RECOGNISING ACHIEVEMENT

Oxford Cambridge and RSA Examinations

Free Standing Mathematics Qualification (Advanced)

ADDITIONAL MATHEMATICS

6993

Specimen Paper

Additional materials: Electronic calculator

TIME 2 hours

Candidate Name	Centre Number	Candidate Number

INSTRUCTIONS TO CANDIDATES

- Write your name, Centre number and candidate number in the spaces above.
- Write your answers, in blue or black ink, in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Read each question carefully and make sure you know what you have to do before starting your answer.

You are expected to use an electronic calculator for this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is **100**.

Question number	For examiner's use only
1	
2	
3	
4	
5	
6	
7	
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9	
10	
11	
12	
13	
14	
TOTAL	

Section A

1 Express $4x^2 + 8x - 21$ in the form $a(x + p)^2 + q$.

Hence find the minimum value of $4x^2 + 8x - 21$. **[6]**

2 Expand $(2 + 3x)^5$ in ascending powers of x, simplifying all the terms. **[4]**

3 A triangle ABC is such that AB = 5 cm, BC = 8 cm and CA = 7 cm.

Show that one angle is 60^o. **[5]**

4 Given that $y = x^3 + 2x - 7$, find $\frac{dy}{dx}$.

Use your result to show that the graph of $y = x^3 + 2x - 7$ has no turning points. **[5]**

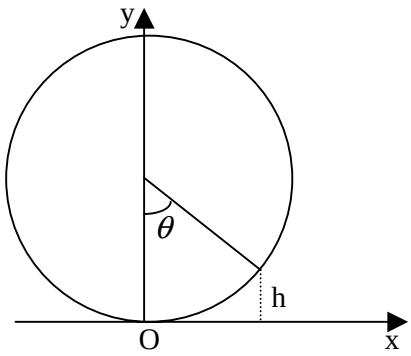
5 The line L goes through the points (1, 2) and (7, 6).

(a) Find the equation of L.

(b) Write down the equation of M, the line through the origin which is perpendicular to L. **[5]**

6 Find the value of $\int_1^2 (x^2 + 3)dx$. **[4]**

7



The 'London Eye' can be considered to be a circular frame of radius 67.5m on the circumference of which are 'capsules' carrying a number of people round the circle. Take a co-ordinate system where O is the base of the circle and Oy is a diameter. At any time after starting off round the frame, the capsule will be at height h metres when it has rotated $\theta°$.

(a) Draw a graph of h against θ. **[2]**

(b) Give an expression for h in terms of θ. **[2]**

(c) Find values of θ when $h = 100$. **[3]**

8 **(a)** Simplify to a single fraction the expression $\frac{x}{x+2} - \frac{6}{x-1}$.

(b) Hence use the quadratic formula to solve the equation $\frac{x}{x+2} - \frac{6}{x-1} = 4$. **[7]**

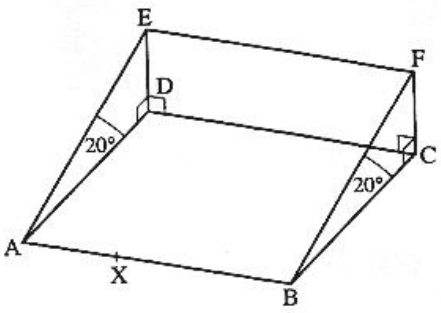
9 Show that $(x-2)$ is a factor of $x^3 - 3x^2 - 4x + 12 = 0$.

Hence solve the equation $x^3 - 3x^2 - 4x + 12 = 0$. **[5]**

10 Prove that if the sides of a triangle can be written as $(n^2 - 1)$, $2n$ and $(n^2 + 1)$ for $n > 1$ then the triangle is right angled. **[4]**

Section B

11



The diagram shows a grass bank which was constructed as part of an assault course.

The rectangle ABCD is horizontal and ABFE is a square inclined at $20°$ to the horizontal such that E is vertically above D and F is vertically above C.

The area of the square ABFE is 1600m^2. X is a point on AB such that AX = 12m.

Calculate

(a) the area of ABCD, **[3]**

(b) the length of the paths XE and XF, **[3]**

(c) the angle between the paths XE and XF, **[3]**

(d) the angle of slope of the path XE with the horizontal. **[3]**

12 A body is falling through a liquid, and the distance fallen is modelled by the formula $s = 48t - t^3$ until it comes to rest, where s centimetres is the distance fallen and t seconds is the time.

(a) Find

(i) the velocity when $t = 1$, **[2]**

(ii) the initial velocity, **[1]**

(iii) the acceleration when $t = 1$, **[3]**

(iv) the time when the body comes to rest, **[2]**

(v) the distance fallen when the body comes to rest. **[2]**

(b) Sketch the velocity/time graph for the period of time until the body comes to rest. **[2]**

13 China cups are packed in boxes of 10. It is known that 1 in 8 are cracked.

Find the probability that in a box of 10, chosen at random,

(a) no cups are cracked, **[3]**

(b) exactly 1 cup is cracked, **[4]**

(c) exactly 2 cups are cracked, **[2]**

(d) at least 3 cups are cracked. **[3]**

14 A bicycle factory produces two models of bicycle, A and B.

Model A requires 20 hours of unskilled and 10 hours of skilled labour.
Model B requires 15 hours of unskilled and 25 hours of skilled labour.

The factory employs 10 unskilled and 8 skilled labourers, each of whom work a 40 hour week.

(a) Suppose the factory makes x model A and y model B per week.

Show that the restriction of unskilled labour results in the inequality

$$4x + 3y \leq 80$$

and find a similar inequality from the restriction on skilled labour. **[5]**

(b) Draw graphs of two lines and shade the feasible region. **[3]**

(c) The factory makes a profit of £40 on model A and £60 on model B.

Write down the objective function. **[2]**

(d) Show that making 15 model A bicycles and 5 model B bicycles is possible and find the number of each that should be made to maximise the profit. **[2]**

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MARK SCHEME

Note: In the following scheme, the following notations apply.

M an M mark is earned for a correct method (or equivalent method) for that part of the question. A method may contain incorrect working, but there must be sufficient evidence that, if correct, it would have given the correct answer.

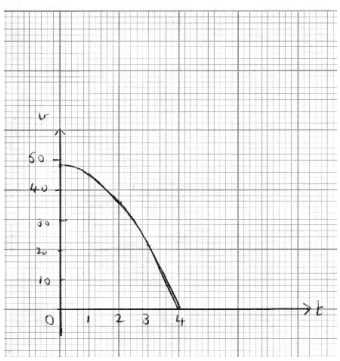
A an A mark is earned for accuracy, but cannot be awarded if the corresponding M mark has not been earned.

B a B mark is an accuracy mark awarded independently of any M mark.

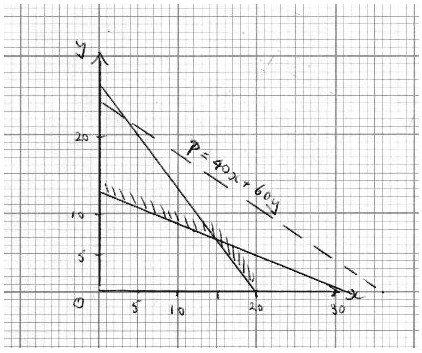
Note: B3,2,1 means that either 1, 2 or 3 B marks can be scored.

Question	Answer	Mark
Section A		
1	$4x^2+8x-21 \equiv 4(x^2+2x)-21$ $\equiv 4(x^2+2x+1)-21-4 \equiv 4(x+1)^2-25$ i.e. $a = 4$, $p = 1$, $q = -25$ Minimum value when $x = -1$; $4x^2+8x-21 = -25$	M1 A1 A1 M1 A2
2	$(2+3x)^5 = 2^5+5.2^4(3x)+10.2^3(3x)^2+10.2^2(3x)^3+5.2(3x)^4+(3x)^5$ $= 32+240x+720x^2+1080x^3+810x^4+243x^5$	M1 (coeffs) A1 (coeffs) A2
3	Cosine rule applied to B: $CosB = \frac{5^2+8^2-7^2}{2.5.8} = \frac{1}{2} \Rightarrow B = 60$ (N.B. Calculation of approximate values for A and C with B = 180 - A - C will gain M2 for correct cosine rule only.)	Selection of angle B1 M2 cos rule A1 A1 or any correct alternative method
4	$y = x^3+2x-7 \Rightarrow \frac{dy}{dx} = 3x^2+2$ $3x^2+2 \neq 0$ for any x, so no turning points.	M1 A1 A1 B1 B1
5(a)	$\frac{y-2}{6-2} = \frac{x-1}{7-1} \Rightarrow \frac{y-2}{2} = \frac{x-1}{3} \Rightarrow 3y-6 = 2x-2 \Rightarrow 3y = 2x+4$	M1 A2
5(b)	Perpendicular line through origin is $2y+3x = 0$	M1 A1
6	$\int_1^2 (x^2+3)dx = \left[\frac{x^3}{3}+3x\right]_1^2 = \left(\frac{8}{3}+6\right)-\left(\frac{1}{3}+3\right) = 5\frac{1}{3}$	M1 A1 A1 A1
7(a)	For graph	B2,1
7(b)	$h = 67.5(1-\cos\theta)$	B2
7(c)	$Cos\theta = 1-\frac{100}{67.5} = -0.4815 \Rightarrow \theta = 118.8$ and 241.2	M1 A1 A1

Question	Answer	Mark
Section A		
8(a)	$\frac{x}{x+2}-\frac{6}{x-1}\equiv\frac{x(x-1)-6(x+2)}{(x+2)(x-1)}\equiv\frac{x^2-7x-12}{(x+2)(x-1)}$	M1 A2
8(b)	$\frac{x}{x+2}-\frac{6}{x-1}=4\Rightarrow x^2-7x-12=4(x^2+x-2)\Rightarrow 3x^2+11x+4=0$	M1 A1
	$x=\frac{-11\pm\sqrt{121-4.3.4}}{6}=\frac{-11\pm\sqrt{73}}{6}=\frac{-11-8.54}{6}$ or $\frac{-11+8.54}{6}$	M1
	$=-3.257$ or -0.409	A1
9	$f(2)=8-12-8+12=0$	B1
	$x^3-3x^2-4x+12=0\Rightarrow(x-2)(x^2-x-6)=0$	M1 A1
	$\Rightarrow(x-2)(x-3)(x+2)=0\Rightarrow x=2,\ 3,\ -2.$	A1 A1
10	If Pythagoras Theorem holds then the triangle is right-angled.	M1
	$(n^2-1)^2+(2n)^2\equiv n^4-2n^2+1+4n^2\equiv n^4+2n^2+1\equiv(n^2+1)^2$	A1
	Since this is true for all $n>1$ the triangle is right-angled for $n>1$.	A2
		Section A Total: 52

Question	Answer	Mark
Section B		
11(a)	AB = AE = 40m AD = AEcos20 = 40cos20 = 37.59 $\Rightarrow$ Area ABCD = AB.AD = 40x37.59 = 1504m^2	B1 M1 A1
11(b)	By Pythagoras $EX = \sqrt{40^2 + 12^2} = \sqrt{1744} = 41.76$ m Similarly $FX = \sqrt{40^2 + 28^2} = \sqrt{2384} = 48.83$ m	M1 A1 A1
11(c)	Cosine Rule: $CosEXF = \dfrac{2384 + 1744 - 1600}{2\sqrt{2384}\sqrt{1744}} = 0.6199 \Rightarrow EXF = 51.7°$	M1 A1 A1
11(d)	The angle of slope requires an extra length, i.e. ED $ED = 40\sin 20$ $\sin x = \dfrac{40\sin 20}{41.76} = 0.3276 \Rightarrow x = 19.1°$	M1 A1 A1
12(a)(i)	$s = 48t - t^3 \Rightarrow v = 48 - 3t^2 \Rightarrow v_1 = 48 - 3 = 45$ cms^{-1}	M1 A1
12(a)(ii)	$v_0 = 48 - 0 = 48$ cms^{-1}	B1
12(a)(iii)	$v = 48 - 3t^2 \Rightarrow a = -6t \Rightarrow a_1 = -6$ cms^{-2}	M1 A1 A1
12(a)(iv)	$v = 48 - 3t^2 \Rightarrow v = 0$ when $3t^2 = 48 \Rightarrow t = 4$ seconds	M1 A1
12(a)(v)	$s = 48t - t^3 \Rightarrow s_4 = 48 \times 4 - 4^3 = 128$ cm	M1 A1
12(b)		B2,1

Question	Answer	Mark
Section B		
13(a)	$\left(\frac{7}{8}\right)^{10} \approx 0.263$	M1 A1 A1
13(b)	$10\left(\frac{1}{8}\right)\left(\frac{7}{8}\right)^{9} \approx 0.376$	M1(Coeff) A1(Coeff) A1 A1
13(c)	$45\left(\frac{1}{8}\right)^{2}\left(\frac{7}{8}\right)^{8} \approx 0.242$	A1(Coeff) A1
13(d)	$1 - P(0) - P(1) - P(2) = 1 -$ **(a)** – **(b)** – **(c)** $= 1 - 0.263 - 0.376 - 0.242 \approx 0.119$ [0.120 from exact answers to **(a), (b), (c)**]	M1 A1 A1

Question	Answer	Mark
Section B		
14(a)	20 hrs of *unskilled* labour per Model A means $20x$ for x bikes. Likewise 15 hrs for Model B means $15y$ for y bikes giving $20x + 15y$. Since the max possible is 10 x 40 hrs = 400 this gives $20x + 15y \leq 400$ i.e. $4x + 3y \leq 80$ For *skilled* labour $10x + 25y \leq 8 \times 40$ i.e. $2x + 5y \leq 64$	B3,2,1 M1 A1
14(b)		B1 + B1 for lines B1 shading
14(c)	$P = 40x + 60y$	M1 A1
14(d)	x y $4x+3y$ $2x+5y$ P 15 5 75 55 i.e. lies in feasible region 16 5 79 57 940 15 6 78 60 960 14 7 77 63 980 so the best solution is 14 Model A and 7 Model B giving a profit of £980.	B2,1
		Section B Total: 48

Total mark available: 100

Assessment Grid

Question	Marks	AO1	AO2	AO3	AO4	AO5	Totals
1	M1 A1 A1 M1 A2	3	 2	 1			6
2	M1 A1 A2	2	2				4
3	B1 M2 A1 A1	 2		1	 2		5
4	M1 A1 A1 B1 B1	3	 2				5
5	M1 A2 M1 A1	1	2 1	 1			5
6	M1 A1 A1 A1	4					4
7	B2 B2 M1 A1 A1		 3	2		2	7
8	M1 A2 M1 A1 M1 A1	3 2	 2				7
9	B1 M1 A1 A1 A1	 2	 2	1			5
10	M1 A1 A2					4	4
11	B1 M1 A1 M1 A1 A1 M1 A1 A1 M1 A1 A1			1	2 3 3	 3	12
12	M1 A1 B1 M1 A1 A1 M1 A1 M1 A1 B2	2 3 2	 2	1 2			12
13	M1 A1 A1 M1 A1 A1 A1 A1 A1 M1 A1 A1		 3	2 3 2		1 1	12
14	B3 M1 A1 B3 M1 A1 B2		3 2	 3	 2	2	12
Totals		29	26	20	12	13	100