1 <br> A1 | Using any means to find $\sqrt{5}$ <br> Includes negative sign. <br> Use of Pythagoras <br> $\operatorname{Sin} \theta$ <br> Includes negative sign |
| :---: | :---: | :---: | :---: | :---: |
|  |  | SC: Allow B1 for $\tan \theta=-1.12$ |  |  |
| 2 |  | $\begin{aligned} & \frac{\mathrm{d} y}{\mathrm{~d} x}=3 x^{2}+5 \\ & \Rightarrow \text { grad tangent }=8 \\ & \Rightarrow \text { grad normal }=-\frac{1}{8} \\ & \Rightarrow y+1=-\frac{1}{8}(x-1) \\ & \Rightarrow 8 y+8=-x+1 \Rightarrow 8 y+x+7=0 \end{aligned}$ | M1 <br> A1 <br> F1 <br> M1 <br> A1 | Attempt at differentiation with at least one term with correct power <br> Dep on use of their normal gradient and correct point Any acceptable form. Acceptable means three terms only |
| 3 | (i) | $\begin{aligned} & 2 x+5 y=2+25 \\ & \Rightarrow 2 x+5 y=27 \end{aligned}$ | $\begin{array}{\|ll\|} \hline \text { M1 } & \\ \text { A1 } & \\ & \mathbf{2} \\ \hline \end{array}$ | Substitute new point to change $c$ <br> If put in form $y=m x+c$ then $m=-0.4$ |
|  |  | SC: B2 from scale drawing only if absolutely correct |  |  |
|  | (ii) | $\begin{aligned} & \text { When } x=3,6+5 y=27 \\ & \Rightarrow 5 y=21 \Rightarrow y=\frac{21}{5} \\ & \Rightarrow p=\frac{21}{5}=4.2 \end{aligned}$ | $\begin{array}{\|ll\|} \hline \text { M1 } & \\ & \\ \text { F1 } & \\ & \\ & \\ \hline \end{array}$ | Substituting $x=3$ into either their equation from (i) or the given equation in (i) <br> Answer must specifically give $p$ |
|  |  | $\mathrm{NB} p=0.2$ comes from using original line. Give M1 A1 for this. |  |  |


| 4 | (i) | $\begin{aligned} \mathrm{AB} & =\sqrt{(5-1)^{2}+(3-1)^{2}} \\ & =\sqrt{4^{2}+2^{2}} \\ & =\sqrt{20}=2 \sqrt{5} \end{aligned}$ | M1 <br> A1 <br> 2 | isw ie ignore any approx value for root. |
| :---: | :---: | :---: | :---: | :---: |
|  |  | NB M1 A0 for 4.47 with no sight of $\sqrt{20}$ |  |  |
|  | (ii) | $\left(\frac{1+5}{2}, \frac{1+3}{2}\right)=(3,2)$ | $\begin{array}{ll} \text { B1 } & \\ & \mathbf{1} \end{array}$ |  |
|  | (iii) | $\begin{gathered} (x \pm a)^{2}+(y \pm b)^{2} \text { with }(a, b) \text { from } \\ (x-a)^{2}+(y-b)^{2} \\ =5 \end{gathered}$ | M1 <br> F1 <br> A1 | Use of equation <br> Their midpoint <br> cao for 5 <br> isw ie ignore any incorrect algebra following a correct equation |
| 5 | (i) | $v^{2}=u^{2}+2 a s \Rightarrow 0=4-2 \times 0.25 s$ $\Rightarrow s=8$ <br> Distance travelled $=8 \mathrm{~m}$ | M1 <br> A1 <br> A1 | Use of right formula(e) Substitution <br> Answer |
|  |  | If $t$ is found first then M1 for any correct equations that lead to finding $s$ <br> Careful also of $4=0+\frac{1}{2} s$, this could be 3 if quoted formula is right. <br> Also of $0=4+\frac{1}{2} s \Rightarrow s=-8$ <br> Both of these M1 for formula only |  |  |
|  | (ii) | $\begin{aligned} & s=u t+\frac{1}{2} a t^{2}=s=3 \times 4-\frac{1}{2} \times 0.25 \times 16 \\ & =12-2=10 \\ & \text { Length of ramp }=10 \mathrm{~m} \end{aligned}$ | M1 <br> A1 <br> A1 <br> 3 |  |
|  |  | NB Anything that uses $v=0$ is M0 |  |  |
| 6 |  | $\begin{aligned} & \frac{\mathrm{d} y}{\mathrm{~d} x}=1-4 x+3 x^{2} \\ & \Rightarrow(y=) x-2 x^{2}+x^{3}(+c) \end{aligned}$ <br> Through ( 2,6 ) $\begin{aligned} & \Rightarrow 6=2-8+8+c \Rightarrow c=4 \\ & \Rightarrow y=x-2 x^{2}+x^{3}+4 \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 <br> 4 | For integrating - increase in power of one in at least two terms <br> Attempt to find $c$ <br> Must be an equation |


| 7 | (i) | $\begin{aligned} & A C^{2}=8^{2}+3^{2}-2 \cdot 8 \cdot 3 \cdot \cos 60 \\ & \quad=73-24=49 \\ & \Rightarrow A C=7 \\ & \Rightarrow \text { Total distance }=18 \mathrm{~km} \end{aligned}$ | M1 <br> A1 <br> A1 <br> F1 <br> 4 | Use of formula <br> AC <br> Total distance |
| :---: | :---: | :---: | :---: | :---: |
|  | (ii) | $\begin{aligned} & \frac{\sin \mathrm{BCA}}{8}=\frac{\sin 60}{9} \\ & \Rightarrow \sin \mathrm{BCA}=\frac{8}{9} \times \sin 60(=0.7698) \\ & \Rightarrow \mathrm{BCA}=50.3^{\circ} \end{aligned}$ | M1 <br> A1 <br> A1 <br> 3 |  |
|  |  | Alternative Scheme: <br> Use of cosine formula twice $\begin{aligned} & \Rightarrow \mathrm{BC}=9.74 \ldots \\ & \Rightarrow \mathrm{BCA}=50.3^{0} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ |  |
| 8 |  | $\begin{aligned} & 2 x+11=x^{2}-x+5 \\ & \Rightarrow x^{2}-3 x-6(=0) \\ & \Rightarrow x=\frac{3 \pm \sqrt{9+24}}{2}=\frac{3 \pm \sqrt{33}}{2} \\ & \quad=4.37 \text { or }-1.37 \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 <br> A1 | Substitute <br> Quadratic <br> Solve <br> Correct substitution <br> Both answers <br> Ignore values for $y$ |
|  |  | Alternative Scheme 1 (relates to last 3 marks) Completion of square: $(x-1.5)^{2}=k$ $\begin{aligned} & x-1.5= \pm \sqrt{8.25} \\ & \Rightarrow x=4.37 \text { or }-1.37 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ | Must contain $\pm$ <br> Must be 2 dp |
|  |  | Alternative Scheme 2: Only 2 marks from last 3 Solving their quadratic by T\&I Both roots | $\begin{array}{\|l} \text { M1 } \\ \text { A1 } \end{array}$ |  |
|  |  | Alternative Scheme 3. Only 4 marks Roots with no working: B2 each | B2,2 |  |
|  |  | Alternative Scheme 4. Only 4 marks <br> Finding a root from the original equations <br> $=$ one of them <br> Finding the second root $=$ the other | $\begin{array}{\|l} \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \end{array}$ |  |
|  |  | Alternative scheme 5. Eliminate $x$. Gives $y^{2}-28 y+163=0$ <br> Gives $y=19.74$ and 8.26 leading to $x$ values | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \end{aligned}$ | Eliminate $x$ Quadratic Solve Both $y$ values Both $x$ values |
|  |  | NB Attempt to solve by graph - M0 |  |  |


| 9 | (i) | $\begin{aligned} & a=4-0.2 t \\ & \Rightarrow v=4 t-0.1 t^{2} \\ & \Rightarrow v_{5}=20-2.5=17.5 \end{aligned}$ <br> Velocity is $17.5 \mathrm{~m} \mathrm{~s}^{-1}$ | M1 <br> A1 <br> A1 <br> 3 | Integrate (increase of po of one in at least one term <br> Ignore $c$ |
| :---: | :---: | :---: | :---: | :---: |
|  | (ii) | $\text { At } t=20, a=0$ ie Maximum velocity | $\begin{array}{\|ll} \hline \text { B1 } & \\ & 1 \end{array}$ |  |
|  | (iii) | $v=4 t-0.1 t^{2}$ $\begin{aligned} \Rightarrow s & =\int_{0}^{20} 4 t-0.1 t^{2} \mathrm{~d} t=\left[2 t^{2}-0.1 \frac{t^{3}}{3}\right]_{0}^{20} \\ & =2 \times 400-0.1 \times \frac{8000}{3}=533.3 \ldots=533 \end{aligned}$ <br> Distance travelled $=533 \mathrm{~m}$ | M1 <br> A1 <br> A1 | Integrate their $v$ from (i) (Increase in power of one term) <br> Ignore $c$ <br> Allow exact answer or 3sf |
| 10 | (i) |  | B2,1 <br> B2,1 | Lines, -1 each error <br> Shading, -1 each error Correct side of line. ft if gradient is the same sign. |
|  | (ii) | $y=2$ | $\begin{array}{\|ll\|} \hline \text { E1 } \\ & \mathbf{1} \\ \hline \end{array}$ | ft their graph |

Section B

| 11 | (i) | $\begin{aligned} & -x^{2}+8 x-9=x^{2}-6 x+11 \\ & \Rightarrow 2 x^{2}-14 x+20=0 \\ & \Rightarrow x^{2}-7 x+10=0 \\ & \Rightarrow(x-5)(x-2)=0 \\ & \Rightarrow x=2,5 \end{aligned}$ <br> Substitute: $x=2 \Rightarrow y=4-12+11=3$ $x=5 \Rightarrow y=25-30+11=6$ | $\begin{array}{\|ll} \hline \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } & \\ \text { A1 } & \\ & \mathbf{5} \end{array}$ | Equate <br> Quadratic <br> Solve: Factorisation needs 2 numbers to multiply to their constant <br> Or one pair, e.g. $(2,3)$ or (5,6) |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Alternative scheme: <br> Completion of square: $(x-3.5)^{2}=k$ $\begin{aligned} & x-3.5= \pm \sqrt{2.25} \\ & \Rightarrow x=5 \text { or } 2 \\ & \Rightarrow y=6 \text { or } 3 \end{aligned}$ | M1 <br> A1 <br> A1 |  |
|  | (ii) | $\begin{aligned} & A=\int_{2}^{5}\left(y_{1}-y_{2}\right) \mathrm{d} x=\int_{2}^{5}\left(-2 x^{2}+14 x-20\right) \mathrm{d} x \\ & =\left[-\frac{2 x^{3}}{3}+7 x^{2}-20 x\right]_{2}^{5} \\ & =\left(-\frac{2 \times 125}{3}+7 \times 25-100\right)-\left(-\frac{16}{3}+28-40\right) \\ & =\left(-\frac{250}{3}+75\right)-\left(-\frac{16}{3}-12\right)=-\frac{234}{3}+87=87-78=9 \end{aligned}$ | A1 <br> M1 <br> A2 <br> M1 <br> A1 | Int between curves <br> $\pm$ Correct expression Integrate their function (not if they divide by 2 ) <br> All terms, -1 for each error <br> Sub into integral Answer |
|  |  | Alternative scheme: $\begin{aligned} A & =\int_{2}^{5}\left(-x^{2}+8 x-9\right) \mathrm{d} x-\int_{2}^{5}\left(x^{2}-6 x+11\right) \mathrm{d} x \\ & =\left[-\frac{x^{3}}{3}+4 x^{2}-9 x\right]_{2}^{5}-\left[\frac{x^{3}}{3}-3 x^{2}+11 x\right]_{2}^{5} \\ & =\left(\left(-\frac{125}{3}+100-45\right)-\left(-\frac{8}{3}+16-18\right)\right) \\ & -\left(\left(\frac{125}{3}-75+55\right)-\left(\frac{8}{3}-12+22\right)\right) \\ & =\left(13 \frac{1}{3}-\left(-4 \frac{2}{3}\right)\right)-\left(21 \frac{2}{3}-12 \frac{2}{3}\right)=18-9 \end{aligned}$ | M1 M1 <br> A1 <br> A1 <br> M1 <br> A1 <br> A1 | Subtracting 2 integrals Integrate either <br> All terms of $y_{1}$ <br> All terms of $y_{2}$ <br> Substitute into either integral <br> For 18 or 9 <br> Final answer |
|  |  | SC $A=\int\left(y_{1}+y_{2}\right) \mathrm{d} x$ M1 integrate and M1 sub only |  |  |


| 12 | (i) | $\begin{aligned} & \frac{100}{\mathrm{BE}}=\sin 30 \\ & \Rightarrow \mathrm{BE}=\frac{100}{\sin 30}=200 \mathrm{~m} \end{aligned}$ | M1 <br> A1 <br> A1 <br> 3 | Fraction right way Correct expression fo BE <br> Or B3 if the special triangle is noticed. |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Alternative scheme: $\begin{aligned} & \frac{100}{\mathrm{BC}}=\tan 30 \Rightarrow \mathrm{BC}=\frac{100}{\tan 30}=173.2 \\ & \mathrm{BE}=\sqrt{100^{2}+173.2^{2}}=200 \end{aligned}$ | M1 <br> A1 <br> A1 | Ratio and Pythagoras <br> Allow not exact |
|  | (ii) | $\begin{aligned} & \text { AE by Pythagoras: } \\ & \mathrm{AE}=\sqrt{500^{2}+200^{2}}=100 \sqrt{29}=538.5 \ldots \\ & \sin A=\frac{100}{538.5} \\ & \Rightarrow A=10.7^{0} \end{aligned}$ | $\begin{array}{\|ll} \hline \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } & \\ \hline \ldots . . . . . . \\ \hline \end{array}$ | soi |
|  |  | Alternative Scheme: $\begin{aligned} & \mathrm{BC}=\sqrt{30000} \approx 173.2 \Rightarrow \mathrm{AC}=\sqrt{280000} \approx 529.2 \\ & \Rightarrow \mathrm{~A}=\tan ^{-1} \frac{100}{\sqrt{280000}}=10.7^{0} \\ & \mathrm{NB} A=10.9^{\circ} \text { comes from } \sin ^{-1} \frac{100}{\sqrt{280000}} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { M1 } \\ & \text { A1 } \end{aligned}$ |  |
|  | (iii) | $\begin{aligned} & \text { Area }=\frac{1}{2} \times 500 \times \text { their } \mathrm{BE} \\ & =50000 \\ & \text { Area }=\frac{1}{2} \times \mathrm{BG} \times \text { their } \mathrm{AE} \\ & \Rightarrow \mathrm{BG}=\frac{2 \times \text { their area }}{\text { their } \mathrm{AE}}=185.7 \ldots . . . \approx 186 \mathrm{~m} \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 <br> A1 <br> 5 |  |
|  |  | Alternative Scheme: <br> Find angle A or E <br> Then $\frac{B G}{500}=\sin \mathrm{A} \Rightarrow \mathrm{BG}=186 \mathrm{~m}$ <br> ie maximum 3 marks. The answer is found, but the question says "Hence" and this is "otherwise". <br> NB If area is attempted but not used then give M1 A1. If area is found after BG is found then do not mark it. | A1 <br> A1 |  |


|  |  | In all parts of this question allow answers to 3sf or 4 dp |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 13 | (a) | The selection is random. Allow anything that implies equal chance of selection | $\begin{array}{\|ll\|} \hline \text { B1 } & \\ \hline \end{array}$ |  |
|  | (b)(i) | $\begin{aligned} \mathrm{P}(\text { all are female })=0.6^{6} & (=0.046656) \\ & =0.0467 \end{aligned}$ | $\begin{array}{ll} \hline \text { M1 } & \\ \text { A1 } & \\ & \\ \hline \end{array}$ | Sight of $0.6^{6}$ |
|  | (ii) | $\begin{aligned} \mathrm{P}(3 \text { of each }) & =\text { Bin coeff } \times 0.6^{3} \times 0.4^{3} \\ & =20 \times 0.6^{3} \times 0.4^{3} \\ & =0.2765 \text { or } 0.276 \end{aligned}$ | $\begin{array}{ll} \hline \text { M1 } & \\ & \\ \text { A1 } & \\ \text { A1 } & \\ & \\ \text { A1 } & \\ & 4 \end{array}$ | One term with binomial coeff 20 (may be implied) Powers (may be implied) |
|  | (iii) | $\begin{aligned} & \mathrm{P}(\text { more females than males })=6,0 \text { or } 5,1 \text { or } 4,2 \\ & =0.6^{6}+6 \times 0.6^{5} \times 0.4+15 \times 0.6^{4} \times 0.4^{2} \\ & =0.04666+0.1866+0.3110 \\ & =0.5443 \\ & \text { Allow } 0.544,0.545,0.5444 \end{aligned}$ | M1 <br> B1 <br> B1 <br> B1 <br> A1 <br> 5 | Add 3 terms Binomial coefficients correct in at least two terms <br> Powers correct in at least two terms At least 2 terms correct. |
|  |  | $\begin{aligned} & \text { Alternative scheme: } \\ & \mathrm{P}(\text { more females than males }) \\ & =1-\mathrm{P}(\text { more males than females or equal numbers }) \\ & =1-\left(0.4^{6}+6 \times 0.4^{5} \times 0.6+15 \times 0.4^{4} \times 0.6^{2}+20 \times 0.4^{3} \times 0.6^{3}\right. \\ & =1-(0.0041+0.0369+0.1382+0.2765) \\ & =0.5443 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { B1 } \\ & \text { B1 } \\ & \text { B1 } \\ & \text { A1 } \end{aligned}$ | Take 4 terms from 1 <br> Binomial coeffs <br> Powers <br> At least 2 terms correct |
|  |  | The terms are: $0.0467,0.1866,0.3110,0.2765,0.1382,0.0369,0.0041$ |  |  |
|  |  | If P (more males than females), treat as MR and -2 If $p=0.4$ and $q=0.6$ then MR -2 (but also 0 for (b)(i) where answer is given!) |  |  |



## Grade Thresholds

Additional Mathematics (6993)
June 2009 Assessment Series
Unit Threshold Marks

| Unit | Maximum <br> Mark | A | B | C | D | E | U |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6993 | 100 | 73 | 63 | 53 | 44 | 35 | 0 |

The cumulative percentage of candidates awarded each grade was as follows:

|  | A | B | C | D | E | U | Total Number of <br> Candidates |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{6 9 9 3}$ | 27.7 | 39.7 | 48.7 | 56.9 | 66.0 | 100 | 9859 |

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# Additional Mathematics 

Report on the Unit
June 2009

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, GCSEs, OCR Nationals, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new syllabuses to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the syllabus content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

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## CONTENTS

## Additional Mathematics FSMQ (6993)

## REPORT ON THE UNIT

Unit PageAdditional Mathematics - 69931
Grade Thresholds ..... 5

## Additional Mathematics - 6993

This year has seen another large increase in the number of candidates sitting the paper. While there are candidates and even the whole cohort from a centre for whom this paper is not suitable, it seems as though the increase is from candidates for whom this paper is appropriate. The mean mark was 44 , up by 5 marks from last year.
Centres are reminded that this is an Advanced FSMQ and therefore has as its starting point a good grade at the higher tier of GCSE. It is evident that there are two types of candidature for whom this paper was not appropriate.
There were candidates who demonstrated a very poor grasp of the work, even of basic GCSE skills. For instance, question 7 deals with the sine and cosine rule and this question could easily be asked in a higher tier GCSE paper. Yet some were unable to even start the question; others could clearly only cope with right-angled triangles. Question 8 required the solution of a quadratic by formula or completion of the square and question 11 the solution by factorisation. A small number of candidates could not cope with either of these. We are concerned that an inappropriate entry of candidates without a good grasp of the higher tier material of GCSE might be a negative experience for such candidates.
There were other candidates who scored well on a limited number of questions and these questions were often well presented. The inference here is that such candidates are mathematically able students who have not covered the syllabus well. The process of entering candidates for this paper for whom there has been no enrichment programme of study (or an ineffective one) could also be a negative experience.
For those who tackled the questions appropriately there seemed to be no difficulty with finishing the paper in time and there were a number of first class scripts seen. However, it seemed as though a number of candidates ran out of time; our perception was that these candidates penalised themselves by inappropriate methods of answering the questions. This specification was written with the intention that it should provide an enrichment programme for students who were going to achieve, or who had already achieved, a good grade at GCSE. Methods of approach which might not be seen or expected in a GCSE paper were expected here, and failure to do so by tackling questions in a rather more standard, and long-winded way would have resulted in a loss of time. Examples of this will be identified in individual questions.

It is appropriate to note two points from the rubric. The first is the statement that answers should normally be given to 3 significant figures. Candidates who consistently wrote answers to more (typically the whole display on their calculator) were penalised by the loss of an accuracy mark at the first place it was seen (answers in Q13 on probability were also accepted to 4 significant figures). The second is that marks will not be awarded unless sufficient working is shown. Candidates who therefore simply wrote down the answer without any working were given the accuracy marks but not the method mark.

A few candidates caused themselves as well as the examiners a few problems in their interpretation of a standard examination demand. In question 14 it was required that candidates sketched a graph of a curve and a graph of a line. Since the intersection was part of the question, it was necessary to sketch (or plot) the functions on the same graph. Consequently the question asked "On the same graph". Unfortunately a number took this instruction to mean "on the same graph as you have drawn for a previous question" and drew these graphs on what they had produced for Q10. No penalty was applied for this except for the ones whose graphs (and shading!) for Q10 obliterated the graphs for Q14 so that the examiner could not see anything worthy of credit.

## Section A

## Q1 Trigonometry and Pythagoras

Most candidates failed to get any marks on this question though some managed a special case allowance of one mark. The crucial problem here was the lack of understanding of the word "exact". Many ignored this demand; others understood it to mean "all the digits on my calculator". Candidates who worked out $\theta$ and then $\tan \theta$ on their calculator seemed unaware that the method could not produce an exact answer. It was disappointing also to see so many candidates giving an angle as an answer, and others not even getting an approximate answer via their calculator, meaning that the lack of understanding extended beyond the word exact.
$\left[\tan \theta=-\frac{\sqrt{5}}{2}\right]$

## Q2 Finding the normal to a curve

Many candidates were able to differentiate and hence find the gradient of the tangent at the required point. Some, however, did not realise that it is not necessary to find the equation of the tangent - one of the places in the paper where time was lost.
$[8 y+x+7=0]$

## Q3 Coordinate geometry of straight lines

It was disappointing to see the approach by most candidates. The "standard" process was to find first the gradient of the given line by expressing $2 x+5 y=7$ in the form $y=m x+c$ and then to substitute the given point to find $c$. Very few realised that any line parallel to $2 x+5 y=7$ would have the form $2 x+5 y=c$ where $c$ could be found by substituting the given point. This is therefore another question where time would have been lost by a significant number of candidates. No extra marks are available, of course, for alternative long-winded methods; there is also the increased chance of making an arithmetic or algebraic error. Many candidates with this approach, for instance, decided that $m=0.4$ or $m=2$.
[(i) $2 x+5 y=27$, (ii) $p=4.2$ ]

## Q4 The circle

The word "exact" appears again here. Most candidates were able to obtain the length of $A B$ $(=\sqrt{20})$ but were unable to resist the temptation to find an approximate value from their calculator. The result of this was to substitute their approximate value for $r$ into the equation for the circle rather than $\frac{1}{2} \sqrt{20}$, which yielded an incorrect equation.
[(i) $\sqrt{20}=2 \sqrt{5}$,
(ii) $(3,2)$, (iii) $\left.(x-3)^{2}+(y-2)^{2}=5\right]$

## Q5 Constant acceleration

There were many "double errors" or "omissions" in part (i) in writing down the correct answer and these were penalised. For instance, some found $t=8$ and thought that that was the answer.
Others substituted into the equation $v^{2}=u^{2}+2$ as, but substituted $a=0.25$ and ignored the resulting negative sign. Others correctly substituted for $a$, but interchanged $u$ and $v$ and again ignored the negative sign. Others substituted $a=0.25$, interchanged $u$ and $v$ and got the correct numerical value for $s$. Here again is a poor method, taking more time away from the candidate by the poor choice of formula - finding first the value of $t$ and then finding $s$.
In part (ii) a number of candidates assumed that $v=0$ which was incorrect. Finding first the actual value of $v$ before using it to find $s$ is a long-winded method.
[(i) 8 m , (ii) 10 m ]

## Q6 Integration to find equation of curve

This was a good source of 4 marks for most candidates, although a proportion failed to use appropriate methods to find $c$.
$\left[y=x^{3}-2 x^{2}+x+4\right]$

## Q7 Cosine and Sine rules

This question also was a good source of marks for even the weakest of candidates though the usual error of taking $73-48 \cos 60=25 \cos 60$ was in evidence!
In part (ii) a considerable loss of time must have been experienced by those candidates who used the cosine rule twice. The first use resulted in a quadratic for $B C$ which had to be solved and then used in a second application of the cosine rule to get the angle.
Unnecessary marks were lost here by even the most able of candidates. Some failed to answer the question completely in part (i), thus losing the last mark. Some also gave the angle in part (ii) to 4 sf or more
[(i) 18 m ,
(ii) $50.3^{\circ}$ ]

## Q8 Intersection of line and quadratic

Most candidates solved this easily, though a few could not remember the quadratic formula. More time was lost here by finding also the $y$ coordinates of the points of intersection, which was not required.

## [4.37 and - 1.37]

## Q9 Variable acceleration

The usual proportion of candidates used constant acceleration formulae in this question including those who got part (i) correct by integration and then reverted to constant acceleration for part (iii). Responses to part (ii) were poor with only a small proportion saying that the velocity was a maximum value (or something similar). Acceleration $=0$ and therefore the car is at rest was often seen.
[(i) $17.5 \mathrm{~m} \mathrm{~s}^{-1}$, (ii) Maximum velocity, (iii) 533 m ]

## Q10 Linear programming

This question also took up more time than it should as the vast majority of candidates were unable to sketch, preferring instead to plot. The feasible region was not as most candidates expected, as a result of which they shaded incorrect regions.
[ (ii) $y=2$ ]

## Section B

## Q11 Areas under graphs

This question often produced a number of marks for even the weaker candidate. Unlike Q8, the $y$ coordinates were required here and a few lost a mark by failing to find them. Those that subtracted the curves before integrating found the answer within a few lines; those that treated the curves separately took rather longer. There were many arithmetic slips, including subtracting the wrong way round and getting a negative answer (the negative sign then being ignored).
[(i) $(2,3)$ and $(5,6)$,
(ii) 9$]$

## Q12 3-D trigonometry

The main difficulty in this question was the failure to set out the working carefully and clearly and it seemed at times that the candidates had got themselves lost!
Many failed to understand what angle was required in part (ii). Unfortunately the calculation of the wrong angle led them to tackle part (iii) the wrong way, and a number failed to find BG by the "hence" method.
[(i) 200 m , (i)
(ii) $10.7^{\circ}$
, (iii) 186 m$]$

## Q13 Binomial distribution

Part (a) required the understanding of the criteria required for the binomial distribution to be appropriate, but sadly only a few candidates wrote anything about randomness. Furthermore, those who did not write about randomness often wrote about there being 100 employees, or even 10 employees, negating a second criterion for the Binomial distribution.
In spite of the fact that many marks were lost in the paper by candidates for failure to write with the appropriate precision, most got part (b)(i) correct, writing enough to convince the examiners that they had not just written down the answer that was given.
Part (b)(ii) required a binomial coefficient and many failed to appreciate this. Some went further to assert that this required a term for three males and a term for three females, equal numbers multiplying and adding. That $20 \times(0.6)^{3} \times(0.4)^{3} \approx(0.6)^{3}+(0.4)^{3}$ is, of course, entirely coincidental.
Part (b)(iii) required the addition of three terms. One can speculate the reasons why candidates should choose to do this part by the " 1 - " method which was, of course, a rather more longwinded way and opened candidates to the error of failing to include the $4^{\text {th }}$ term of males and females in equal numbers.
[(ii) 0.276 , (iii) 0.544]

## Q14 Roots of cubic

The idea of this question was to lead candidates via three different examples in three different ways to conclude that cubic equations of this form only had one root. Many were able to do so, in spite of losing their way in some of the parts of the question.
In part (a), if the candidate drew something other than a cubic (a quadratic was the usual alternative seen) or if the curve was not extended into the negative quadrants then the conclusion that there was only one root did not follow.
In part (b)(i) something needed to be said about $\frac{\mathrm{d} y}{\mathrm{~d} x}$ such as " $=0$ has no roots" or "always
positive" for full marks and in part (ii) "crosses $x$-axis once" was not good enough, some reference to part (i) being necessary.
Likewise in part (c) the result that $f(2)=0$ or $(x-2)$ is a factor was often not written precisely enough.
[(c)(ii) $\left.(x-2)\left(x^{2}+2 x+5\right)=0\right]$

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