OXFORD CAMBRIDGE AND RSA EXAMINATIONS
FREE-STANDING MATHEMATICS QUALIFICATION
Advanced Level

ADDITIONAL MATHEMATICS **6993**

Summer 2003

Friday **13 JUNE 2003** Morning 2 hours

Additional materials:
Answer booklet
Graph paper

TIME 2 hours

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a scientific or graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given correct to three significant figures where appropriate.
- The total number of marks for this paper is 100.

This question paper consists of 5 printed pages and 3 blank pages.

Section A

1 Solve simultaneously the equations $y = x + 6$ and $y = x^2 - x + 3$. [4]

2 (i) Show that there is a stationary point at $(1, 9)$ on the curve $y = x^3 - 6x^2 + 9x + 5$ and determine the nature of this stationary point. [5]

(ii) Find the coordinates of the other stationary point and hence sketch the curve. [2]

3 The gradient function of a curve is given by $\frac{dy}{dx} = 2 + 2x - x^2$. Find the equation of the curve given that it passes through the point (3, 10). [4]

4 Find the four values of θ in the range $0° \leqslant \theta \leqslant 360°$ that satisfy the equation $\sin 2\theta = 0.5$. [4]

5 (i) By drawing suitable graphs on the same axes, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$3x + 4y \leqslant 24$$
$$3x + y \leqslant 15$$
$$x \geqslant 0$$
$$y \geqslant 0$$

[5]

(ii) Find the maximum value of $2x + y$ subject to these conditions. [2]

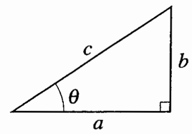
6 (i) Expand $(2 + x)^7$ in ascending powers of x up to and including the term in x^3. [4]

(ii) Use your expansion with an appropriate value of x to find an approximate value of 1.99^7. Give your answer to 4 decimal places.

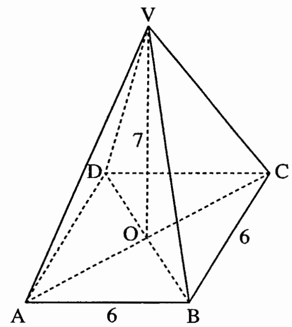
Show your working clearly, giving the numerical value of each term.

[Just writing down the value of 1.99^7 from your calculator will earn no marks.] [3]

7 Use the given triangle to prove that, for $0° < \theta < 90°$, $1 + \tan^2\theta = \dfrac{1}{\cos^2\theta}$. [3]



8 A pyramid ABCDV has a square, horizontal, base ABCD of side 6 cm. The vertex V is vertically above the centre of the base O. The pyramid has height 7 cm.



Find the angle that the sloping edge VA makes with the horizontal. [5]

9 The function $f(x)$ is defined by $f(x) = x^3 + 2x^2 - 5x - 6$.

(i) Show that when $f(x)$ is divided by $(x - 3)$ the remainder is 24. [2]

(ii) Show that $(x - 2)$ is a factor of $f(x)$. [1]

(iii) Hence solve the equation $f(x) = 0$. [4]

10 A car, which is initially travelling at $20\,\text{m s}^{-1}$, accelerates uniformly at $1.2\,\text{m s}^{-2}$.

Find

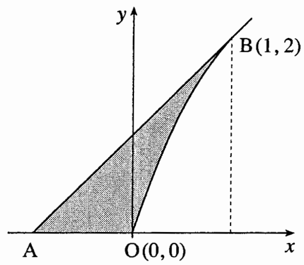
(i) the speed after 5 seconds, [2]

(ii) the distance travelled in this time. [2]

[Turn over

Section B

11 The shaded region on the diagram shows a boat's sail. The units are metres and, referred to the axes shown, the coordinates of O and B are $(0, 0)$ and $(1, 2)$ respectively. OB is part of the curve $y = 3x - x^2$. The tangent to the curve at B meets the x-axis at A.



Find

(i) $\dfrac{dy}{dx}$ for the curve $y = 3x - x^2$, [2]

(ii) the equation of the tangent at B, [3]

(iii) the coordinates of the point A, [1]

(iv) the area of the sail. [6]

12 **(a)** A quality control officer inspects a large batch of electric light bulbs which are in packs of 8. He chooses a pack at random and tests all the bulbs to see if they are working. On this day 10% of the bulbs are faulty.

Find the probability that in the pack chosen

(i) none is faulty, [2]

(ii) two or more are faulty. [5]

(b) If the officer finds no faulty bulbs, he accepts the whole batch. If he finds two or more faulty bulbs he rejects the whole batch. If he finds one faulty bulb then he chooses a second pack at random and accepts the whole batch only if this second pack has no faulty bulbs.

Find the probability that the whole batch is accepted. [5]

13 There are 10 tonnes of potatoes in a large container. Bags of potatoes of nominal mass 5 kg are filled from this container. The potatoes are not all the same size and it is not possible to make the bags exactly 5 kg. [1 tonne = 1000 kg.]

(i) If all bags could be made with a mass of exactly 5 kg, how many bags would be filled from the container? [1]

The bags could be too light by up to x kg or too heavy by up to x kg.

(ii) State, in terms of x, the largest and smallest number of bags that can be filled from the container. [2]

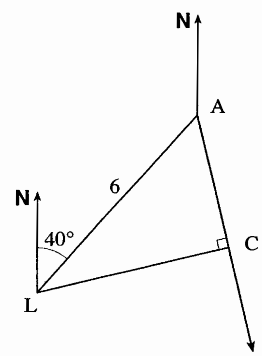
(iii) Given that the largest number of bags is 100 more than the smallest number of bags, write down an equation in x and show that it simplifies to $x^2 + 200x - 25 = 0$. [6]

(iv) Solve this equation and hence work out the largest and smallest mass of a bag of potatoes. [3]

14 At 1200 a ship is at a point A on a bearing of 040° from a lighthouse L and at a distance of 6 nautical miles.

The ship is moving on a bearing of 170° at 21 knots. [1 knot is a speed of 1 nautical mile per hour.]

C is the point where the ship is nearest to the lighthouse.



Not to scale

(i) Show that the angle LAC = 50°. [1]

(ii) Find the distance LC and the time when the ship is at the point C. [6]

(iii) At what time is the ship on a bearing of 110° from the lighthouse? [5]

Mark Scheme

OCR

RECOGNISING ACHIEVEMENT

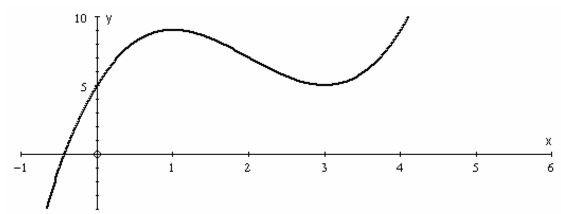
FSMQ Additional Mathematics

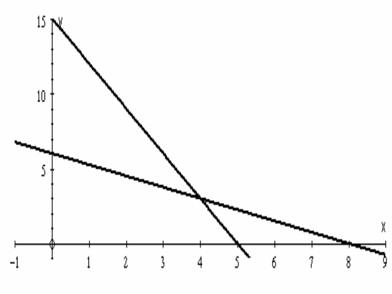
6993

Mark Scheme, June 2003

6993 Additional Mathematics Summer 2003 Solutions

Section A

1	$x+6=x^2-x+3 \Rightarrow x^2-2x-3=0$ $\Rightarrow (x-3)(x+1)=0 \Rightarrow (3, 9),\ (-1, 5)$ Alternatively: eliminate y giving $y^2 - 14y + 45 = 0$ **4**	M1 A1 A1 A1	Or A1 for 3, -1 A1 for 9, 5
2	(i) $\frac{dy}{dx}=3x^2-12x+9=0$ when $x^2-4x+3=0$ $\Rightarrow (x-3)(x-1)=0 \Rightarrow x=1 \Rightarrow y=9$ $\frac{d^2y}{dx^2}=6x-12<0$ when $x=1 \Rightarrow$ maximum (Alternatively B1 B1 for +ve on left and -ve on right of stationary point.) **5** (ii) Other stationary point is (3, 5)  **2** **7**	M1 Diff A1 getting 0 B1 getting 9 DM1 A1 B1 for other stationary point F1 for sketch (General shape)	 Dep. On 1st M1 Conforms to 2nd stationary point.
3	$\frac{dy}{dx}=2+2x-x^2 \Rightarrow y=2x+x^2-\frac{x^3}{3}\ (+c)$ Through (3, 10) $\Rightarrow 6+9-9+c=10 \Rightarrow c=4$ $\Rightarrow y=2x+x^2-\frac{x^3}{3}+4$ **4**	M1 (ignore c) A1 all correct M1 for c F1	Increase in powers evident
4	$\sin 2\theta=0.5 \Rightarrow 2\theta=30$ and 150 and also $2\theta=390$ and also $2\theta=510$ $\Rightarrow \theta=15, 75, 195, 255$ **4**	M1 Solving A1 for 15 F2,1 remaining 3, -1 each error	

5	 (i) $3x+4y=24$ $3x+\ \ y=15$ meet at (4, 3) **5** (ii) Max value of $2x+y$ is 11 **2** **7**	B1 B1 each line B1 B1 each shading B1 for (4,3) M1 F1 for 11	Ignore $y \geq$ Award this where used or seen
6	(i) $(2+x)^7 = 2^7 + 7.2^6x + 21.2^5x^2 + 35.2^4x^3 + \ldots$ $= 128 + 448x + 672x^2 + 560x^3 + \ldots..$ **4** (ii) Substitute $x = -0.01$ $\Rightarrow 1.99^7 = 128 - 4.48 + 0.0672 - 0.000560 + \ldots$ **3** $= 123.5866$ **7**	B1 powers (x and 2s) B1 coefficients B2 all terms, (B1 1 error) M1 substitute F1 terms A1 final answer	 0.01 or -0.01
7	L.H.S. = $1 + \tan^2\theta = 1 + \frac{b^2}{a^2} = \frac{a^2+b^2}{a^2} = \frac{c^2}{a^2} = \frac{1}{\cos^2\theta}$=R.H.S **3**	B1 B1 B1	Starting with end equation B2 only
8	$AB = 6 \Rightarrow AC = 6\sqrt{2} \approx 8.485 \Rightarrow AO = 3\sqrt{2} \approx 4.243$ $\Rightarrow \tan\theta = \frac{7}{3\sqrt{2}} \approx 1.650 \Rightarrow \theta = 58.8^0$ **5**	M1 A1 B1 Correct angle M1 A1	Application of Pythagoras in ABC Must be angle in correct triangle
9	(i) $f(x) = x^3 + 2x^2 - 5x - 6 \Rightarrow f(3) = 27 + 18 - 15 - 6 = 24$ **2** (ii) f(2) = 8 + 8 −10 − 6 = 0 **1** (iii) f(x) = $(x-2)(x^2+4x+3) = (x-2)(x+1)(x+3)$ $\Rightarrow$ f(x)=0 $\Rightarrow (x-2)(x+1)(x+3) = 0 \Rightarrow x = -3, -1, 2$ **4** **7**	M1 A1 B1 M1 A1 A1 A1	 For each linear term. Or use of factor theorem.
10	(i) $v = u + at \Rightarrow v = 20 + 1.2 \times 5 = 26\text{ms}^{-1}$ **2** (ii) $s = ut + \frac{1}{2}at^2 \Rightarrow s = 100 + 0.6 \times 25 = 115\text{m}$ **2** **4**	M1 A1 M1 A1	-1 max penalty for incorrect or missing units

Section B

11	(i) $y = 3x - x^2 \Rightarrow \frac{dy}{dx} = 3 - 2x.$	**2**	M1 A1	
	(ii) When $x = 1, g = 1 \Rightarrow y - 2 = 1(x-1) \Rightarrow y = x + 1$	**3**	F1 M1 A1	
	(iii) When $y = 0,\ x = -1 \Rightarrow$ A(-1,0)	**1**	F1	
	(iv) Area under curve = $\int_0^1 (3x - x^2)\,dx$		M1	Must have correct limits
	$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{3}{2} - \frac{1}{3} = \frac{7}{6}$ m^2		A1 A1	
	Area triangle = $\frac{1}{2}.2.2 = 2$ m^2		F1	
	Total Area = $2 - \frac{7}{6} = \frac{5}{6}$ m^2	**6**	M1 F1	
		12		
12	**(a) (i)** $\left(\frac{9}{10}\right)^8 = 0.430$	**2**	M1 A1	
	(ii) $1 - \left(\frac{9}{10}\right)^8 - 8.\left(\frac{9}{10}\right)^7\left(\frac{1}{10}\right)$ $= 1 - 0.4305 - 0.3826 = 0.187$	**5**	M1 (1 -) A1 (2 terms) B1 (powers) B1 (coeff) A1 (ans)	Alt: M1 other terms A1 7 terms B1 powers B1 coeffs
	(b) (a)(i) + (a)(i) $\times$ P(1 faulty) = 0.4305 + 0.4305 $\times$ 0.3826 = 0.595	**5**	F1 (1st term) M1 F1 (2nd term) M1(add) A1 (ans)	Their (a)(i) Mult of 2 terms 3 or 4 sig figs
		12		

13	(i) $\frac{10000}{5} = 2000$	**1**	B1	Sp… case… with 1… and follow… through.
	(ii) $\frac{10000}{5-x}$, $\frac{10000}{5+x}$	**2**	B1 B1	
	(iii) $\frac{10000}{5-x} - \frac{10000}{5+x} = 100$		M1 A1	Accept either order
	$\Rightarrow 10000\left((5+x)-(5-x)\right) = 100\left(25 - x^2\right)$		M1 A2,1	(even if wrong way round)
	$\Rightarrow x^2 + 200x - 25 = 0$	**6**	A1	
				Clearing fractions
	(iv) $\Rightarrow x = \frac{-200 \pm \sqrt{200^2 + 100}}{2} \approx 0.125$		M1 A1	
	$\Rightarrow$ largest = 5.125, smallest 4.875	**3**	A1	
		12		
14	(i) Angle LAC = 40 + 10 = 50	**1**	B1	
	(ii) LC = 6sin50 = 4.596		M1 A1	
	AC = 6cos50 = 3.857		A1	
	$\Rightarrow$ Time = $\frac{3.857}{21} = 0.1837\text{hrs} \approx 11\text{mins}$		M1 A1	
	$\Rightarrow 1211$	**6**	F1	
	(iii) $\frac{AB}{\sin 70} = \frac{6}{\sin(180 - 70 - 50)}$		M1 A1	
	$\Rightarrow AB = 6 \times \frac{\sin 70}{\sin 60} \approx 6.510$		A1	
	$\Rightarrow$ Time = $\frac{6.510}{21} = 0.31\text{hrs} \approx 18.6\text{mins}$	**5**	M1 F1 (allow 19 mins)	
	$\Rightarrow 1219$			
	Alt. (iii)			
	Bearing = 110^0, Angle ALC = 40^0.			M1
	If position is B then angle CLB = 30^0			A1 (for 30)
	CB = LCTan30 = 2.65			A1
	Time = $\frac{2.65}{21} \times 60 = 8\,\text{mins}$			M1
	1211 + 8 = 1219	**12**		F1

Examiner's Report

Report on 6993. Additional Mathematics Summer 2003.

This is the first year of this specification. It replaced the old Additional Mathematics syllabuses, 8645 (the old O&C "traditional paper" and 8647, the MEI Additional Mathematics syllabus) and was geared to the same sort of candidature. The regulations for an Advanced FSMQ, as stated clearly in the specification, is that the appropriate starting point is a good grade at Higher Tier. Many of the candidates had clearly started from here, and had found that the course did not challenge them in quite the same way. There were many high scores including some full marks which was most pleasing to see. However, it was equally clear that many candidates had not started from the appropriate place. Many candidates not only failed to demonstrate any understanding of the extension material but failed to demonstrate the sort of understanding of some Higher Tier topics. Centres are encouraged to seek advice, if necessary, to find the most appropriate course for their students.
Because it is the first year of this specification the comments below are rather more full than they might be; it is hoped that teachers will use this report to inform them of requirements and standards expected.

1 This is a typical case in point. The question required candidates top find the intersection of a curve and a line and this is a standard Higher Tier topic. As such it should have been a relatively easy start to the paper. Yet many failed to show any understanding of the basic algebraic skills required to make the substitution in order to obtain the quadratic to be solved.
Some substituted for x, giving a quadratic in y. While this clearly results in the correct answer, it is rather more complicated to do. Many candidates failed to find the values of the other variable. Solving simultaneously means find the pairs (x, y) and is the same question as "find the coordinates of the intersection of the curve and the line".

[(-1, 5), (3, 9)]

2 Many candidates failed to show that the stationary point was at (1, 9), demonstrating simply that there was a turning point when $x = 1$. The "nature" of the turning point was done by either the second derivative, the "direction" of the gradient around the turning point or, having established $y = 9$ at the turning point to find values of y close either side of $x = 1$. This last method requires the verification of $y = 9$ when $x = 1$ which, as stated above, was often omitted.
The "sketch" was often rather too sloppy for the mark. Candidates are expected to show intercepts on the axes and the turning points for their mark.

[(ii) (3, 5)]

3 A simple integration question which was well answered; only a small minority of candidates failed to calculate the constant of integration. A typical indication of an inappropriate entry was the candidate who took the "m" of $y = mx + c$ to be $2 + 2x - x^2$, writing the equation as
$y = (2 + 2x - x^2)x + c$.

[$y = 2x + x^2 - \frac{x^3}{3} + 4$]

4 Many students managed $2\theta = 30 \Rightarrow \theta = 15$, but failed to find the other roots.The info that there were four roots should have given a clue but a number of candidates ignored it! Rather more worrying were the candidates who wrote $\sin 2\theta = 0.5 \Rightarrow \sin\theta = 0.25 \Rightarrow \theta = 14.5^0$ or even worse, $\sin 2\theta = 0.5 \Rightarrow \theta = \dfrac{0.5}{\sin 2} = \dfrac{0.5}{0.0349} = 14.32^0$

[15^0, 75^0, 195^0, 255^0]

5 The first part of this question was, in general, well done, although some candidates were unable to draw the lines properly. The maximum value of $2x + y$ was not so well done; the good candidates realised that it occurred at the intersection while others found the value at all corners in order to deduce. For this short question in Section A it was not a requirement to justify that the point of intersection was indeed (4, 3) or that this was the point that maximised the objective function.

[(ii) 11 at (4, 3)]

6 The binomial expansion had not been covered by many; some tried to multiply it all out while others wrote incorrect algebra. When a numeric value is to be found "without use of the calculator" the values of all terms was expected to be seen. While no candidate simply wrote down the answer (which would have come from the calculator) many wrote down incorrect arithmetic (for instance all terms positive) and then the right answer. Candidates who are clever enough to take short cuts should remember to write down everything in situations like this.

[(i) $128 + 448x + 672x^2 + 560x^3 + ...$ (ii) 123.5866]

7 This question was an A grade discriminator!! Those candidates who were able to do the question usually started with the equality given, manipulating both sides to give something that they knew to be true (i.e. Pythagoras) rather than start with one side of the identity and to manipulate to give the other side.

8 There were some surprising answers here! Many candidates got it right but by a very roundabout way. The intention was to find AO and then in the triangle VOA use the tan ratio to give the answer. Many did not do this. A common alternative seen was to find the hypotenuse by Pythagoras and then use the sin or cos ratio, thus involving an extra step. Another was to find AV and CV (deduced correctly to be equal) and then to apply the cosine formula on triangle AVC.

[58.8^0]

9 This is a case where all working needs to be seen, even though it is very simple for the able candidate. Examiners need to see the substitution of $x = 3$ into the function to yield 24.
Some did part (iii) by attempting to divide by $(x - 2)$ while others found the other roots by trail and error. Examiners did not report seeing any candidate using the logic of trying only certain numbers due to the fact that the produce of the three roots had to be 6.
Some lost the last mark by factorising f(x) only without solving the equation f(x) = 0.

[(iii) $x = 2, -1, -3$]

10 Part (i) was often logically deduced without any clear statement of one of the standard formulae, but (ii) caused extra problems. Some effectively used $s = \frac{(u+v)}{2}t$ for each second to achieve the right answer. There was not much evidence that these formulae were well known. We expected to see the units given in these answers.

[(i) 26 ms^{-1}, (ii) 115m]

11 This question was often well done. The weaker candidates were able to differentiate the function to find the gradient function but then failed to obtain the equation of the tangent. The weakest, of course, were unable even to differentiate.

Because of the limits of each area the method of using $\int_a^b (y_1 - y_2)\,\mathrm{d}x$ was clearly not appropriate.

We were surprised that so many integrated the equation of the tangent to finds the are under the line instead of seeing it as a triangle.

$\left[\text{(i)}\ \frac{\mathrm{d}y}{\mathrm{d}x} = 3 - 2x. \quad \text{(ii)}\ y = x + 1 \quad \text{(iii)}\ (-1,0) \quad \text{(iv)}\ 2 - \frac{7}{6} = \frac{5}{6}\ \text{m}^2\right]$

12 This was a standard binomial distribution question set in previous Additional Mathematics papers and many completed it well and with the minimum of fuss (though some lost a mark through not correcting answers to 3 significant figures as required by the rubric – some gave far too many and others approximated prematurely). The same proportion of candidates completing the question failed to involve the binomial coefficients. Many weak candidates misread the question and worked on $\left(\frac{1}{8}\right)^{10}$ or similar, incorrect, terms.

[(i) 0.430, (ii) 0.187 (iii) 0.595]

13 Unfortunately many candidates did not know the connection between tonnes and kilograms though compensatory marks were available. In spite of this many were able to set up the equation $\frac{10000}{5-x} - \frac{10000}{5+x} = 100$. The main problem arose over the algebraic manipulation resulting in the quadratic equation $x^2 + 230x - 25 = 0$. Needless to say, incorrect algebra leading to the correct answer resulted in no marks! Candidates need to be aware that examiners cannot be fooled, for when the answer is given we look very carefully at all the steps that lead to it.
Some were able to re-enter the question and simply solve the quadratic equation.

[(i)2000, (ii) $\frac{10000}{5-x}, \frac{10000}{5+x}$, (iv) 5.125 kg, 4.875 kg]

14 Justification of the angle of 50^0 was often not done and lengths were sometimes correcte soon. Unfortunately, some candidates did not read the question properly and, having worked out as required, then used that length to work out the "time" rather than understand that, although it was not asked for, it was the length AC that was required. Two methods were seen for the last part; some worked out the "extra" bit, getting an angle of 30^0 in a right-angled triangle and adding the time on, while others started from the beginning again with a triangle that required the use of the sine rule.

[(ii) LC = 4.596 km, time = 1211 (iii) 1219]