RECOGNISING ACHIE

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS 6993

Summer 2006

Thursday **15 JUNE 2006** Afternoon 2 hours

Additional materials:
16 page answer booklet
Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Additional sheets of graph paper should be securely attached to your answer booklet.
- Final answers should be given correct to three significant figures where appropriate.

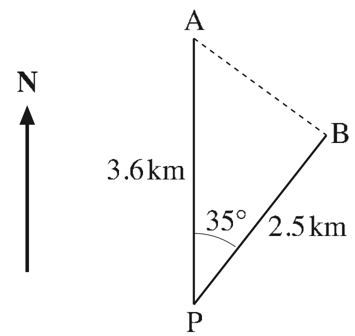
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 100.

Section A

1 Find $\int_1^3 (x^2 + 3)\,\mathrm{d}x$. [4]

2 Adam and Beth set out walking from a point P. After one hour Adam is 3.6 kilometres due north of P and Beth is 2.5 kilometres from P on a bearing of 035°.



Not to scale

Fig. 2

Calculate how far they are apart at this time. Give your answer correct to 2 significant figures. [4]

3 Calculate the values of x in the range $0° < x < 360°$ for which $\sin x = 2\cos x$. [4]

4 **(i)** Find the distance between the points $(2, 3)$ and $(7, 9)$. [2]

(ii) Hence find the equation of the circle with centre $(2, 3)$ and passing through the point $(7, 9)$. [2]

5 Solve the inequality $x^2 + 4x > 5$. [5]

6 A curve has gradient given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 2$. The curve passes through the point $(3, 0)$. Find the equation of the curve. [5]

7 **(i)** Show that the two lines whose equations are given below are parallel.

$$y = 4 \quad 2x$$
$$4x + 2y = 5$$
[2]

(ii) Find the equation of the line which is perpendicular to these two lines and which passes through the point $(1, 6)$. [3]

8 **(i)** By drawing suitable graphs on the same axes, indicate the region for which the fol[lowing] inequalities hold. You should shade the region which is **not** required.

$$3x + 2y \leqslant 18$$
$$y \leqslant 3x$$
$$y \geqslant 0$$
[5]

(ii) Find the maximum value of $x + 2y$ subject to these conditions. [2]

9 You are given that $f(x) = x^3 - 4x^2 + x + 6$.

(i) Find the remainder when $f(x)$ is divided by $(x - 1)$. [1]

(ii) Show that $(x - 3)$ is a factor of $f(x)$. [2]

(iii) Hence solve the equation $f(x) = 0$. [4]

10 Find the coordinates of the points of intersection of the line $y = 5 - 2x$ with the curve $y = x^2 - 4x - 11$, giving your answers correct to 2 decimal places. [7]

Section B

11 It is known that 65% of all people living in the UK went abroad for a holiday last year.

A random sample of 5 people living in the UK was chosen.

Find the probability that

(i) all 5 went abroad for a holiday last year, [1]

(ii) exactly 4 went abroad for a holiday last year, [3]

(iii) at least 2 went abroad for a holiday last year. [4]

An additional random sample of 5 people living in the UK was chosen.

(iv) Find the probability that in the 10 people chosen altogether, exactly 8 went abroad for a holiday last year. [4]

12 A train normally travels between two points A and D at a constant speed of 30 metres per second. The distance AD is 12 kilometres.

(i) Find the time taken for the train to travel between A and D at $30\,\mathrm{m\,s^{-1}}$. [1]

Between A and D there are two other points, B and C, which are placed such that AB $= 2$km, BC $= 6$km and CD $= 4$km. On one day there is a speed restriction of $10\,\mathrm{m\,s^{-1}}$ between B and C.

The train decelerates uniformly from $30\,\mathrm{m\,s^{-1}}$ at A to $10\,\mathrm{m\,s^{-1}}$ at B. It travels the distance BC at $10\,\mathrm{m\,s^{-1}}$. The train then accelerates uniformly from $10\,\mathrm{m\,s^{-1}}$ at C to $30\,\mathrm{m\,s^{-1}}$ at D.

Find

(ii) the time taken to travel from A to B, [2]

(iii) the acceleration over the distance CD, [3]

(iv) the extra time taken in travelling from A to D as a result of the speed restriction. [6]

13 Fig. 13.1 shows a solid block which is in the shape of a pyramid. The horizontal base, ABC a square with side 20 cm and the vertex, V, is 15 cm vertically above the centre, O, of the sq base. N is the midpoint of AB.

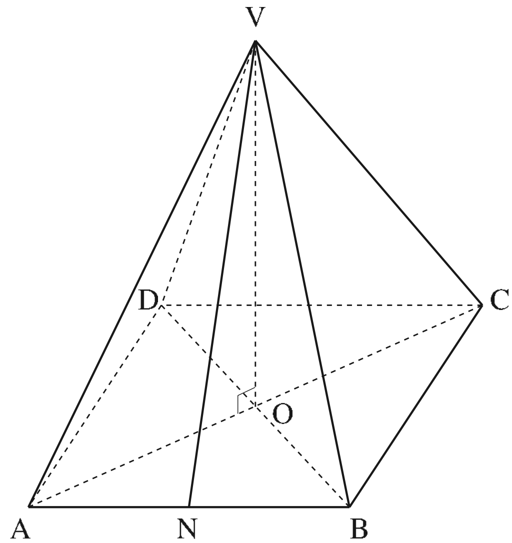


Fig. 13.1

(i) Calculate the length of the diagonal AC. [2]

(ii) Show that the length of the edge AV is $\sqrt{425}$ cm. [2]

(iii) Calculate the angle that the edge AV makes with the base. [2]

(iv) Calculate the length VN. [2]

M is the point on VB such that AM is perpendicular to VB as shown in Fig. 13.2.

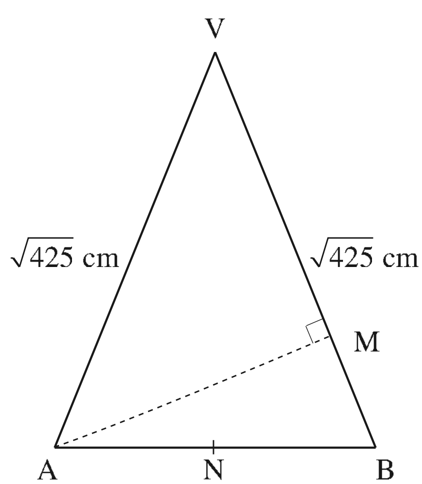


Fig 13.2

(v) Calculate the area of triangle VAB. Hence or otherwise calculate the distance AM. [4]

14

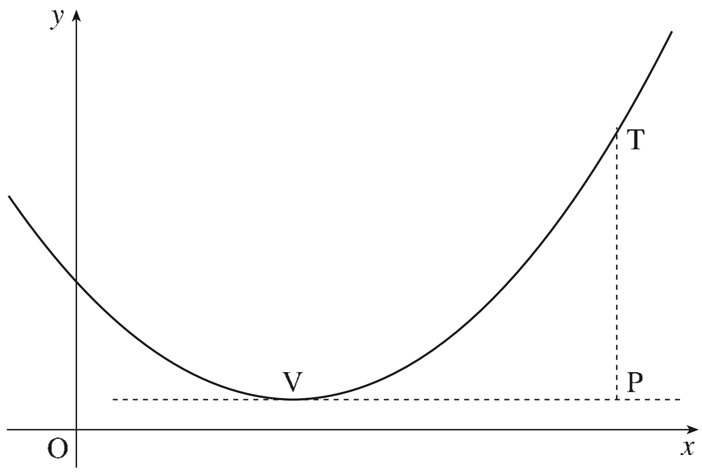


Fig. 14

Fig. 14 shows the quadratic curve $y = x^2 - 4x + 5$.

V$(2, 1)$ is the minimum point of the curve.

T$(5,10)$ is a point on the curve.

The line VP is the tangent to the curve at V and TP is perpendicular to this line.

(i) Write down the coordinates of P. [1]

(ii) Find the coordinates of M, the midpoint of VP. [2]

(iii) Find the equation of the tangent to the curve at T. [4]

(iv) Show that the tangent to the curve at T passes through the point M. [2]

(v) Use the result in part **(iv)** to suggest a way of drawing a tangent to a point on a quadratic curve without involving calculus. [3]

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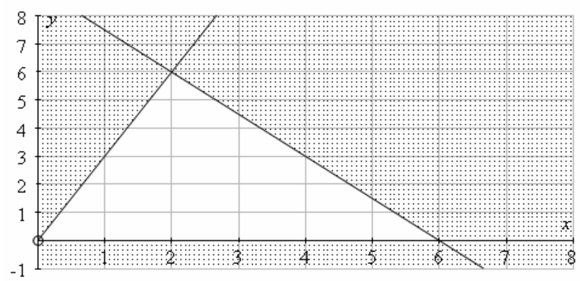
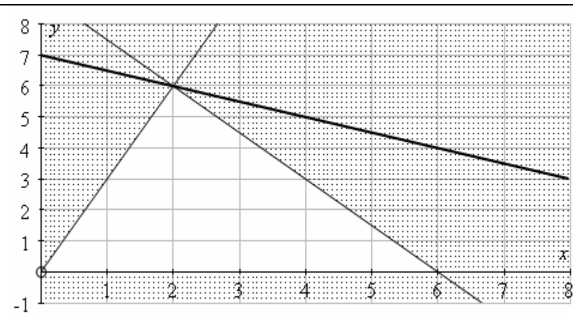
Q.		Answer	Mark	Notes
Section A				
1		$\int_1^3 (x^2+3)\,dx = \left[\frac{x^3}{3}+3x\right]_1^3$ $=\left(\frac{27}{3}+9\right)-\left(\frac{1}{3}+3\right)$ $=18-3\frac{1}{3}=14\frac{2}{3}$ Accept 14.7 but not 14.6	M1 A1 DM1 A1 **4**	Attempt to integrate Both terms correct Substitute 3 and 1 and subtract Sp. Case: Allow M1 even if one of the values is not as per question.
2		Cosine rule: $AB^2 = 2.5^2+3.6^2-2\times2.5\times3.6\times\cos 35$ $=4.465.....$ $\Rightarrow AB = 2.1$ (km) Alternatively: Find sides of rt angled triangle putting East line across triangle, then use trig and Pythagoras is OK	M1 A1 A1 A1 **4**	Attempt at cosine rule plus sub I their formula (May have + and/or no 2) May be implied A0 if correct ans not given to 2.s.f. (This counts as the tfsf penalty for the paper.)
3		$\sin x = 2\cos x \Rightarrow \tan x = 2$ $\Rightarrow x = 63.4,$ Add 180 243 (allow 243.4) Alternative: $\sin x = 2\cos x \Rightarrow \sin^2 x = 4\cos^2 x \Rightarrow 1-\cos^2 x = 4\cos^2 x$ $\Rightarrow \cos^2 x = \frac{1}{5} \Rightarrow \cos x = \pm\frac{1}{\sqrt{5}} \Rightarrow x = 63.4$ Sorting which quadrant for other root $\Rightarrow$ Add 180 243 (allow 243.4)	B1 B1 M1 F1 **4**	 for 63.4 For adding 180 (and nothing else Not if extras are in range. B1 B1 M1 F1
4	(i)	$d=\sqrt{(7-2)^2+(9-3)^2}$ $=\sqrt{25+36}=\sqrt{61}$ $(=7.81)$	M1 A1 **2**	
	(ii)	$(x-2)^2+(y-3)^2=61$	M1 F1 **2**	Correct LHS = something

5		$x^2+4x>5 \Rightarrow x^2+4x-5>0$ $\Rightarrow (x+5)(x-1)>0$ Both positive or both negative $\Rightarrow x>1$ or $x<-5$ Sp. Case Sub $x = 1$ gives f(x) = 0 gives $x > 1$ B1 Sub $x = -5$ gives f(x) = 0 gives $x < -5$ B1	M1 M1 A1 A1 A1 **5**	Get Quad Factorise or ske LHS: Allow $(x+2)^2>9$ Can be obtained from sketch or drawn on a number line on sketch.
6		$\frac{\mathrm{d}y}{\mathrm{d}x}=2x+2 \Rightarrow y=x^2+2x+c$ Sub: $0=9+6+c \Rightarrow c=-15$ $\Rightarrow y=x^2+2x-15$	M1 A1 A1 DM1 A1 **5**	Integrate For x^2+2x Includes c Must be $y = \ldots\ldots$
7	**(i)**	First line $y = -2x + 4$ 2nd line: $y = -2x + {}^5/_2$ Therefore same gradients (Alt. Try to solve and get impossibility such as 8 = 5)	B1 B1 **2**	Both values seen clearly to be -2 Comment
	(ii)	Perpendicular line has gradient $\frac{1}{2}$ $\Rightarrow y-6=\frac{1}{2}(x-1) \Rightarrow 2y-x=11$	M1 DM1 A1 **3**	For negative reciprocal (must be numeric) in any equivalent form
8	**(i)**		B1 B1 B1 B1 B1 **5**	$3x + 2y \leq 18$ Shading $y = 3x$ Shading $y \geq 0$ shading -1 if triangle shaded
	(ii)	 Point required is intersection this is (2, 6) giving 14	M1 A1 **2**	Includes attempt to work out $x + 2y$

9	**(i)**	f(1) = 1 – 4 + 1 + 6 = 4	B1 **1**	
	(ii)	f(3) = 27 – 36 + 3 + 6 = 0 i.e. (x – 3) is a factor	M1 A1 **2**	Substitute or long division. If latter then we must see $x^3 - 3x^2$
	(iii)	$x^3 - 4x^2 + x + 6 = 0$ $\Rightarrow (x-3)(x^2 - x - 2) = 0$ $\Rightarrow (x-3)(x-2)(x+1) = 0$ $\Rightarrow x = -1, 2, 3$ Attempt to get quadratic can be by "trial" or long division Alt: test for a root consistent with 6 M1 Get one root A1 Get the other root A1 Give answer F1	M1 A1 A1 F1 **4**	Attempt to factorise Quadratic Factors But only if integers i.e. x = -1, ± 2
10		Substitute: $y = 5 - 2x \Rightarrow 5 - 2x = x^2 - 4x - 11$ $\Rightarrow x^2 - 2x - 16 = 0$ $\Rightarrow x = 5.12, -3.12$ $\Rightarrow (5.12, -5.25), (-3.12, 11.25)$ Special cases: Graphs B1 B1 M1(acknowledging that the answer is where they meet) Max: 3 If no graph but points are given then B1, B1 for each pair. N.B. It is possible to eliminate x to give a quadratic is y. This is $y^2 - 16y - 109 = 0$	M1 A1 M1 A1 F1 M1 A1 **7**	Correct sub For quadratic eqn Solve Each x Correct pairing

Q.		Answer	Mark	Notes
Section B				
11	**(i)**	$(0.65)^5 = 0.1160$	B1 **1**	
	(ii)	$5(0.65)^4(0.35) = 0.3124$	B1 B1 B1 **3**	$(0.65)^4(0.35)$ $5\times$ Ans
	(iii)	$1-(0.35)^5-5(0.35)^4(0.65)$ $=1-0.00525-0.0488$ $=0.9460$ Misread Sp. Case: Adding 0, 1 and 2 M1 A1 A1 A0 but – 1 misread. Alt: Add terms = P(2) + P(3) + P(4) + P(5) Add 4 binomial = 0.1812 + 0.3364 terms M1 + 0.31324 + 0.1160 Powers A1 Coeffs A1 =0.946 Ans A1	M1 A1 A1 A1 **4**	1 – 2 or 3 binomial terms Powers (all correct) Coeff ans
	(iv)	$\binom{10}{8}(0.65)^8(0.35)^2 = 0.1757$	M1 A1 A1 A1 **4**	Binomial term with at least sum of powers = 10 Powers Coeff Ans

Q.		Answer	Mark	Notes
12	**(i)**	$t = \frac{12000}{30} = 400$ sec	B1 **1**	
	(ii)	$s = \frac{(u+v)}{2}t \Rightarrow t = \frac{2\times 2000}{10+30} = 100\text{sec}$	M1 A1 **2**	
	(iii)	$v^2 = u^2 + 2as \Rightarrow 30^2 = 10^2 + 2a.4000$ $\Rightarrow a = \frac{800}{8000} = \frac{1}{10} = 0.1\text{ms}^{-2}$	M1 A1 A1 **3**	For any valid const accel formula Credit 2 for t_3 if seen here.
	(iv)	$s = \frac{(u+v)}{2}t \Rightarrow t = \frac{2\times 4000}{10+30} = 200\text{sec}$ For 2nd part: $s = vt \Rightarrow t = \frac{6000}{10} = 600\text{sec}$ $\Rightarrow$ total time = 900 sec. Original time = 400 sec so loss is 500 secs	M1 A1 M1 A1 M1 F1 **6**	Can be given in (ii) if seen. For $t_1 + t_2 + t_3$ – their (i)

N.B. Using km throughout counts as misread.
Using 100 m = 1 km is also a misread.
Make sure they are consistently wrong throughout the paper. If not, then deduct the appropriate marks

13	**(i)**	$AC = \sqrt{20^2 + 20^2} = 20\sqrt{2} \approx 28.3$	M1 A1 **2**	
	(ii)	$AV = \sqrt{15^2 + 200} = \sqrt{425}$	M1 A1 **2**	Must be clear (N.B. Ans given)
	(iii)	Angle $VAO = \tan^{-1}\dfrac{15}{14.14} \approx 46.7^0$	M1 A1 **2**	Using half (i) and tan
	(iv)	$VN = \sqrt{425-100} = \sqrt{325} \approx 18.0$ $OR = \sqrt{15^2 + 10^2} = \sqrt{325} \approx 18.0$	M1 A1 **2**	
	(v)	Area $= \frac{1}{2}AB.VN = \frac{1}{2}20\sqrt{325} \approx 180.2....$ Area $= \frac{1}{2}AM.VB = \frac{1}{2}AM\sqrt{425}$ $\Rightarrow \frac{1}{2}20\sqrt{325} = \frac{1}{2}AM\sqrt{425}$ $\Rightarrow AM = \dfrac{20\sqrt{325}}{\sqrt{425}} \approx 17.5$ N.B. Candidates might find AM by other means and then find the area of the triangle using AM. This is acceptable	M1 A1 M1 A1 **4**	Either form Finding angle B and then AM from triangle AMB M1A1 And then area can be found M1 A1

14	**(i)**	P(5, 1)	B1 **1**	
	(ii)	M is $\left(\dfrac{2+5}{2}, 1\right) = \left(3\frac{1}{2}, 1\right)$	M1 F1 **2**	2 + their (i), divided by 2
	(iii)	$y = x^2 - 4x + 5 \Rightarrow \dfrac{dy}{dx} = 2x - 4$ When $x = 5$, $g = 6 \Rightarrow y - 10 = 6(x-5) \Rightarrow y = 6x - 20$ $\Rightarrow y = 6x - 20$	M1 A1 DM1 A1 **4**	Differentiate finding g and using eqn of line
	(iv)	Substitute the coordinates of M into line When $x = 3.5$, $y = 6 \times 3.5 - 20 = 21 - 20 = 1$	DM1 A1 **2**	Only if line and point are correct!
	(v)	Find the minimum point and draw a line parallel to the x-axis Drop a perpendicular from T to this line at P. Find the midpoint of VP, M The tangent goes through T and M.	B1 B1 B1 **3**	

Free Standing Mathematics Qualification, Advanced Level.
6993 Additional Mathematics

Summer 2006
Chief Examiner's Report

The number of candidates for this specification continues to rise, with an entry nearly 15% up from last year and almost double the entry for the first examination in 2003.
We were pleased to see a large number of very good scripts - in more than one centre the total candidature recorded a mark of over 80%. However, it is still disappointing to find a number of centres for which this specification is clearly not appropriate. The specification clearly states that the specification is suitable for those gaining a good grade at GCSE - typically A*, A or B. The specification is designed to be an enrichment programme for Higher Tier students and it is therefore inappropriate for an entry for students at any other level.

The rubric states that answers should be given to 3 significant figures where appropriate. In past years this has resulted in marks being deducted for the following reasons
Answers being approximated to less than 3 significant figures, particularly the answers in the binomial probability question
Angles being given to 2 or more decimal places
Lengths being given to a large number of significant figures, usually resulting from candidates writing down the total display on their calculator.

The "appropriateness" of this procedure should be evident in questions 2 (where 2 significant figures was demanded) 4, 10, 11 and 13. In general, we adopted a policy of deducting a mark for this where it was first seen and only once throughout the paper.

Section A

Q1 (Calculus)

Better candidates had few problems, though the "integration" of the second term to give $\frac{3^2}{2}$ was often seen. Even those who got the integration correct failed to complete the arithmetic correctly; typically we saw $\frac{3^3}{3} = 3$.

Q2 (Cosine rule)
There was an alternative method of course, which was to draw a line East -West from B, calculating the sides of the two resulting right-angled triangles. This was a typical situation where candidates lost time due to working through a process that was rather longer than the expected method.
Of those who used the cosine rule, some failed to remember the formula properly and many failed to give the answer to 2 significant figures as required.
A large number of candidates also left their answer as 4.465… which is a^2, in spite of writing the formula correctly, and so lost the last accuracy mark for failing to take the square root.

Q3 (Trigonometry)
This was attempted by a variety of methods, most leading to inaccurate values. Trial and improvement should be discouraged with this work as it is both time consuming and unnecessary. Most who obtained the first value were also able to give the second and only a very few found values in other quadrants.

Q4 (Coordinate geometry of the circle)
While the vast majority of candidates were able to evaluate the distance between two points, dealing with the equation of a circle which did not have its centre at the origin was not at all well known.

Q5 (Inequalities)
About a third of candidates did not understand that they had to factorise a quadratic function to proceed with the question. Most of the remainder were able to deal with the correct factorisation, but unable to complete the inequality. A common answer was $x > 1$ and $x > -5$.

Q6 (Calculus)
Some omitted the constant of integration then spuriously tried to compensate thereafter. Only a few replaced the m in the general equation of the line by the function of x given as the gradient function.

Q7 (Coordinate geometry)
There were two acceptable methods. The first was to write both equations in the form $y = mx + c$ and to comment that the coefficient of x, which is the gradient, is the same for both lines. Of those who did this a large number said that the gradient was $-2x$. The other method was to claim that two lines are parallel if they do not intersect and attempts to find the point of intersection by solving simultaneously would, for two parallel lines, produce an impossibility (typically $8 = 5$). This is quite subtle and unfortunately we were not convinced in most cases that candidates knew this and were trying to develop this argument. They solved simultaneously (perhaps because they did not know what else to do) and then could not cope with the apparent mess into which they were getting. The gradient of the perpendicular line seemed to be well known and those who found -2 as the common gradient used $\frac{1}{2}$ as the gradient of a perpendicular line successfully to complete the question. Of those who wrote the gradient of the given lines as $-2x$ some then wrote the gradient of a perpendicular line as $\frac{1}{2x}$. Some successfully completed the question, and so we put this down to sloppy notation but others got themselves confused.

Q8 (Linear Programming)
In general this question was well done. Common errors that led to the loss of one or more marks were:

- The incorrect shading for the inequality $y \leq 3x$ which not only led to the incorrect answer but encouraged candidates to shade incorrectly also the domain $y \geq 0$, shading instead the region for which $x \geq 0$.
- The drawing of the line $y = \frac{1}{3}x$
- The final answer left as (2, 6).

Q9 (Polynomials)
It was clear that answers to this question were more than usually centre-dependent, in that in some centres hardly any candidate got it right and in many centres practically every candidate obtained full marks.
Most candidates were able to justify that $(x - 3)$ was a factor by using the factor theorem (though many did not say so, simply showing that f(3) = 0 with no comment) but a significant number of these did not seem to know the remainder theorem and obtained the answer to (i) by long division. Rather more candidates than last year gave the full solution to the equation, though some did still give $x = -1, 2$ as the answer.

Q10 (Intersection of line and curve)
A few candidates failed to substitute properly and their algebraic manipulation let them down. Most were able to solve their quadratic equation, however. Once again, marks were lost, often by very good candidates, by failing to read the question. In this question the y values were required as well.

Section B

Q11 (Binomial distribution)
Most candidates knew what to do but there were the expected few who failed to write terms which had consistent powers or coefficients. A surprising number worked with the probabilities 0.65 and 0.45 or even 0.25.

Q12 (Constant acceleration)
There were very few candidates who were unable to make any headway with this question. However, the constant acceleration formulae were not well known; many used $u = 0$ throughout and also many failed to use average speed during the sections of deceleration and acceleration.
For those who used a formula requiring a time in (iii) the two marks allocated to this in the mark scheme (and on the paper) in (iv) were awarded when seen in (iii). For these candidates the allocation of marks to the sections was 1, 2, 5, 4.
A number of candidates used kilometres and some also took 100 m = 1 km. If one of these errors had been consistent throughout the question it would have been possible to treat it as a misread, but unfortunately many of these candidates used incorrect units or conversion inconsistently, dealing with it correctly in some parts but not in others.

Q13 (3-D trigonometry)
This question was possibly the best of the section B questions, perhaps because it was nearest to being part of the GCSE syllabus. In (ii) many answers were unconvincing. Candidates should be

clear that when a question says "show" then no fudging or omission of working is acceptab[le.] [In] this case also it was not acceptable to take an approximate value to be rounded to the given va[lue.]
A handful found the wrong angle in (iii). Others used their angle in (iii) in part (v). Generally though, apart from (v), this was popular and an easy source of marks for most of the candidates. In some cases this was the only significant source of marks.
The straightforward method of answering (v) was not adopted by most candidates who chose a rather more complicated route to get to the answers. Finding AM in order to evaluate the area was accepted.

Q14 (Calculus of curves)
Better candidates had few problems and seemed to do the whole problem in a few lines.
The majority were able to score full marks in (i) and (ii). Some differentiated in (iii) then stopped, others read ahead and worked out the equation of the line TM rather than the tangent. Some of the descriptions in (v) were vague, but attempts to describe what had been done in this specific case as a general process were credited.

FSMQ Advanced Additional Mathematics 6993
June 2006 Assessment Session

Unit Threshold Marks

Unit	Maximum Mark	A	B	C	D	E	U
6993	100	79	67	56	45	34	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
6993	35.2	48.1	57.3	65.7	75.3	100	4381